Digital Signal Processing 2024S – Assignment 1

Analogue Signals and Systems

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Exercise 1 *Complex Numbers*

With $\operatorname{atan2}(y, x) = \arctan\left(\frac{y}{x}\right)$ for $x > 0$ and $\operatorname{atan2}(y, x) = \arctan\left(\frac{y}{x}\right) + \pi$ otherwise.

- $$c_2 = \frac{\sqrt{2}}{2}e^{-\frac{j3\pi}{4}} = \frac{\sqrt{2}}{2}\cos\left(-\frac{3\pi}{4}\right) + j\frac{\sqrt{2}}{2}\sin\left(-\frac{3\pi}{4}\right) = -\frac{1}{2} - j\frac{1}{2}$$
$$c_4 = c_1 + c_2 = -5 - \frac{1}{2} + j\left(3 - \frac{1}{2}\right)$$

- $$c_1 = -5 + j3 = \sqrt{-5^2 + 3^2} \cdot e^{j\operatorname{atan2}(3,-5)} = 5.8310 \cdot e^{j2.6012}$$
$$c_5 = c_1 \cdot c_2 = \left(5.8310\frac{\sqrt{2}}{2}\right) e^{j\left(2.6012 + \frac{-3\pi}{4}\right)} = 4.1231e^{j0.245}$$

- $$c_6 = |c_3|^2 = \sqrt{\left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2}^2 = 1$$

- $$c_7 = \arg(c_3) = \operatorname{atan2}\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right) = 0.7854$$

- TODO
- TODO

Exercise 2 *Fourier Transform*

TODO: more rigorous
TODO: diagram!!!

$$x(t) = \hat{X}\cos(2\pi f_0 t) = \hat{X}\frac{e^{j2\pi f_0 t} + e^{-j2\pi f_0 t}}{2} = \frac{\hat{X}}{2}e^{j2\pi f_0 t} \qquad + \frac{\hat{X}}{2}e^{-j2\pi f_0 t}$$
$$X(f) = \frac{\hat{X}}{2}\delta(f - f_0) + \frac{\hat{X}}{2}\delta(f + f_0)$$

Exercise 3 *Time Shift and Phase*

a) In general, we can formulate ϕ_i as

$$\begin{aligned} 2\pi f_i t + \phi_i &= 2\pi f_i (t-\tau) \\ \phi_i &= 2\pi f_i t - 2\pi f_i \tau - 2\pi f_i t \\ \phi_i &= -2\pi f_i \tau \end{aligned}$$

And thus for $\tau = 0.1s$ we have $\phi_1 = -0.2\pi$ and $\phi_2 = -\frac{1}{15}\pi$.
We verify that this corresponds to the "shift theorem" by applying it to Y_i and ensuring that the results are as expected.

$$\begin{aligned} X_1(f) &= -\frac{j}{2}\delta(f-f_1) + \frac{j}{2}\delta(f+f_1) \\ Y_1(f) &= \left(-\frac{j}{2}\delta(f-f_1) + \frac{j}{2}\delta(f+f_1)\right) e^{-j2\pi f 0.1} \\ &= -\frac{j}{2}e^{-j0.2\pi f}\delta(f-f_1) + \frac{j}{2}e^{-j0.2\pi f}\delta(f+f_1) \end{aligned}$$

Since $\delta(t)$ is 0 for all $t \neq 0$, only $f = f_1$ and $f = -f_1$ will affect our result. Given $f_1 = 1\text{Hz}$ we can reformulate the above to

$$Y_1(f) = \begin{cases} -\frac{j}{2}e^{-j0.2\pi}\delta(0), & \text{if } f = f_1 \\ \frac{j}{2}e^{j0.2\pi}\delta(0), & \text{if } f = -f_1 \\ 0, & \text{otherwise} \end{cases}$$

where we observe that the exponent matches our calculated ϕ_1.
We can do the same for $Y_2(f)$, where we obtain

$$\begin{aligned} Y_2(f) &= -\frac{j}{2}e^{-j2\pi f 0.1}\delta(f-f_2) + \frac{j}{2}e^{-j2\pi f 0.1}\delta(f+f_2) \\ Y_2(f) &= \begin{cases} -\frac{j}{2}e^{-j\frac{1}{15}\pi}\delta(0), & \text{if } f = f_2 \\ \frac{j}{2}e^{j\frac{1}{15}\pi}\delta(0), & \text{if } f = -f_2 \\ 0, & \text{otherwise} \end{cases} \end{aligned}$$

and again see that the exponent matches our calculated ϕ_2.

b) See Figures 1 and 2.

Exercise 4 *Linearity and Time Invariance*

TODO

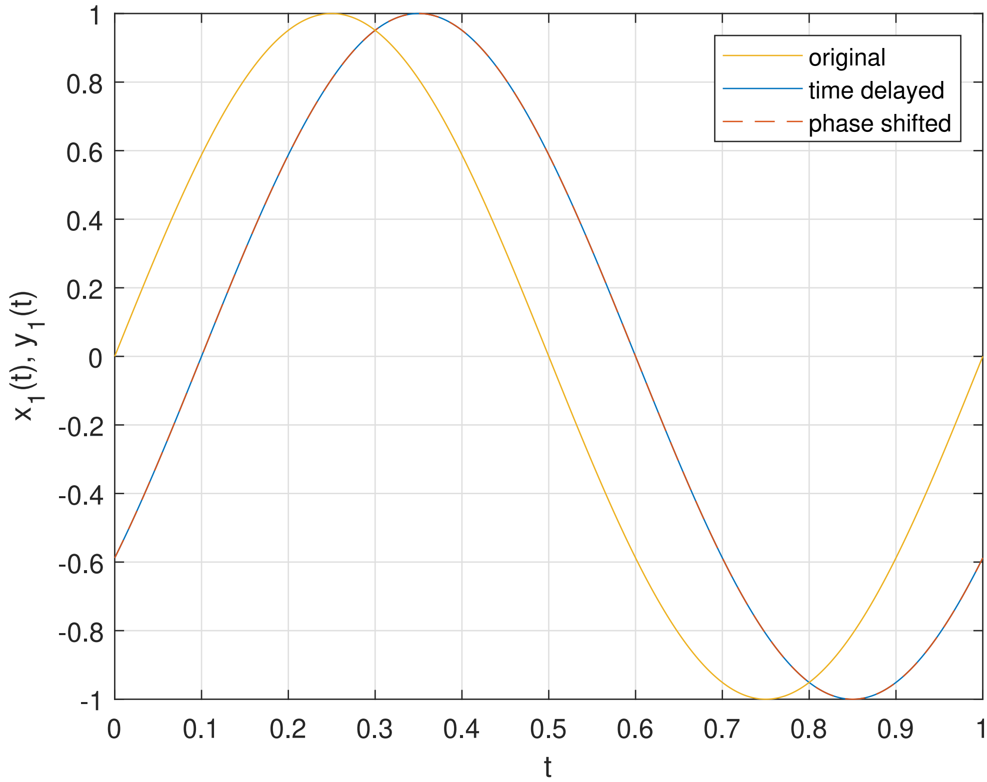

Figure 1: Signals for $f_1 = 1\text{Hz}$

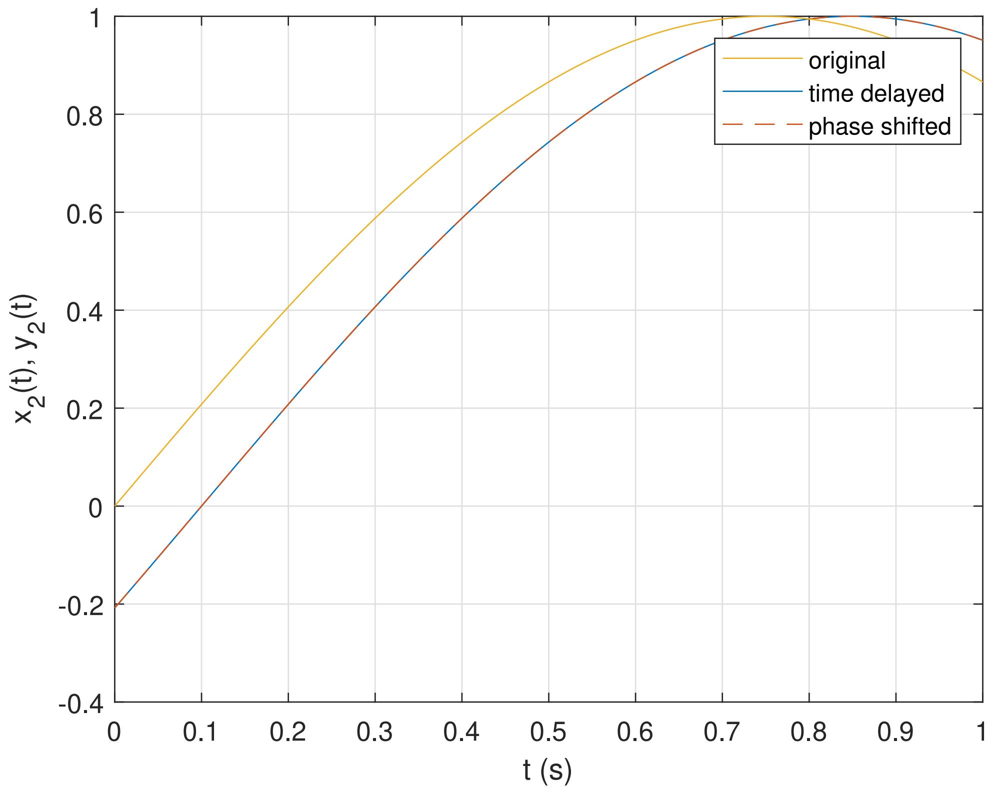

Figure 2: Signals for $f_2 = 3\text{Hz}$