Physics Notes
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Contents

Chapter 1

1D Kinematics

1.1 Coordinate System in 1D

Coords in 1D:

- Set origin
- Set axis
  - Set positive coordinate system
- Unit vector
  - You only need one because it only has one dimension

Every point in 1D space has a point $\hat{\mathbf{i}} \rightarrow$ point p_1 has a vector $\hat{\mathbf{i}}_1$ and point p_2 has a vector $\hat{\mathbf{i}}_2$
$\hat{\mathbf{i}}_1 = \hat{\mathbf{i}}_2 \rightarrow |\hat{\mathbf{i}}| = 1$

1.2 Position Vector in 1D

$x(t)$ is the position function that changes according to time.
Position vector is $\vec{r}(t) = x(t)\hat{\mathbf{i}}$
$x(t)$ is the component of the potition vector
$x(t)$ changes sign depending on location $\rightarrow x(t) > 0$ when the object is positive of the origin, $x(t) = 0$ when the object is on the vector, and $x(t) < 0$ when the object is negative of the origin
Direction of the vector is determined by the sign of the component $(x(t))$ times the component $(\hat{\mathbf{i}})$

1.3 Displacement Vector in 1D

The position after time Δt if the object has been moving at a constant velocity is $r(t + \Delta t) = x(t + \Delta t)\hat{\mathbf{i}}$
Displacement vector for $[t, t + \Delta t]$ is

$$\Delta\vec{r} \equiv \vec{r}(t + \Delta t) - \vec{r}(t) = (x(t + \Delta t) - x(t))\hat{\mathbf{i}}$$
$$\Delta\vec{r} = \Delta x\hat{\mathbf{i}}$$

Δx is the component of the displacement vector, where a $\Delta x > 0$ means a displacement in the positive x direction, a $\Delta x = 0$ meaning no net change in position, and a $\Delta x < 0$ meaning a change in position in the negative x direction

1.4 Avg. Velocity in 1D

Average velocity depends on the time interval
Ex. for interval $[t, \Delta t]$:

$$\vec{v}_{avg} \equiv \frac{\Delta \vec{r}}{\Delta t} = \frac{\Delta x}{\Delta t}\hat{\mathbf{\imath}}$$

where $\frac{\Delta x}{\Delta t}$ is the component of velocity

1.5 Instantaneous Velocity in 1D

How do we find velocity at specific time t_1?
Consider avg. velocity over time interval $[t_1, t_{1+\Delta t}] \longrightarrow \vec{v}_{avg}$ = slope of the line.
As we shrink Δt, we find the slope changes. If we consider $\lim_{x \to 0} \frac{\Delta x}{\Delta t}$, we will get to the slope of the tangent line at time t_1
Thus we have:

$$\vec{v}(t_1) = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t}\hat{\mathbf{\imath}}$$
$$= \lim_{\Delta t \to 0} (\frac{x(t_1 + \Delta t) - x_1(t)}{\Delta t})\hat{\mathbf{\imath}}$$

$\vec{v}(t_1)$ is the instantaneous at time $t = t_1$
More generally, $\vec{v}(t) = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t}\hat{\mathbf{\imath}} \longrightarrow \vec{v}(t) = \frac{dx}{dt}\hat{\mathbf{\imath}}$

Chapter 2

1D Kinematics - Acceleration

2.1 Intro to Acceleration

Acceleration is the change in velocity over time $\longrightarrow \frac{d\vec{v}}{dt}$
For an object in freefall, $\vec{F}_{grav} = m\vec{a}$ where $\vec{a}$ is the gravitational constant

Note:-

Freefall is where an object is under the influence of the gravitational force $\vec{F}_g$

2.2 Acceleration in 1D

$\Delta\vec{v} = (v(t + \Delta t) - v(t))\hat{\mathbf{i}}$ on the interval $[t, \Delta t]$
Now we find instantaneous acceleration:

$$\vec{a}(t) = \lim_{\Delta t \to 0} \frac{\Delta \vec{v}}{\Delta t} = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t}\hat{\mathbf{i}}$$

Visually, $\vec{a}(t)$ is the slope of the tangent line of the plot of $\vec{v}(t)$ vs t
Componently, $\vec{a}(t) = a_x(t)\hat{\mathbf{i}} = \frac{dv_x}{dt}\hat{\mathbf{i}}$

Chapter 3

2D Kinematics

3.1 Coordinate System and Position Vector in 2D

To represent 2D motion in vectors, one first needs to set up a coordinate system. For any arbitrary point p_1, there'll be unit vectors $\hat{\mathbf{i}}_1$ and $\hat{\mathbf{j}}_1$.

Note:-

The Cartesian coordinate system is interesting in that no matter what point we're at, the unit vectors are all the same $\longrightarrow$ we just have $\hat{\mathbf{i}}$ and $\hat{\mathbf{j}}$

Now we find the position vector, $\vec{r}(t)$, a vector from the origin to the position of the object. We can reprecent $\vec{r}_t$ as two vectors, $x(t)$ and $y(t)$.
Thus $\vec{r}(t) = x(t)\hat{\mathbf{i}} + y(t)\hat{\mathbf{j}}$

3.2 Instantaneous Velocity in 2D

To find velocity, we want to find $\lim_{\Delta t \to 0} \vec{r}(t)$
Graphically, $\lim_{\Delta t \to 0} \vec{r}(t)$ is direcly tangent to the position at time t.
Given $\vec{r}(t) = x(t)\hat{\mathbf{i}} + y(t)\hat{\mathbf{j}}$, we can find velocity by:

$$\vec{v} = \lim_{\Delta t \to 0} \frac{\Delta \vec{r}}{\Delta t} = (\lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t}\hat{\mathbf{i}}) + (\lim_{\Delta t \to 0} \frac{\Delta y}{\Delta t}\hat{\mathbf{j}})$$
$$\vec{v} = \frac{d\vec{r}}{dt} = \frac{dx}{dt}\hat{\mathbf{i}} + \frac{dy}{dt}\hat{\mathbf{j}}$$
$$\vec{v} = v_x\hat{\mathbf{i}} + v_y\hat{\mathbf{j}}$$

where $(v_x, v_y) = (\frac{dx}{dt}, \frac{dy}{dt})$

Note:-

The speed of the velocity is $v = |(v_x^2 + v_y^2)^{\frac{1}{2}}|$

We can find the direction of velocity as such:

$$\tan\theta = \frac{v_y}{v_x}$$
$$\theta = \arctan\frac{v_y}{v_x}$$

3.3 Instantaneous Acceleration in 2D

$\vec{a}(t) - \frac{d\vec{v}}{dt} = \lim_{\Delta t \to 0} \frac{\Delta \vec{v}}{\Delta t}$

$$\vec{v} = v_x \hat{\mathbf{i}} + v_y \hat{\mathbf{j}} \longrightarrow \vec{a} = \frac{d\vec{v}}{dt} = \frac{dv_x}{dt}\hat{\mathbf{i}} + \frac{dv_y}{dt}\hat{\mathbf{j}}$$

$$\vec{a} = \frac{d^2x}{dt^2}\hat{\mathbf{i}} + \frac{d^2y}{dt^2}$$

$$\vec{a} = a_x \hat{\mathbf{i}} + a_y \hat{\mathbf{j}}$$

$$a_x = \frac{d^2x}{dt^2} = \frac{dv_x}{dt}, a_y = \frac{d^2y}{dt^2} = \frac{dv_y}{dt}$$

$a = |(a_x^2 + a_y^2)^{\frac{1}{2}}|$

3.4 Projectile Motion

Projectile motion is the motion of an object while under the influence of gravity.
Need to apply Newtons 2nd Law in order to analyze the kinematics.

$\hat{\mathbf{j}}$:

$\vec{F}$	$m\vec{a}$
$-mg$	ma_y

Applying Newtons 2nd Law, we can set $-mg = ma_y \longrightarrow -g = a_y$

$\hat{\mathbf{i}}$: $0 = ma_x \longrightarrow a_x = 0$
Thus, we have:

$$v_y(t) = v_{y0} - gt \tag{3.1}$$

$$y(t) = y_0 + v_{y0}t - \frac{1}{2}gt^2$$

$$v_x(t) = v_{x0} \tag{3.2}$$

$$x(t) = x_0 + v_{x0}t$$

$$x_0 = 0 \longrightarrow x = v_{x0}t$$

$$t = x/v_{x0}$$

$$y(x) = y_0 + \frac{v_{y0}x}{v_{x0}} - \frac{1}{2}g\frac{x^2}{v_{x0}} \tag{3.3}$$

Chapter 4

Newtons First and Second Laws

Newtons First Law tells us about the motion of isloated bodies - where the net force is 0. States that an isolated body moves in a straight line at a const velocity.
Ex: Isloated body at rest will remain at rest as long as its undisturbed.