

1. The two truth constants are \top and \perp . Variables are “propositions that have no further internal structure”. Clauses are disjunctions of the former. Thus,
 - (a) contains the three clauses $(x \vee y)$, $(z \vee \neg z)$ and (x) (an unary clause), and three variables, x , y and z .
 - (b) contains the four clauses $(a \vee b \vee \neg c)$, $(\top \vee \neg c)$, $(a \vee b \vee c)$ and $(a \vee b \vee \neg \perp \vee \neg a)$ three variables a , b and c as well as the truth constants \top and \perp .
 - (c) contains the two clauses $(\text{box} \vee \text{diamond})$ and $(\neg \text{box} \vee \neg \text{diamond})$ and two variables box and diamond .
2. The truth tables for formulas (a) – (g) are presented in Truth Table (a) – (g).

a	\perp	$(a \vee \perp)$
0	0	0
1	0	1

Truth Table (a): Satisfiable with $\{a\}$.

a	\top	$(a \vee \top)$
0	1	1
1	1	1

Truth Table (b): Satisfiable with $\{a, \neg a\}$.

a	$\neg a$	$(a \vee \neg a)$
0	1	1
1	0	1

Truth Table (c): Satisfiable with $\{a, \neg a\}$.

3. The conjunction $A \wedge B$ is defined to be true iff A and B are true. It follows that $B \wedge A$ is true iff B and A are true. Thus, the order of the arguments is irrelevant — conjunction is commutative.

For the special case where $A = B$, the statement $A \wedge B$ will always evaluate to A . (This follows from the definition above since in that case it can effectively be reduced to: “The conjunction $A \wedge B$, where $A = B$, is defined to be true iff A is true.”) Thus $A \wedge A$ is equal to A — conjunction is idempotent.

Consider $A \wedge (B \wedge C)$. Following the definition above it is true iff A and $(B \wedge C)$ are true. Recursively following the definition leads to it being true iff A is true and iff B and C are true. This can be shortened to it being true iff A , B and C are true. Since $(A \wedge B) \wedge C$ would lead to the same statement, conjunction is associative.

a	b	$C_{1d} = (a \vee \neg b)$	$C_{2d} = (\neg a \vee b)$	$C_{1d} \wedge C_{2d}$
0	0	1	1	1
0	1	0	1	0
1	0	1	0	0
1	1	1	1	1

Truth Table (d): Satisfiable with $\{\neg a, \neg b\}$, among others.

a	b	$C_{1e} = (\neg a \vee \neg b)$	$C_{1d} \wedge C_{2d}$	$C_{2e} = (a \vee b)$	$C_{1e} \wedge C_{1d} \wedge C_{2d} \wedge C_{2e}$
0	0	1	1	0	0
0	1	1	0	1	0
1	0	1	0	1	0
1	1	0	1	1	0

Truth Table (e): Not satisfiable.

4. The formula $(a \wedge b) \vee c$ has 5 models.
5. The given formula can be expanded to

$$(l_{11} \vee l_{12} \vee l_{13}) \wedge (l_{21} \vee l_{22} \vee l_{23}) \wedge (l_{31} \vee l_{32} \vee l_{33}),$$

or alternatively to

$$\{\{l_{11}, l_{12}, l_{13}\}, \{l_{21}, l_{22}, l_{23}\}, \{l_{31}, l_{32}, l_{33}\}\}$$

a	b	c	$C_{1f} = (a \vee b \vee c)$	C_{2d}	$C_{2f} = (c \vee a)$	$C_{1f} \wedge C_{2d} \wedge C_{2f}$
0	0	0	0	1	0	0
0	0	1	1	1	1	1
0	1	0	1	1	0	0
0	1	1	1	1	1	1
1	0	0	1	0	1	0
1	0	1	1	0	1	0
1	1	0	1	1	1	1
1	1	1	1	1	1	1

Truth Table (f): Satisfiable with $\{\neg a, \neg b, c\}$, among others.

a	b	c	d	f	\perp	C_{1f}	C_{2d}	$C_{1g} = (c \vee d \vee f)$	$C_{2g} = (\neg d \vee \neg f)$	$C_{1f} \wedge C_{2d} \wedge C_{1g} \wedge C_{2g} \wedge \perp$
0	0	0	0	0	0	0	1	1	1	0
0	0	0	0	1	0	0	1	1	1	0
0	0	0	1	0	0	0	1	1	1	0
0	0	0	1	1	0	0	1	1	0	0
0	0	1	0	0	0	1	1	1	1	0
0	0	1	0	1	0	1	1	1	1	0
0	0	1	1	0	0	1	1	1	1	0
0	0	1	1	1	0	1	1	1	0	0
0	1	0	0	0	0	1	1	1	1	0
0	1	0	0	1	0	1	1	1	1	0
0	1	0	1	0	0	1	1	1	1	0
0	1	0	1	1	0	1	1	1	0	0
0	1	1	0	0	0	1	1	1	1	0
0	1	1	0	1	0	1	1	1	1	0
0	1	1	1	0	0	1	1	1	1	0
0	1	1	1	1	0	1	1	1	0	0
1	0	0	0	0	0	1	1	0	1	0
1	0	0	0	1	0	1	1	0	1	0
1	0	0	1	0	0	1	1	0	1	0
1	0	0	1	1	0	1	1	0	0	0
1	0	1	0	0	0	1	1	0	1	0
1	0	1	0	1	0	1	1	0	1	0
1	0	1	1	0	0	1	1	0	1	0
1	0	1	1	1	0	1	1	0	0	0
1	1	0	0	0	0	1	1	1	1	0
1	1	0	0	1	0	1	1	1	1	0
1	1	0	1	0	0	1	1	1	1	0
1	1	0	1	1	0	1	1	1	0	0
1	1	1	0	0	0	1	1	1	1	0
1	1	1	0	1	0	1	1	1	1	0
1	1	1	1	0	0	1	1	1	1	0
1	1	1	1	1	0	1	1	1	1	0
1	1	1	1	1	1	1	1	1	0	0

Truth Table (g): Not satisfiable.