Title:

An Introduction to Simulation and modeling

CS543

(HW1)

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1. Problem Definition:

In this study, first we evaluated the behavior of a random number generator in order to check the generated numbers are indeed uniform or not. To achieve this goal, we used the Chi-square test. The chi-square test is used to determine whether there is a significant difference between the expected frequencies and the observed frequencies in one or more categories.

In second part, an M/M/1 single server queue is simulated by the use of c programming language. In this study, average number of customers and average waiting in system are measured and compared with analytical results. In this part, the server goes to idle mode if there is no customer in the queue.

In the last part, an M/M/1 single server queue with vacation time is simulated. In this part, when the queue is empty, the server goes to a vacation time. The same as previous part, average number of customers and average waiting time in system are measured for different values of vacation time means.

2. Evaluation of Random Number Generator :

400 uniform random numbers is generated. The histogram of generated numbers is illustrated in Figure 1.

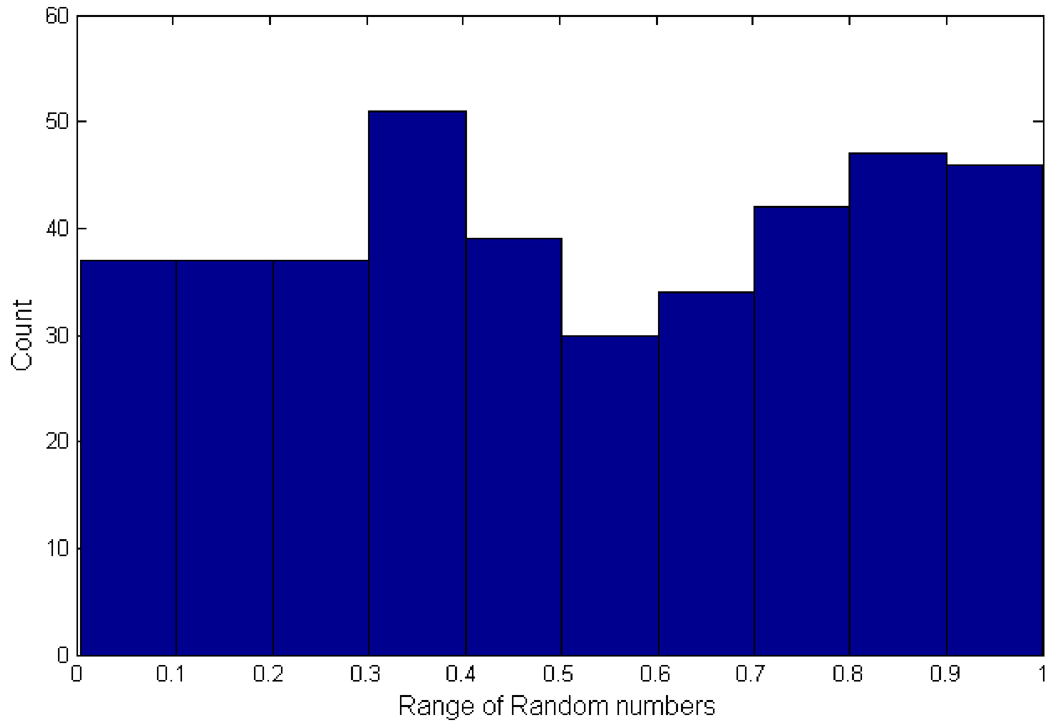

Figure 1. The histogram of generated uniform random numbers.

The observed and expected values of number of random numbers in each of the bins are as follows:

Number of Observed values	Number of expected values
37	40
37	40
37	40
51	40
39	40
30	40
34	40
42	40
47	40
46	40

Based on the following equation Chi-square value is equal to

$$Y = \sum_i (E_i - O_i)^2 / E_i$$
$$Y = 9.35$$

Based on the above result, Y < 16.919 and we cannot reject the H_0 hypothesis.

The mean value of generated random numbers is 0.5142. The mean value of uniform distribution can be calculated as follows:

U (0, 1) ---- mean = ½*(0+1) = 0.5

It can be seen that the real mean value and calculated mean value has a small difference to each other.

3. Simulation and evaluation of M/M/1 queue

In this part, a single server queue is simulated and analyzed. Simulation parameters are listed in table 1.

Table 1. Simulation Parameters

Simulation Parameter	Value
Number of customers	2000
Service rate	1
Arrival rates	0.1,0.2,0.3,0.4,0.5,0.6,0.7,0,0.9
Number of runs	10

In the following subsections, the simulation results are compared with analytical results. The amount of average waiting time and number of customers in system can be calculated analytically by the use of following equations:

$$N = \frac{\lambda}{\mu - \lambda} + \frac{\lambda}{\mu} \qquad (1)$$

$$T = \frac{1}{\mu - \lambda} \qquad (2)$$

Where N and T are average number of customers and average waiting time in the system respectively.

1.1 ρ Vs. Average number in system

The simulation results are illustrated in Figure 2. As we can see, as long as ρ is increased average number of customers in the system is increased as well. In order to change the value of ρ, the arrival rate should be changed when your service rate is fixed. When you increase the arrival rate, you have more customers in the system that the server should service them. In addition, the average number of customers in the system is calculated based on equation number 1 and compared with the simulation results. It can be seen in Figure 2 that the produced results in both of simulation and analytical approaches are approximately are the same.

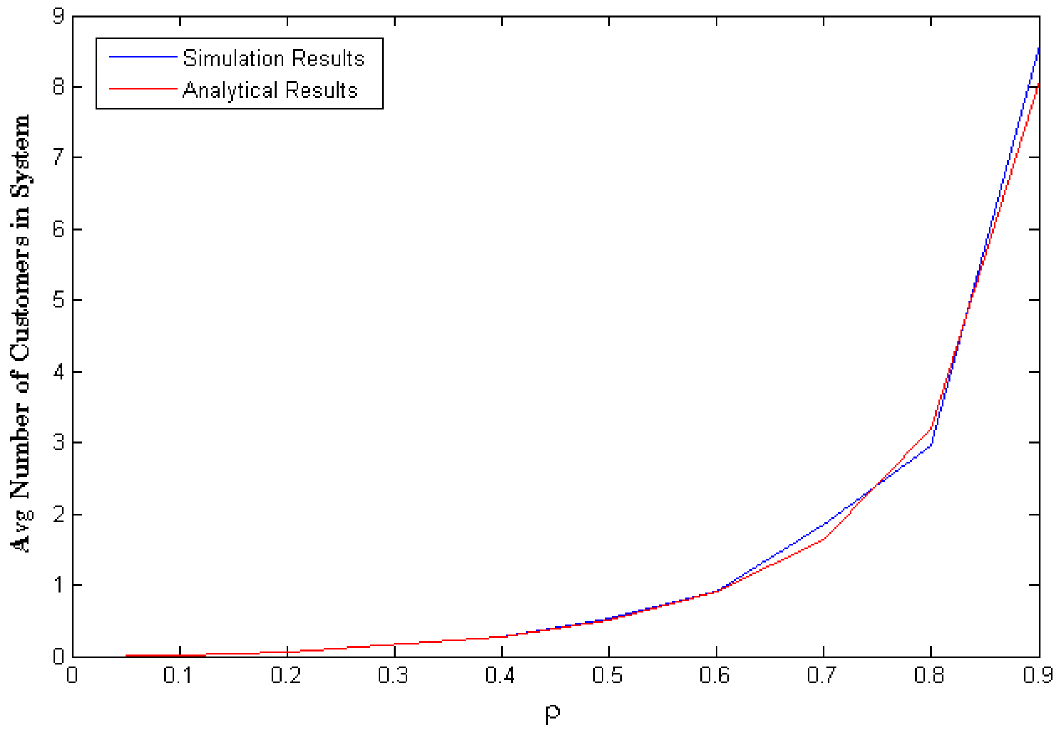

Figure 2. Average number of customers in system vs. utilization

2.1 ρ vs. average waiting time in system

The simulation results are shown in Figure 3. As we can see, when the ρ is increased, average waiting time in the system is increased as well. It is because of you have more customers in system to service consequently; the customers should wait more in the queue to get service.

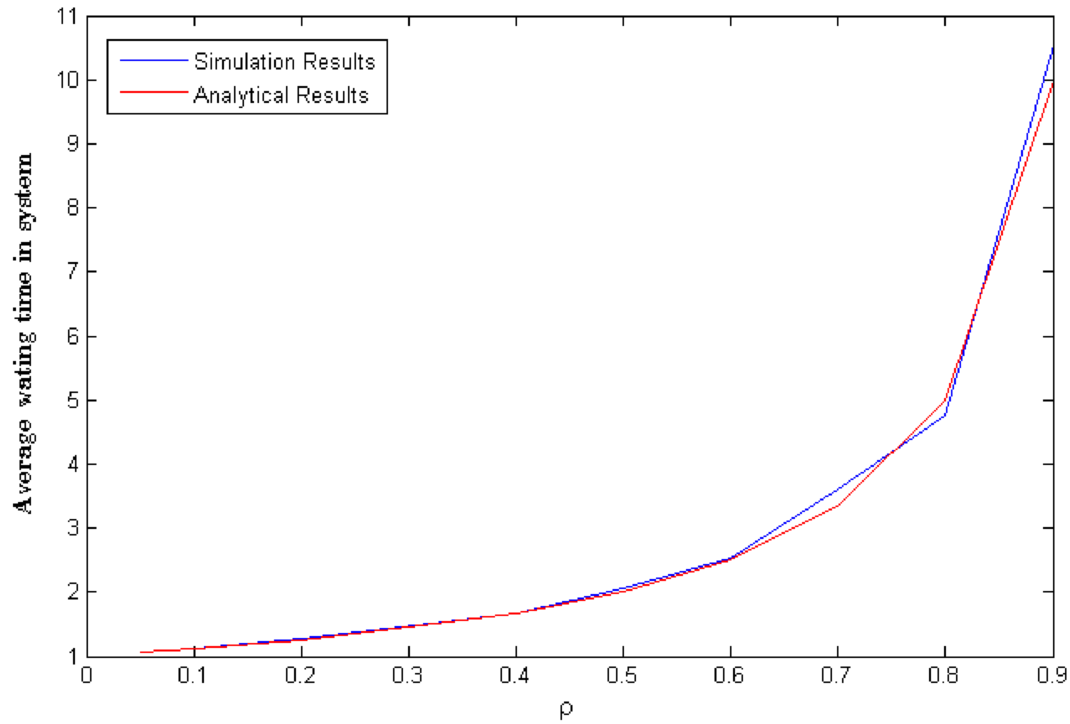

Figure 3. Average waiting time in system vs. utilization

4. Simulation and evaluation M/M/1 with vacation time

Simulation Parameters are listed in Table 2.

Simulation Parameter	Value
Number of customers	2000
Service rate	1
Arrival rates	0.9
Number of runs	10
k	1..10

The simulation results are shown in Figures 4 and 5. Figure 4 shows the average number of customers vs. k. In figure 4, we can see that when the mean vacation time is increased, the average number of customers in system is increased as well. The same trend is existence for average waiting time in the system. This matter is originated from this fact that when your server is on vacation for a longer period of time, the customers should wait for a longer period of time to get service.

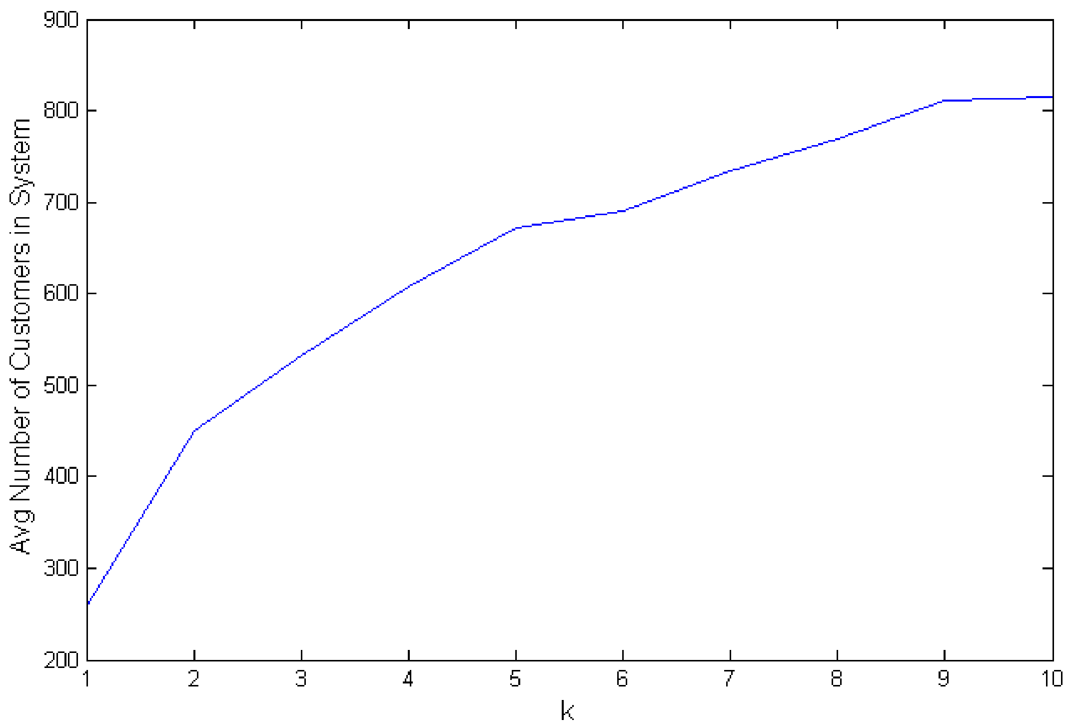

Figure 4. Average number of customers in system vs. k

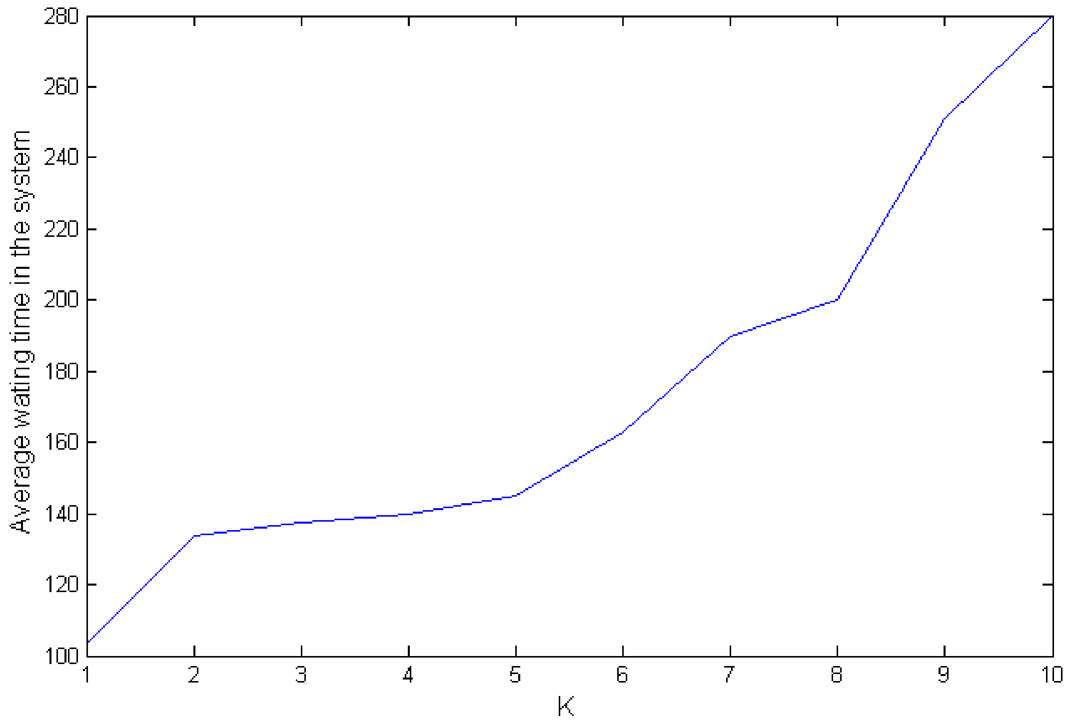

Figure 3. Average waiting time in system vs. k

5. Conclusion

In this report, we tried to evaluate the goodness of a random number generator by the use of Chi-square test. In addition, a single server queue without vacation and with vacation is simulated.