The Deutsch-Josza Algorithm

Essentials We will describe quantum computation in terms of circuits as opposed to TMs. These circuits are time-reversible. Wires contain qubits, which are either $|0\rangle$, $|1\rangle$ or both (then we call the configuration a *superposition*). Superposition collapse when they are inspected.

Minimal Circuit A prototypical quantum circuit has four phases:

1. Encode input bitstring (0, 1) into qubits ($|0\rangle$, $|1\rangle$)
2. Enter superpositions
3. Apply some Boolean circuit
4. Revert superpositions
5. Measure output wires (once again 0, 1)

I didn't write down the full example, but suffice to say that we were able to construct a quantum circuit which determines whether a 1-bit function is constant or balanced using a single query. A conventional approach would naturally need two.

Deutsch-Josza The above generalizes to n-bit functions $\{0,1\}^n \to \{0,1\}$, meaning that there exists a quantum circuit to determine whether such a function is balanced or constant *in a single query*. A conventional approach would require $O(2^n)$ function queries (worst case).

Although it's somewhat contrived, the algorithm demonstrates an exponential quantum advantage. Other problems that can be approached with similar ideas include factoring (Shor), solving linear systems, machine learning, ...