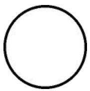
TIPE : RÉSEAUX DE NEURONES ÉLECTRIQUES ANALOGIQUES

n° de candidat : 32538

Problématique : Peut-on construire un système électrique qui reproduit fidèlement les caractéristiques d'un réseau de neurones et qui soit capable d'apprentissage automatique ?

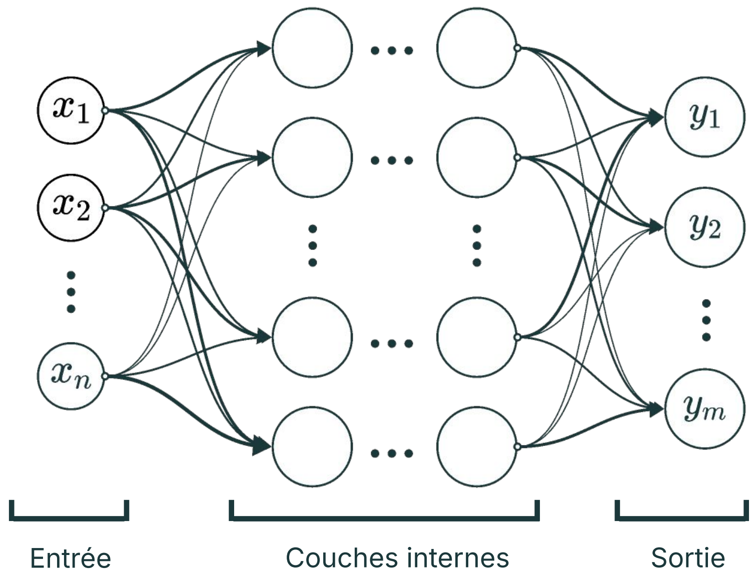
Sommaire

Étude théorique

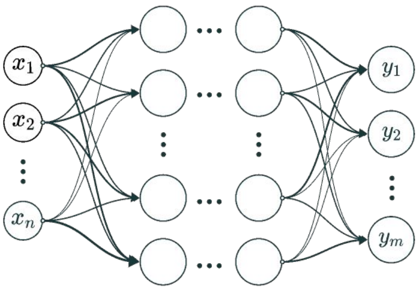


: Neurone

: Poids



Étude théorique

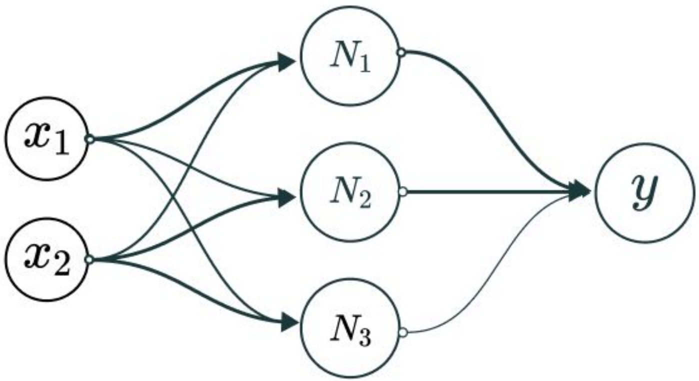


$$\forall i \in [[2, p-1]],$$

$$\begin{pmatrix} N_1^{(i+1)} \\ N_2^{(i+1)} \\ \vdots \\ N_l^{(i+1)} \end{pmatrix} = f\left(\begin{pmatrix} \omega_{11}^{(i)} & \omega_{12}^{(i)} & \omega_{13}^{(i)} & \dots & \omega_{1n}^{(i)} \\ \omega_{21}^{(i)} & \omega_{22}^{(i)} & \omega_{23}^{(i)} & \dots & \omega_{2n}^{(i)} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \omega_{l1}^{(i)} & \omega_{l2}^{(i)} & \omega_{l3}^{(i)} & \dots & \omega_{ln}^{(i)} \end{pmatrix} \times \begin{pmatrix} N_1^{(i)} \\ N_2^{(i)} \\ \vdots \\ N_l^{(i)} \end{pmatrix} + \begin{pmatrix} b_1^{(i)} \\ b_2^{(i)} \\ \vdots \\ b_l^{(i)} \end{pmatrix} \right)$$

$$\begin{pmatrix} y_1 \\ y_2 \\ \vdots \\ y_m \end{pmatrix} = f\left(\begin{pmatrix} \omega_{11}^{(p)} & \omega_{12}^{(p)} & \omega_{13}^{(p)} & \dots & \omega_{1n}^{(p)} \\ \omega_{21}^{(p)} & \omega_{22}^{(p)} & \omega_{23}^{(p)} & \dots & \omega_{2n}^{(p)} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \omega_{l1}^{(p)} & \omega_{l2}^{(p)} & \omega_{l3}^{(p)} & \dots & \omega_{ln}^{(p)} \end{pmatrix} \times \begin{pmatrix} N_1^{(p)} \\ N_2^{(p)} \\ \vdots \\ N_l^{(p)} \end{pmatrix} + \begin{pmatrix} b_1^{(p)} \\ b_2^{(p)} \\ \vdots \\ b_m^{(p)} \end{pmatrix} \right)$$

Étude théorique

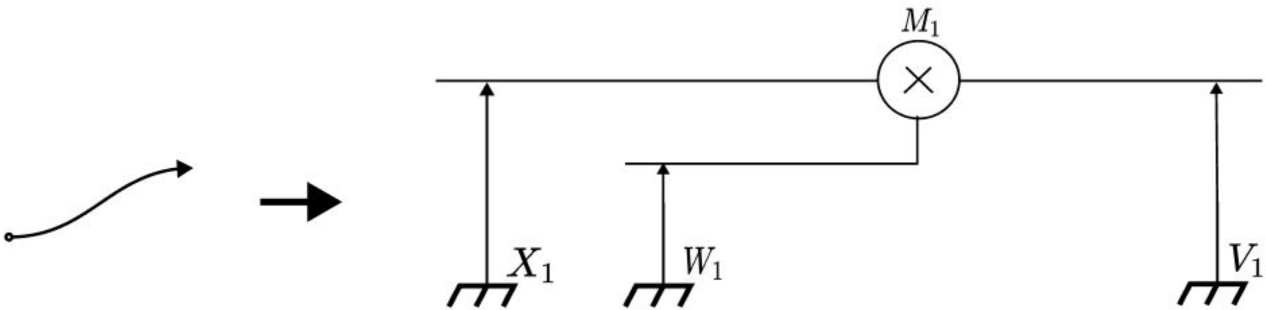


$$\forall i \in [[1, 3]],$$

$$N_i = \tanh(w_{1i}x_1 + w_{2i}x_2 + b_i)$$

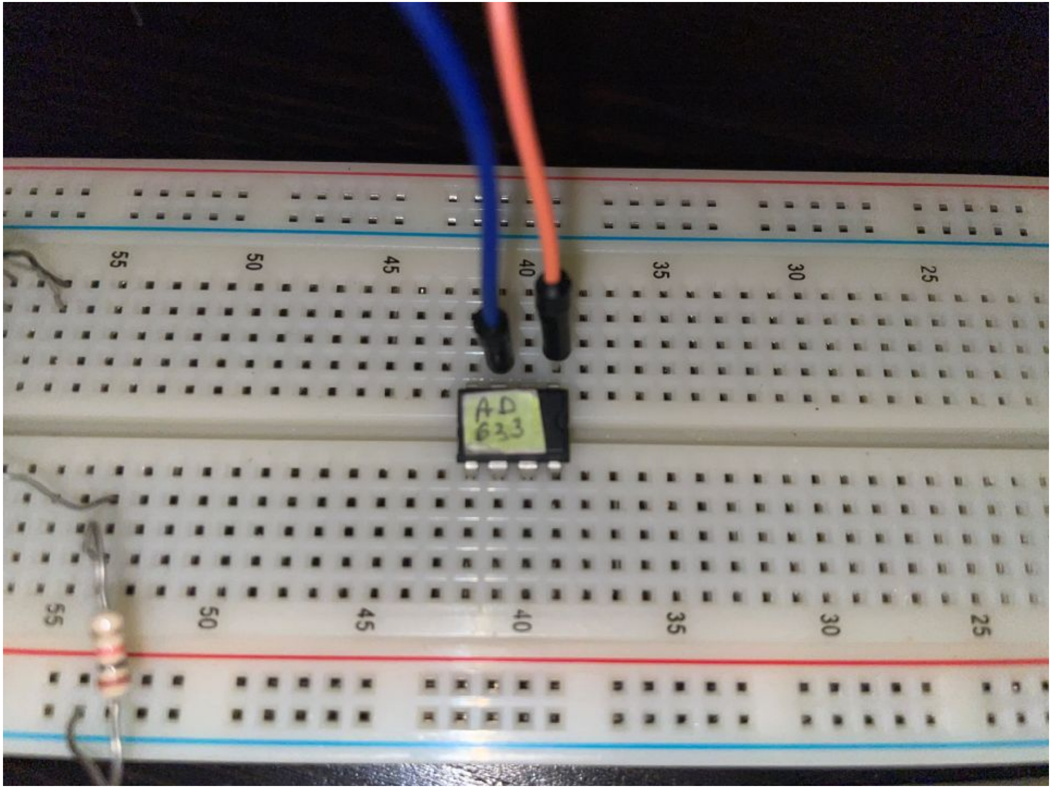
$$y = w_1 N_1 + w_2 N_2 + w_3 N_3 \ + \ b_4$$

Analogie électrique

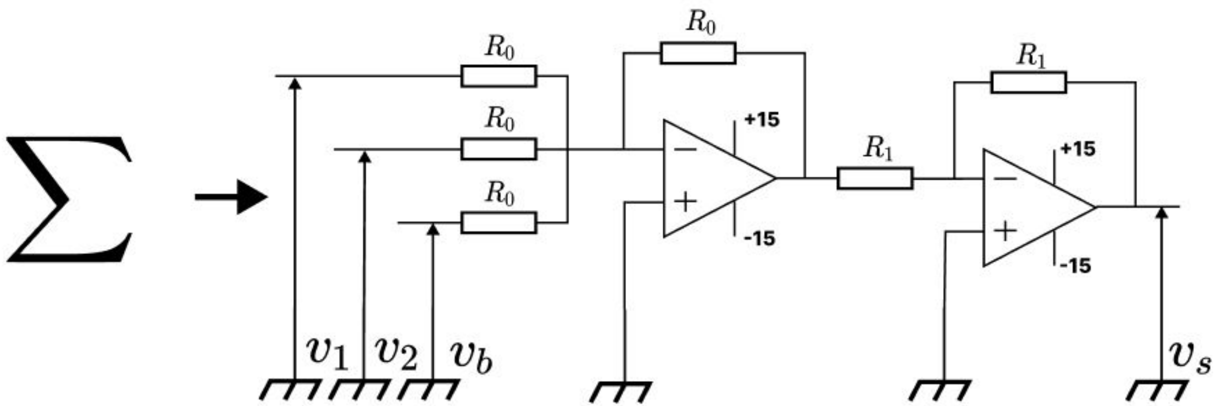


$$V_1 = X_1 \times W_1$$

Analogie électrique

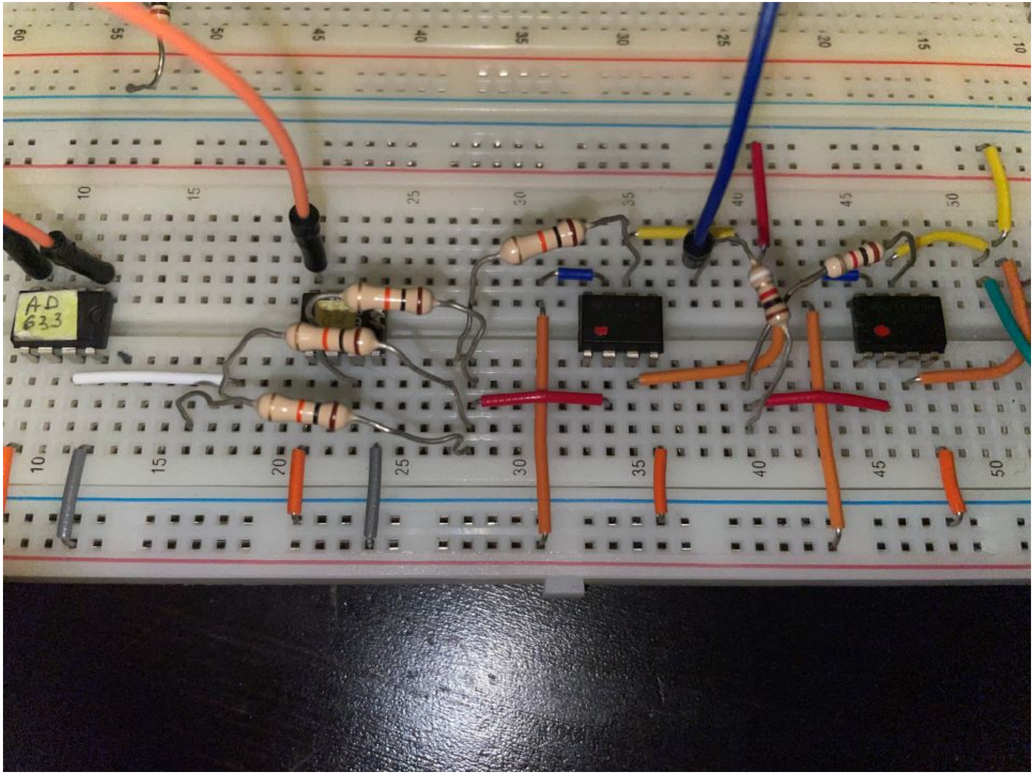


Analogie électrique

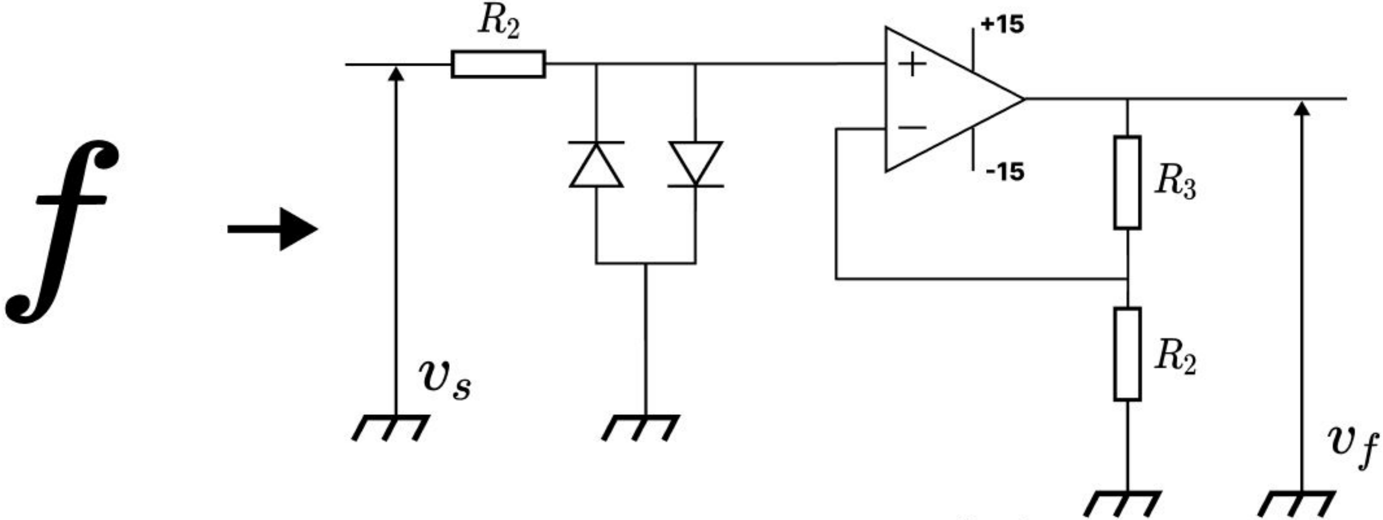


$$v_s = v_1 + v_2 + v_b$$

Analogie électrique

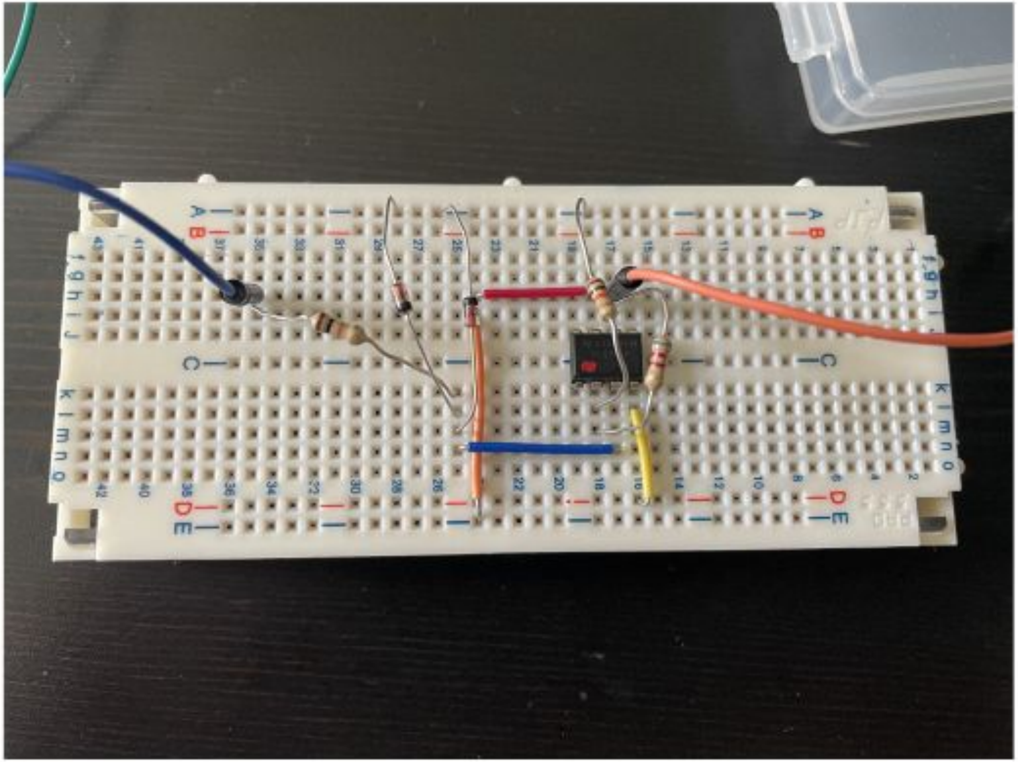


Analogie électrique

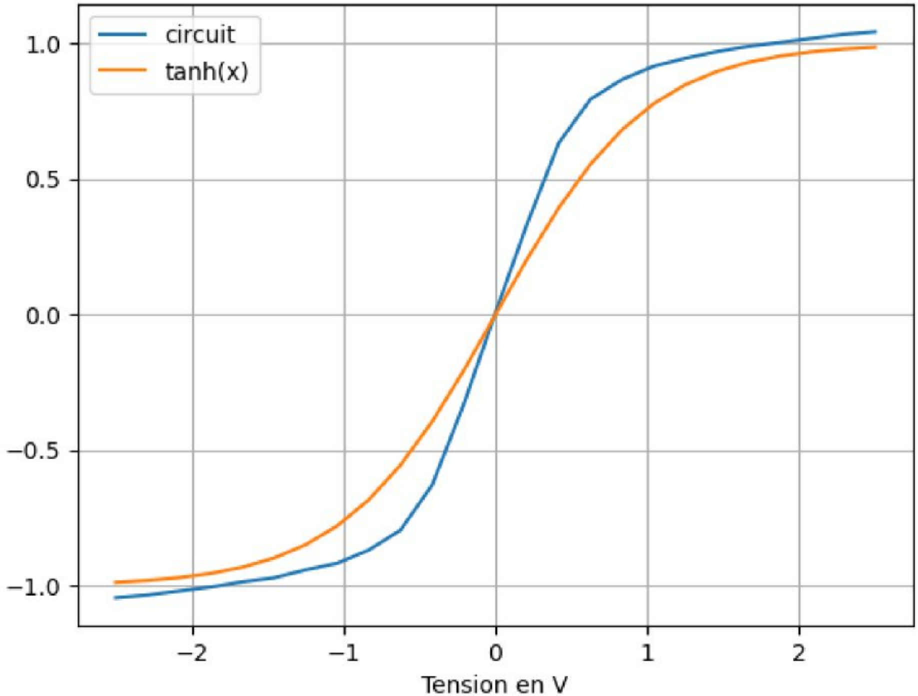


$$v_f = \tanh(v_s)$$

Analogie électrique



Analogie électrique



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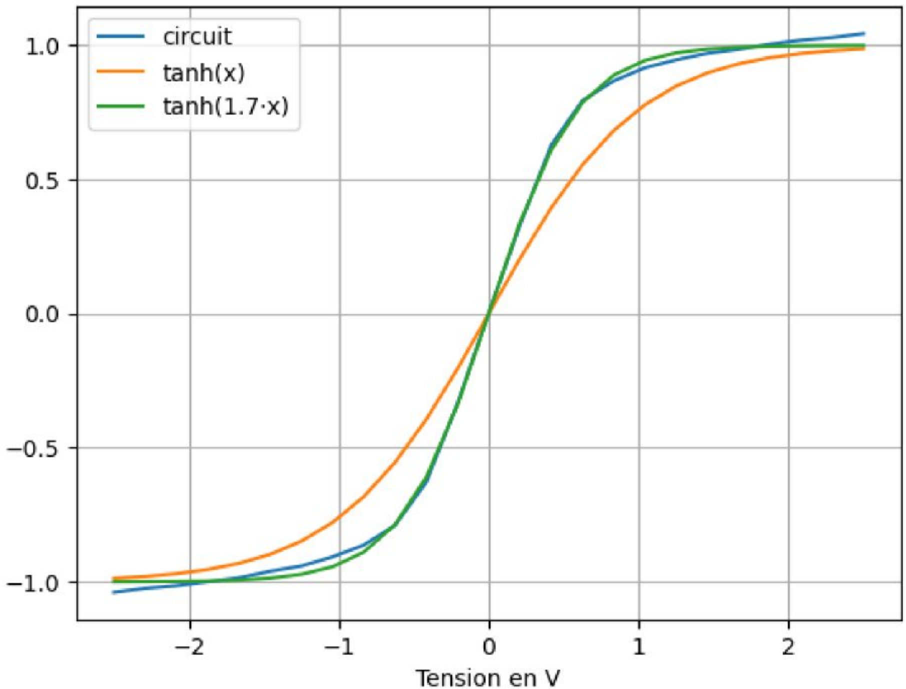

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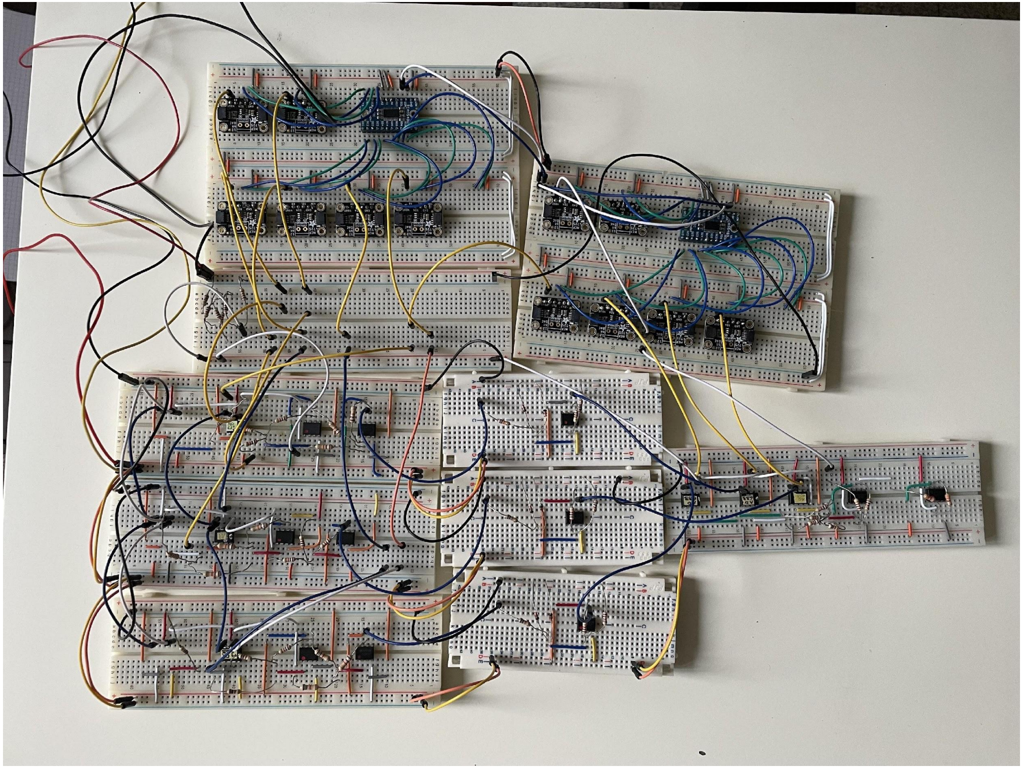
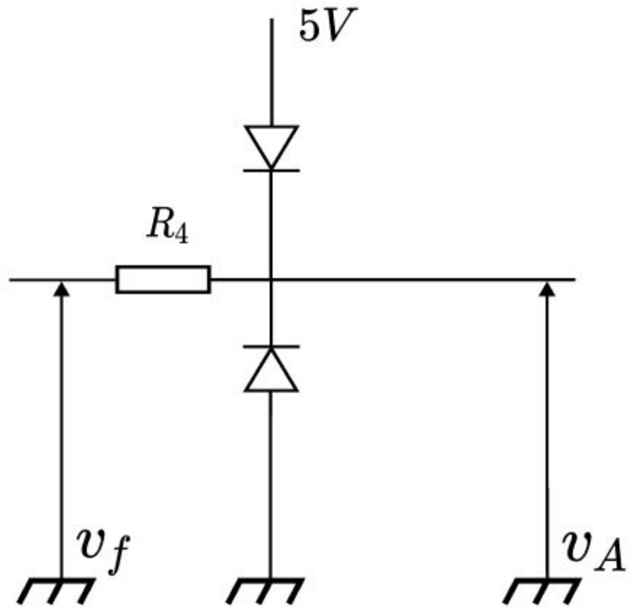
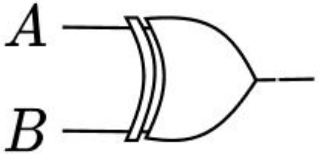


$clampage\ de\ v_f\ tel\ que:$

$$0V \leq v_f \leq 5V$$



Porte logique XOR

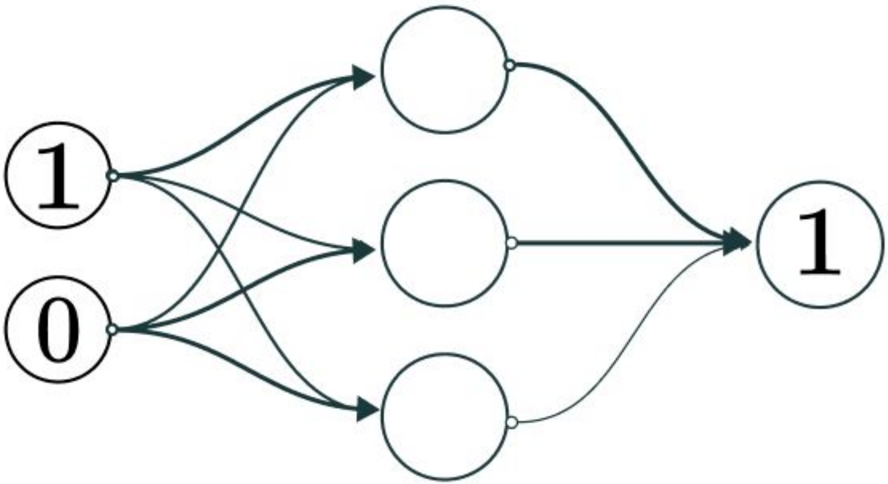


$$\bar{A}B + A\bar{B}$$

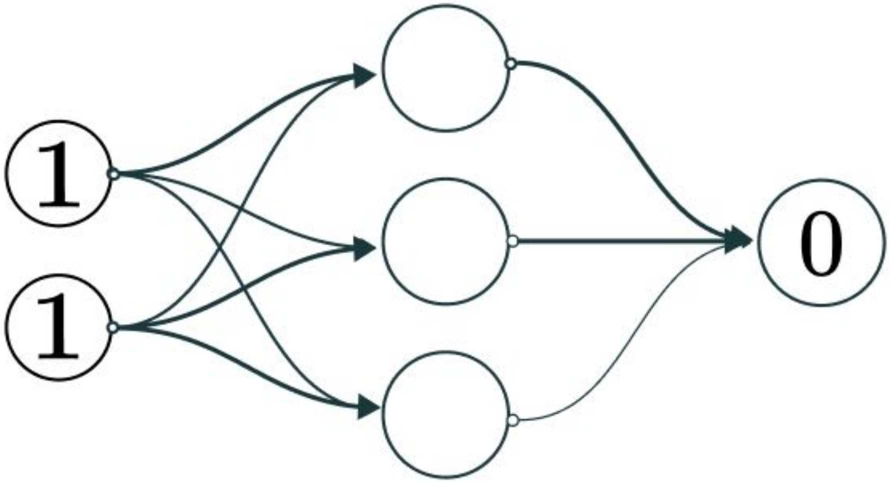
Table de vérité

A	B	$A \oplus B$
0	0	0
0	1	1
1	0	1
1	1	0

Porte logique XOR



Porte logique XOR



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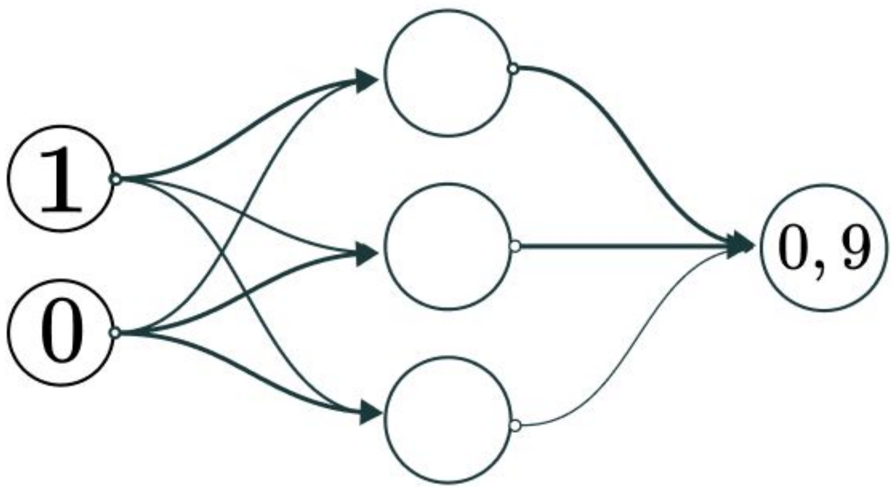


Porte logique XOR

$\hat{y_k}$ **: Valeur étiquetté**

y_k **: Valeur calculé**

$$E(1, 0.9) = (1 - 0.9)^2$$

$$Erreur\ quadratique\ moyenne$$

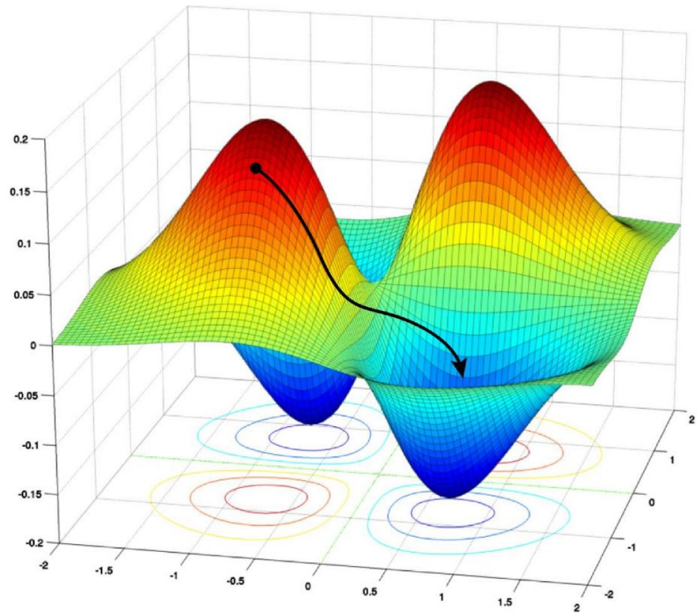
$$E(\hat{y_k}, y_k) = \frac{1}{n} \sum_{k=0}^{n} (\hat{y_k} - y_k)^2$$

Descente de gradient

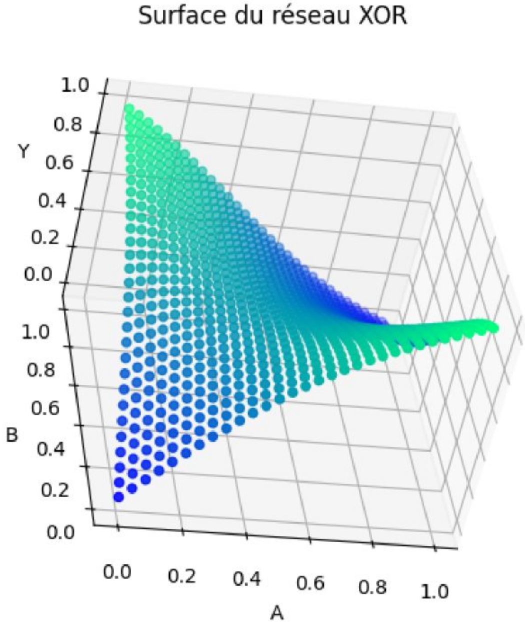
Calculer ∇E

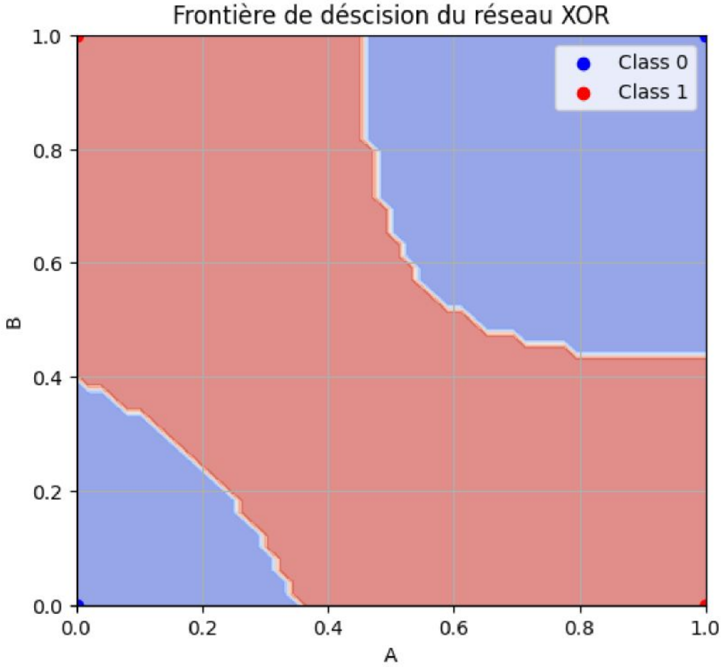
Avancer d'un certain pas dans la direction de $-\nabla E$

Recommencer

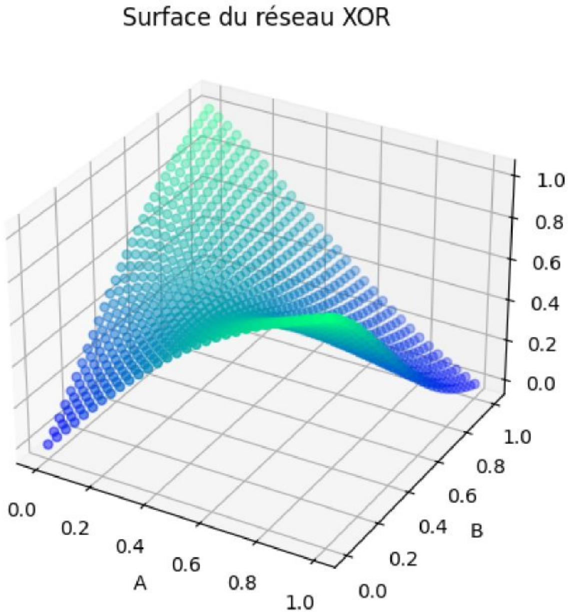


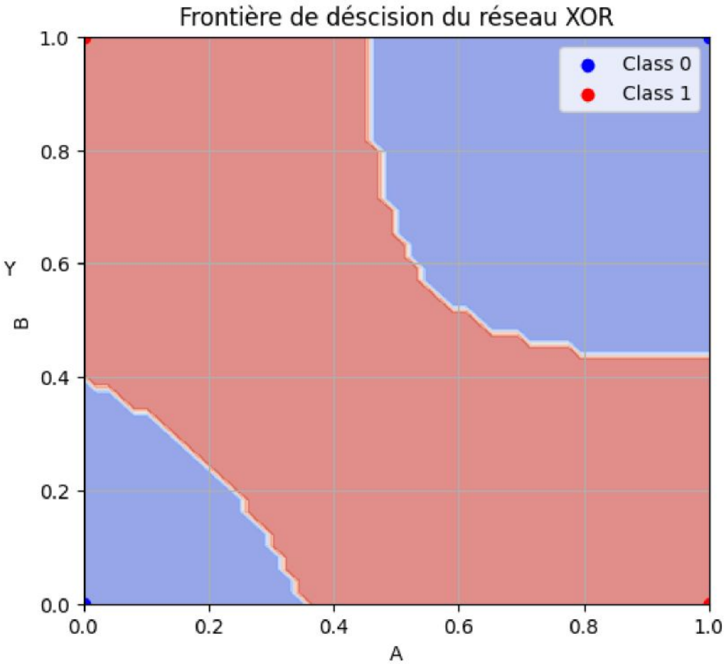
Résultats expérimentaux XOR



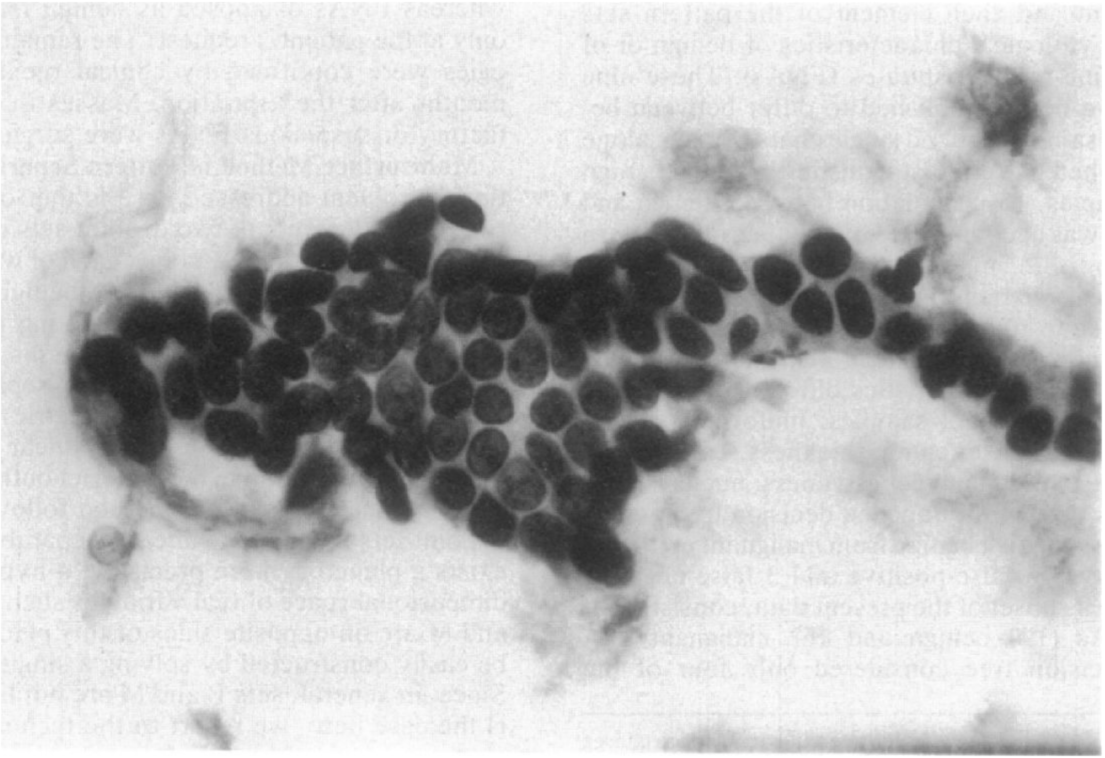


Résultats expérimentaux XOR

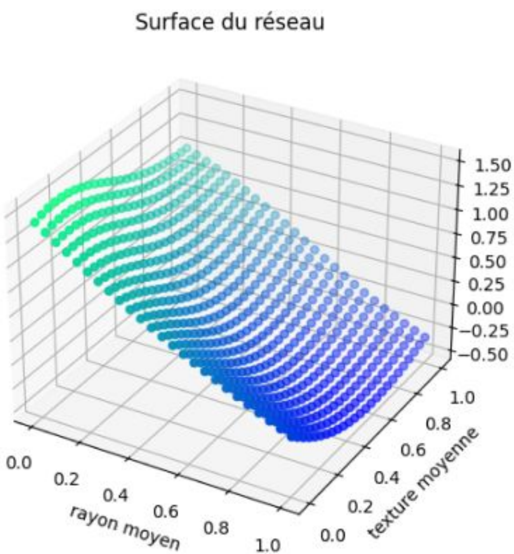


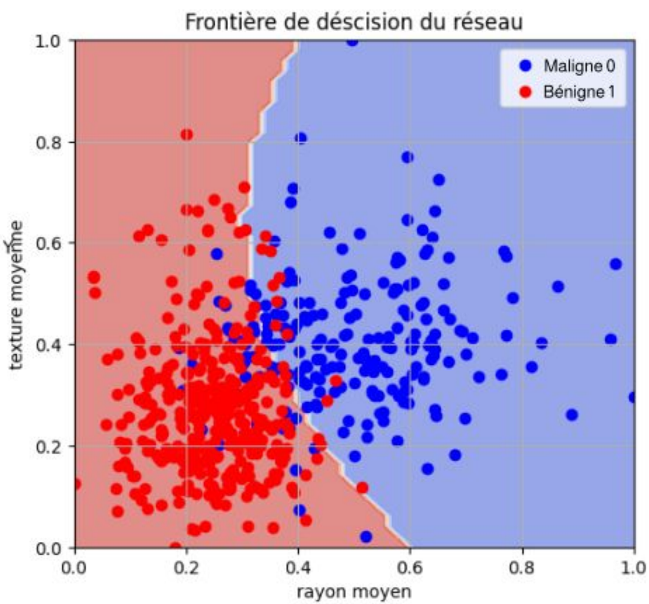


Expérimentation

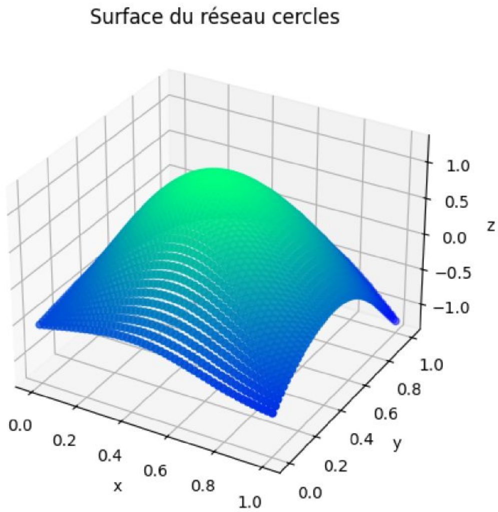


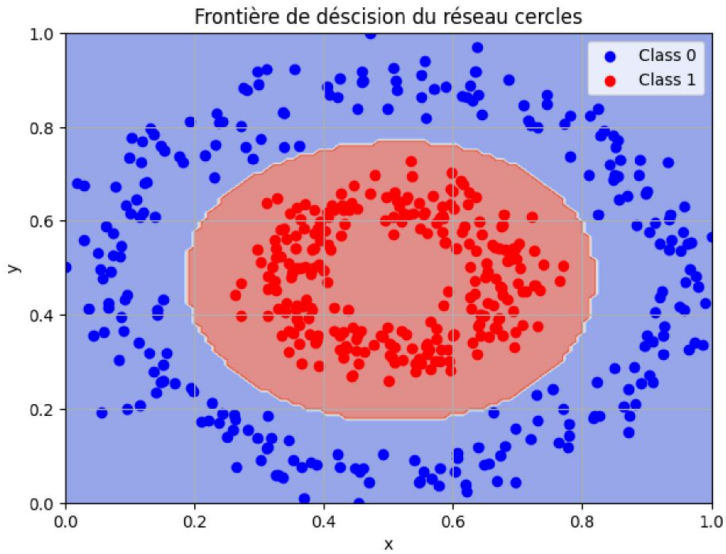
Expérimentation





Résultats réseau cercle





CONCLUSION ET LIMITES

Bibliographie

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[3] KURT HORNIK : Multilayer Feedforward Networks are Universal Approximators : Neural Networks, 2:359 366, 1989 : https://cognitivemedium.com/magic_paper/assets/Hornik.pdf

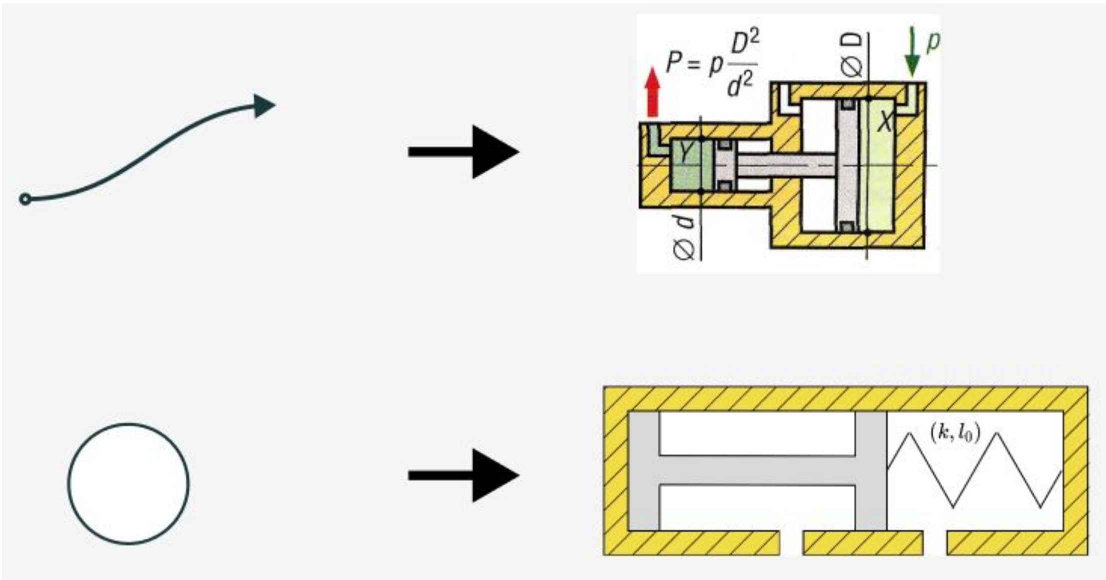
[4] JEAN-CHRISTOPHE ORLIANGES, YOUNES EL MOUSTAKIME, AURELIAN CRUNTEANU STANESCU, RICARDO CARRIZALES JUAREZ, OIHAN ALLEGRET : Retour vers le perceptron - fabrication d'un neurone synthétique à base de composants électroniques analogiques simples : Université de Limoge, 2024 : https://www.unilim.fr/journees-interdisciplinarite/761

Theorem 2.4

For every squashing function Ψ, every r, and every probability measure μ on (R^r, B^r), $\Sigma^r(\Psi)$ is uniformly dense on compacta in C^r and ρ_μ-dense in M^r. $\square$

In other words, standard feedforward networks with only a single hidden layer can approximate any continuous function uniformly on any compact set and any measurable function arbitrarily well in the ρ_μ metric, regardless of the squashing function Ψ (continuous or not), regardless of the dimension of the input space r, and regardless of the input space environment μ. Thus, Σ networks are also universal approximators.

Annexes



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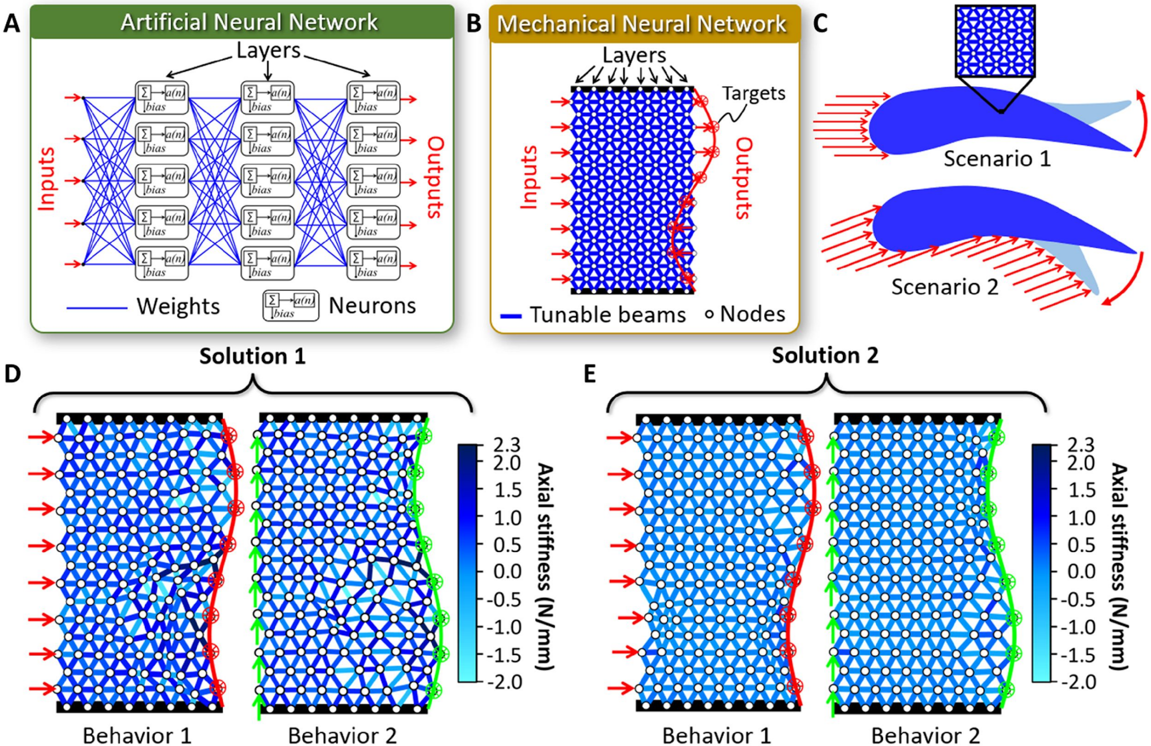
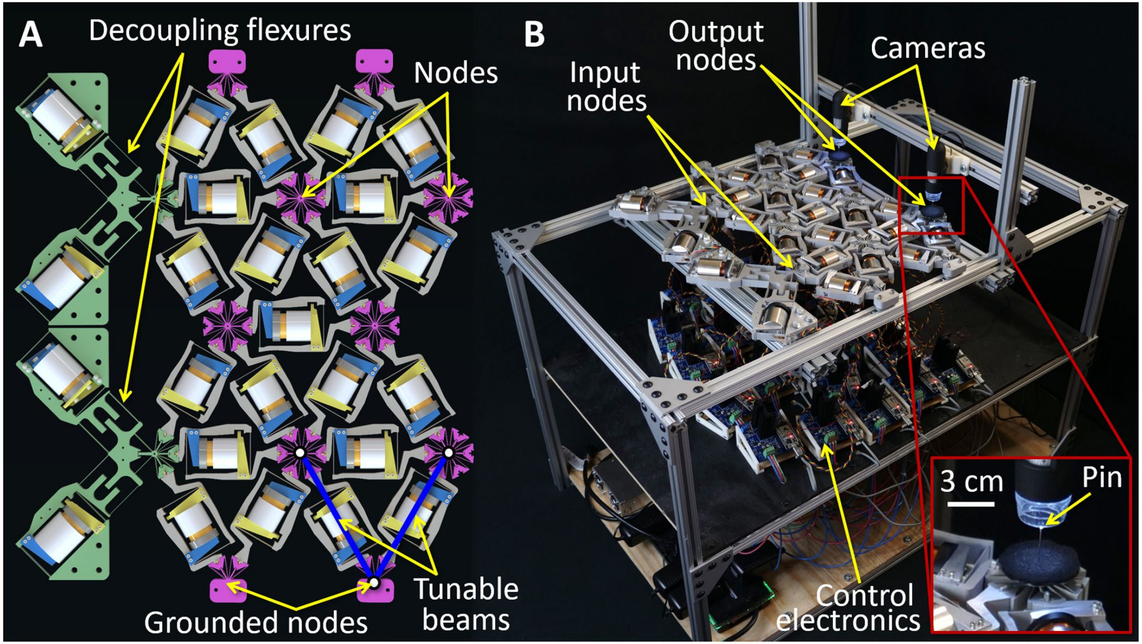

A
Artificial Neural Network
Layers
Inputs
Outputs
Weights
Neurons
B
Mechanical Neural Network
Layers
Targets
Inputs
Outputs
Tunable beams
Nodes
C
Scenario 1
Scenario 2
D
Solution 1
Behavior 1
Behavior 2
Axial stiffness (N/mm)
2.3
2.0
1.5
1.0
0.5
0.0
-0.5
-1.0
-1.5
-2.0
E
Solution 2
Behavior 1
Behavior 2
Axial stiffness (N/mm)

Annexes


A
Decoupling flexures
Nodes
Grounded nodes
Tunable beams
B
Input nodes
Output nodes
Cameras
Control electronics
3 cm
Pin