Project 1 Tutorial
CSE 473/573 Fall 2021

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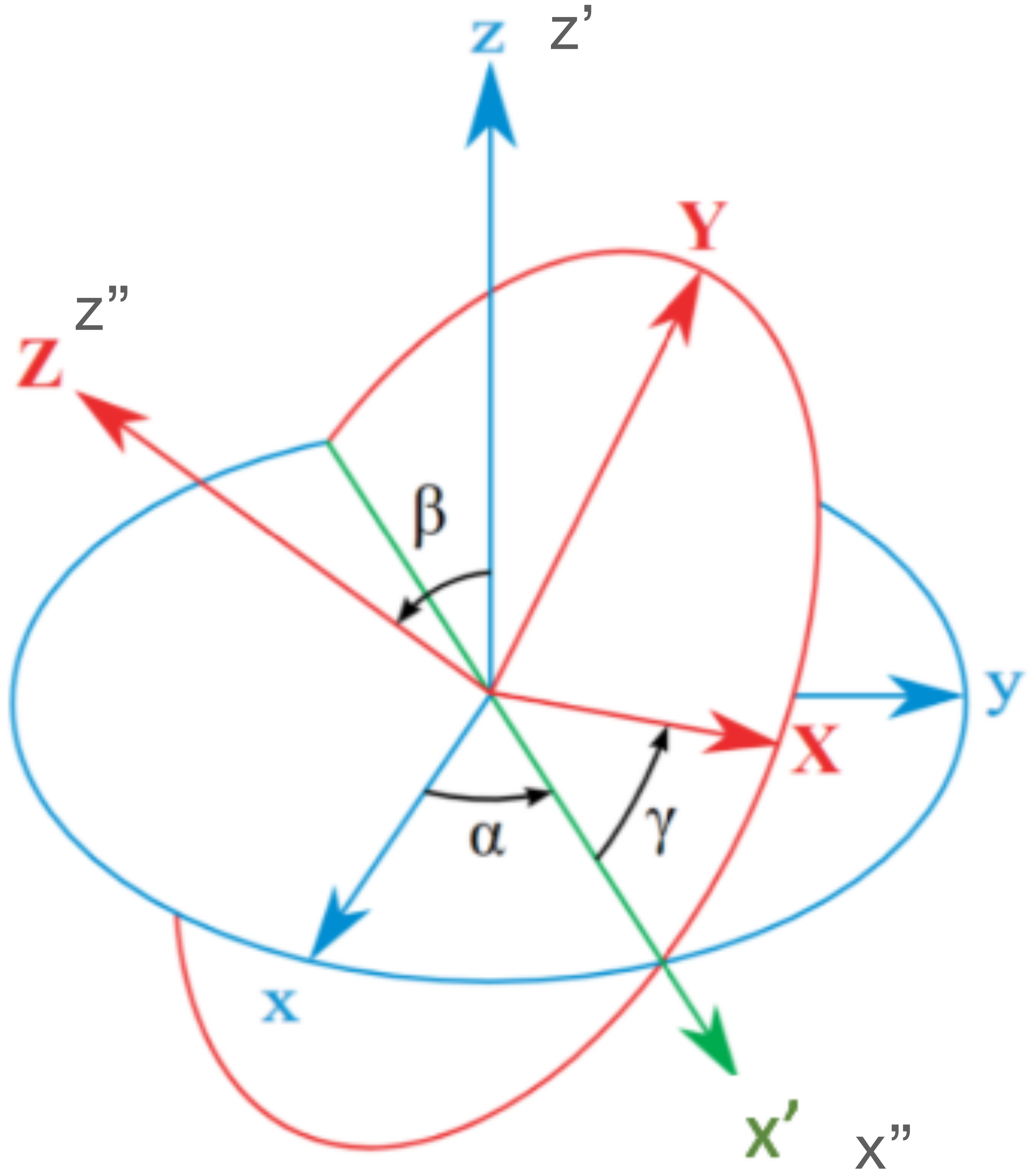
Rotation Visualization

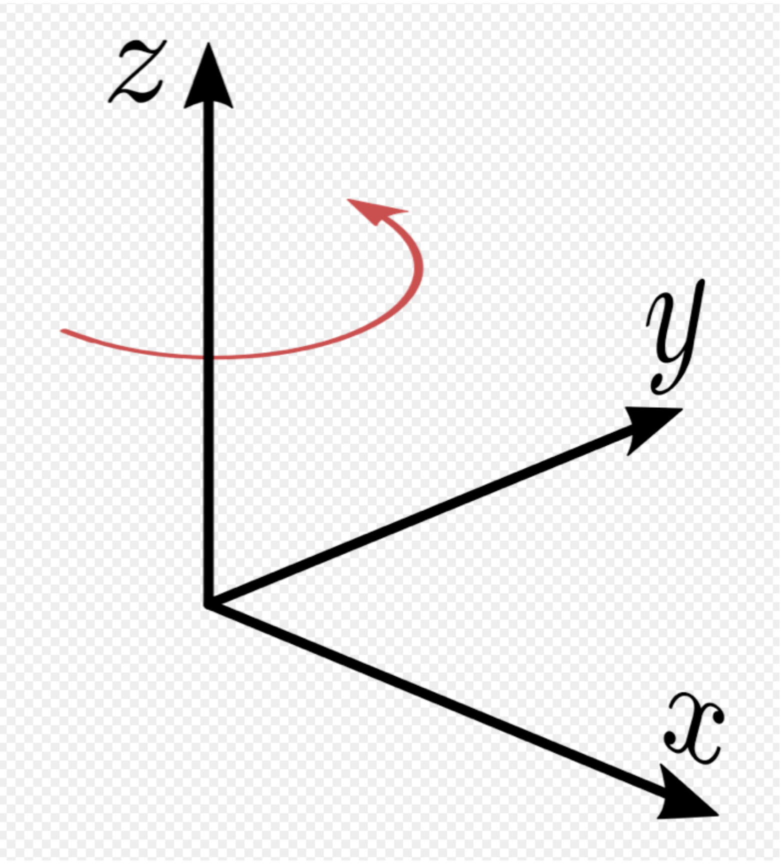
Figure 1 illustrates the transformation from coordinate xyz to XYZ: 1)rotate around z axis with $\alpha = 45°$ to get $x'y'z'$ axis; 2) rotate around x' axis with $\beta = 30°$ to get $x''y''z''$; 3) rotate around z'' axis with $\gamma = 60°$ to get XYZ.

- Design a program to get the rotation matrix from xyz to XYZ.
- Design a program to get the rotation matrix from XYZ to xyz.



- **from xyz rotate around z to get x'y'z': z' is same as z**

Basic Rotations

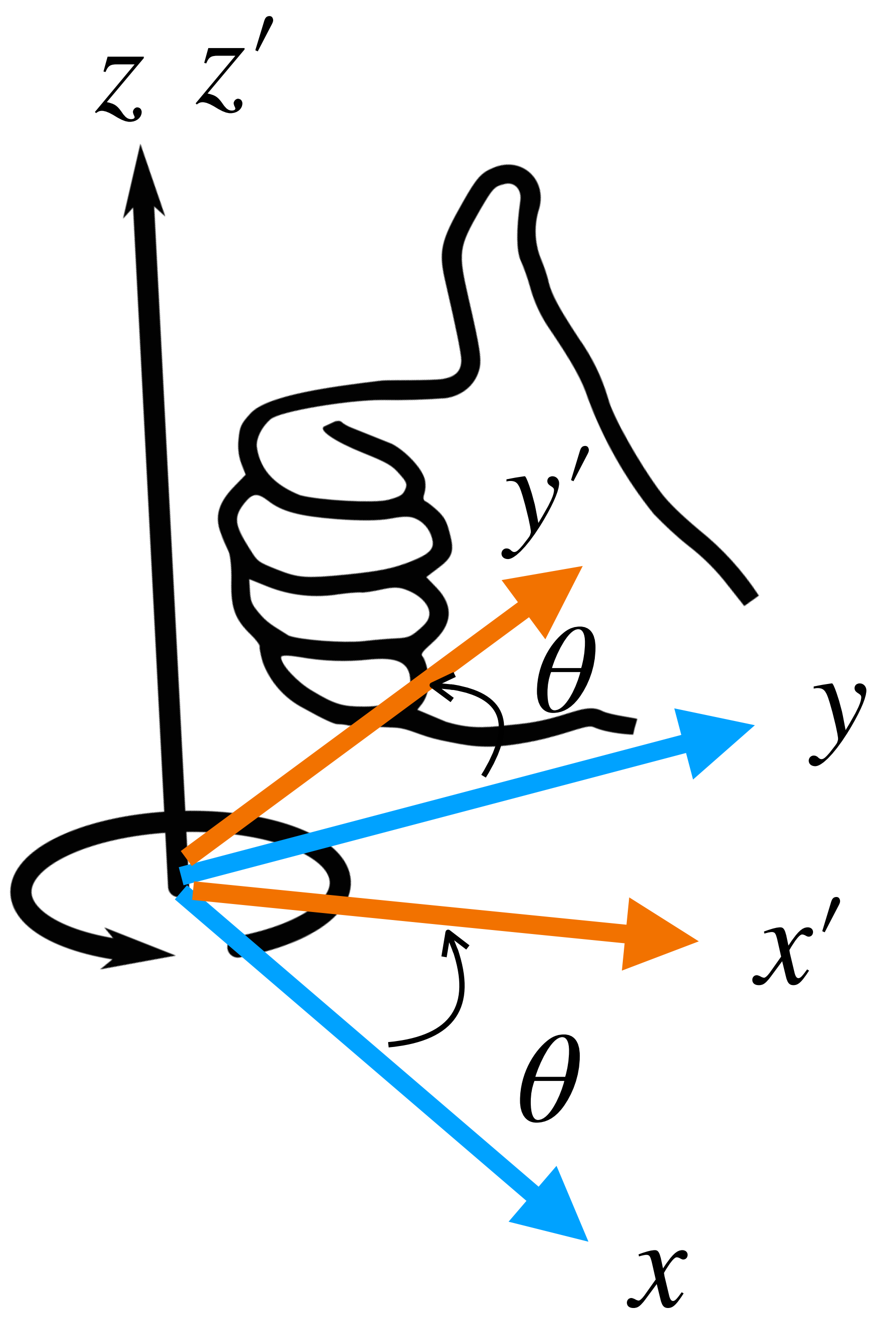


- **The following three basic rotation matrices rotate vectors by an angle θ about the x, y, or z-axis, in three dimensions, using the right-hand rule.**

$$R_x(\theta) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{bmatrix}$$

$$R_y(\theta) = \begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix}$$

$$R_z(\theta) = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$



https://en.wikipedia.org/wiki/Rotation_matrix

https://en.wikipedia.org/wiki/Right-hand_rule

Rotation Sequence

$$R_X(\theta_x) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & cos\theta_x & sin\theta_x \\ 0 & -sin\theta_x & cos\theta_x \end{pmatrix}$$

$$R_Y(\theta_y) = \begin{pmatrix} cos\theta_y & 0 & -sin\theta_y \\ 0 & 1 & 0 \\ sin\theta_y & 0 & cos\theta_y \end{pmatrix}$$

$$R_Z(\theta_z) = \begin{pmatrix} cos\theta_z & sin\theta_z & 0 \\ -sin\theta_z & cos\theta_z & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$P_{rotated} = RP \qquad P = \begin{pmatrix} p_x \\ p_y \\ p_z \end{pmatrix}$$

- Consider the rotation about X, then Y', then Z"

$$P_0 \xrightarrow{R_X} P_1 \xrightarrow{R_Y} P_2 \xrightarrow{R_Z} P_{final}$$

$$P_{final} = R_Z(\theta_z)R_Y(\theta_y)\boxed{R_X(\theta_x)P_0} = R_Z(\theta_z)\boxed{R_Y(\theta_y)\boxed{P_1}} = R_Z(\theta_z)\boxed{P_2}$$

$$P_{final} = R_{total}P_0$$

https://www1.udel.edu/biology/rosewc/kaap686/notes/matrices_rotations_2.pdf

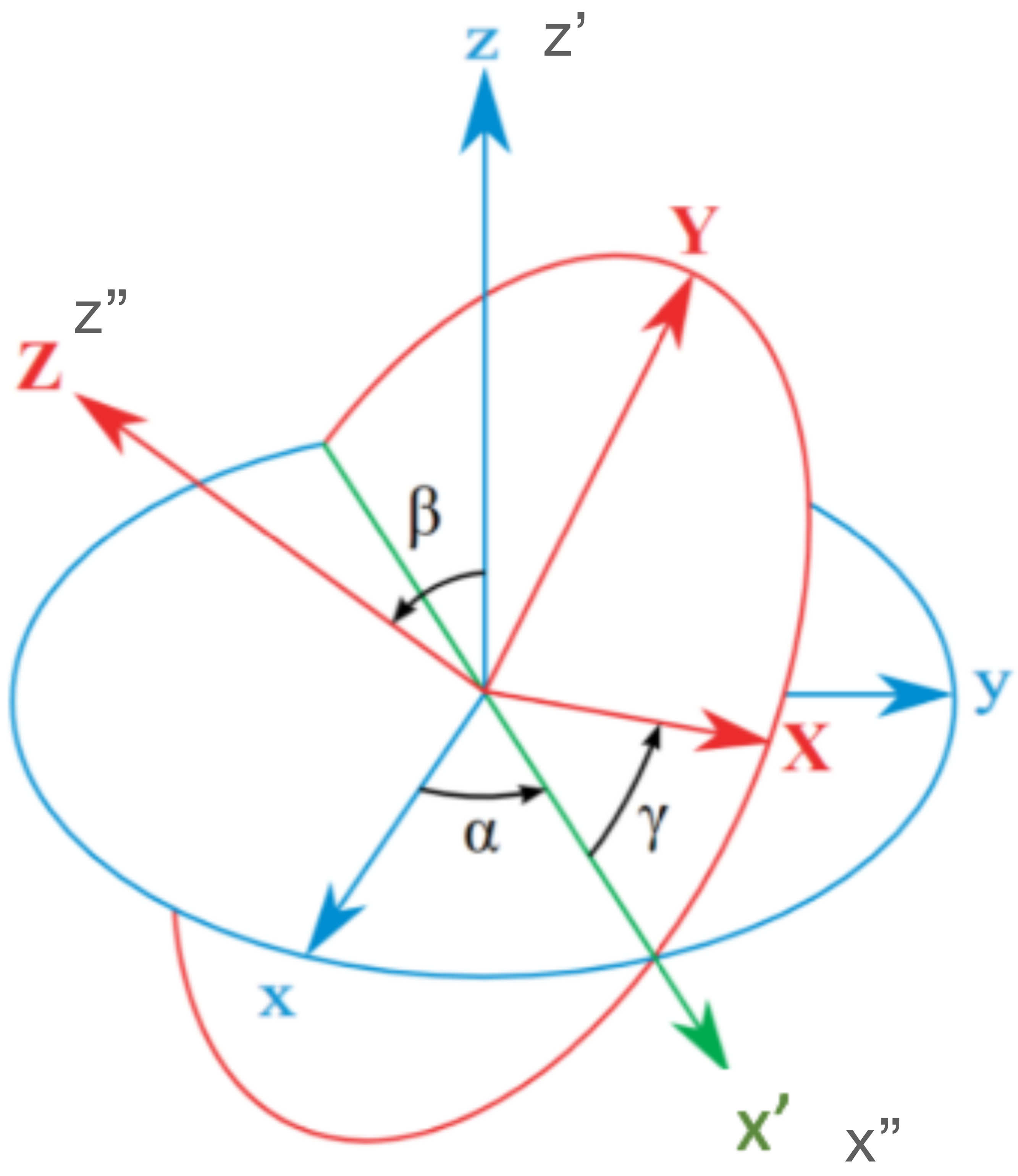
Rotation Matrix

Figure 1 illustrates the transformation from coordinate xyz to XYZ: 1)rotate around z axis with $\alpha = 45°$ to get $x'y'z'$ axis; 2) rotate around x' axis with $\beta = 30°$ to get $x''y''z''$; 3) rotate around z'' axis with $\gamma = 60°$ to get XYZ.

- Design a program to get the rotation matrix from xyz to XYZ.
- Design a program to get the rotation matrix from XYZ to xyz.

$$R = R_3(\theta_3) \cdot R_2(\theta_2) \cdot R_1(\theta_1)$$

$$R^{-1} = R_1(-\theta_1) \cdot R_2(-\theta_2) \cdot R_3(-\theta_3)$$



- **Understand the rotation visualization**
- **Get individual rotation matrix based on right-hand rule**
- **Ensemble individual rotation matrix and get total rotation matrix**

Calibration

- 1. Get world/image coordinate for 32 points
- 2. Build equation (Preliminary 2)

$$\begin{bmatrix} X_w^1 & Y_w^1 & Z_w^1 & 1 & 0 & 0 & 0 & 0 & -x^1X_w^1 & -x^1Y_w^1 & -x^1Z_w^1 & -x^1 \\ 0 & 0 & 0 & 0 & X_w^1 & Y_w^1 & Z_w^1 & 1 & -y^1X_w^1 & -y^1Y_w^1 & -y^1Z_w^1 & -y^1 \\ & & & & & & \vdots & & & & & \\ X_w^n & Y_w^n & Z_w^n & 1 & 0 & 0 & 0 & 0 & -x^nX_w^n & -x^nY_w^n & -x^nZ_w^n & -x^n \\ 0 & 0 & 0 & 0 & X_w^n & Y_w^n & Z_w^n & 1 & -y^nX_w^n & -y^nY_w^n & -y^nZ_w^n & -y^n \end{bmatrix} \cdot \begin{bmatrix} m_{11} \\ m_{12} \\ m_{13} \\ m_{14} \\ m_{21} \\ m_{22} \\ m_{23} \\ m_{24} \\ m_{31} \\ m_{32} \\ m_{33} \\ m_{34} \end{bmatrix} = \mathbf{0}, \qquad (8)$$

Updated in the new project1.PDF

- 3. Solve $Ax = 0$ where $|x| = 1$ (Preliminary 3)
- 4. Solve λ where $m = \lambda \cdot x$ (Refer to the attributes of rotation matrix in Preliminary 1)
- 5. Get m and calculate $f_x, f_y o_x, o_y$ (Preliminary 1)

Typo in previous PDF

$$\begin{aligned} sx &= m_{11}X_w + m_{12}Y_w + m_{13}Z_w + m_{14}, \\ sy &= m_{21}X_w + m_{22}Y_w + m_{23}Z_w + m_{24}, \\ s &= m_{31}X_w + m_{32}Y_w + m_{33}Z_w + m_{34}. \end{aligned}$$

$$x = \frac{sx}{s} = \frac{m_{11}X_w + m_{12}Y_w + m_{13}Z_w + m_{14}}{m_{31}X_w + m_{32}Y_w + m_{33}Z_w + m_{34}}$$

$$y = \frac{sy}{s} = \frac{m_{21}X_w + m_{22}Y_w + m_{23}Z_w + m_{24}}{m_{31}X_w + m_{32}Y_w + m_{33}Z_w + m_{34}}$$

Corrected Equation 8

$$\begin{bmatrix} X_w^1 & Y_w^1 & Z_w^1 & 1 & 0 & 0 & 0 & 0 & -x^1X_w^1 & -x^1Y_w^1 & -x^1Z_w^1 & -x^1 \\ 0 & 0 & 0 & 0 & X_w^1 & Y_w^1 & Z_w^1 & 1 & -y^1X_w^1 & -y^1Y_w^1 & -y^1Z_w^1 & -y^1 \\ & & & & & & \vdots & & & & & \\ & & & & & & \vdots & & & & & \\ X_w^n & Y_w^n & Z_w^n & 1 & 0 & 0 & 0 & 0 & -x^nX_w^n & -x^nY_w^n & -x^nZ_w^n & -x^n \\ 0 & 0 & 0 & 0 & X_w^n & Y_w^n & Z_w^n & 1 & -y^nX_w^n & -y^nY_w^n & -y^nZ_w^n & -y^n \end{bmatrix} \cdot \begin{bmatrix} m_{11} \\ m_{12} \\ m_{13} \\ m_{14} \\ m_{21} \\ m_{22} \\ m_{23} \\ m_{24} \\ m_{31} \\ m_{32} \\ m_{33} \\ m_{34} \end{bmatrix} = \mathbf{0}, \qquad (8)$$