Time Hierarchy Theorem

Informally, the theorem states that given more time, a Turing machine can solve more problems. Formally, if $f(n)$ is a time-honest function then

$$\mathsf{DTIME}(f(n)) \subsetneq \mathsf{DTIME}\left(f(n)^2\right),$$

with the understanding that this can be restated in more general terms (as in the lecture notes).

As suggested, I will prove a weaker version, showing that $\mathsf{DTIME}(f(n))$ is smaller than (thus a strict subset of) $\mathsf{DTIME}(f(2n+1)^3)$. Following a hint from the lecture notes, let A be the language that contains all TMs which accept an input after at most $f(|x|)$ steps.