

- The two truth constants are  $\top$  and  $\perp$ . Variables are “propositions that have no further internal structure”. Clauses are disjunctions of the former. Thus,
  - contains the three clauses  $(x \vee y)$ ,  $(z \vee \neg z)$  and  $(x)$  (an unary clause), and three variables,  $x$ ,  $y$  and  $z$ .
  - contains the four clauses  $(a \vee b \vee \neg c)$ ,  $(\top \vee \neg c)$ ,  $(a \vee b \vee c)$  and  $(a \vee b \vee \neg \perp \vee \neg a)$  three variables  $a$ ,  $b$  and  $c$  as well as the truth constants  $\top$  and  $\perp$ .
  - contains the two clauses  $(\text{box} \vee \text{diamond})$  and  $(\neg \text{box} \vee \neg \text{diamond})$  and two variables  $\text{box}$  and  $\text{diamond}$ .
- The truth tables for formulas (a) – (g) are presented in Truth Table (a) – (g).

$a$	$\perp$	$(a \vee \perp)$
0	0	0
1	0	1

Truth Table (a): Satisfiable with  $\{a\}$ .

$a$	$\top$	$(a \vee \top)$
0	1	1
1	1	1

Truth Table (b): Satisfiable with  $\{a, \neg a\}$ .

$a$	$\neg a$	$(a \vee \neg a)$
0	1	1
1	0	1

Truth Table (c): Satisfiable with  $\{a, \neg a\}$ .

- The conjunction  $A \wedge B$  is defined to be true iff  $A$  and  $B$  are true. It follows that  $B \wedge A$  is true iff  $B$  and  $A$  are true. Thus, the order of the arguments is irrelevant — conjunction is commutative.

For the special case where  $A = B$ , the statement  $A \wedge B$  will always evaluate to  $A$ . (This follows from the definition above since in that case it can effectively be reduced to: “The conjunction  $A \wedge B$ , where  $A = B$ , is defined to be true iff  $A$  is true.”) Thus  $A \wedge A$  is equal to  $A$  — conjunction is idempotent.

Consider  $A \wedge (B \wedge C)$ . Following the definition above it is true iff  $A$  and  $(B \wedge C)$  are true. Recursively following the definition leads to it being true iff  $A$  is true and iff  $B$  and  $C$  are true. This can be shortened to it being true iff  $A$ ,  $B$  and  $C$  are true. Since  $(A \wedge B) \wedge C$  would lead to the same statement, conjunction is associative.

$a$	$b$	$C_{1d} = (a \vee \neg b)$	$C_{2d} = (\neg a \vee b)$	$C_{1d} \wedge C_{2d}$
0	0	1	1	1
0	1	0	1	0
1	0	1	0	0
1	1	1	1	1

Truth Table (d): Satisfiable with  $\{\neg a, \neg b\}$ , among others.

$a$	$b$	$C_{1e} = (\neg a \vee \neg b)$	$C_{1d} \wedge C_{2d}$	$C_{2e} = (a \vee b)$	$C_{1e} \wedge C_{1d} \wedge C_{2d} \wedge C_{2e}$
0	0	1	1	0	0
0	1	1	0	1	0
1	0	1	0	1	0
1	1	0	1	1	0

Truth Table (e): Not satisfiable.

4. The formula  $(a \wedge b) \vee c$  has 5 models.
5. The given formula can be expanded to

$$(l_{11} \vee l_{12} \vee l_{13}) \wedge (l_{21} \vee l_{22} \vee l_{23}) \wedge (l_{31} \vee l_{32} \vee l_{33}),$$

or alternatively to

$$\{\{l_{11}, l_{12}, l_{13}\}, \{l_{21}, l_{22}, l_{23}\}, \{l_{31}, l_{32}, l_{33}\}\}$$

$a$	$b$	$c$	$C_{1f} = (a \vee b \vee c)$	$C_{2d}$	$C_{2f} = (c \vee a)$	$C_{1f} \wedge C_{2d} \wedge C_{2f}$
0	0	0	0	1	0	0
0	0	1	1	1	1	1
0	1	0	1	1	0	0
0	1	1	1	1	1	1
1	0	0	1	0	1	0
1	0	1	1	0	1	0
1	1	0	1	1	1	1
1	1	1	1	1	1	1

Truth Table (f): Satisfiable with  $\{\neg a, \neg b, c\}$ , among others.

$a$	$b$	$c$	$d$	$f$	$\perp$	$C_{1f}$	$C_{2d}$	$C_{1g} = (c \vee d \vee f)$	$C_{2g} = (\neg d \vee \neg f)$	$C_{1f} \wedge C_{2d} \wedge C_{1g} \wedge C_{2g} \wedge \perp$
0	0	0	0	0	0	0	1	1	1	0
0	0	0	0	1	0	0	1	1	1	0
0	0	0	1	0	0	0	1	1	1	0
0	0	0	1	1	0	0	1	1	0	0
0	0	1	0	0	0	1	1	1	1	0
0	0	1	0	1	0	1	1	1	1	0
0	0	1	1	0	0	1	1	1	1	0
0	0	1	1	1	0	1	1	1	0	0
0	1	0	0	0	0	1	1	1	1	0
0	1	0	0	1	0	1	1	1	1	0
0	1	0	1	0	0	1	1	1	1	0
0	1	0	1	1	0	1	1	1	0	0
0	1	1	0	0	0	1	1	1	1	0
0	1	1	0	1	0	1	1	1	1	0
0	1	1	1	0	0	1	1	1	1	0
0	1	1	1	1	0	1	1	1	0	0
1	0	0	0	0	0	1	1	0	1	0
1	0	0	0	1	0	1	1	0	1	0
1	0	0	1	0	0	1	1	0	1	0
1	0	0	1	1	0	1	1	0	0	0
1	0	1	0	0	0	1	1	0	1	0
1	0	1	0	1	0	1	1	0	1	0
1	0	1	1	0	0	1	1	0	1	0
1	0	1	1	1	0	1	1	0	0	0
1	1	0	0	0	0	1	1	1	1	0
1	1	0	0	1	0	1	1	1	1	0
1	1	0	1	0	0	1	1	1	1	0
1	1	0	1	1	0	1	1	1	0	0
1	1	1	0	0	0	1	1	1	1	0
1	1	1	0	1	0	1	1	1	1	0
1	1	1	1	0	0	1	1	1	1	0
1	1	1	1	1	0	1	1	1	0	0
1	1	1	1	1	1	1	1	1	0	0

Truth Table (g): Not satisfiable.