

- The two truth constants are \top and \perp . Variables are “propositions that have no further internal structure”. Clauses are disjunctions of the former. Thus,
 - contains the three clauses $(x \vee y)$, $(z \vee \neg z)$ and (x) (an unary clause), and three variables, x , y and z .
 - contains the four clauses $(a \vee b \vee \neg c)$, $(\top \vee \neg c)$, $(a \vee b \vee c)$ and $(a \vee b \vee \neg \perp \vee \neg a)$ three variables a , b and c as well as the truth constants \top and \perp .
 - contains the two clauses $(\text{box} \vee \text{diamond})$ and $(\neg \text{box} \vee \neg \text{diamond})$ and two variables box and diamond .
- The truth tables for formulas (a) – (g) are presented in Truth Table (a) – (g).

| a | \perp | $(a \vee \perp)$ |
|-----|---------|------------------|
| 0 | 0 | 0 |
| 1 | 0 | 1 |

Truth Table (a): Satisfiable with $\{a\}$.

| a | \top | $(a \vee \top)$ |
|-----|--------|-----------------|
| 0 | 1 | 1 |
| 1 | 1 | 1 |

Truth Table (b): Satisfiable with $\{a, \neg a\}$.

| a | $\neg a$ | $(a \vee \neg a)$ |
|-----|----------|-------------------|
| 0 | 1 | 1 |
| 1 | 0 | 1 |

Truth Table (c): Satisfiable with $\{a, \neg a\}$.

- The conjunction $A \wedge B$ is defined to be true iff A and B are true. It follows that $B \wedge A$ is true iff B and A are true. Thus, the order of the arguments is irrelevant — conjunction is commutative.

For the special case where $A = B$, the statement $A \wedge B$ will always evaluate to A . (This follows from the definition above since in that case it can effectively be reduced to: “The conjunction $A \wedge B$, where $A = B$, is defined to be true iff A is true.”) Thus $A \wedge A$ is equal to A — conjunction is idempotent.

Consider $A \wedge (B \wedge C)$. Following the definition above it is true iff A and $(B \wedge C)$ are true. Recursively following the definition leads to it being true iff A is true and iff B and C are true. This can be shortened to it being true iff A , B and C are true. Since $(A \wedge B) \wedge C$ would lead to the same statement, conjunction is associative.

| a | b | $C_{1d} = (a \vee \neg b)$ | $C_{2d} = (\neg a \vee b)$ | $C_{1d} \wedge C_{2d}$ |
|-----|-----|----------------------------|----------------------------|------------------------|
| 0 | 0 | 1 | 1 | 1 |
| 0 | 1 | 0 | 1 | 0 |
| 1 | 0 | 1 | 0 | 0 |
| 1 | 1 | 1 | 1 | 1 |

Truth Table (d): Satisfiable with $\{\neg a, \neg b\}$, among others.

| a | b | $C_{1e} = (\neg a \vee \neg b)$ | $C_{1d} \wedge C_{2d}$ | $C_{2e} = (a \vee b)$ | $C_{1e} \wedge C_{1d} \wedge C_{2d} \wedge C_{2e}$ |
|-----|-----|---------------------------------|------------------------|-----------------------|--|
| 0 | 0 | 1 | 1 | 0 | 0 |
| 0 | 1 | 1 | 0 | 1 | 0 |
| 1 | 0 | 1 | 0 | 1 | 0 |
| 1 | 1 | 0 | 1 | 1 | 0 |

Truth Table (e): Not satisfiable.

4. The formula $(a \wedge b) \vee c$ has 5 models.
5. The given formula can be expanded to

$$(l_{11} \vee l_{12} \vee l_{13}) \wedge (l_{21} \vee l_{22} \vee l_{23}) \wedge (l_{31} \vee l_{32} \vee l_{33}),$$

or alternatively to

$$\{\{l_{11}, l_{12}, l_{13}\}, \{l_{21}, l_{22}, l_{23}\}, \{l_{31}, l_{32}, l_{33}\}\}$$

| a | b | c | $C_{1f} = (a \vee b \vee c)$ | C_{2d} | $C_{2f} = (c \vee a)$ | $C_{1f} \wedge C_{2d} \wedge C_{2f}$ |
|-----|-----|-----|------------------------------|----------|-----------------------|--------------------------------------|
| 0 | 0 | 0 | 0 | 1 | 0 | 0 |
| 0 | 0 | 1 | 1 | 1 | 1 | 1 |
| 0 | 1 | 0 | 1 | 1 | 0 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 0 | 0 | 1 | 0 | 1 | 0 |
| 1 | 0 | 1 | 1 | 0 | 1 | 0 |
| 1 | 1 | 0 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 |

Truth Table (f): Satisfiable with $\{\neg a, \neg b, c\}$, among others.

| a | b | c | d | f | \perp | C_{1f} | C_{2d} | $C_{1g} = (c \vee d \vee f)$ | $C_{2g} = (\neg d \vee \neg f)$ | $C_{1f} \wedge C_{2d} \wedge C_{1g} \wedge C_{2g} \wedge \perp$ |
|-----|-----|-----|-----|-----|---------|----------|----------|------------------------------|---------------------------------|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 0 |
| 0 | 0 | 0 | 0 | 1 | 0 | 0 | 1 | 1 | 1 | 0 |
| 0 | 0 | 0 | 1 | 0 | 0 | 0 | 1 | 1 | 1 | 0 |
| 0 | 0 | 0 | 1 | 1 | 0 | 0 | 1 | 1 | 0 | 0 |
| 0 | 0 | 1 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 0 |
| 0 | 0 | 1 | 0 | 1 | 0 | 1 | 1 | 1 | 1 | 0 |
| 0 | 0 | 1 | 1 | 0 | 0 | 1 | 1 | 1 | 1 | 0 |
| 0 | 0 | 1 | 1 | 1 | 0 | 1 | 1 | 1 | 0 | 0 |
| 0 | 1 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 0 |
| 0 | 1 | 0 | 0 | 1 | 0 | 1 | 1 | 1 | 1 | 0 |
| 0 | 1 | 0 | 1 | 0 | 0 | 1 | 1 | 1 | 1 | 0 |
| 0 | 1 | 0 | 1 | 1 | 0 | 1 | 1 | 1 | 0 | 0 |
| 0 | 1 | 1 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 0 |
| 0 | 1 | 1 | 0 | 1 | 0 | 1 | 1 | 1 | 1 | 0 |
| 0 | 1 | 1 | 1 | 0 | 0 | 1 | 1 | 1 | 1 | 0 |
| 0 | 1 | 1 | 1 | 1 | 0 | 1 | 1 | 1 | 0 | 0 |
| 1 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 1 | 0 |
| 1 | 0 | 0 | 0 | 1 | 0 | 1 | 1 | 0 | 1 | 0 |
| 1 | 0 | 0 | 1 | 0 | 0 | 1 | 1 | 0 | 1 | 0 |
| 1 | 0 | 0 | 1 | 1 | 0 | 1 | 1 | 0 | 0 | 0 |
| 1 | 0 | 1 | 0 | 0 | 0 | 1 | 1 | 0 | 1 | 0 |
| 1 | 0 | 1 | 0 | 1 | 0 | 1 | 1 | 0 | 1 | 0 |
| 1 | 0 | 1 | 1 | 0 | 0 | 1 | 1 | 0 | 1 | 0 |
| 1 | 0 | 1 | 1 | 1 | 0 | 1 | 1 | 0 | 0 | 0 |
| 1 | 1 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 0 |
| 1 | 1 | 0 | 0 | 1 | 0 | 1 | 1 | 1 | 1 | 0 |
| 1 | 1 | 0 | 1 | 0 | 0 | 1 | 1 | 1 | 1 | 0 |
| 1 | 1 | 0 | 1 | 1 | 0 | 1 | 1 | 1 | 0 | 0 |
| 1 | 1 | 1 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 0 |
| 1 | 1 | 1 | 0 | 1 | 0 | 1 | 1 | 1 | 1 | 0 |
| 1 | 1 | 1 | 1 | 0 | 0 | 1 | 1 | 1 | 1 | 0 |
| 1 | 1 | 1 | 1 | 1 | 0 | 1 | 1 | 1 | 1 | 0 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 |

Truth Table (g): Not satisfiable.