Binary relations R operating on a set A are said to be asymmetric, antisymmetric or irreflexive according to the following definitions for arbitrary but fixed R and A.

$$asymmetric(R, A) \Longleftrightarrow \forall x, y \in A : R(x, y) \Rightarrow \neg R(y, x) \tag{1}$$

$$antisymmetric(R, A) \iff \forall x, y \in A : (R(x, y) \land R(y, x)) \Rightarrow (x = y)$$
 (2)

$$irreflexive(R, A) \iff \forall x \in A : \neg R(x, x)$$
 (3)

We show that, again for arbitrary but fixed R and A, the implication

$$asymmetric(R, A) \Rightarrow (antisymmetric(R, A) \land irreflexive(R, A))$$

holds by assuming that asymmetric(R, A) is true and showing that antisymmetric(R, A) and irreflexive(R, A) then hold.

It is elementary that, for arbitrary but fixed $x, y \in A$ the statement $R(x, y) \wedge R(y, x)$ in (2) is a contradiction. R(x, y) implies that $\neg R(y, x)$, therefore $R(x, y) \wedge R(y, x)$ can never be true for fixed x, y under the assumption of (1). Since the left-hand side of the implication in (2) is always false the statement will always be true. We have thus shown that antisymmetric(R, A) holds.

If R(x,x) were true for arbitrary but fixed $x \in A$ it would lead to $R(x,x) \Rightarrow \neg R(x,x)$ being false. Since that is a contradiction (we know that (1) holds for our R and A) we conclude that $\neg R(x,x)$. We have thus shown that irreflexive(R,A) holds.