Title:

An Introduction to Simulation and Modeling

HW5

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Fall 2014

1- For this part, I have used tandem queue that I have implemented before (HW2). In addition, in order to calculate confidence interval and relative precision, I have used excel and Matlab . In Matlab environment, confplot tool is used in order to plot calculated confidence Interval. In addition a piece of code is written in order to calculate relative precision vs. different number of runs (both of codes are in the folder and you can use them if you like).

In addition, average waiting time in the system is calculated in analytical form in order to compare with simulation results based on following formula:

$$\text{Avg\_wating\_time\_system} = \sum_{i=1}^{8} \frac{1}{\mu_i - \lambda}$$

As you can see in Figure 1, both of simulation results and analytical results are in the confidence boundaries which shows that implemented tandem queue behaves properly.

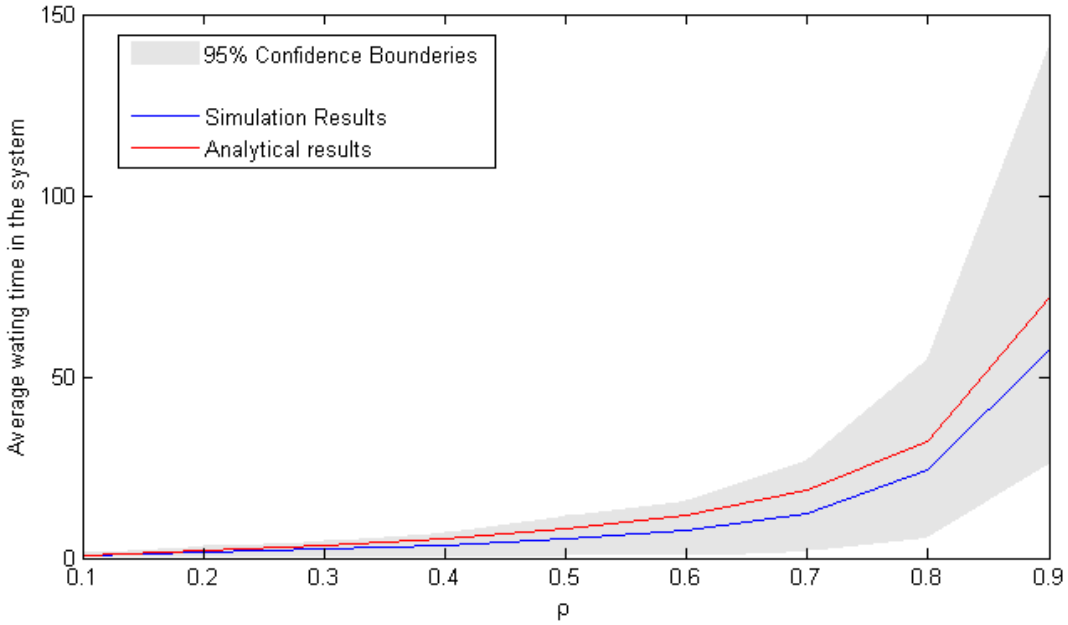

Figure 1. Confidence Interval for average waiting time in the system based on normal distribution

The effects of number of runs on relative precession is illustrated in Figures 2 and 3 (We have shown results only for two different values of service time). Simulation Results show that, 60 numbers of runs is needed in order to reach 0.1 precession value.

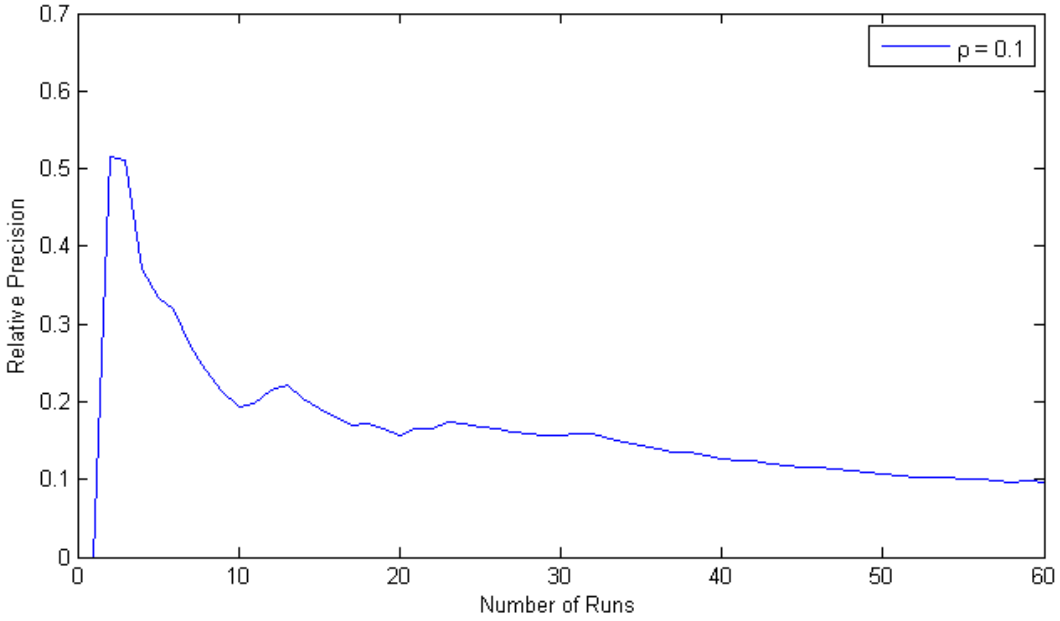

Figure 2. Relative precision vs. different number of runs

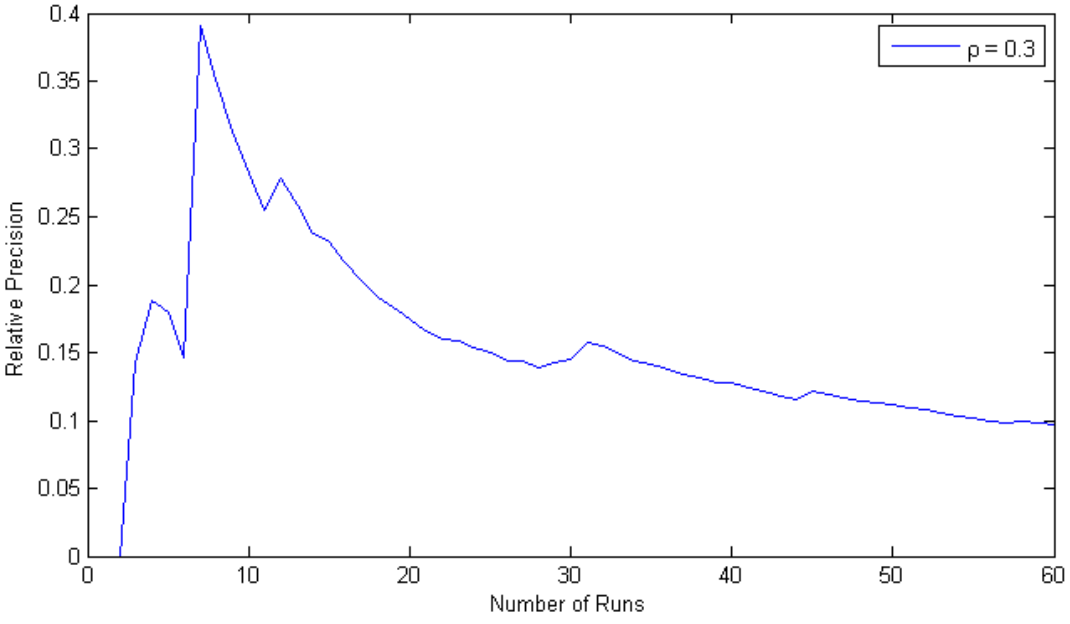

Figure 3. Relative precision vs. different number of runs

2- For in this case a small modification is applied in my source code (For generation of uniform random variables). Following algorithm is used in order to calculate confidence interval based on authentic method. (Two different folders are in the files which are named U and 1-U. these are source codes which I have used for simulation and producing results)

X = average waiting time in the System

X1, X2 ,……X60 is generate based on following random variables: U1,U2,……U60

X1',X2',X3',……X60' is generate based on following random variables: 1-U1, 1-U2, 1-U3 ….. 1-U60

After that average of each pairs is calculated and used for CI calculation.

Y1=(X1 + X1')/2

Y2=(X2 + X2')/2

…..

Y60 = (X60 + X60')/2

Calculated confidence Interval is calculated for average waiting time in the system is illustrated in Figure 4.

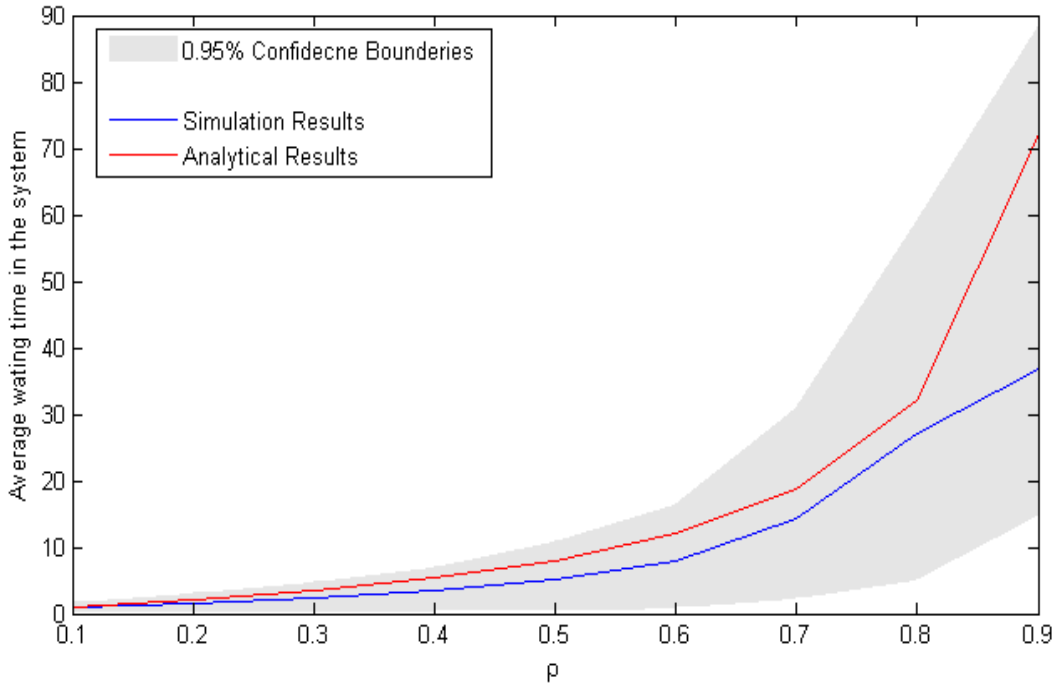

Figure 4. Confidence Interval for average waiting time in the system based on authentic method

In addition, relative precision values show that after around 40 runs we can reach to relative precision of 0.1 by using authentic method. The results are shown in Figures 5 and 6. This issue originated form this fact that variance is decreased be making simulation runs dependent to each other.

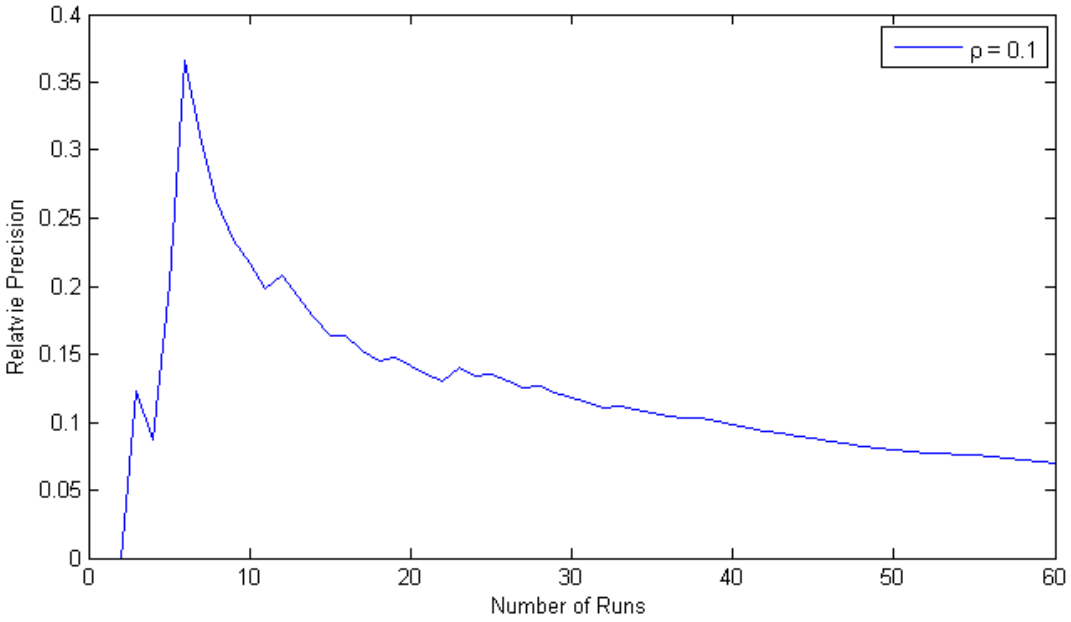

Figure 5. Relative precision by using authentic method

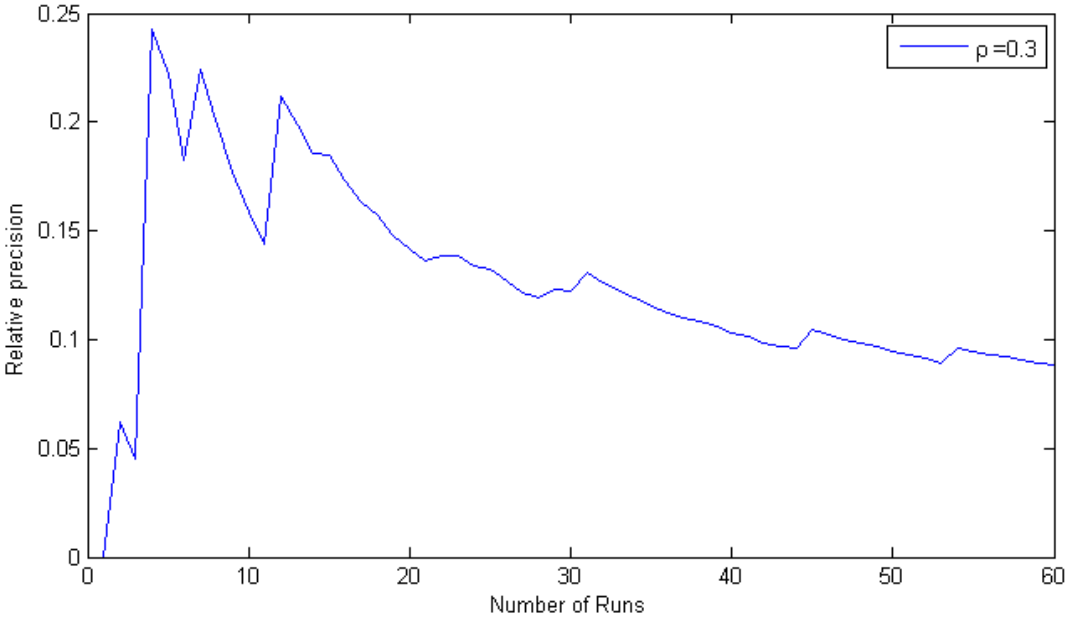

Figure 6. Relative precision by using authentic method

3- In this part, X is average waiting time in the system, and Y is total service time for over all of the queues.

Our estimation is calculated based on following formula:

$$\overline{X} + \hat{c} \times (\mathrm{Y} - \mu_Y)$$

Where

$$\hat{c} = -\frac{Cov(X, Y)}{Var(Y)}$$

Calculated confidence Interval is calculated for average waiting time in the system is illustrated in Figure 7. Calculated values show that in this method CI is decreased when compared with previous methods which show that control variate method has a better performance when compared with previous methods.

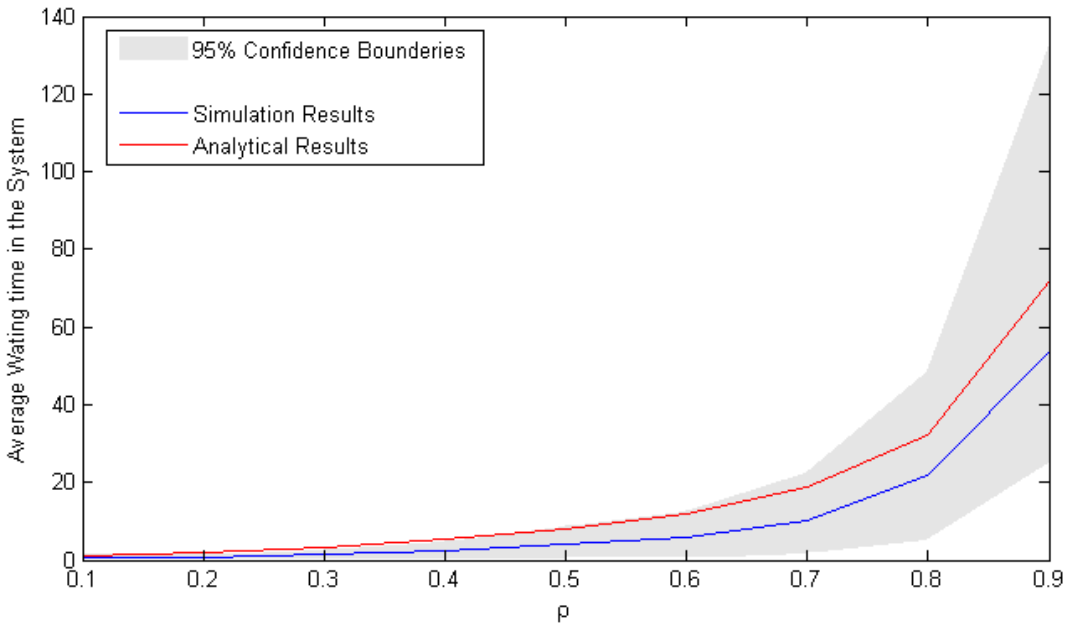

Figure 7. Confidence Interval for average waiting time in the system based on control variate method.

In addition relative precision vs. number of runs for control variate method is illustrated in Figure 8 and 9.

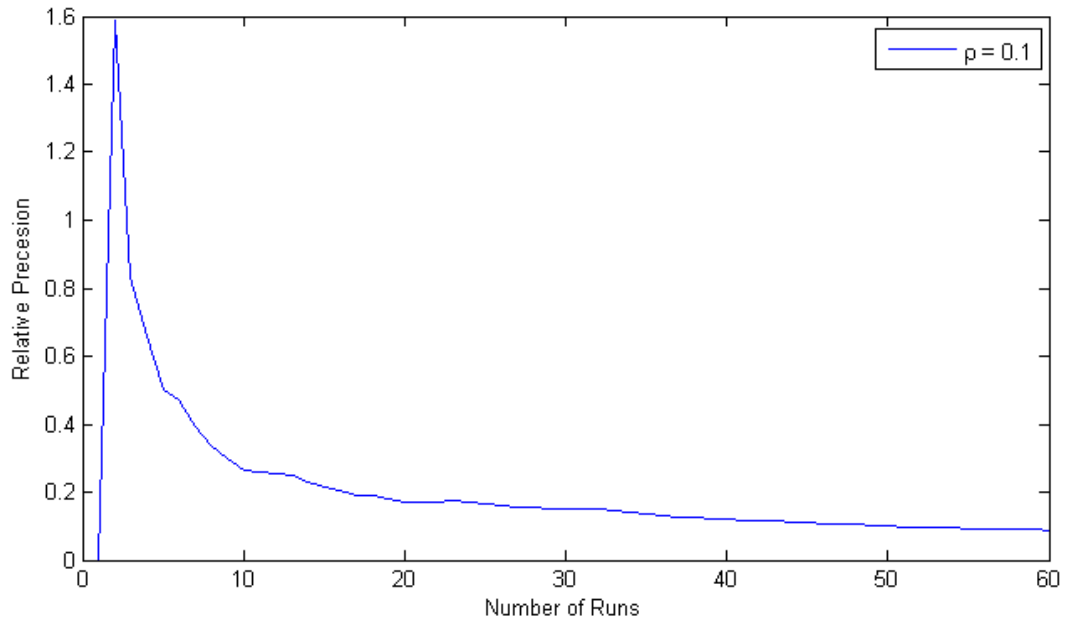

Figure 8. Relative precision by using variate method

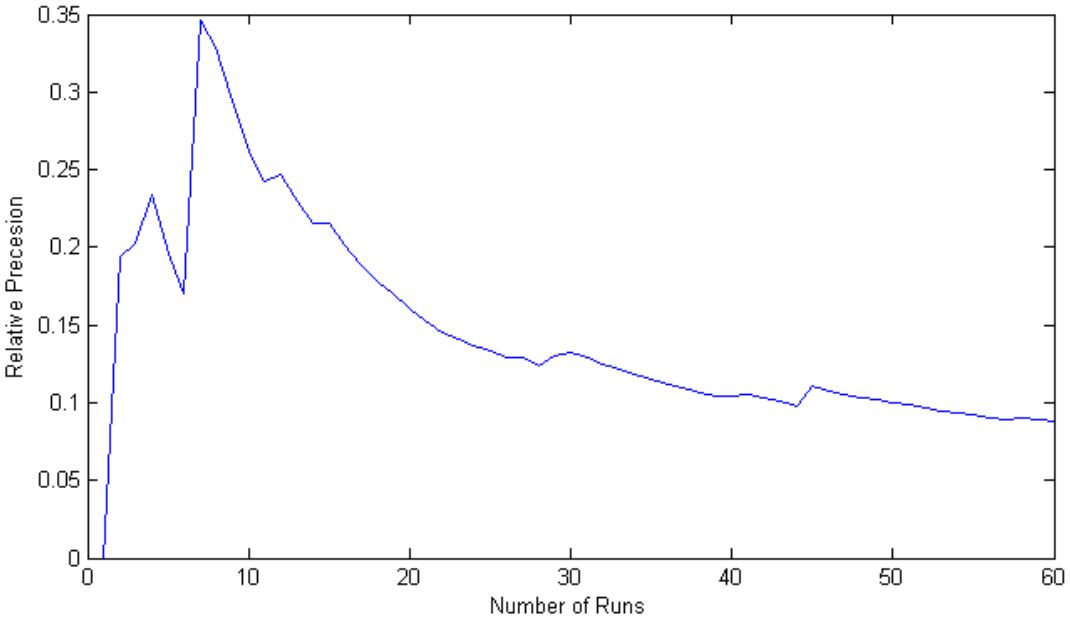

Figure 9. Relative precision by using variate method

Conclusion:

In this homework, we have tried to compute confidence Interval for average waiting time in the a tandem queue system. In addition, variance reduction methods are examined and results are reported.