数字图象处理

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http://www.imageprocessingplace.com/root_files_V3/students/students.htm

第4章　频率域滤波

第4章　频率域滤波：图像变换

动机：为了有效和快速地对图象进行处理，常常需要将原定义在图象空间的图象以某种形式转换到另外一些空间（频率域空间）并加工，最后再转换回图象空间以得到所需的效果。

第4章　频率域滤波

4.1　离散傅立叶变换(DFT)

4.2　频率域滤波基础

4.3　频率域滤波器平滑图像

4.4　频率域滤波器锐化图像

4.1 傅立叶变换基础

- 傅里叶级数（FS）
  - 任何周期函数都可以表示为不同频率的正弦或余弦之和的形式，每个正弦项和余弦项都乘以不同的系数
- 傅里叶变换（FT）
  - 任何非周期函数（其曲线下的面积是有限的）可以表示为正弦和/或余弦乘以加权函数的积分来表示

FIGURE 4.1 The function at the bottom is the sum of the four functions above it. Fourier's idea in 1807 that periodic functions could be represented as a weighted sum of sines and cosines was met with skepticism.

4.1 傅立叶变换基础

一维傅立叶变换及其反变换

$$F(u)=\int_{-\infty}^{\infty} f(x)e^{-j2\pi ux}dx \qquad F(u,v)=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)e^{-j2\pi(ux+vy)}dxdy$$

$$F(x)=\int_{-\infty}^{\infty} f(u)e^{j2\pi ux}du \qquad F(x,y)=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(u,v)e^{j2\pi(ux+vy)}dudv$$

离形形式：

$$F(u) = \frac{1}{M}\sum_{x=0}^{M-1} f(x)\, e^{-j2\pi ux/M} \qquad x = 0,1,2\cdots M-1$$

$$f(x) = \sum_{u=0}^{M-1} F(u)\, e^{j2\pi ux/M} \qquad u = 0,1,2\cdots M-1$$

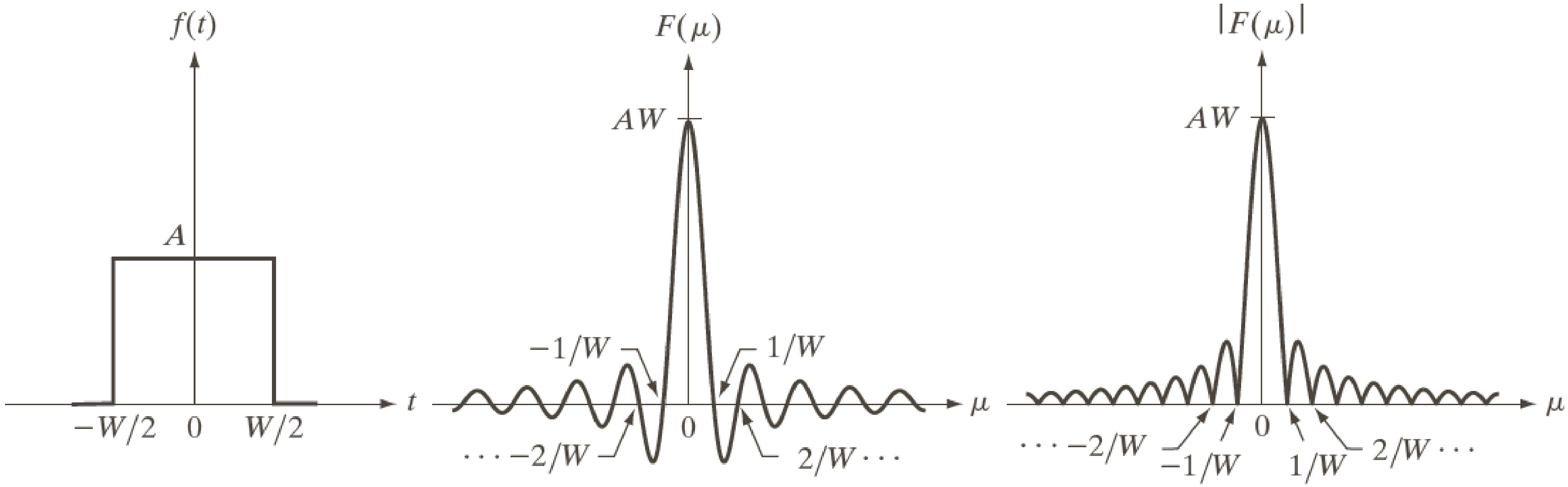
频域→不同的频域成份，可以表示成极坐标形式：

$$F(u)=|F(u)|e^{-j\phi(u)}$$

$$|F(u)|=\left[R^2(u)+I^2(u)\right]^{\frac{1}{2}}$$

$$\phi(u)=arctg\left[\frac{2(u)}{R(u)}\right]$$

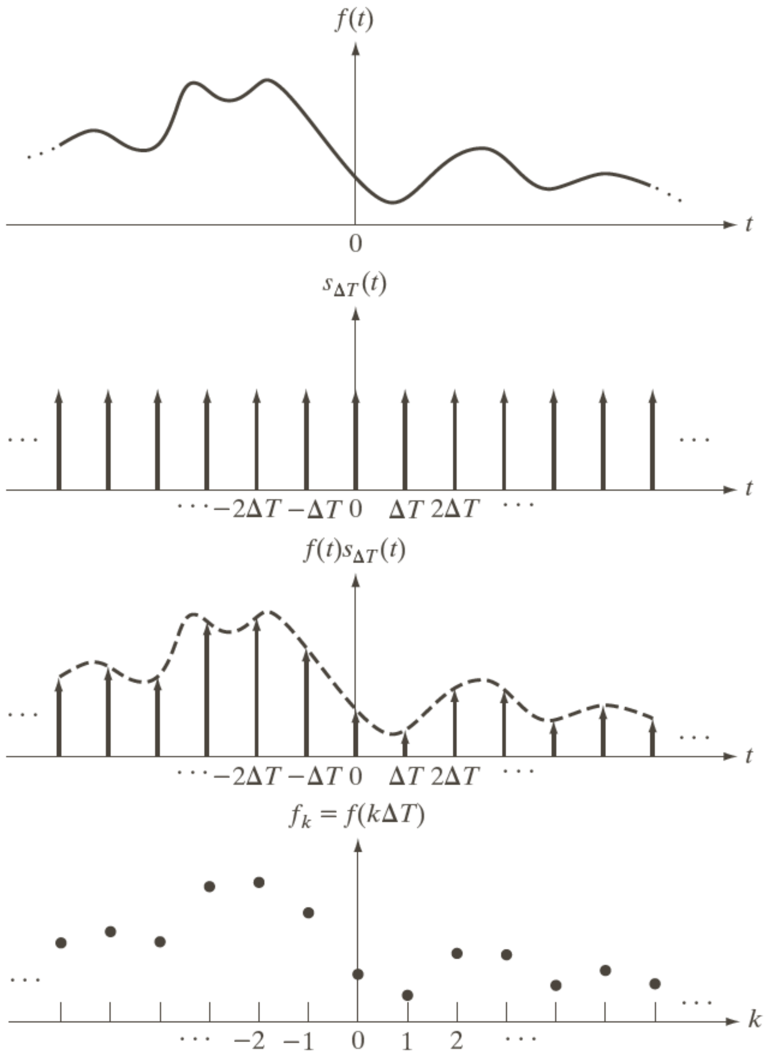
4.1 傅立叶变换基础



a b c

FIGURE 4.4 (a) A simple function; (b) its Fourier transform; and (c) the spectrum. All functions extend to infinity in both directions.

取样：连续函数转换为离散序列

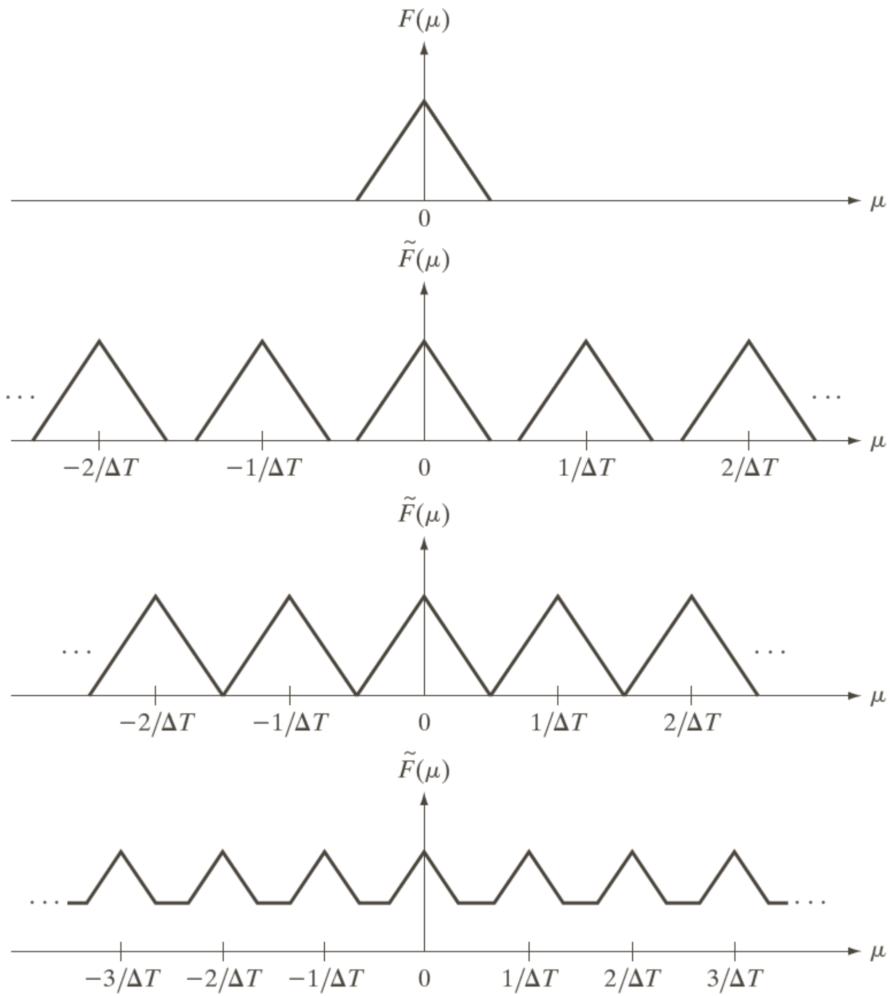


a
b
c
d

FIGURE 4.5
(a) A continuous function. (b) Train of impulses used to model the sampling process. (c) Sampled function formed as the product of (a) and (b). (d) Sample values obtained by integration and using the sifting property of the impulse. (The dashed line in (c) is shown for reference. It is not part of the data.)

取样函数的傅里叶变换

奈奎斯特采样率：
完全等于信号最高频率的两倍的取样率。



a
b
c
d

FIGURE 4.6 (a) Fourier transform of a band-limited function. (b)–(d) Transforms of the corresponding sampled function under the conditions of over-sampling, critically-sampling, and under-sampling, respectively.

混淆

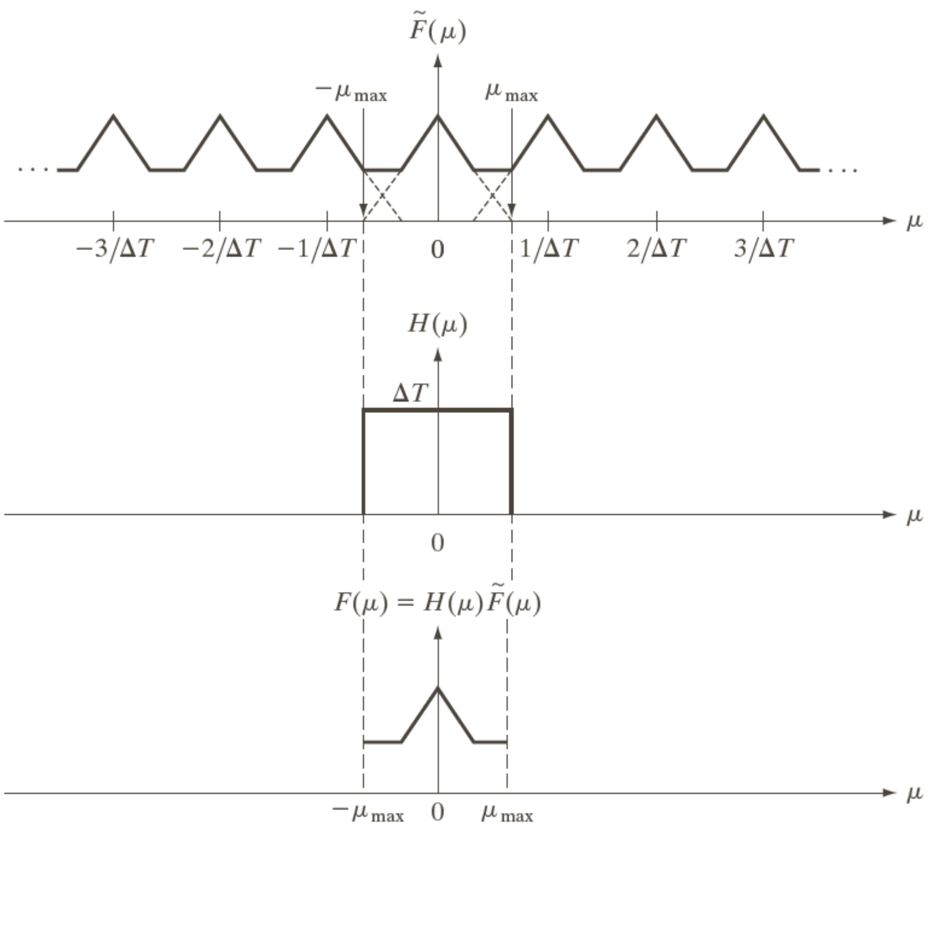
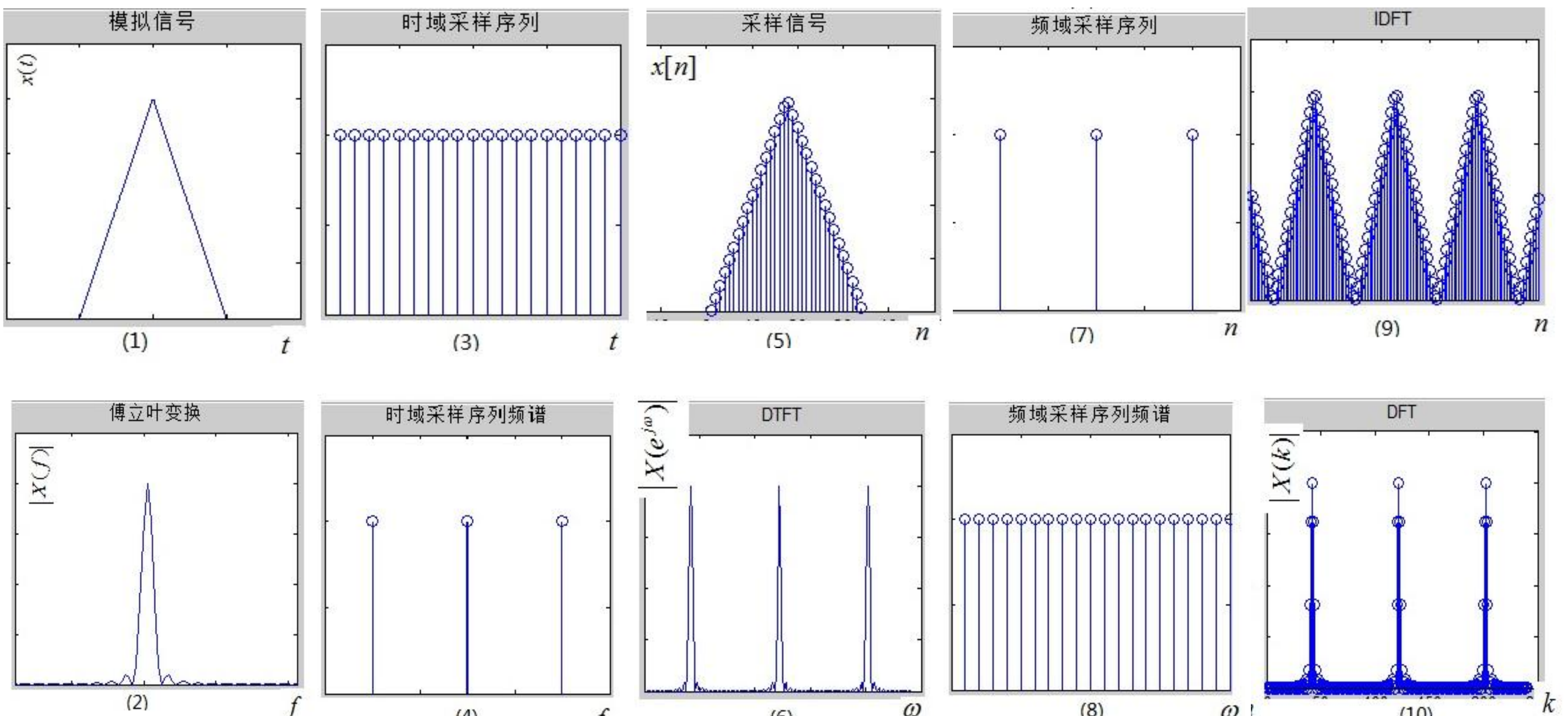


FIGURE 4.9 (a) Fourier transform of an under-sampled, band-limited function. (Interference from adjacent periods is shown dashed in this figure). (b) The same ideal lowpass filter used in Fig. 4.8(b). (c) The product of (a) and (b). The interference from adjacent periods results in aliasing that prevents perfect recovery of $F(\mu)$ and, therefore, of the original, band-limited continuous function. Compare with Fig. 4.8.

4.1 傅立叶变换基础

- 傅里叶变换（FT）、离散时间傅里叶变换（DTFT）和离散傅里叶变换（DFT）之间的关系
  - DFT：对一个周期取样



4.1 离散傅立叶变换(DFT)

二维DFT及其反变换

一个图像关于的函数的离散

$$F(u,v)=\frac{1}{MN}\sum_{x=0}^{M-1}\sum_{y=0}^{N-1}f(x,y)e^{-j2\pi(ux/M+vy/N)}$$

$$u=0.1.2.\cdots M-1$$

$$v=0.1.2.\cdots N-1$$

$$f(x,y)=\sum_{u=0}^{M-1}\sum_{v=0}^{N-1}F(u,v)e^{j2\pi(ux/M+vy/N)}$$

$$x=0.1.2.\cdots M-1$$

$$y=0.1.2.\cdots N-1$$

同理：

谱：$|F(u,v)|=\left[K^2(x,y)+I^2(x,y)\right]^{\frac{1}{2}}$

相角：$F(u,v)=arctg\left[\frac{I(u,v)}{R(u,v)}\right]$

功率谱：

$$P(u,v)=|F(u,v)|^2=R^2(u,v)+I^2(u,v)$$

可以证明：

$$\wp\left[f(x,y)(-1)^{x+y}\right]=F(u-\frac{M}{2},v-\frac{N}{2})$$

原点被放置在 $u=\frac{M}{2},v=\frac{N}{2}$ 上

$$F(0,0)=MN\frac{1}{MN}\sum_{x=0}^{M-1}\sum_{y=0}^{N-1}f(x,y)=MN\bar{f}$$

$\bar{f}$ 为图象平均灰度。

数字图像中的混淆

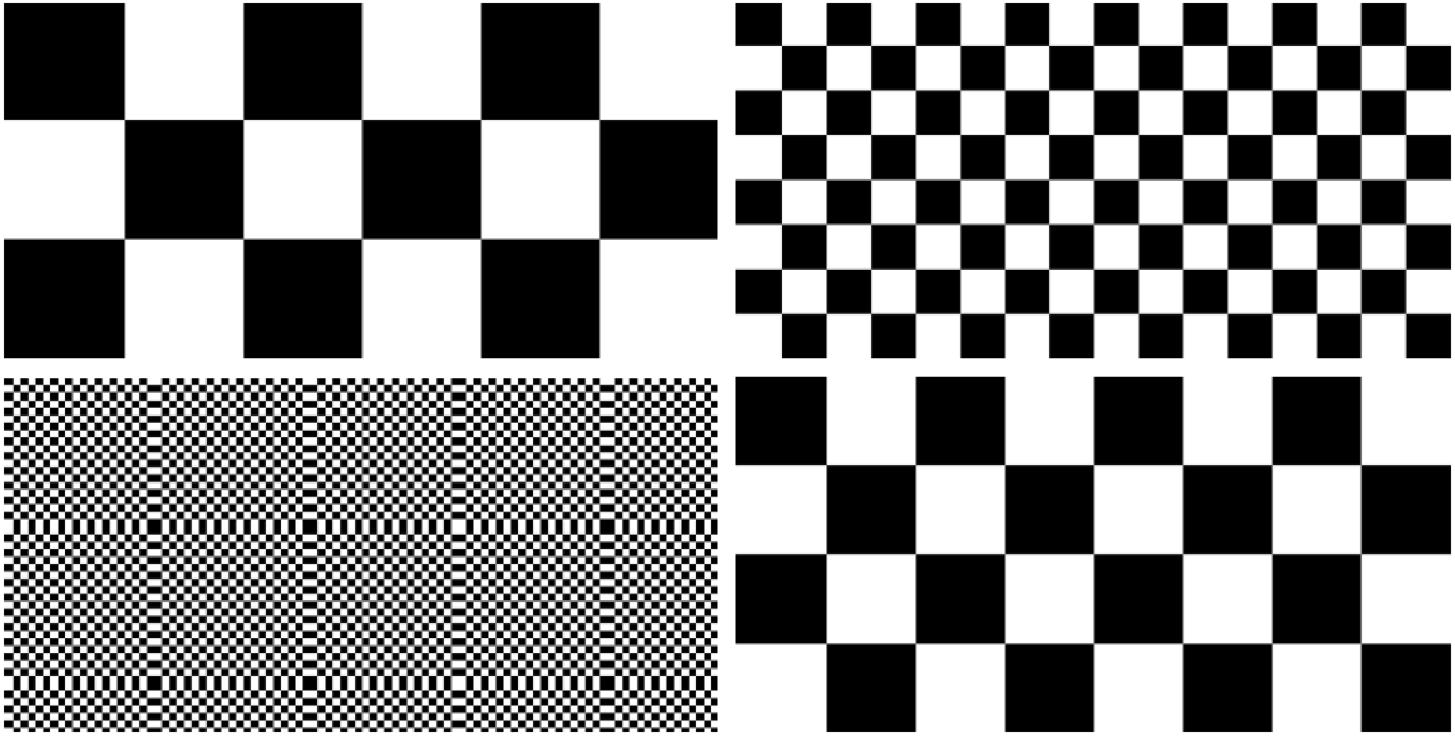


FIGURE 4.16 Aliasing in images. In (a) and (b), the lengths of the sides of the squares are 16 and 6 pixels, respectively, and aliasing is visually negligible. In (c) and (d), the sides of the squares are 0.9174 and 0.4798 pixels, respectively, and the results show significant aliasing. Note that (d) masquerades as a "normal" image.

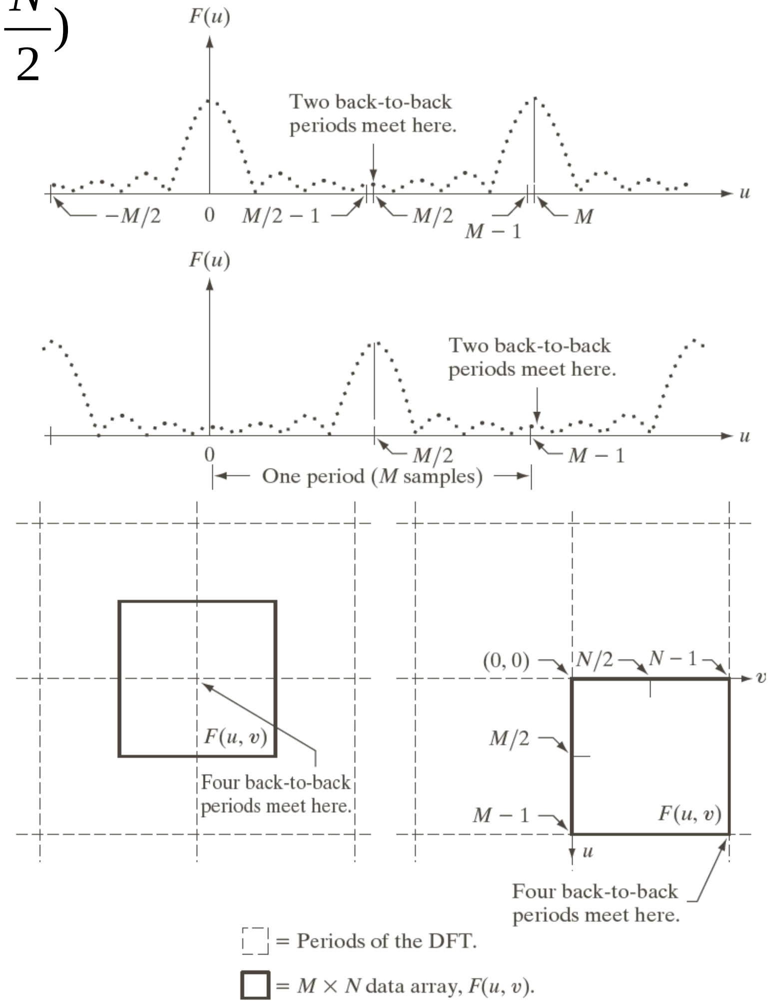
数字图像中的混淆

a b c

FIGURE 4.17 Illustration of aliasing on resampled images. (a) A digital image with negligible visual aliasing. (b) Result of resizing the image to 50% of its original size by pixel deletion. Aliasing is clearly visible. (c) Result of blurring the image in (a) with a 3×3 averaging filter prior to resizing. The image is slightly more blurred than (b), but aliasing is not longer objectionable. (Original image courtesy of the Signal Compression Laboratory, University of California, Santa Barbara.)

2D DFT的性质：周期性

$$\wp\left[f(x,y)(-1)^{x+y}\right]=F(u-\frac{M}{2},v-\frac{N}{2})$$



a
b
c d

FIGURE 4.23 Centering the Fourier transform. (a) A 1-D DFT showing an infinite number of periods. (b) Shifted DFT obtained by multiplying $f(x)$ by $(-1)^x$ before computing $F(u)$. (c) A 2-D DFT showing an infinite number of periods. The solid area is the $M \times N$ data array, $F(u, v)$, obtained with Eq. (4.5-15). This array consists of four quarter periods. (d) A Shifted DFT obtained by multiplying $f(x, y)$ by $(-1)^{x+y}$ before computing $F(u, v)$. The data now contains one complete, centered period, as in (b).

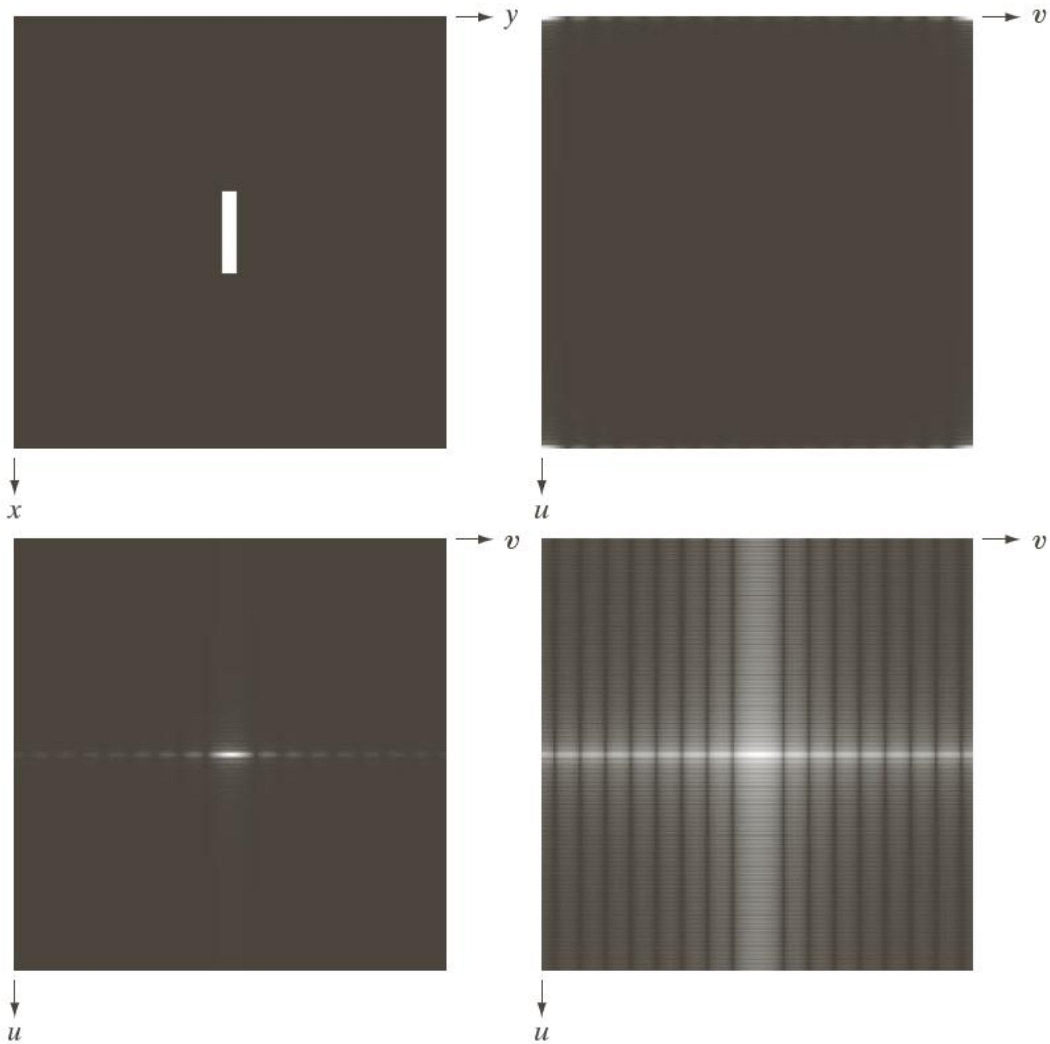
2D DFT及其反变换的一些性质

	Spatial Domain[†]		Frequency Domain[†]
1)	$f(x, y)$ real	$\Leftrightarrow$	$F^*(u, v) = F(-u, -v)$
2)	$f(x, y)$ imaginary	$\Leftrightarrow$	$F^*(-u, -v) = -F(u, v)$
3)	$f(x, y)$ real	$\Leftrightarrow$	$R(u, v)$ even; $I(u, v)$ odd
4)	$f(x, y)$ imaginary	$\Leftrightarrow$	$R(u, v)$ odd; $I(u, v)$ even
5)	$f(-x, -y)$ real	$\Leftrightarrow$	$F^*(u, v)$ complex
6)	$f(-x, -y)$ complex	$\Leftrightarrow$	$F(-u, -v)$ complex
7)	$f^*(x, y)$ complex	$\Leftrightarrow$	$F^*(-u - v)$ complex
8)	$f(x, y)$ real and even	$\Leftrightarrow$	$F(u, v)$ real and even
9)	$f(x, y)$ real and odd	$\Leftrightarrow$	$F(u, v)$ imaginary and odd
10)	$f(x, y)$ imaginary and even	$\Leftrightarrow$	$F(u, v)$ imaginary and even
11)	$f(x, y)$ imaginary and odd	$\Leftrightarrow$	$F(u, v)$ real and odd
12)	$f(x, y)$ complex and even	$\Leftrightarrow$	$F(u, v)$ complex and even
13)	$f(x, y)$ complex and odd	$\Leftrightarrow$	$F(u, v)$ complex and odd

[†]Recall that x, y, u, and v are *discrete* (integer) variables, with x and u in the range $[0, M - 1]$, and y, and v in the range $[0, N - 1]$. To say that a complex function is *even* means that its real *and* imaginary parts are even, and similarly for an odd complex function.

TABLE 4.1 Some symmetry properties of the 2-D DFT and its inverse. $R(u, v)$ and $I(u, v)$ are the real and imaginary parts of $F(u, v)$, respectively. The term *complex* indicates that a function has nonzero real and imaginary parts.

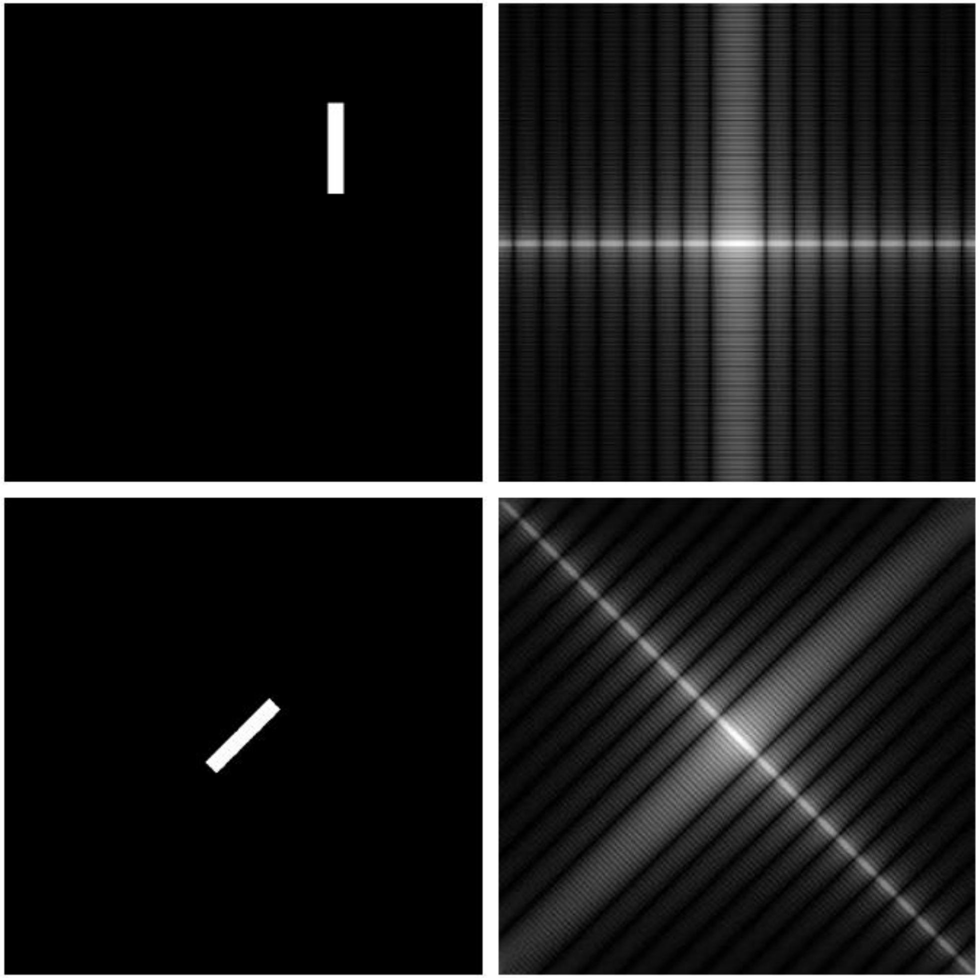
数字图像的DFT变换



a b
c d

FIGURE 4.24
(a) Image. (b) Spectrum showing bright spots in the four corners. (c) Centered spectrum. (d) Result showing increased detail after a log transformation. The zero crossings of the spectrum are closer in the vertical direction because the rectangle in (a) is longer in that direction. The coordinate convention used throughout the book places the origin of the spatial and frequency domains at the top left.

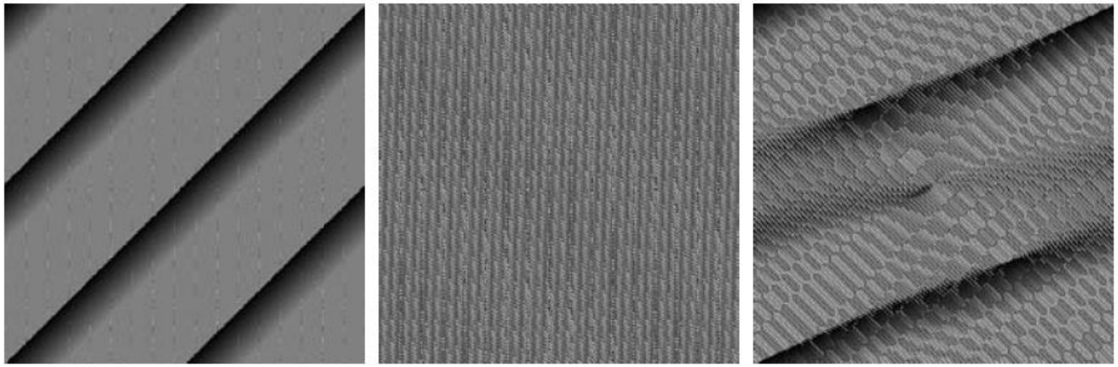
数字图像的DFT变换



a b
c d

FIGURE 4.25
(a) The rectangle in Fig. 4.24(a) translated, and (b) the corresponding spectrum. (c) Rotated rectangle, and (d) the corresponding spectrum. The spectrum corresponding to the translated rectangle is identical to the spectrum corresponding to the original image in Fig. 4.24(a).

数字图像的DFT变换



a b c

FIGURE 4.26 Phase angle array corresponding (a) to the image of the centered rectangle in Fig. 4.24(a), (b) to the translated image in Fig. 4.25(a), and (c) to the rotated image in Fig. 4.25(c).

4.1 离散傅立叶变换(DFT)

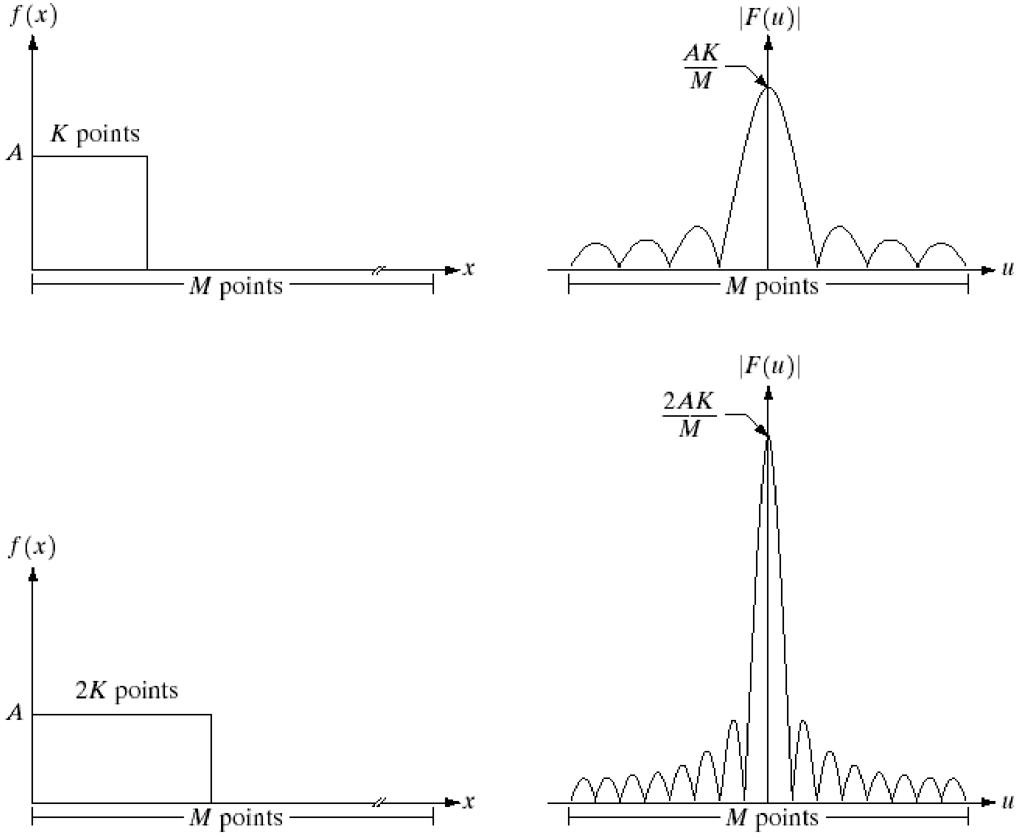
二维DFT及其反变换

如果 $f(x,y)$ 为实数，其傅氏变换为对称的： $F(u,v)=F(-u,-v)$

同样，空间域和频率域抽样点之间的关系： $|F(u,v)|=|F(-u,-v)|$

$$\Delta u = \frac{1}{M\Delta x}$$

$$\Delta v = \frac{1}{N\Delta y}$$



a b
c d

FIGURE 4.2 (a) A discrete function of M points, and (b) its Fourier spectrum. (c) A discrete function with twice the number of nonzero points, and (d) its Fourier spectrum.

4.1 离散傅立叶变换(DFT)

二维DFT及其反变换

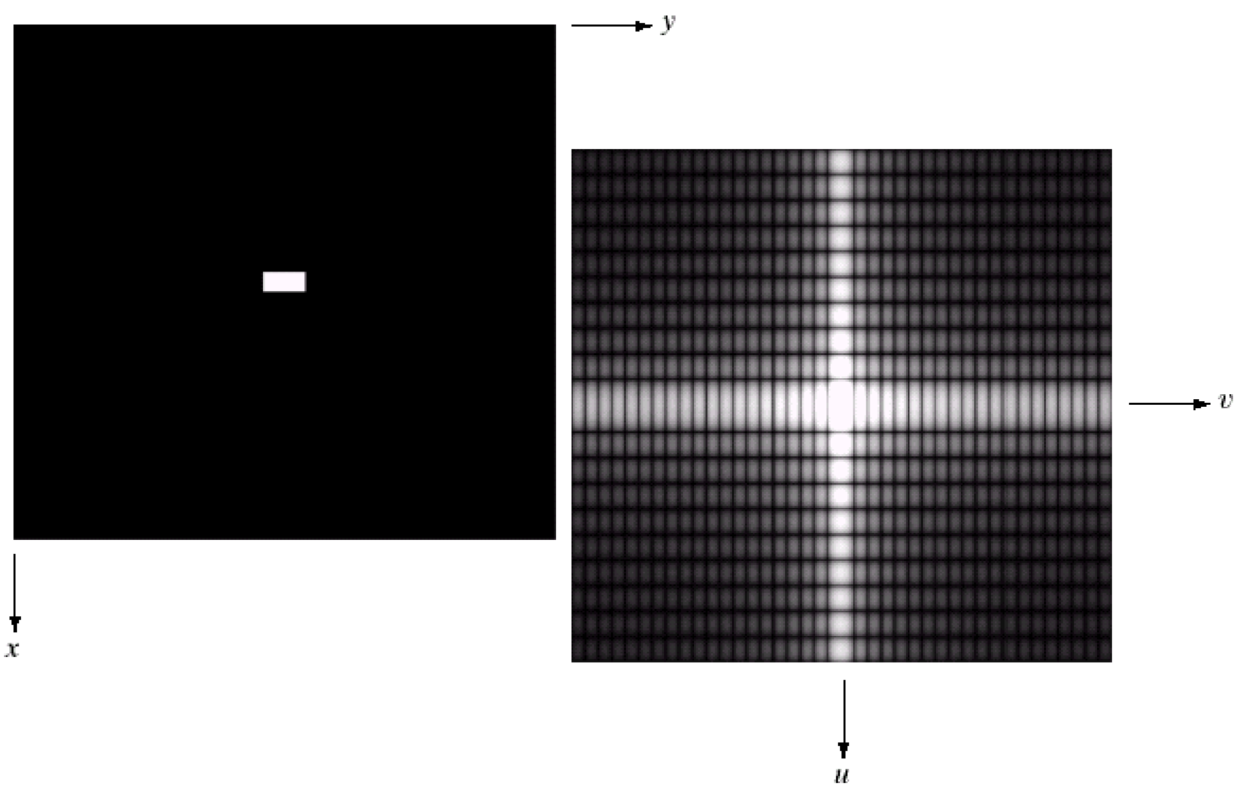
下图实例：

中心化，矩形宽高化为 2 : 1

反映到频域轴亮点间距恰好相反

a b

FIGURE 4.3
(a) Image of a 20 × 40 white rectangle on a black background of size 512 × 512 pixels.
(b) Centered Fourier spectrum shown after application of the log transformation given in Eq. (3.2-2). Compare with Fig. 4.2.



基于二维DFT频谱与相位的反变换

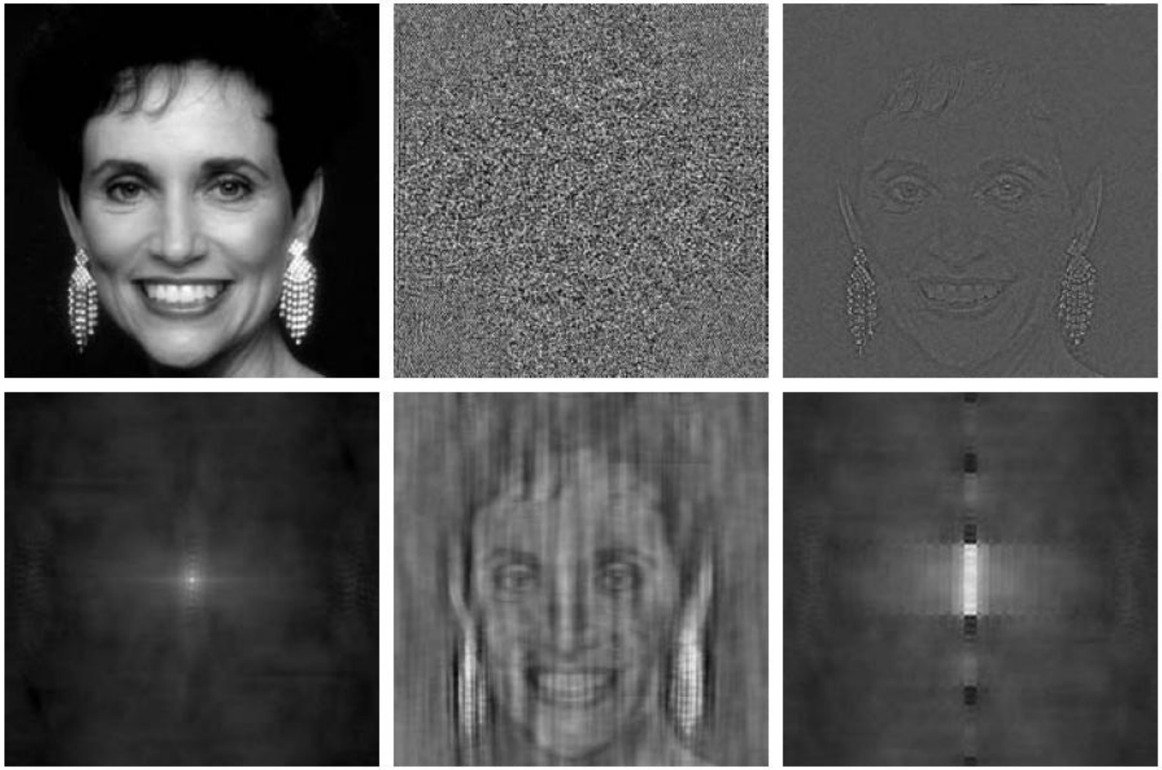


FIGURE 4.27 (a) Woman. (b) Phase angle. (c) Woman reconstructed using only the phase angle. (d) Woman reconstructed using only the spectrum. (e) Reconstruction using the phase angle corresponding to the woman and the spectrum corresponding to the rectangle in Fig. 4.24(a). (f) Reconstruction using the phase of the rectangle and the spectrum of the woman.

二维DFT的性质

- 线性 $f_1(x,y)+f_2(x,y)$ $F_1(u,v)+F_2(u,v)$
- 比例 $f\ (ax,by)$ $\frac{1}{ab}F(\frac{u}{a},\frac{v}{b})$
- 平移 $f(x-a,y-b)$ $e^{-j2\pi(au+bv)}F(u,v)$

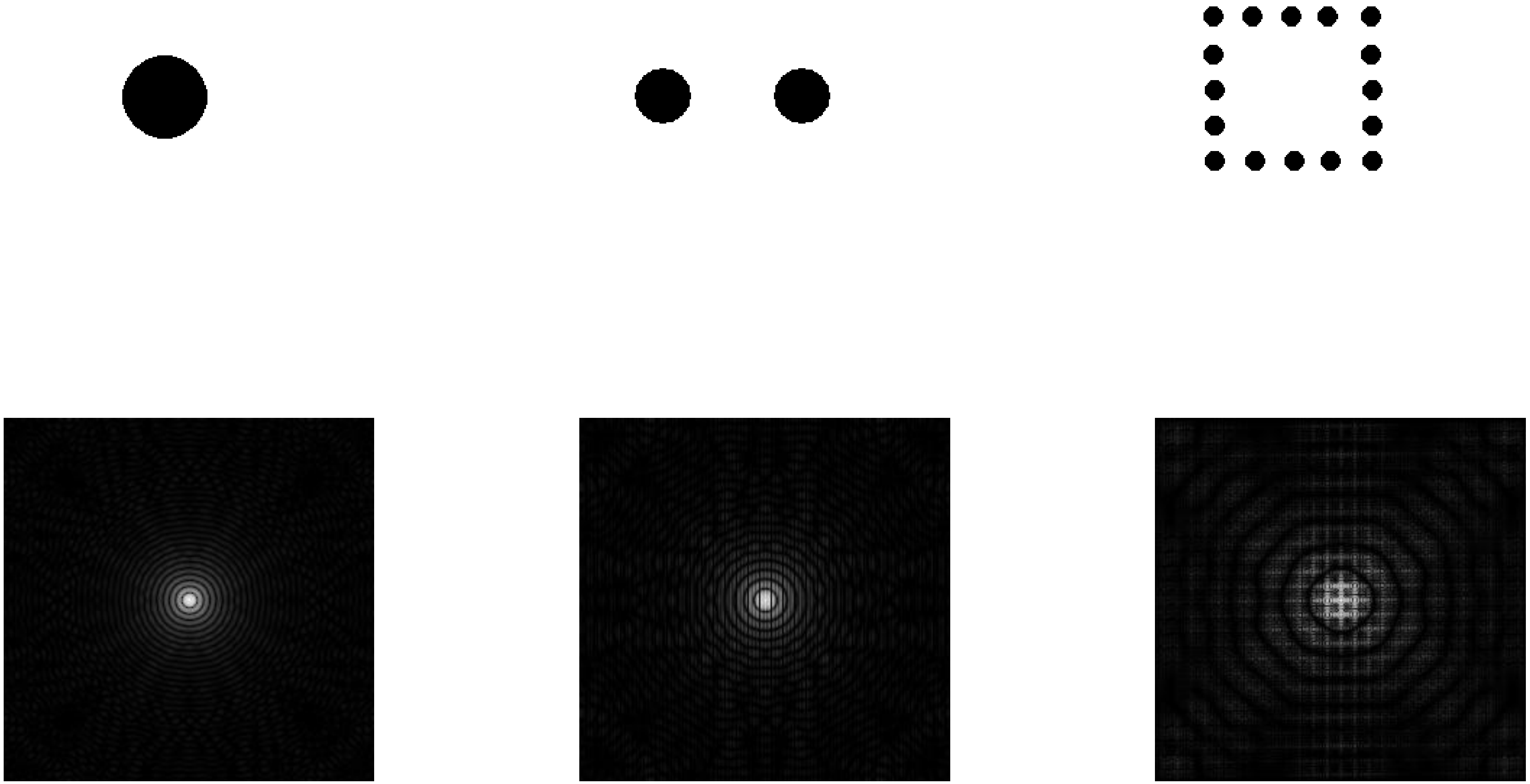
  $e^{j2\pi(cx+dy)}f(x,y)$ $F(u-c,v-d)$
- 卷积 $f_1(x,y)*f_2(x,y)$ $F_1(u,v)F_2(u,v)$

  $f_1(x,y)f_2(x,y)$ $F_1(u,v)*F_2(u,v)$
- 旋转 $f(x\cos\theta+y\sin\theta,-x\sin\theta+ycon\theta)$

  $F(u\cos\theta+v\sin\theta,-u\sin\theta+vcon\theta)$

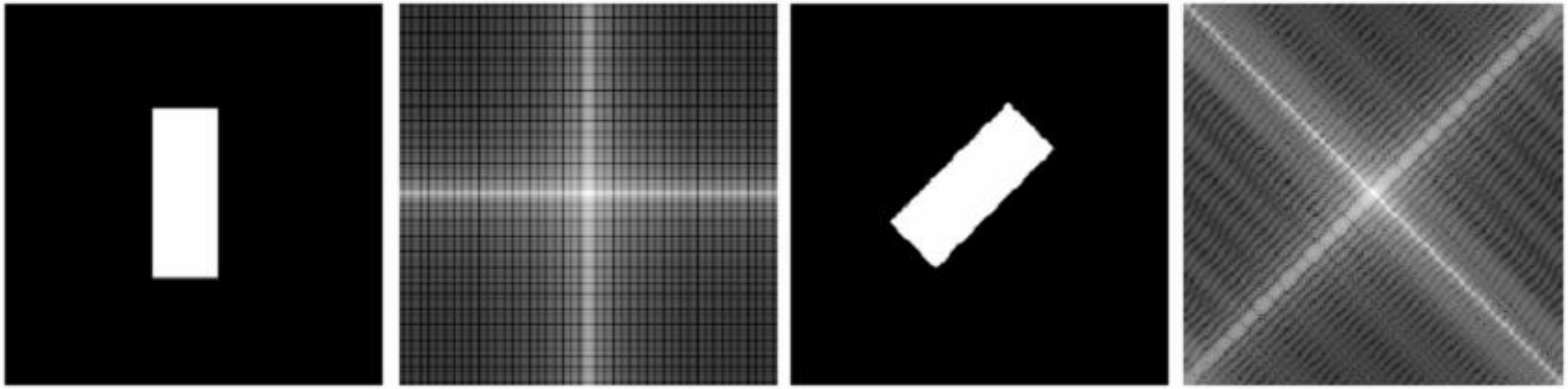
4.1 离散傅立叶变换(DFT)

线性叠加及尺度变化

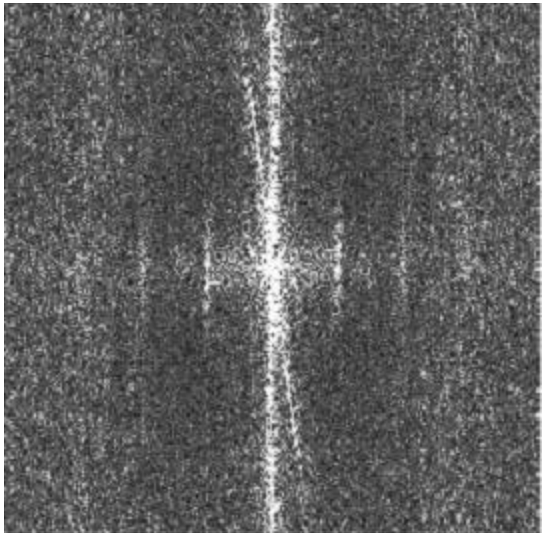


4.1 离散傅立叶变换(DFT)

旋转性

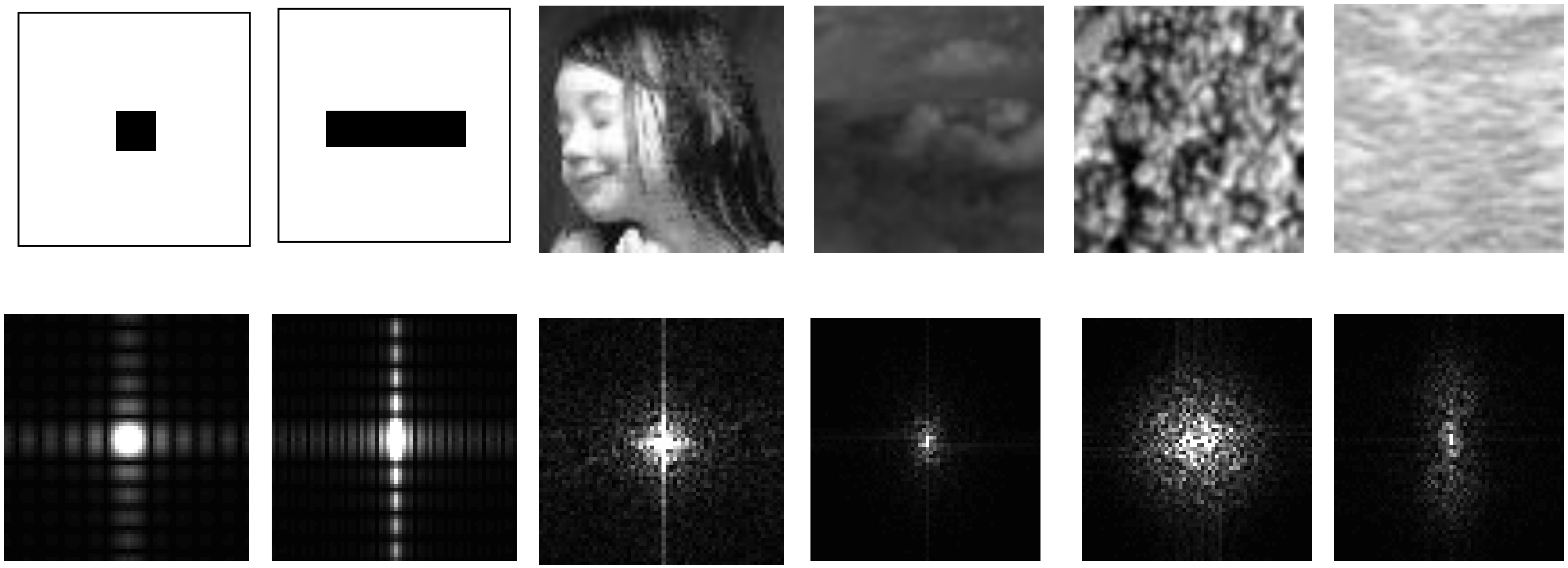


4.1 离散傅立叶变换(DFT)

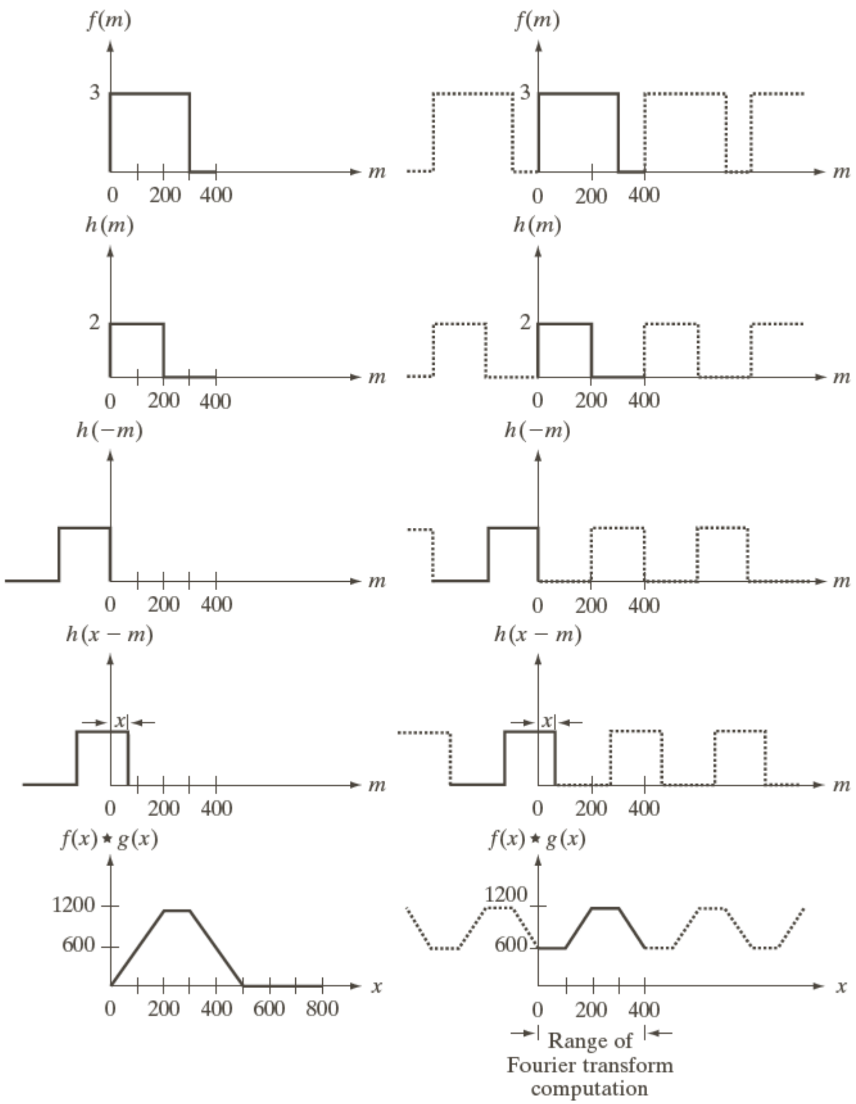


4.1 离散傅立叶变换(DFT)

典型图象的频谱



空域卷积 vs. DFT



a f
b g
c h
d i
e j

FIGURE 4.28 Left column: convolution of two discrete functions obtained using the approach discussed in Section 3.4.2. The result in (e) is correct. Right column: Convolution of the same functions, but taking into account the periodicity implied by the DFT. Note in (j) how data from adjacent periods produce wraparound error, yielding an incorrect convolution result. To obtain the correct result, function padding must be used.

DFT定义及相关表达式小结

Name	Expression(s)
1) Discrete Fourier transform (DFT) of $f(x, y)$	$F(u, v) = \sum_{x=0}^{M-1}\sum_{y=0}^{N-1} f(x, y)\, e^{-j2\pi(ux/M+vy/N)}$
2) Inverse discrete Fourier transform (IDFT) of $F(u, v)$	$f(x, y) = \frac{1}{MN}\sum_{u=0}^{M-1}\sum_{v=0}^{N-1} F(u, v)\, e^{j2\pi(ux/M+vy/N)}$
3) Polar representation	$F(u, v) = \lvert F(u, v)\rvert e^{j\phi(u,v)}$
4) Spectrum	$\lvert F(u, v)\rvert = \left[R^2(u, v) + I^2(u, v)\right]^{1/2}$ $R = \text{Real}(F);\quad I = \text{Imag}(F)$
5) Phase angle	$\phi(u, v) = \tan^{-1}\left[\frac{I(u, v)}{R(u, v)}\right]$
6) Power spectrum	$P(u, v) = \lvert F(u, v)\rvert^2$
7) Average value	$\bar{f}(x, y) = \frac{1}{MN}\sum_{x=0}^{M-1}\sum_{y=0}^{N-1} f(x, y) = \frac{1}{MN}F(0, 0)$
8) Periodicity (k_1 and k_2 are integers)	$F(u, v) = F(u + k_1M, v) = F(u, v + k_2N)$ $= F(u + k_1M, v + k_2N)$ $f(x, y) = f(x + k_1M, y) = f(x, y + k_2N)$ $= f(x + k_1M, y + k_2N)$
9) Convolution	$f(x, y) \bigstar h(x, y) = \sum_{m=0}^{M-1}\sum_{n=0}^{N-1} f(m, n)h(x - m, y - n)$
10) Correlation	$f(x, y) \star h(x, y) = \sum_{m=0}^{M-1}\sum_{n=0}^{N-1} f^*(m, n)h(x + m, y + n)$
11) Separability	The 2-D DFT can be computed by computing 1-D DFT transforms along the rows (columns) of the image, followed by 1-D transforms along the columns (rows) of the result. See Section 4.11.1.
12) Obtaining the inverse Fourier transform using a forward transform algorithm.	$MNf^*(x, y) = \sum_{u=0}^{M-1}\sum_{v=0}^{N-1} F^*(u, v)e^{-j2\pi(ux/M+vy/N)}$ This equation indicates that inputting $F^*(u, v)$ into an algorithm that computes the forward transform (right side of above equation) yields $MNf^*(x, y)$. Taking the complex conjugate and dividing by MN gives the desired inverse. See Section 4.11.2.

第4章　频率域滤波

4.1　离散傅立叶变换(DFT)

4.2　频率域滤波基础

4.3　频率域滤波器平滑图像

4.4　频率域滤波器锐化图像

4.2 频率域滤波基础

频率域滤波基本步骤：

1、 $(-1)^{x+y} \times$ 原图像

2、 $F(u,v)$

3、 $H(u,v) \times F(u,v)$

4、反DEF

5、实部

6、用 $(-1)^{x+y} \times (5)$结果。

被滤波图像$= \wp^{-1}[G(u,v)]$

$$G(u,v) = H(u,v)F(u,v)$$

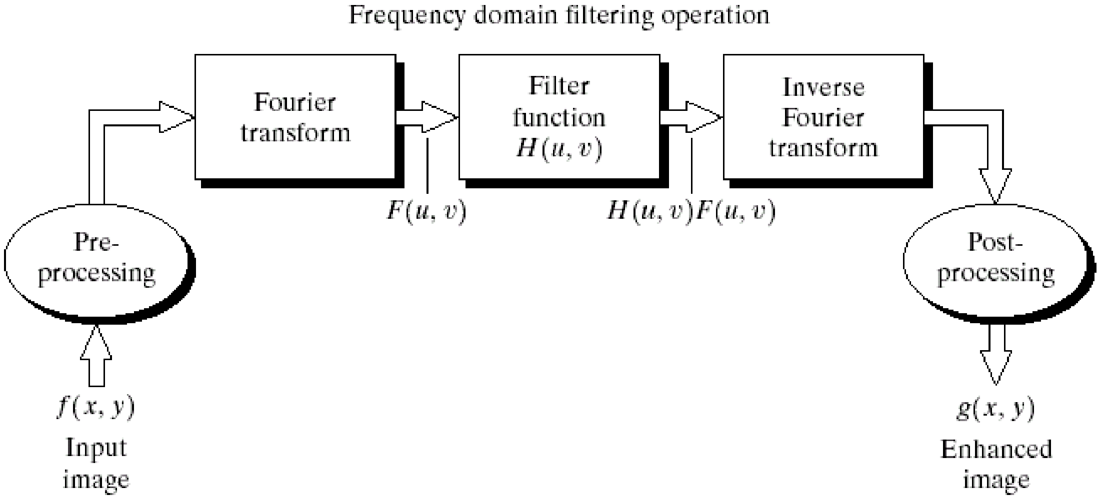
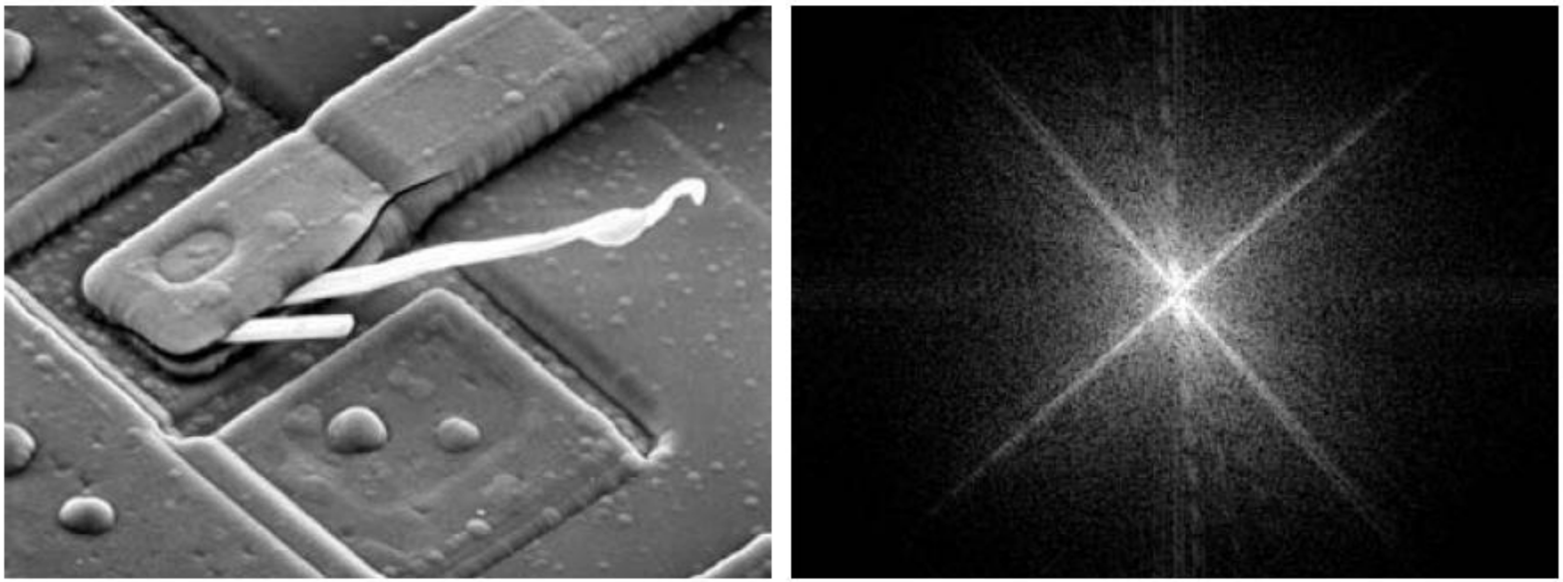


FIGURE 4.5 Basic steps for filtering in the frequency domain.

图像空域与频域的定性分析



a b

FIGURE 4.29 (a) SEM image of a damaged integrated circuit. (b) Fourier spectrum of (a). (Original image courtesy of Dr. J. M. Hudak, Brockhouse Institute for Materials Research, McMaster University, Hamilton, Ontario, Canada.)

4.2 频率域滤波基础

一些基本的滤波器及其性质：

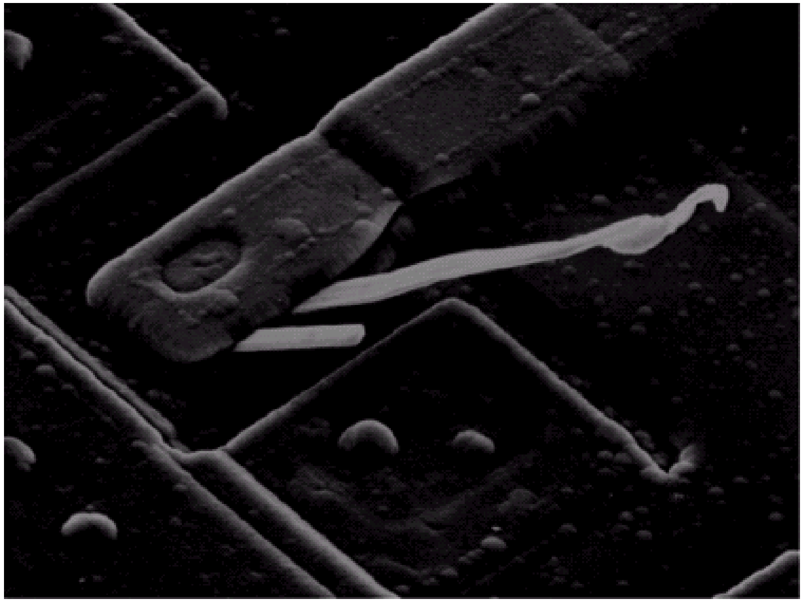
陷波滤波器：$F(0,0) \;\rightarrow\;$ 平均值

如果想使其平均灰度为0，则可：

$$H(u,v)=\begin{cases}0 & (u,v)=(\frac{M}{2},\frac{N}{2})\\ 1 & \text{其它}\end{cases}$$

具体见如下图示：

FIGURE 4.6 Result of filtering the image in Fig. 4.4(a) with a notch filter that set to 0 the $F(0, 0)$ term in the Fourier transform.



4.2 频率域滤波基础

高通与低通滤波

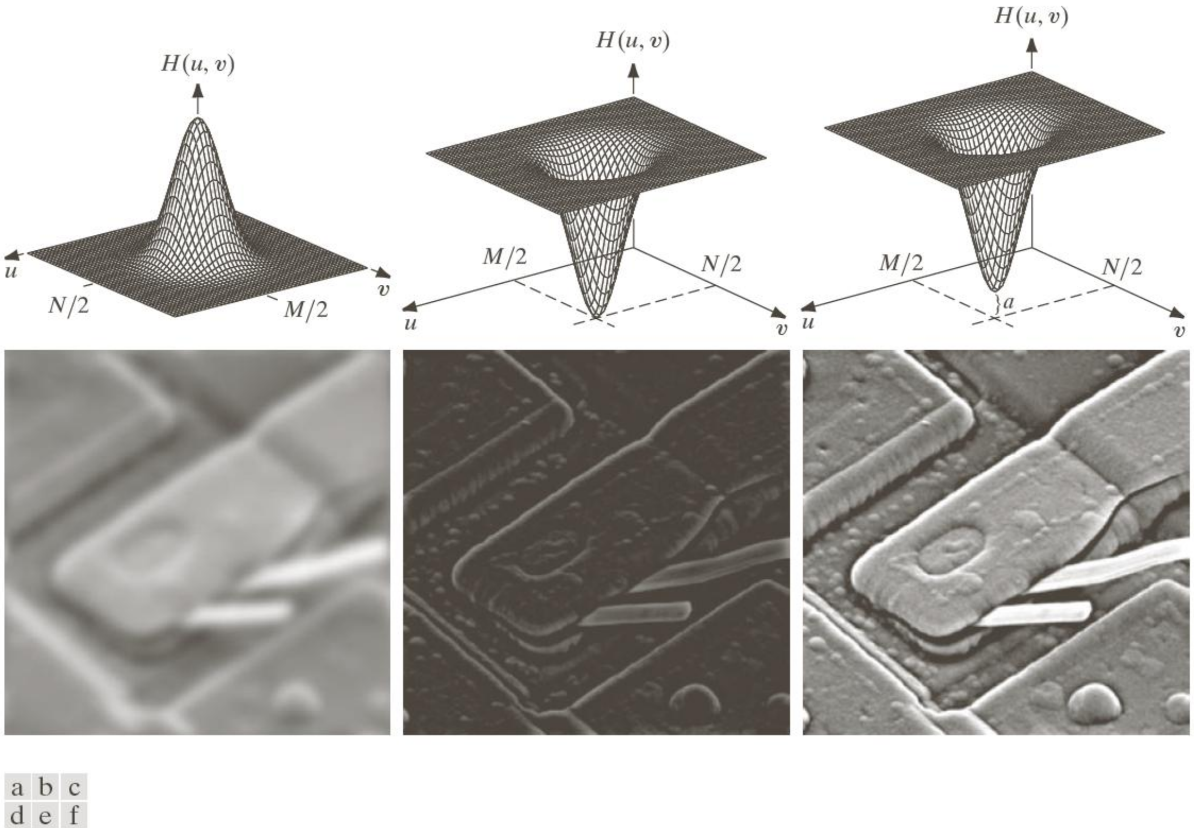
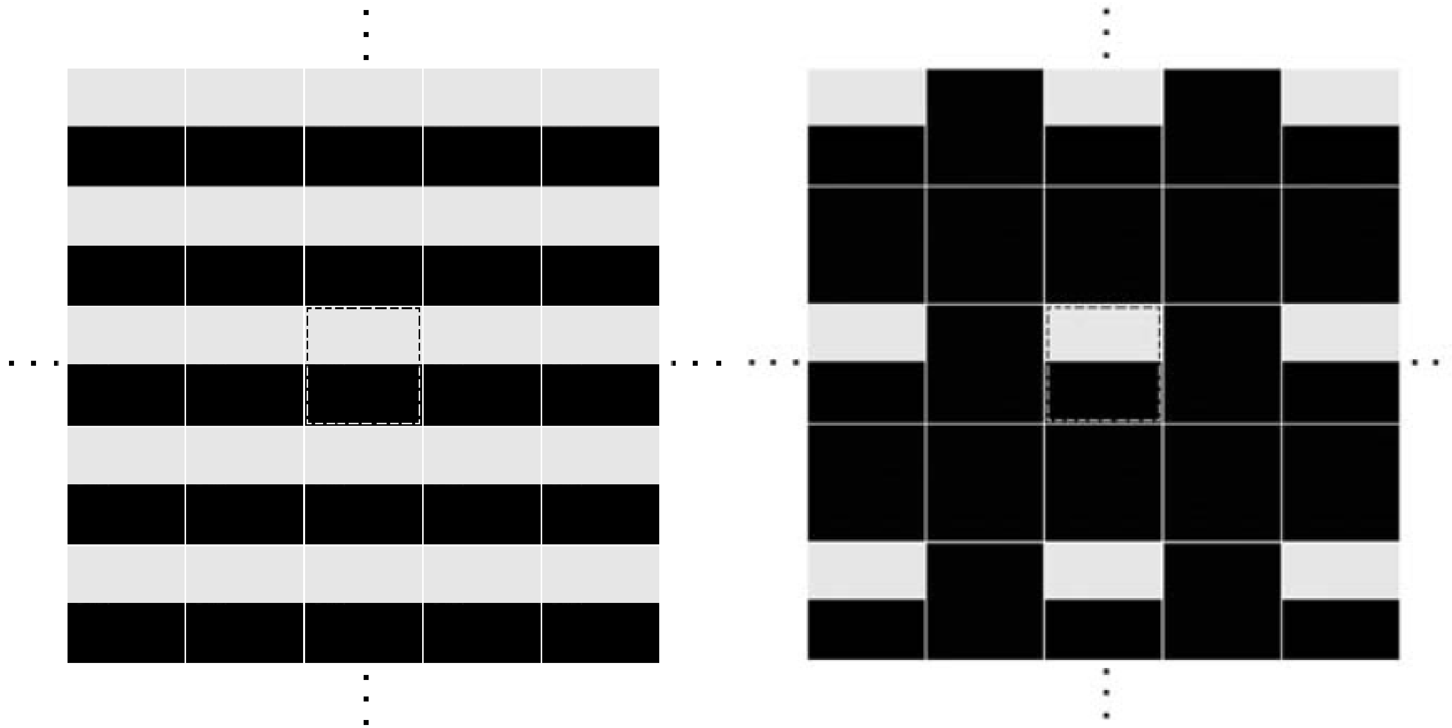


FIGURE 4.31 Top row: frequency domain filters. Bottom row: corresponding filtered images obtained using Eq. (4.7-1). We used $a = 0.85$ in (c) to obtain (f) (the height of the filter itself is 1). Compare (f) with Fig. 4.29(a).

使用DFT时二维图像固有的周期性



a b

FIGURE 4.33 2-D image periodicity inherent in using the DFT. (a) Periodicity without image padding. (b) Periodicity after padding with 0s (black). The dashed areas in the center correspond to the image in Fig. 4.32(a). (The thin white lines in both images are superimposed for clarity; they are not part of the data.)

空间和频率域滤波间的对应

空间域滤波和频率域滤波之间的对应关系

基本联系就是卷积定理

离散卷积：

$$f(x,y)*h(x,y)=\frac{1}{MN}\sum_{m=0}^{M-1}\sum_{n=0}^{N-1}f(m,n)h(x-m,y-n)$$

$$f(x,y)*h(x,y)\Leftrightarrow F(u,v)H(u,v)$$

$$f(x,y)h(x,y)\Leftrightarrow F(u,v)*H(u,v)$$

在 (x_0,y_0) 处，强度为A的冲激函数 $A\delta(x-x_0,y-y_0)$,

并且：

$$\sum_{x=0}^{M-1}\sum_{y=0}^{N-1}S(x,y)A\delta(x-x_0,y-y_0)=AS(x_0,y_0)$$

把 $A\delta(x-x_0,y-y_0)$ 看作 $-M\times N$ 大小的图像，在(x_0,y_0)有值，其它全为0。

$$\sum_{x=0}^{M-1}\sum_{y=0}^{N-1}S(x,y)\delta(x,y)=S(0,0)$$

空间和频率域滤波间的对应

单位冲激的傅氏变换：

$$F(u,v)=\frac{1}{MN}\sum_{x=0}^{M-1}\sum_{y=0}^{N-1}\delta(x,y)\varsigma^{-2\pi(ux/M+vy/N)}$$

$$=\frac{1}{MN}$$

可得到： $f(x,y)=\delta(x,y),$

令

$$f(x,y)*h(x,y)=\frac{1}{MN}\sum_{m=0}^{M-1}\sum_{n=0}^{N-1}\delta(m,n)h(x-m,y-n)$$

$$=\frac{1}{MN}h(x,y)$$

可以输入 $\delta(x,y)$ 出 $h(x,y)$

结论：空间域和频率域中的滤波器组成了一个傅立叶变换对。一般空间域更适合使用小的滤波器。

空间和频率域滤波间的对应

空间域滤波和频率域滤波之间的对应关系

例如：高斯滤波函数

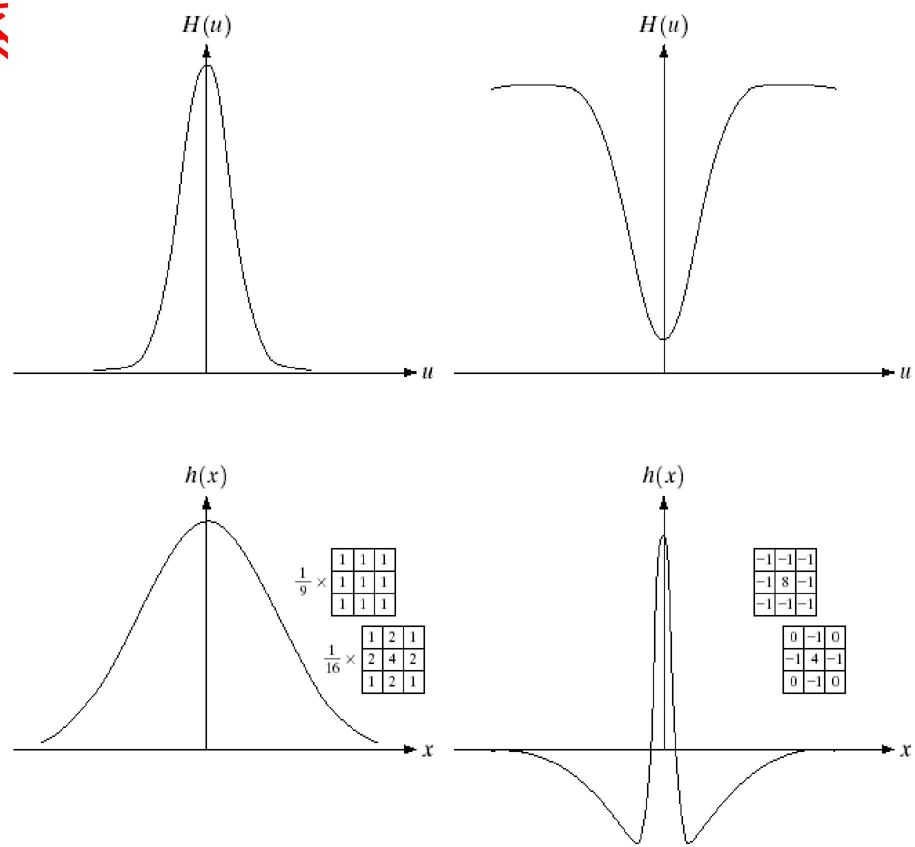
$$H(u)=Ae^{-u^2/2\sigma^2}$$

σ 为标准差：

$$h(x)=\sqrt{2\pi}\sigma Ae^{-2\pi^2\sigma^2x^2}$$

傅立叶变换及其反变换因为实数。

曲线形状如右图所示：



a b
c d

FIGURE 4.9 (a) Gaussian frequency domain lowpass filter. (b) Gaussian frequency domain highpass filter. (c) Corresponding lowpass spatial filter. (d) Corresponding highpass spatial filter. The masks shown are used in Chapter 3 for lowpass and highpass filtering.

第4章　频率域滤波

4.1　离散傅立叶变换(DFT)

4.2　频率域滤波基础

4.3　频率域滤波器平滑图像

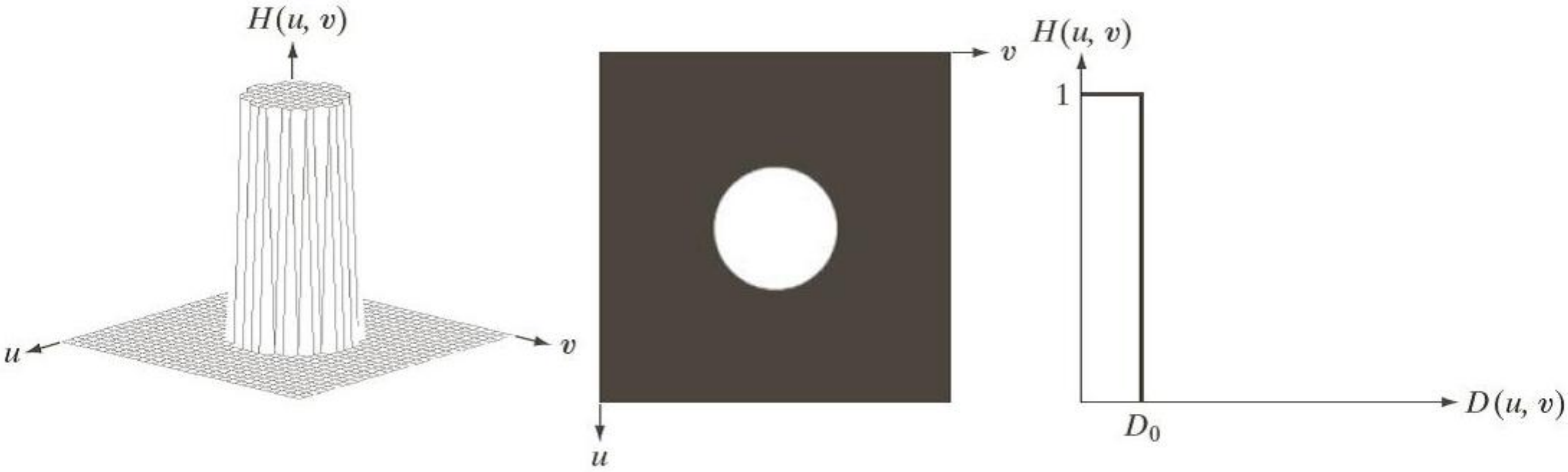
4.4　频率域滤波器锐化图像

4.3 频率域滤波平滑

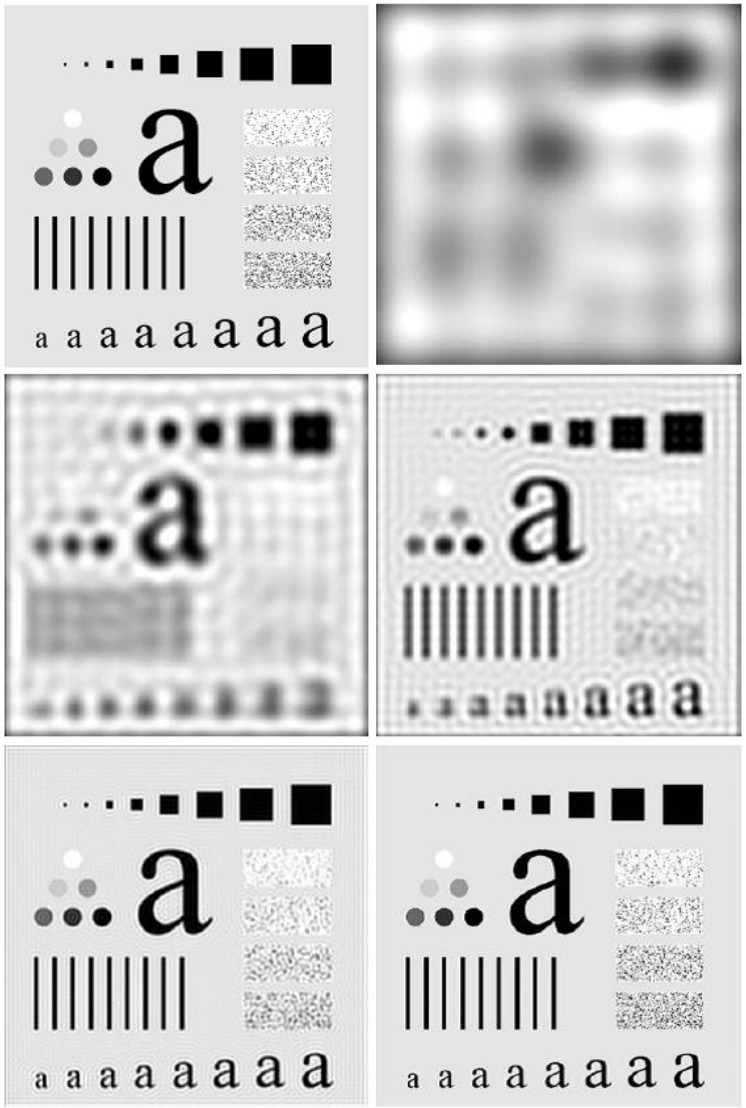
理想低通滤波器

$$H(u,v)=\begin{cases}1 & D(u,v)\le D_0\\ 0 & D(u,v)>D_0\end{cases}$$

其中，D(u,v)是频率域中点(u,v)与频率矩形中心的距离



理想低通滤波器示例

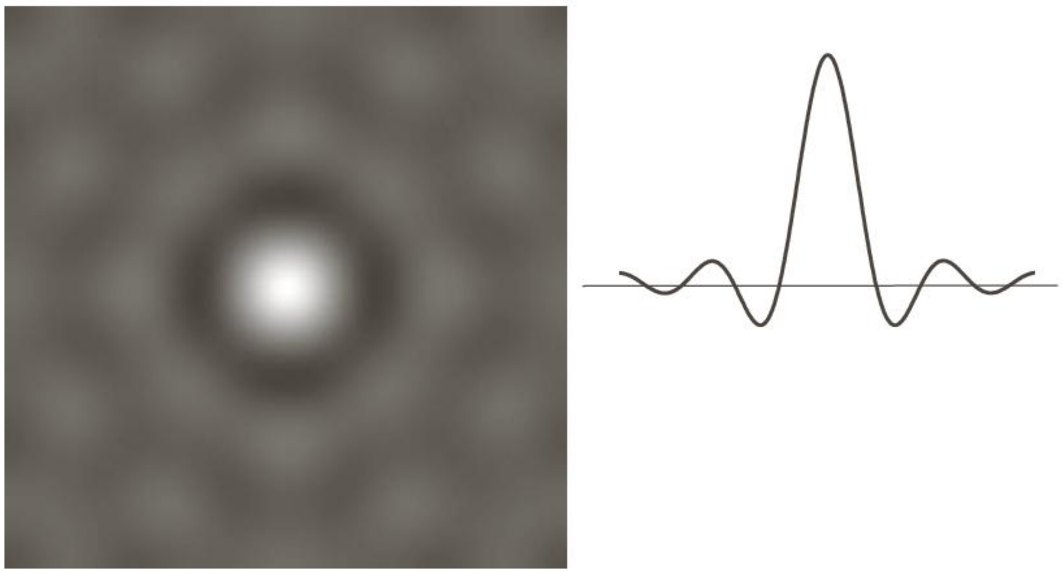


a b
c d
e f

(a) 原图

(b)-(f) 使用理想低通滤波器，截止频率设置10,30,60,160和460。这些滤波器移除的功率分别为总功率的13%，6.9%，4.3%，2.2%和 0.8%。

振铃效应解释



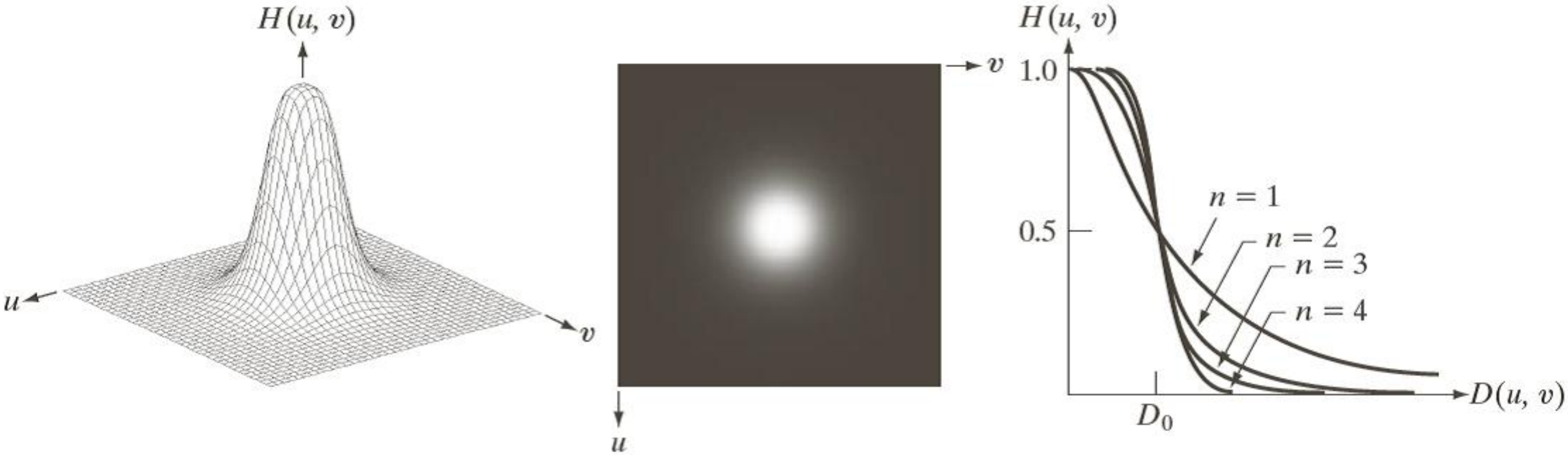
a b

FIGURE 4.43 (a) Representation in the spatial domain of an ILPF of radius 5 and size 1000 × 1000. (b) Intensity profile of a horizontal line passing through the center of the image.

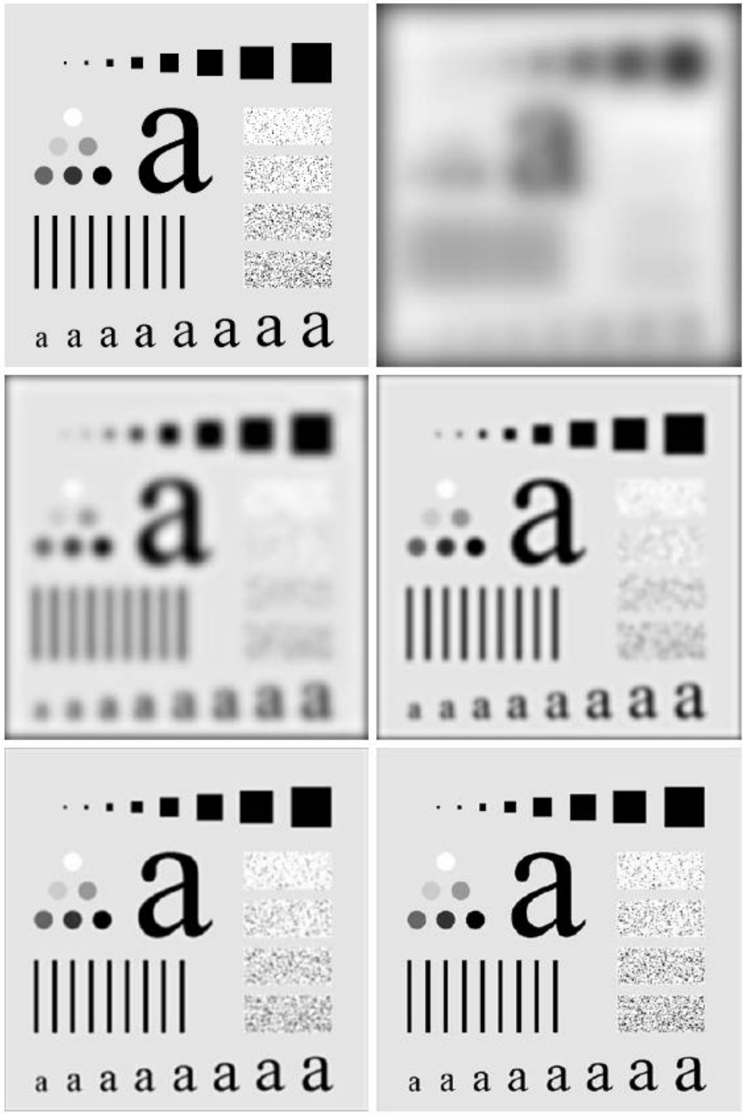
4.3 频率域滤波平滑

- n 阶布特沃斯低通滤波器

$$H(u,v)=\frac{1}{1+[D(u,v)/D_0]^{2n}}$$



布特沃斯滤波器示例



a	b
c	d
e	f

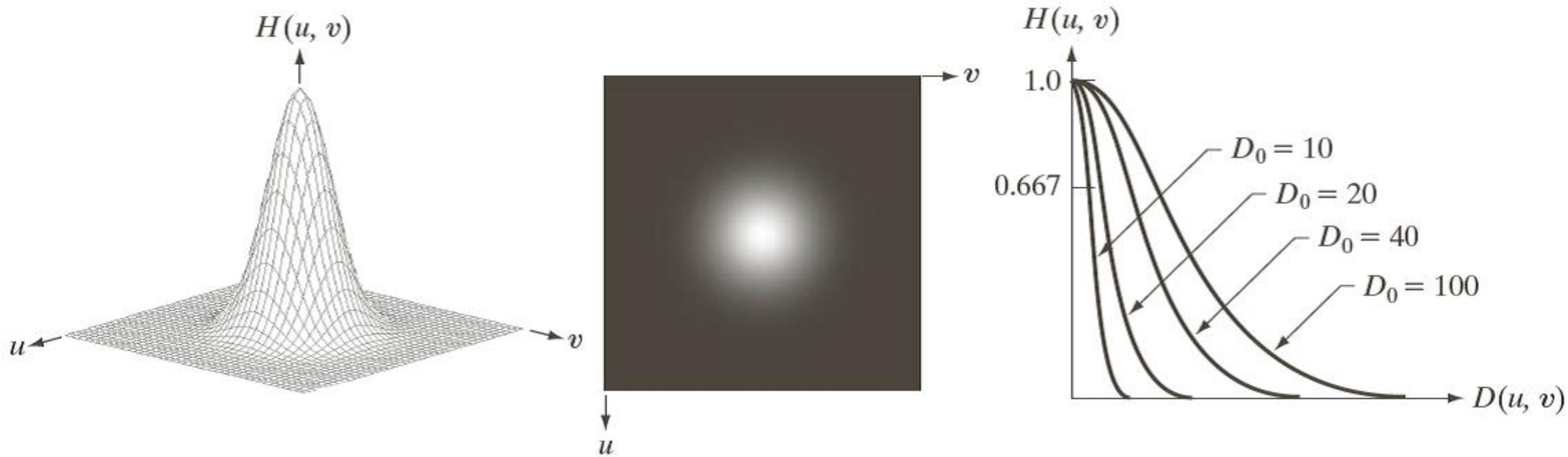
(a) 原图

(b)-(f) 使用二阶布特沃斯低通滤波器的结果，截止频率仍为10，30，60，160和460。

4.3 频率域滤波平滑

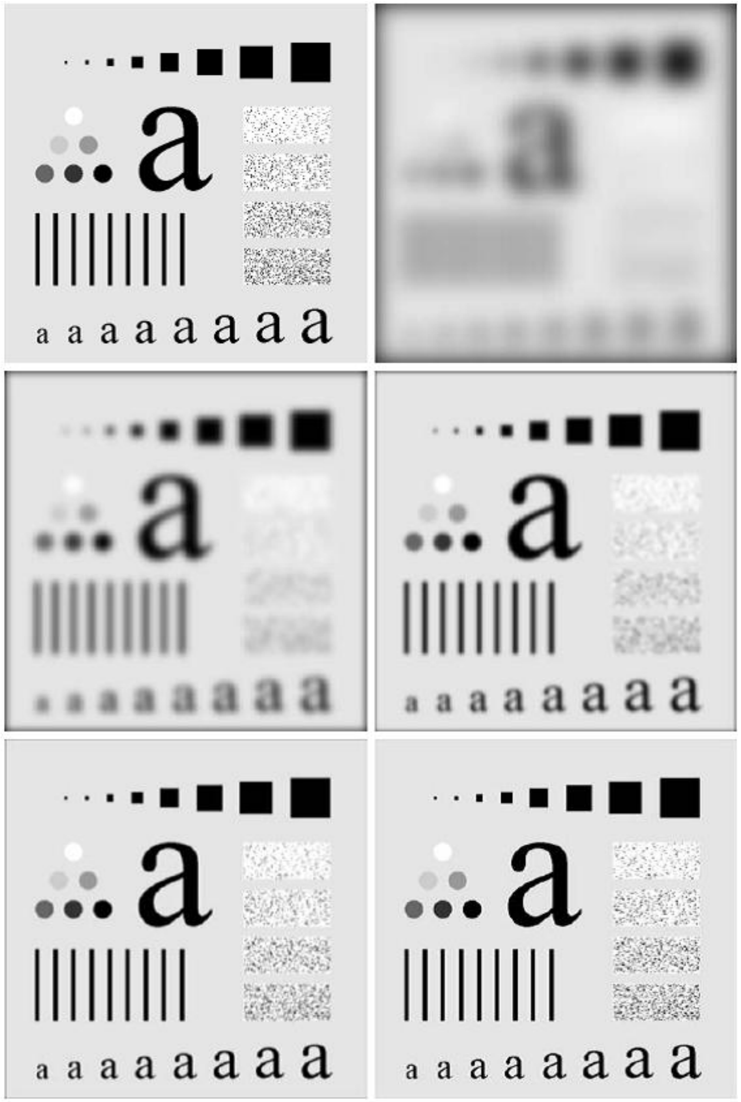
高斯低通滤波器

$$H(u,v)=e^{-D^2(u,v)/2D_0^2}$$



高斯低通滤波器（GLPF）的傅里叶反变换也是高斯的，因此通过上式的IDFT得到的空间高斯滤波器没有振铃。

高斯低通滤波器示例



a b
c d
e f

(a) 原图

(b)-(f) 使用高斯低通滤波器的结果，截止频率仍为10，30，60，160和460。

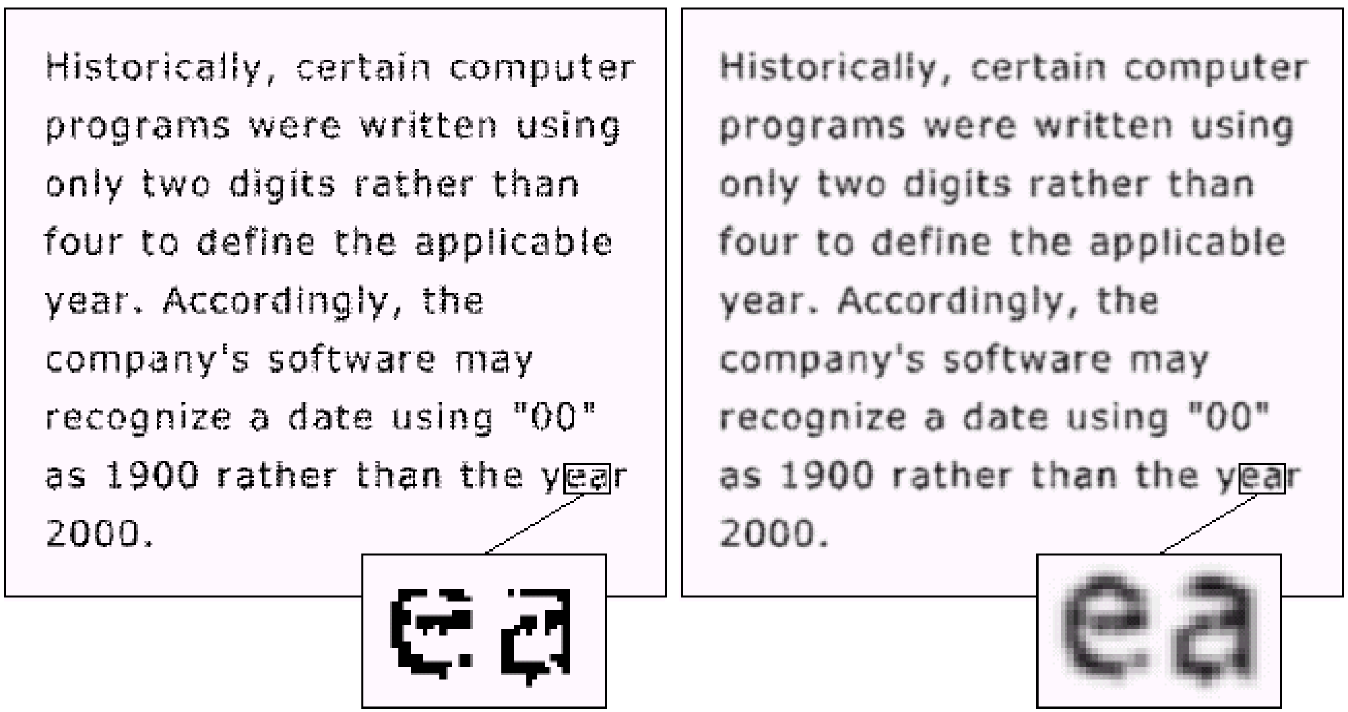
低通滤波的其他例子

1. 字符识别：

下图：断裂现象

a b

FIGURE 4.19 (a) Sample text of poor resolution (note broken characters in magnified view). (b) Result of filtering with a GLPF (broken character segments were joined).



低通滤波的其他例子

2. 印刷和出版业：预处理

下图：减少面部细纹

a b c

FIGURE 4.20 (a) Original image (1028 × 732 pixels). (b) Result of filtering with a GLPF with $D_0 = 100$. (c) Result of filtering with a GLPF with $D_0 = 80$. Note reduction in skin fine lines in the magnified sections of (b) and (c).

低通滤波的其他例子

3. 卫星和航空图像：

下图：墨西哥湾和佛罗里达图像存在“扫描线”（用高斯低通来处理）

a b c

FIGURE 4.21 (a) Image showing prominent scan lines. (b) Result of using a GLPF with $D_0 = 30$. (c) Result of using a GLPF with $D_0 = 10$. (Original image courtesy of NOAA.)

第4章　频率域滤波

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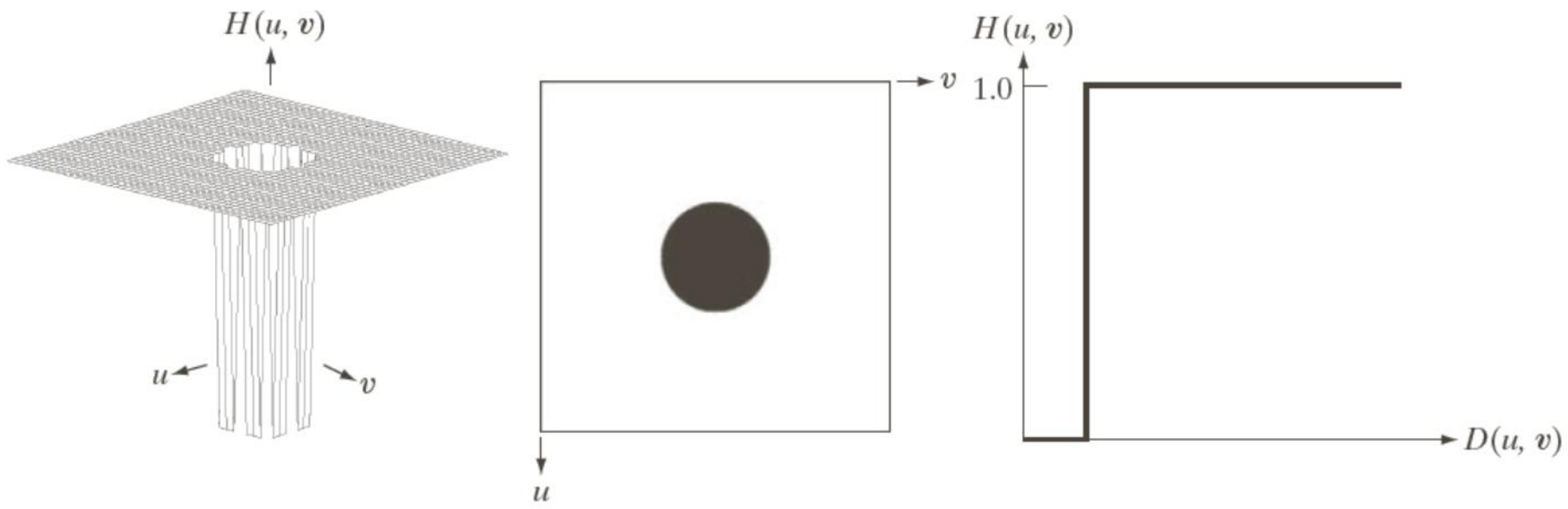
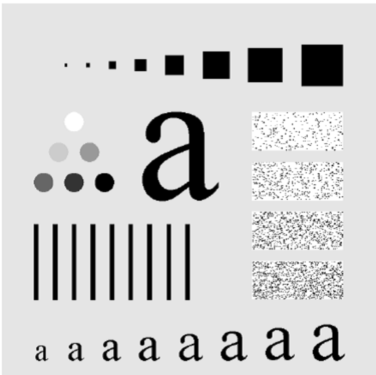
4.4 频率域滤波锐化

理想高通滤波器

$$H(u,v)=\begin{cases}0 & D(u,v)\le D_0\\ 1 & D(u,v)>D_0\end{cases}$$



理想高通滤波器示例



原图

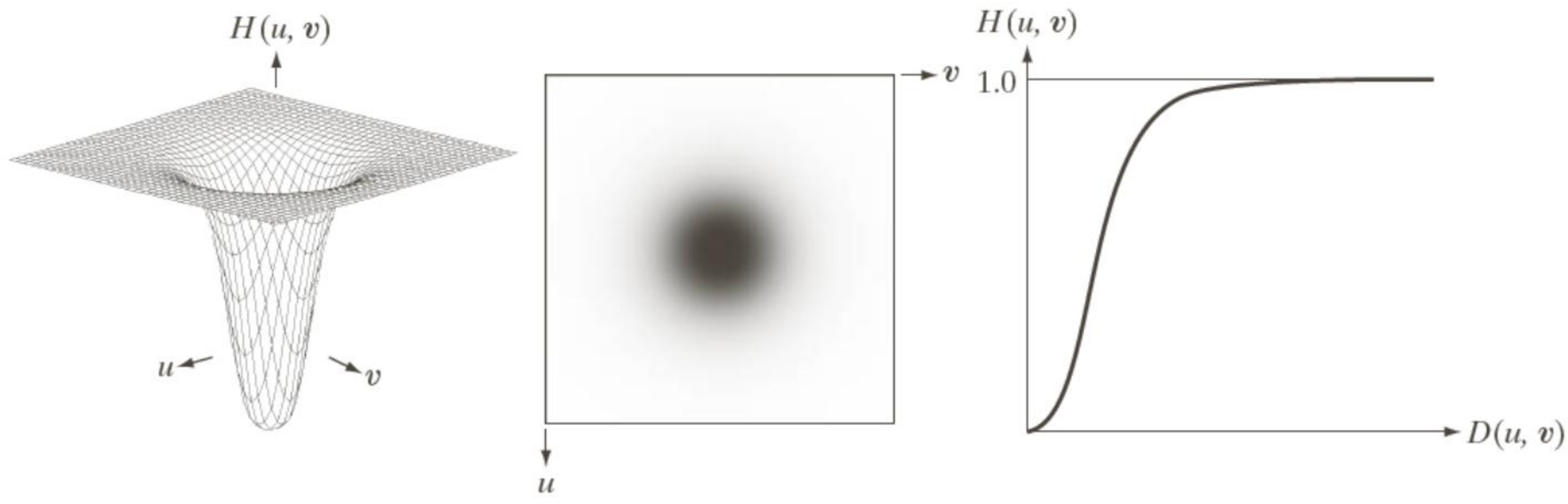
a b c

$D_0 = 30, 60, 100$ 的理想高通滤波器结果

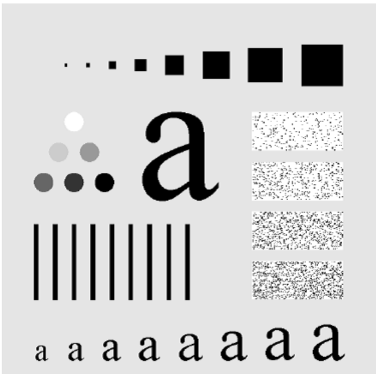
4.4 频率域滤波锐化

布特沃斯高通滤波器

$$H(u,v)=\frac{1}{1+[D_0/D(u,v)]^{2n}}$$



布特沃斯高通滤波器示例



原图

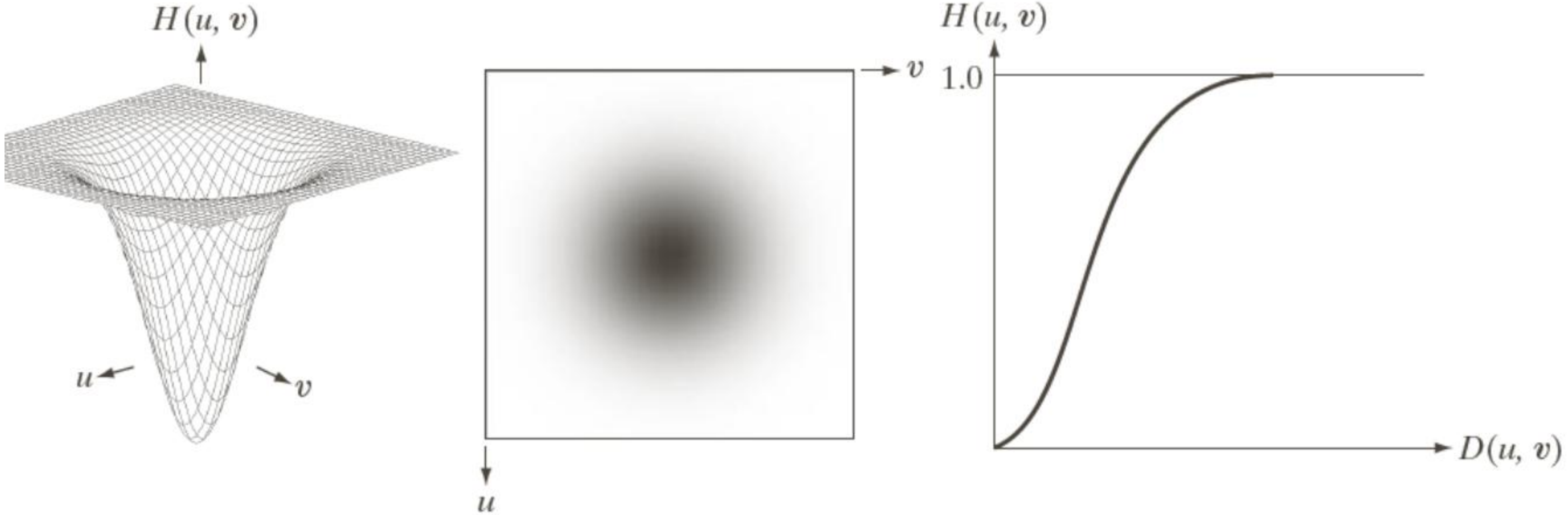
a b c

$D_0 = 30, 60, 100$ 的2阶布特沃斯高通滤波器结果

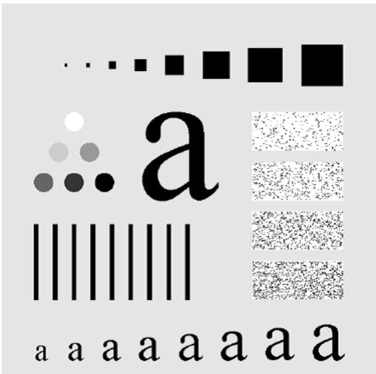
4.4 频率域滤波锐化

高斯高通滤波器

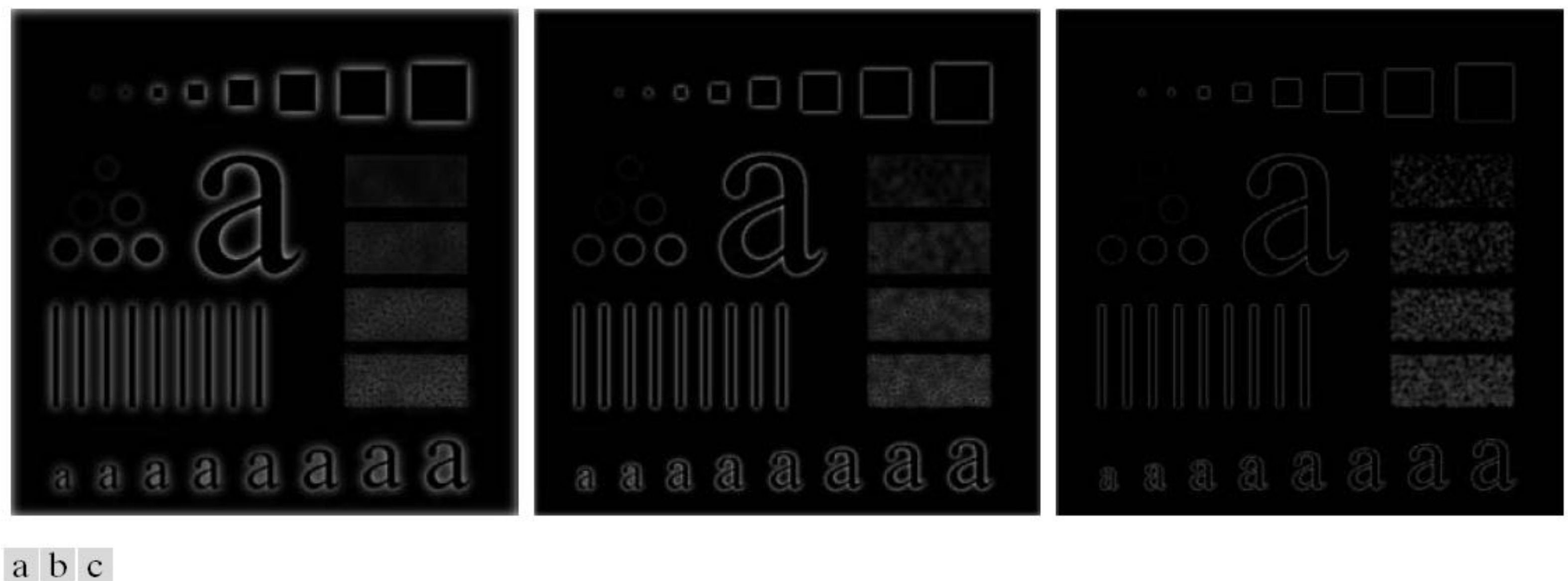
$$H(u,v) = 1 - e^{-D^2(u,v)/2D_0^2}$$



高斯高通滤波器示例



原图



$D_0 = 30, 60, 100$ 的高斯高通滤波器结果

4.4 频率域滤波锐化：同态滤波

- 一幅图像$f(x,y)$可以表示为照射分量和反射分量的乘积。

$$f(x,y) = i(x,y)r(x,y)$$

- 然而上式不能用来直接对两部分分量分别进行操作，因为两个函数乘积的傅立叶变换是不可分的。

$$\mathrm{F}\{f(x,y)\} \neq \mathrm{F}\{i(x,y)\}\mathrm{F}\{r(x,y)\}$$

- 我们对图像函数两边取对数，则可以将两个分量分开。

$$z(x,y) = \mathbf{ln}\, f(x,y) = \mathbf{ln}\, i(x,y) + \mathbf{ln}\, r(x,y)$$

$$\mathrm{F}\{\mathbf{ln}\, f(x,y)\} = \mathrm{F}\{\mathbf{ln}\, i(x,y)\} + \mathrm{F}\{\mathbf{ln}\, r(x,y)\}$$

同态滤波步骤

1. 两边取对数： $f(x,y) = \ln i(x,y) + \ln r(x,y)$
2. 两边取付氏变换： $F(u,v) = I(u,v) + R(u,v)$
3. 用一频域函数 $H(u,v)$处理 $F(u,v)$：

$$H(u,v)F(u,v) = H(u,v)\,I(u,v) + H(u,v)\,R(u,v)$$

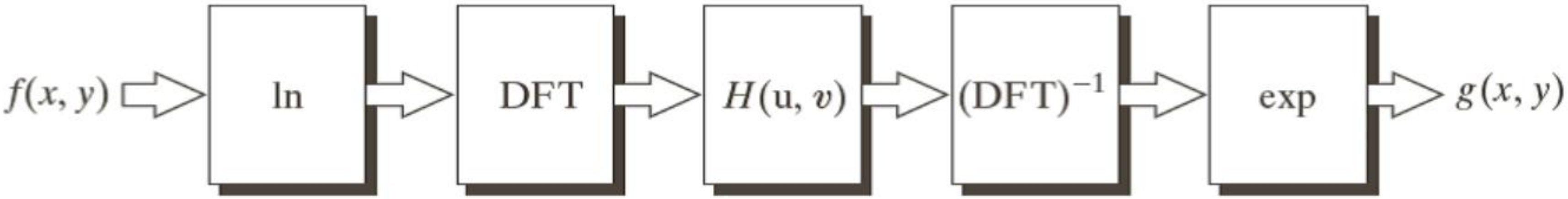
4. 反变换到空域：

$$s(x,y) = i'(x,y) + r'(x,y)$$

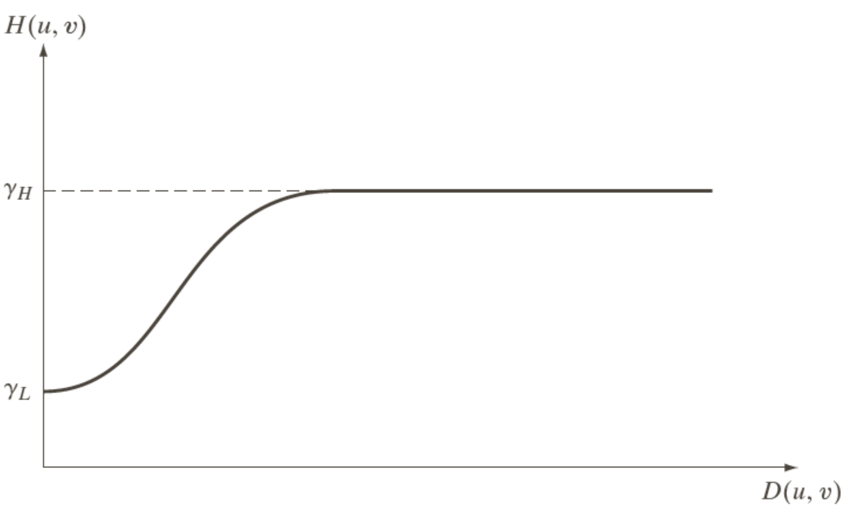
5. 两边取指数：

$$g(x,y) = exp\{i'(x,y)\} + exp\{r'(x,y)\}$$

同态滤波步骤及滤波器径向剖面图



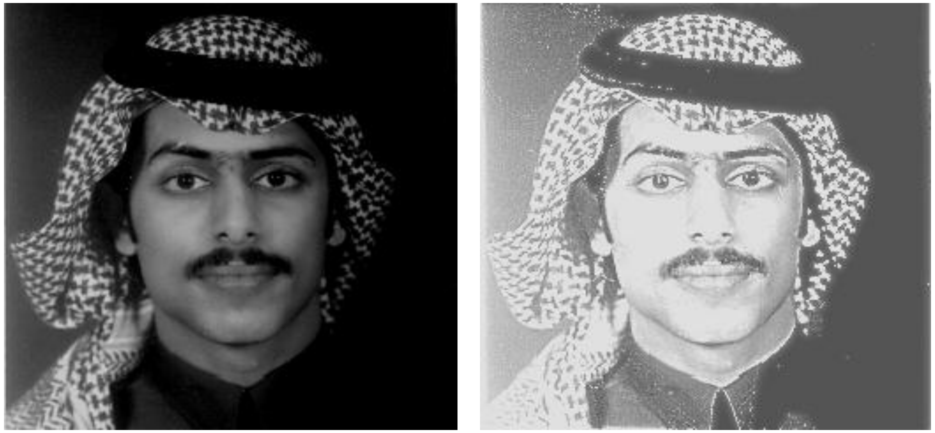
同态滤波器函数径向剖面图：滤波器函数趋向于衰减低频的贡献，而增强高频的贡献。

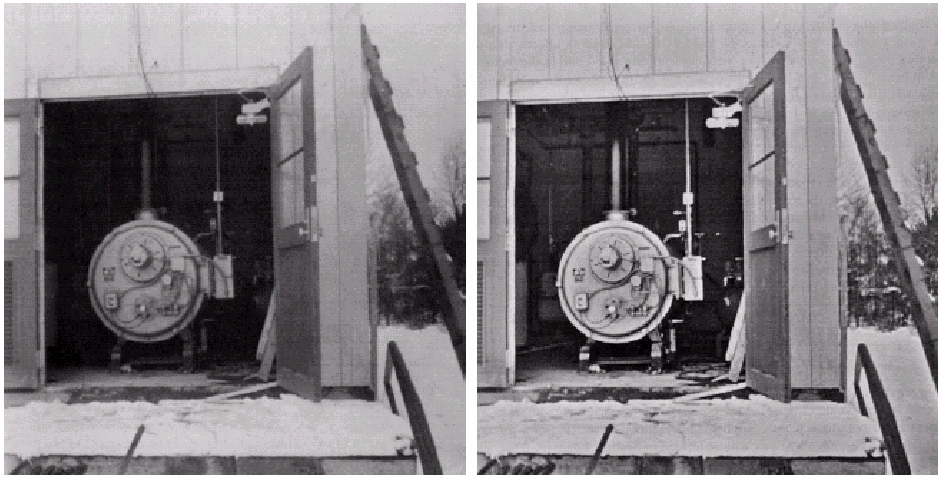


同态滤波示例

- **特点：**能消除乘性噪声，能同时压缩图象的整体动态范围和增加图象中相邻区域间的对比度

- **示例一：**





同态滤波示例

同态滤波示例二：