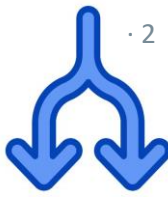




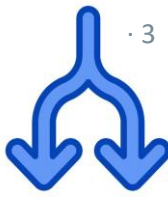
Practical Concurrent and Parallel Programming V

Performance Measurements

Jørgen Staunstrup



- Performance measurements: motivation and introduction
- Pitfalls (and avoiding them)
- Calculating means and variance (efficiently)
- Measurements of thread overhead
- Algorithms for parallel computing



- **Performance measurements: motivation and introduction**
- Pitfalls (and avoiding them)
- Calculating means and variance (efficiently)
- Measurements of thread overhead
- Algorithms for parallel computing

Motivations for Concurrency

From Week01



Inherent: User interfaces and other kinds of input/output

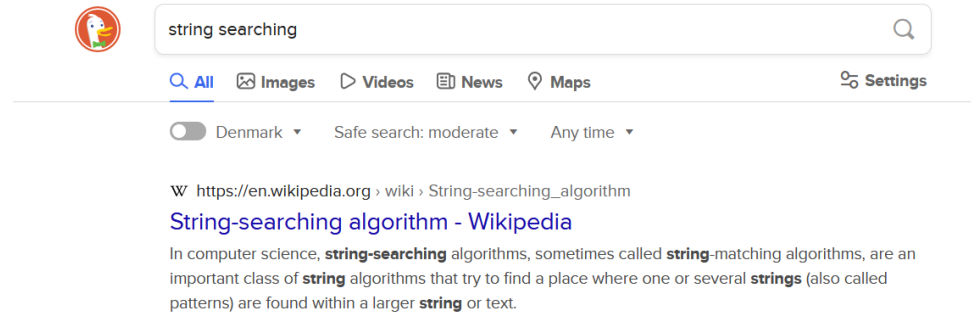
Exploitation: Hardware capable of simultaneously executing multiple streams of statements

Hidden: Enabling several programs to share some resources in a manner where each can act as if they had sole ownership

Motivation 1: Time consuming computations



Searching in a (large) text



<https://www.geeksforgeeks.org/applications-of-string-matching-algorithms/>

Computing prime numbers

2, 3, 5, 7, 11, 13, 17, 19, 23,
29, 31, 37, 41, 43, 47, 53, 59,
61, 67, 71, 73, 79, 83, 89, 97,
...

Cornerstone of all computer security

<https://science.howstuffworks.com/math-concepts/prime-numbers.htm>



Thread creation is expensive ?

The Java tutorials say that creating a Thread is expensive. But why exactly is it expensive? What exactly is happening when a Java Thread is created that makes its creation expensive? I'm taking the statement as true, but I'm just interested in mechanics of Thread creation in JVM.

Thread lifecycle overhead. Thread creation and teardown are not free. The actual overhead

But how expensive ?

~ 600 ns to create (on this laptop)

~ 20 times more time than creating a simple object

40000 ns to start a thread !!! (on this laptop)

Today: How to get such numbers !

(Performance) Measurements



Key in many sciences (experiments, observations, predictions, ...)

A bit of statistics

A bit of numerical analysis

A bit of computer architecture (cores, caches, number representation,)

Code for measuring execution time

Based on Microbenchmarks in Java and C# by Peter Sestoft (see [benchmarkingNotes.pdf](#) in material for this week)

All numbers in these slides were measured in August 2021 on a:

Intel Core i5-1035G4 CPU @ 1.10GHz, 4 Core(s), 8 Logical Processor(s)

- Performance measurements: motivation and introduction
- **Pitfalls (and avoiding them)**
- Calculating means and variance (efficiently)
- Measurements of thread overhead
- Algorithms for parallel computing

Example: measuring a (simple) function



```
private static double multiply(int i) {  
    double x = 1.1 * (double)(i & 0xFF);  
    return x * x * x * x * x * x * x * x * x * x * x * x *  
        * x * x * x * x * x * x * x * x * x * x * x * x;  
}
```

```
cd code-exercises/week05exercises  
gradle -PmainClass=exercises05.Measurement
```

```
start= System.nanoTime();  
multiply(126465);  
end= System.nanoTime();  
  
System.out.println(end-start+" ns");
```

Try to do this, what
result do you get?

3600 ns

1400 ns

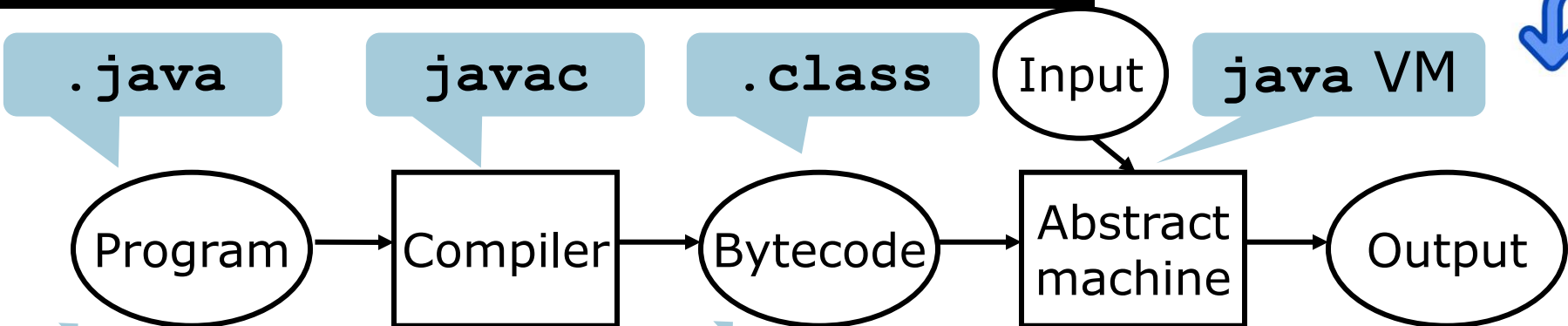
1500 ns

~ 2 - 5 ns

What is going on?

Java compiler and virtual machine

· 10



```
for (int i=0; i<n; i++)  
    sum += sqrt(arr[i]);
```

```
21 iconst_0  
22 istore 5  
24 iload 5  
26 iload 2  
27 if_icmpge      46  
30 dload 3  
31 aload 1  
32 iload 5  
34 daload  
35 invokestatic  Math.sqrt: (D)D  
38 dadd  
39 dstore 3  
40 iinc 5, 1  
43 goto 24
```

JVM

JIT (Just In Time)

```
19 xorl %ebx,%ebx  
1b jmp 3a  
1d leal 0x00(%ebp),%ebp  
20 fldl 0xec(%ebp)  
23 cmpl %ebx,0xc(%edi)  
26 jbe 49  
2c leal  
0x10(%edi,%ebx,8),%eax  
...
```

x86

Benchmarking note

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Microbenchmarks in Java and C#

Peter Sestoft (sestoft@itu.dk)

IT University of Copenhagen, Denmark

Version 0.8.0 of 2015-09-16

A goldmine of good advice

Accompanying code: [Benchmark.java](#)

On PCPP GitHub (week05)

Abstract: Sometimes one wants to measure the speed of software, for instance, to measure whether a

```
class Benchmark {
    public static void main(String[] args) { new Benchmark(); }

    public Benchmark() {
        // SystemInfo();
        // Mark0();
        // Mark1();
        ...
        Mark6("multiply", i -> multiply(i));
        ...
        // SortingBenchmarks();
        ...
    }
}
```

How to use Benchmark



Go to the directory for Week 5

Name	Date modified	Type
code-exercises	08-09-2022 14:04	File folder
code-lecture	08-09-2022 14:04	File folder
benchmarkingNotes.pdf	03-08-2020 07:43	Foxit PDF Re...
exercices05.pdf	08-09-2022 00:44	Foxit PDF Re...

```
cd code-exercises/week05exercises
gradle -PmainClass=exercices05.Benchmark run
```

What do you see?

```
# OS:    windows 10; 10.0; amd64
# JVM:   Oracle Corporation; 1.8.0_181
# CPU:   Intel64 Family 6 Model 126 Stepping 5, GenuineIntel; ...
# Date:  2022-09-08T14:10:56+0200
...

```

Example: measuring a simple function

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```
private static double multiply(int i) {  
    double x = 1.1 * (double)(i & 0xFF);  
    return x * x * x * x * x * x * x * x * x * x * x * x  
        * x * x * x * x * x * x * x * x * x * x * x * x;  
}
```

```
public static double Mark2() {  
    Timer t = new Timer();  
    int count = 100_000_000;  
    double dummy = 0.0;  
    for (int i=0; i<count; i++)  
        dummy += multiply(i);  
    double time = t.check() * 1e9 / count;  
    System.out.printf("%6.1f ns%n", time);  
    return dummy;  
}
```

```
# OS:    Windows 10; 10.0; amd64  
# JVM:   Oracle Corporation; 17.0.2  
# CPU:   Intel64 Family 6 Model 126 Stepping 5, GenuineIntel; 8 "cores"  
# Date:  2023-09-04T08:45:32+0200  
11.9 ns
```

The Timer class (in Benchmark.java)



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A simple Timer class for Java

Works on all platforms (Linux, MacOS, Windows)

```
public class Timer {  
    private long start, spent = 0;  
    public Timer() { play(); }  
    public double check()  
    { return (System.nanoTime()-start+spent)/1e9; }  
    public void pause() { spent += System.nanoTime()-start; }  
    public void play() { start = System.nanoTime(); }  
}
```

What does 1e9 mean?

In what time unit do we get the results?

Automating multiple runs (Mark3)



Results will usually vary

```
public static double Mark3() {  
    int n = 10;  
    int count = 100_000_000;  
    double dummy = 0.0;  
    for (int j=0; j<n; j++) {  
        Timer t = new Timer();  
        for (int i=0; i<count; i++)  
            dummy += multiply(i);  
        double time = t.check() * 1e9 / count;  
        System.out.printf("%6.1f ns%n", time);  
    }  
    return dummy;  
}
```

```
24.6 ns  
24.6 ns  
24.5 ns  
24.6 ns  
24.4 ns  
24.3 ns  
24.5 ns  
24.4 ns  
24.7 ns  
24.6 ns
```

What is the running time?



What should you report as the result, when the observations are:

30.7 ns 30.3 ns 30.1 ns 30.7 ns 30.5 ns 30.4 ns 30.9 ns 30.3 ns 30.5 ns 30.8 ns ?

Mean: 30.4 ns

What if they are:

30.7 ns 100.2 ns 30.1 ns 30.7 ns 20.2 ns 30.4 ns 2.0 ns 30.3 ns 30.5 ns 5.4 ns ??

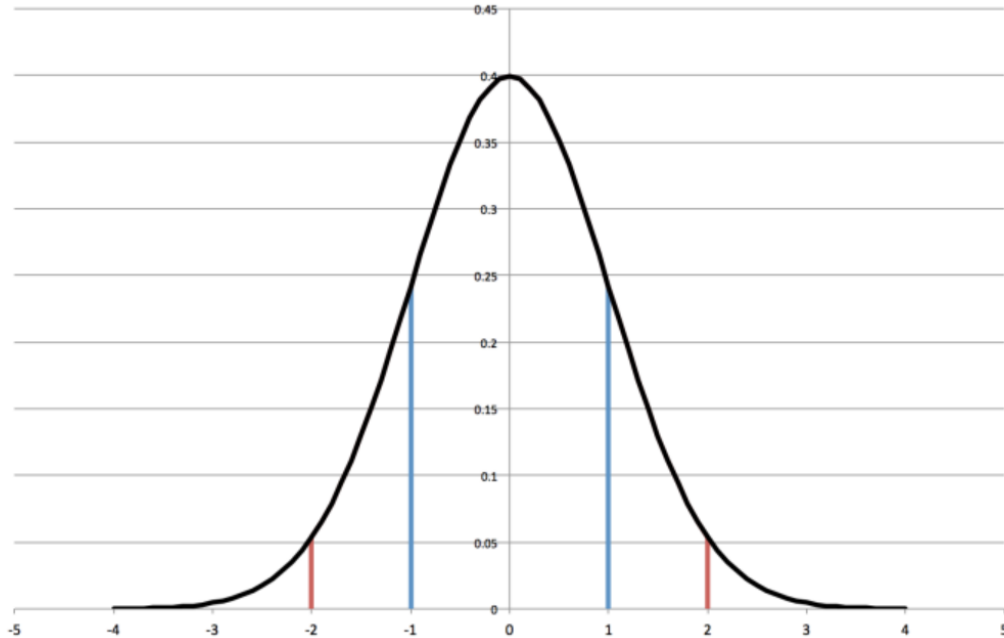
Mean: 31.0 ns

- Performance measurements: motivation and introduction
- Pitfalls (and avoiding them)
- **Calculating means and variance (efficiently)**
- Measurements of thread overhead
- Algorithms for parallel computing

Normal distribution



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Measuring physical properties

Your exam grades

Course evaluations

Fabrication faults

Running time of Java code

...

Mark5 - computes mean and variance



```
public static double Mark5() {
    int n = 10, count = 1, totalCount = 0;
    double dummy = 0.0, runningTime = 0.0, st = 0.0, sst = 0.0;
    do {
        count *= 2;
        st = sst = 0.0;
        for (int j=0; j<n; j++) {
            Timer t = new Timer();
            for (int i=0; i<count; i++) dummy += multiply(i);
            runningTime = t.check();
            double time = runningTime * 1e9 / count;
            st += time;
            sst += time * time;
            totalCount += count;
        }
        double mean = st/n, sdev = Math.sqrt((sst - mean*mean*n)/(n-1));
        System.out.printf("%6.1f ns +/- %8.2f %10d%n", mean, sdev, count);
    } while (runningTime < 0.25 && count < Integer.MAX_VALUE/2);
    return dummy / totalCount;
}
```

Mark5 - computes mean and variance

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```
public static double Mark5() {
    int n = 10, count = 1, totalCount = 0;
    double dummy = 0.0, runningTime = 0.0, st = 0.0, sst = 0.0;
    do {
        count *= 2;
        st = sst = 0.0;
        for (int j=0; j<n; j++) {
            Timer t = new Timer();
            for (int i=0; i<count; i++) dummy += multiply(i);
            runningTime = t.check();
            double time = runningTime * 1e9 / count;
            st += time;
            sst += time * time;
            totalCount += count;
        }
        double mean = st/n, sdev = Math.sqrt((sst - mean*mean*n)/(n-1));
        System.out.printf("%6.1f ns +/- %8.2f %10d%n", mean, sdev, count);
    } while (runningTime < 0.25 && count < Integer.MAX_VALUE/2);
    return dummy / totalCount;
}
```

```
public static double Mark5() {
    int n = 10, count = 1, totalCount = 0;
    double dummy = 0.0, runningTime = 0.0, st = 0.0, sst = 0.0;
    do {
        count *= 2;
        st = sst = 0.0;
        for (int j=0; j<n; j++) {
            Timer t = new Timer();
            for (int i=0; i<count; i++) dummy += multiply(i);
            runningTime = t.check();
            double time = runningTime * 1e9 / count;
            st += time;
            sst += time * time;
            totalCount += count;
        }
        double mean = st/n, sdev = Math.sqrt((sst - mean*mean*n)/(n-1));
        System.out.printf("%6.1f ns +/- %8.2f %10d%n", mean, sdev, count);
    } while (runningTime < 0.25 && count < Integer.MAX_VALUE/2);
    return dummy / totalCount;
}
```

Parameterizing function to be measured

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Yes?

```
private static double multiply(int i) {  
    . . .  
}
```

Java: `multiply(i)` is a number

Java: `i -> multiply(i)` is a function

<https://docs.oracle.com/javase/tutorial/java/javaOO/lambdaexpressions.html>

```
Mark6( . . . , i -> multiply(i));
```

Mark6 - introduce a functional argument

· 24



The function **f** is benchmarked

```
public static double Mark6(String msg, IntToDoubleFunction f) {
    int n = 10, count = 1, totalCount = 0;
    double dummy = 0.0, runningTime = 0.0, st = 0.0, sst = 0.0;
    do {
        count *= 2;
        st = sst = 0.0;
        for (int j=0; j<n; j++) {
            Timer t = new Timer();
            for (int i=0; i<count; i++) dummy += f.applyAsDouble(i);
            runningTime = t.check();
            double time = runningTime * 1e9 / count;
            st += time; sst += time * time; totalCount += count;
        }
        double mean = st/n, sdev = Math.sqrt((sst - mean*mean*n)/(n-1));
        System.out.printf("%-25s %15.1f ns %10.2f %10d%n", msg, mean, sdev, count);
    } while (runningTime < 0.25 && count < Integer.MAX_VALUE/2);
    return dummy / totalCount;
}

public interface IntToDoubleFunction { double applyAsDouble(int i); }

Mark6("multiply", i -> multiply(i));
```

←
lambda

Mark6 - introduce a functional argument



```
public static double Mark6(String msg, IntToDoubleFunction f) {
    int n = 10, count = 1, totalCount = 0;
    double dummy = 0.0, runningTime = 0.0, st = 0.0, sst = 0.0;
    do {
        count *= 2;
        st = sst = 0.0;
        for (int j=0; j<n; j++) {
            Timer t = new Timer();
            for (int i=0; i<count; i++) dummy += f.applyAsDouble(i);
            runningTime = t.check();
            double time = runningTime * 1e9 / count;
            st += time; sst += time * time; totalCount += count;
        }
        double mean = st/n, sdev = Math.sqrt((sst - mean*mean*n)/(n-1));
        System.out.printf("%-25s %15.1f ns %10.2f %10d%n", msg, mean, sdev, count);
    } while (runningTime < 0.25 && count < Integer.MAX_VALUE/2);
    return dummy / totalCount;
}

public interface IntToDoubleFunction { double applyAsDouble(int i); }

Mark6("multiply", i -> multiply(i));
```


Example use of Mark6



```
Mark6("multiply", i -> multiply(i));
```

multiply	595.0 ns	1407.81	2
multiply	147.5 ns	90.10	4
multiply	212.5 ns	152.53	8
multiply	170.6 ns	59.44	16
multiply	201.9 ns	157.69	32
multiply	60.8 ns	34.55	64
multiply	65.1 ns	59.83	128
multiply	54.3 ns	14.85	256
...			
multiply	24.6 ns	0.75	524288
multiply	24.6 ns	0.88	1048576
multiply	24.9 ns	2.71	2097152
multiply	24.3 ns	0.85	4194304
multiply	24.2 ns	0.72	8388608
multiply	25.0 ns	1.38	16777216

Mark7 - printing only final values



```
public static double Mark7(String msg, IntToDoubleFunction f) {  
    ...  
    do {  
        ...  
    } while (runningTime < 0.25 && count < Integer.MAX_VALUE/2);  
    double mean = st/n, sdev = Math.sqrt((sst - mean*mean*n)/(n-1));  
    System.out.printf("%-25s %15.1f %10.2f %10d%n", msg, mean, sdev, count);  
    return dummy / totalCount;  
}
```

Timing prime calculation

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```
private static boolean isPrimeS(int n) {  
    int k = 2;  
    while (k < n && n % k != 0) k++;  
    return n >= 2 && k >= n;  
}
```

Does it matter which one
we use?

```
private static boolean isPrime(int n) {  
    int k = 2;  
    while (k * k <= n && n % k != 0) k++;  
    return n >= 2 && k * k > n;  
}
```

- Performance measurements: motivation and introduction
- Pitfalls (and avoiding them)
- Calculating means and variance (efficiently)
- **Measurements of thread overhead**
- Algorithms for parallel computing

Thread creation



```
Mark7("Thread create",  
    i -> {  
        Thread t = new Thread(() -> {  
            for (int j=0; j<1000; j++)  
                ai.getAndIncrement();  
        });  
        return t.hashCode(); // to confuse compiler to not optimize  
    });
```

Takes 700 ns

What are we really measuring?

Slow or fast?

Creating an object

· 33



A thread is an object, so let us start finding the cost of creating a simple object.

```
class Point {  
    public final int x, y;  
    public Point(int x, int y) { this.x = x; this.y = y; }  
}
```

```
Mark7("hashCode()", i -> myPoint.hashCode());
```

```
Mark7("Point creation",  
    i -> {  
        Point p = new Point(i, i);  
        return p.hashCode();  
    });
```

hashCode() 3 ns

Point creation 50 ns

So object creation is: ~ 47 ns

Thread creation ~ 650ns

Thread create + start



· 34

```
Mark6("Thread create start",
    i -> {
        Thread t = new Thread(() -> {
            for (int j=0; j<1000; j++)
                ai.getAndIncrement();
        });
        t.start();
        return t.hashCode();
    });
```

What are we really measuring?

Thread create + start



```
Mark6("Thread create start",
      i -> {
        Thread t = new Thread(() -> {
          for (int j=0; j<1000; j++)
            ai.getAndIncrement();
        });
        t.start();
        return t.hashCode();
      });
```

For loop not included, why?

Thread create + start

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```
Mark6("Thread create start",
    i -> {
        Thread t = new Thread(() -> {
            for (int j=0; j<1000; j++) //most iterations not done
                ai.getAndIncrement(); // Why?
        });
        t.start();
        return t.hashCode();
    });
```

Takes ~ 47000 ns

- So, a lot of work goes into starting a thread
- Even after creating it
- Note: does not include executing the loop

Never create threads for small computations !!!

- Performance measurements: motivation and introduction
- Pitfalls (and avoiding them)
- Calculating means and variance (efficiently)
- Measurements of thread overhead
- **Algorithms for parallel computing**



Quicksort: <https://www.chrislaux.com/quicksort.html>

```
private static void qsort(int[] arr, int a, int b) {
    if (a < b) {
        int i = a, j = b;
        int x = arr[(i+j) / 2];
        do {
            while (arr[i] < x) i++;
            while (arr[j] > x) j--;
            if (i <= j) { swap(arr, i, j); i++; j--; }
        } while (i <= j);
        qsort(arr, a, j); qsort(arr, i, b);
    }
}
```

see SearchAndSort.java in week 05 material

Prime counting: <https://www.dcode.fr/prime-number-pi-count>

```
long count = 0;
final int from = 0, to = range;
for (int i=from; i<to; i++) if (isPrime(i)) count++;
```

Multithreaded version of CountPrimes



· 40

2, 3, 4, 5,



thread0



thread1

range

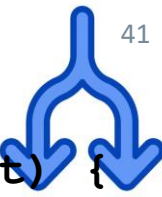


threadN

Code for exercises week05: [TestCountPrimesThreads.java](#)

Java code for TestCountTimesThreads

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```
private static long countParallelN(int range, int threadCount) {  
    final int perThread= range / threadCount;  
    final LongCounter lc= new LongCounter();  
    Thread[] threads= new Thread[threadCount];  
    for (int t=0; t<threadCount; t++) {  
        final int from= perThread * t,  
            to= (t+1==threadCount) ? range : perThread * (t+1);  
        threads[t]= new Thread()  
            > {for (int i=from; i<to; i++)  
                if (isPrime(i)) lc.increment();  
            });  
    }  
    for (int t=0; t<threadCount; t++) threads[t].start();  
    try { for (int t=0; t<threadCount; t++) threads[t].join();  
        } catch (InterruptedException exn) { }  
    return lc.get();  
}
```



countSequential		5922958.0	ns	289879.33
countParallel	1	7107236.6	ns	448417.55
countParallel	2	6069944.7	ns	802224.61
countParallel	3	3621185.5	ns	152693.03
countParallel	4	3124067.0	ns	640480.51
countParallel	5	3699514.7	ns	364428.77
countParallel	6	4114074.2	ns	642562.19
countParallel	7	2049595.7	ns	26888.15
countParallel	8	1801465.6	ns	12532.85
countParallel	9	1793099.1	ns	11017.57
countParallel	10	1798921.4	ns	11541.43
countParallel	11	1807408.3	ns	9763.61

Good or bad?



countParallel	1	7107236.6	ns	448417.55
countParallel	2	6069944.7	ns	802224.61
countParallel	3	3621185.5	ns	152693.03
countParallel	4	3124067.0	ns	640480.51

...

2, 3, 4, 5,



thread0



thread1

range



threadN

Is this good or bad, and why?

Breaking the task into smaller pieces/tasks



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2, 3, 4, 5,



range



When a thread is done with one task, it gets a new task
until all tasks are done



```
new Thread(runnable1).start();  
...  
new Thread(runnable2).start();  
...  
new Thread(runnable3).start();
```

Threads are expensive !

```
ExecutorService pool;
```

```
pool.execute(runnable1);  
...  
pool.execute(runnable2);  
...  
pool.execute(runnable2);
```

Reuse of threads

<https://howtodoinjava.com/java/multi-threading/java-fixed-size-thread-pool-executor-example/>

Prime counter task (skeleton)

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```
public class countPrimesTask implements Runnable {  
    private final int low;  
    private final int high;  
    private final ExecutorService pool;  
  
    @Override public void run() {  
  
        int mid= low+(high-low)/2;  
        pool.submit( new countPrimesTask(low, mid, pool) );  
        pool.submit( new countPrimesTask(mid+1, high, pool) );  
  
    }  
}
```

Shortcomings:

1. How to stop?
2. Will create too many "small" tasks
3. Returning result (# primes)

Reducing the number of tasks

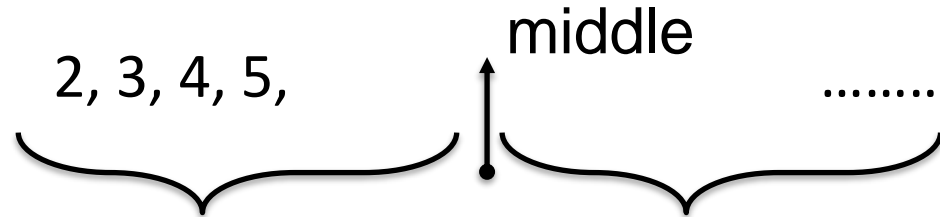


2, 3, 4, 5,

range



range > threshold: split task



range <= threshold: count the number of primes



@Override

```
public void run() {  
    if ((high-low) < threshold) {  
        for (int i=low; i<=high; i++)    if (isPrime(i)) lc.increment();  
    } else {  
        int mid= low+(high-low)/2;  
        pool.submit(new countPrimesTask(lc, low, mid, pool, threshold) );  
        pool.submit(new countPrimesTask(lc, mid+1, high, pool, threshold) );  
    }  
}
```

Shortcomings:

1. How to stop?
- ~~2. Will create too many "small" tasks~~
3. Returning result (# primes)

More next week

Counting the primes

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```
public class countPrimesExecutor {
    ..
    private final LongCounter lc; // Global shared variable !!!
    ...                          // requires atomicity
}

public class countPrimesTask implements Runnable {
    ...
    public void run() {
        ...
        for (int i=low; i<=high; i++)
            if (isPrime(i)) lc.increment();
        ...
    }
}
```

Could `lc` become a bottleneck?

Counting the primes



```
public class countPrimesExecutor {  
    ..  
    private final LongCounter lc; // Global shared variable !!!  
    ...                          // requires atomicity  
  
}  
  
public class countPrimesTask implements Runnable {  
    ...  
    public void run() {  
        ...  
        for (int i=low; i<=high; i++)  
            if (isPrime(i)) lc.increment();  
        ...  
    }  
}
```

Exercise 5.2 ?

Thread vs Executor



Counting primes in the range 2..1_000_000

Sequential	1.2 Sec
Threads (4)	0.5 Sec
Executor	0.4 Sec

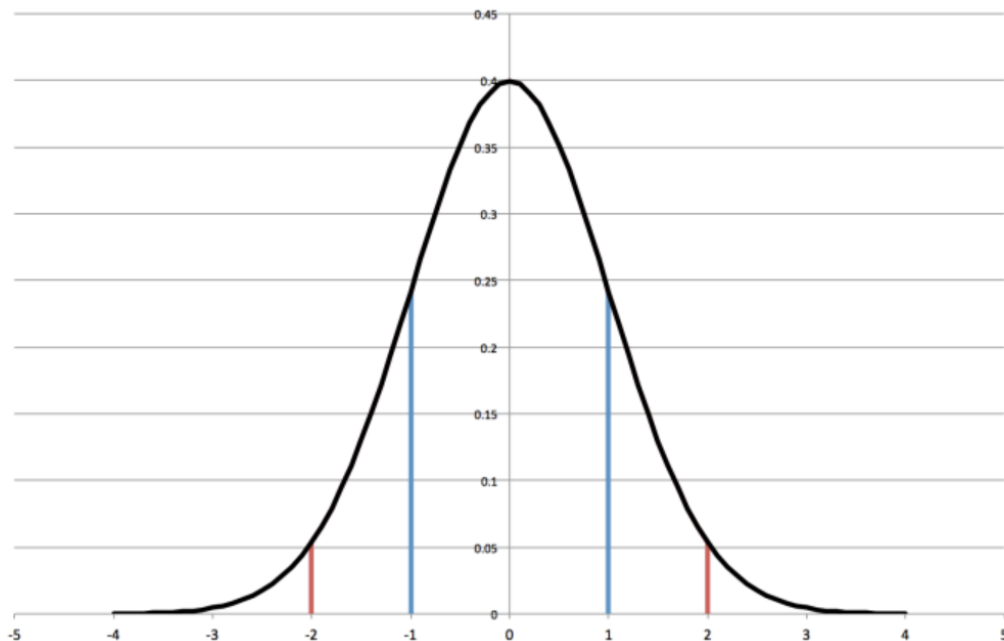
More on executors next week

- Performance measurements: motivation and introduction
- Pitfalls (and avoiding them)
- **Calculating means and variance (efficiently)**
- Measurements of thread overhead
- Algorithms for parallel computing

Normal distribution



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Measuring physical properties

Your exam grades

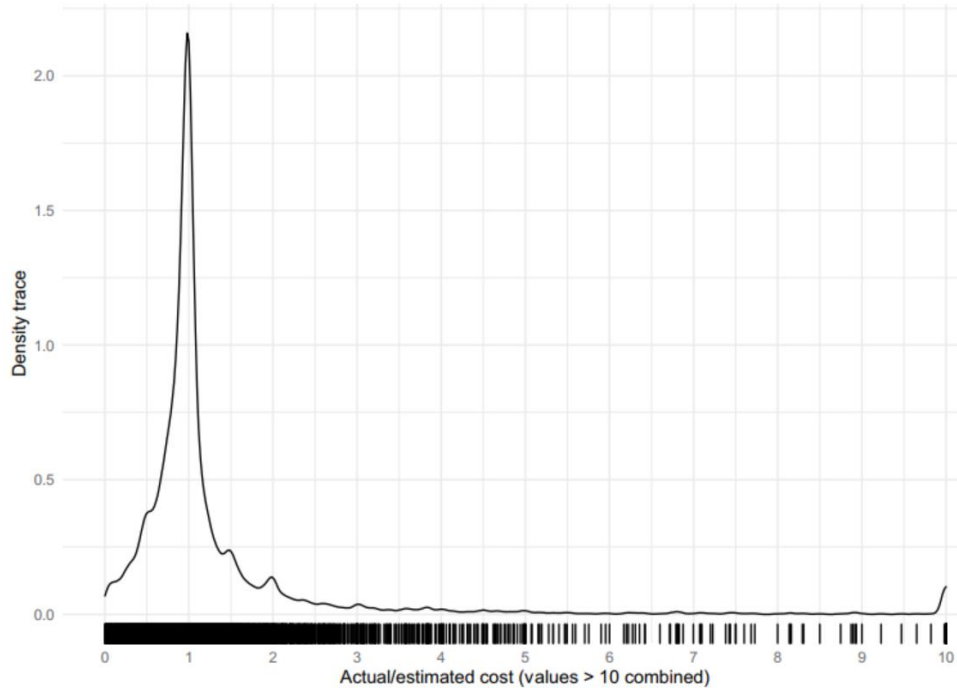
Course evaluations

Fabrication faults

Running time of Java code

...

But there are exceptions



Source: Bent Flyvbjerg, Alexander Budzier, Jong Seok Lee, Mark Keil, Daniel Lunn & Dirk W. Bester (2022) The Empirical Reality of IT Project Cost Overruns: Discovering A Power-Law Distribution, Journal of Management Information Systems, 39:3, 607-639, DOI: 10.1080/07421222.2022.2096544

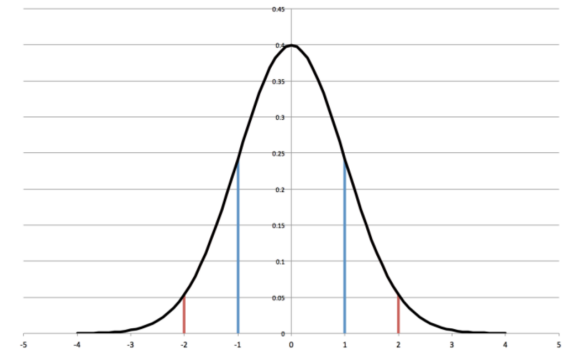
Standard deviation/variance

$$\mu = \frac{1}{n} \sum_{j=1}^n t_j$$

Mean



Benchmark note p6



Standard deviation/variance



$$\mu = \frac{1}{n} \sum_{j=1}^n t_j$$

$$\sigma = \sqrt{\frac{1}{n-1} \sum_{j=1}^n (t_j - \mu)^2}$$

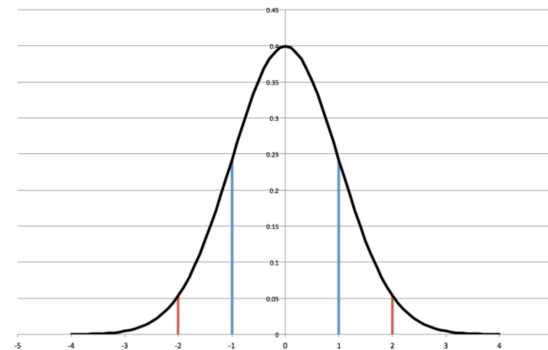
Mean

Standard deviation

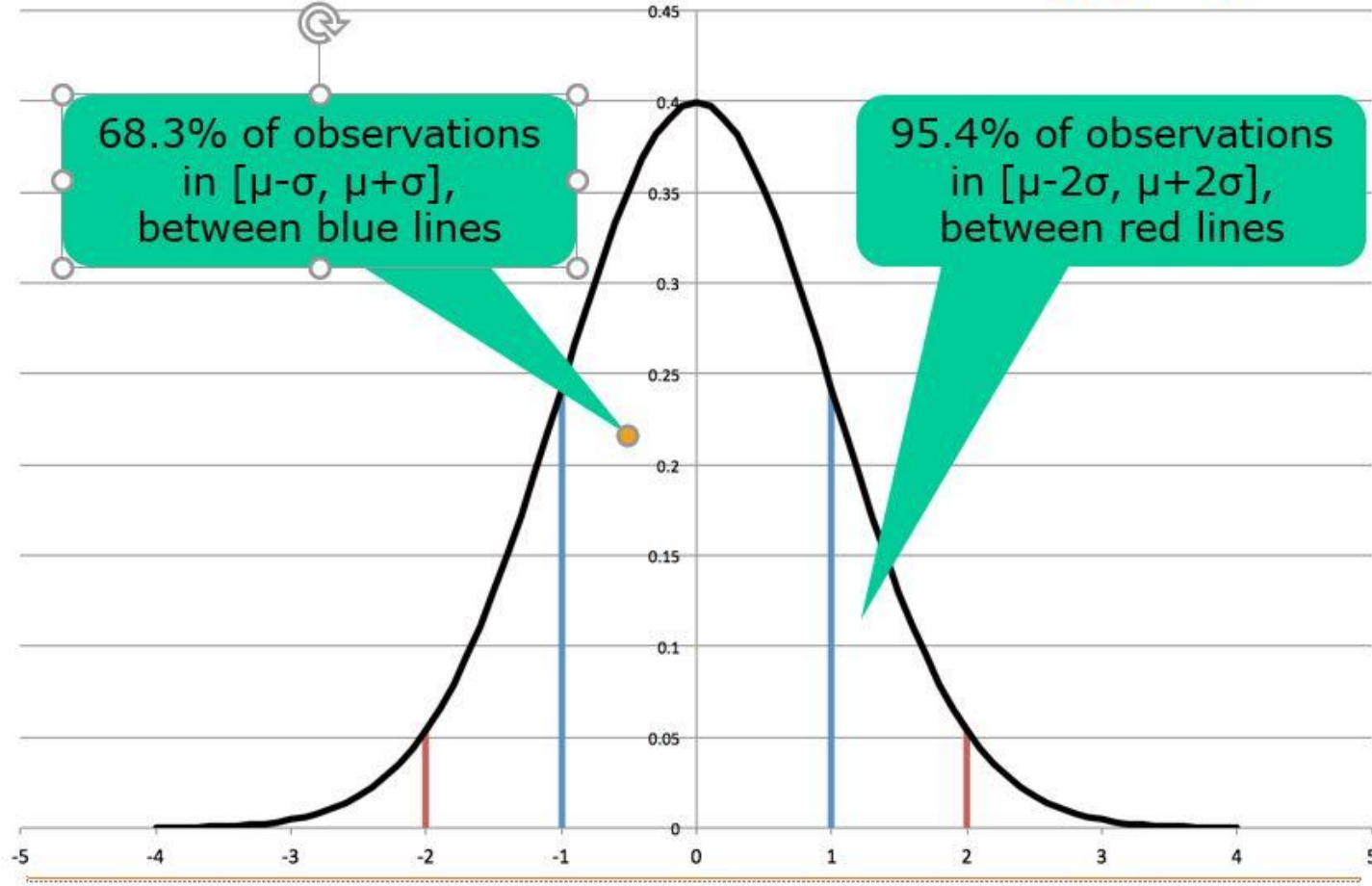
Benchmark note p6

30.7 ns 30.3 ns 30.1 ns 30.7 ns 50.2 ns 30.4 ns 30.9 ns 30.3 ns 30.5 ns 30.8 ns ??

Mean: 32.5 ns Standard deviation: 6.2



Normal distribution



What should you report as the result, when the observations are:

30.7 ns 30.3 ns 30.1 ns 30.7 ns 50.2 ns 30.4 ns 30.9 ns 30.3 ns 30.5 ns 30.8 ns ??

Mean: 32.5 ns Standard deviation: 6.2

50.2 is an outlier

because there is a probability of less than 4.6 % that 50.2 is a correct observation

Computing the variance

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$$\mu = \frac{1}{n} \sum_{j=1}^n t_j$$

$$\sigma = \sqrt{\frac{1}{n-1} \sum_{j=1}^n (t_j - \mu)^2}$$

Requires two passes through the data

$$\sigma^2 = \frac{1}{n(n-1)} (n \sum_{j=1}^n t_j^2 - (\sum_{j=1}^n t_j)^2)$$

Can be done in one pass (on-line alg.)

```
for (int j=0; j<n; j++) {  
    Timer t = new Timer();  
    for (int i=0; i<count; i++)  
        ...  
    double time = t.check() * 1e9 / count;  
    st += time;  
    sst += time * time;  
}  
double mean = st/n, sdev = Math.sqrt((sst - mean*mean*n) / (n-1));  
System.out.printf("%6.1f ns +/- %6.3f%n", mean, sdev);
```

The two formulas give the same result



$$\mu = \frac{1}{n} \sum_{j=1}^n t_j$$

$$\sigma = \sqrt{\frac{1}{n-1} \sum_{j=1}^n (t_j - \mu)^2}$$

$$\sigma = \sqrt{\frac{1}{n-1} \sum_{j=1}^n (t_j^2 + \mu^2 - 2t_j\mu)}$$

$$\sigma^2 = \frac{1}{n-1} \sum_{j=1}^n (t_j^2 + \mu^2 - 2t_j\mu)$$

$$\sigma^2 = \frac{1}{n-1} (\sum_{j=1}^n t_j^2 + \sum_{j=1}^n (\mu^2 - 2t_j\mu))$$

$$\sigma^2 = \frac{1}{n-1} (\sum_{j=1}^n t_j^2 + n\mu^2 - 2\mu \sum_{j=1}^n t_j)$$

$$\sigma^2 = \frac{1}{n-1} (\sum_{j=1}^n t_j^2 + n\mu^2 - 2\mu n\mu)$$

$$\sigma^2 = \frac{1}{n-1} (\sum_{j=1}^n t_j^2 - n\mu^2)$$

$$\sigma^2 = \frac{1}{n(n-1)} (n \sum_{j=1}^n t_j^2 - \mu^2)$$

$$\sigma^2 = \frac{1}{n(n-1)} (n \sum_{j=1}^n t_j^2 - (\frac{1}{n} \sum_{j=1}^n t_j)^2)$$



Formula in Benchmark note

See exercises05.pdf

also https://en.wikipedia.org/wiki/Algorithms_for_calculating_variance



Formula used in code (one pass algorithm)

$$\sigma^2 = \frac{1}{n(n-1)} \left(n \sum_{i=1}^n x_i^2 - \left(\sum_{i=1}^n x_i \right)^2 \right)$$

```
int n = 10;
...
for (int j=0; j<n; j++) {
    Timer t = new Timer();
    for (int i=0; i<count; i++)
        ...
    double time = t.check() * 1e9 / count;
    st += time;
    sst += time * time;
}
double mean = st/n, sdev = Math.sqrt((sst - mean*mean*n) / (n-1));
System.out.printf("%6.1f ns +/- %6.3f%n", mean, sdev);
```

Beware: $\text{sst} - \text{mean} * \text{mean} * n$

can be a very small number

Beware of cancellation when subtracting numbers that are close to each other:

28 significant digits

$$\begin{array}{r} 1010101000010110110001110101.111 \\ -1010101000010110110001110001.100 \\ \hline 0000000000000000000000000100.011 \end{array}$$

3 significant digits

<https://blog.demofox.org/2017/11/21/floating-point-precision>

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