



数字图象处理

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第5章 图像复原与重建

- 数字图象恢复与增强的目的类似，也是旨在改善图象的质量.
- 但恢复是力求保持图象的本来面目，即以保真原则为其前提，这是区别于增强的基本不同点。因而恢复时要了解图象质量下降的物理过程，找出或估计其物理模型。恢复的过程就是沿着质量下降的逆过程来重现原始图象.



第5章 图像复原与重建

- 5.1 图像退化/复原过程的模型
- 5.2 噪声模型
- 5.3 只存在噪声的复原——空间滤波
- 5.4 用频率域消除周期噪声
- 5.5 线性、位置不变的退化
- 5.6 估计退化函数
- 5.7 逆滤波
- 5.8 最小均方误差（维纳）滤波



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5.8 最小均方误差（维纳）滤波

5.1 图像退化/复原过程的模型

□ 线性位移不变系统，加性噪声

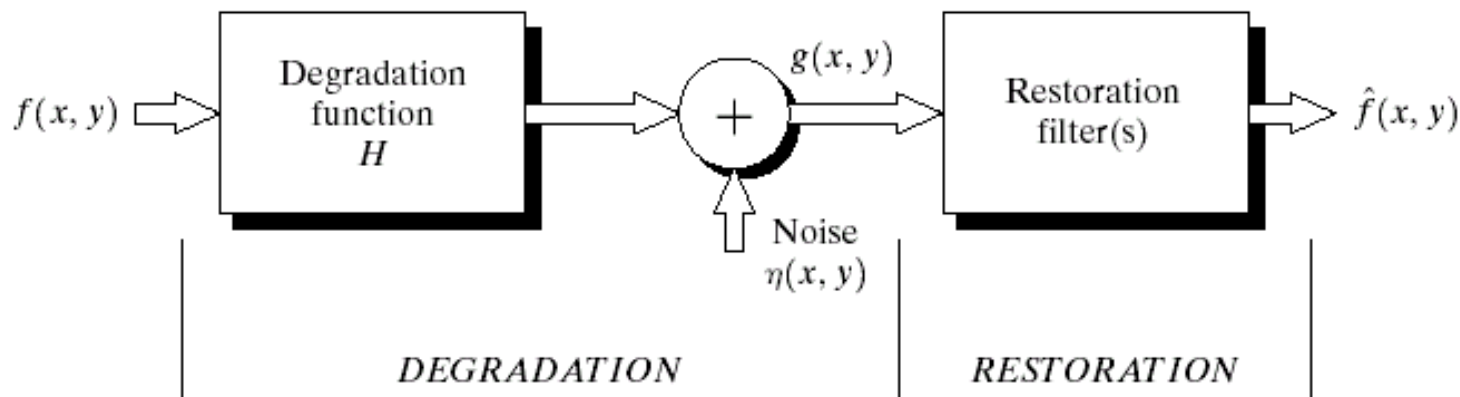


FIGURE 5.1 A model of the image degradation/restoration process.

$$g(x, y) = h(x, y) * f(x, y) + \eta(x, y)$$

$$G(u, v) = H(u, v)F(u, v) + N(u, v)$$



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5.2 噪声模型

□ 噪声的产生

- 图像获取：CCD相机
- 图像传输：无线传输

□ 噪声的空域特性和频域特性

- 空域特性：噪声的直方图分布
- 频域特性：噪声在Fourier频谱上的分布

□ 几种典型的噪声模型

- 高斯噪声 (Gaussian noise), 瑞利噪声(Rayleigh noise), 厄兰噪声(Erlang noise), 指数噪声(Exponential noise), 均匀噪声(Uniform noise), 椒盐噪声(salt-and-pepper),

几种典型的噪声模型

$$p(z) = \frac{1}{\sqrt{2\pi}\sigma} e^{-(z-\mu)^2/2\sigma^2} \quad (\text{高斯噪声})$$

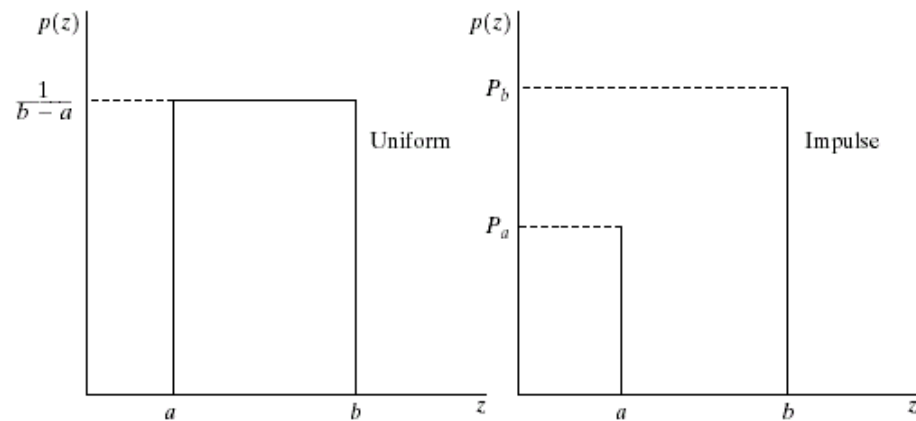
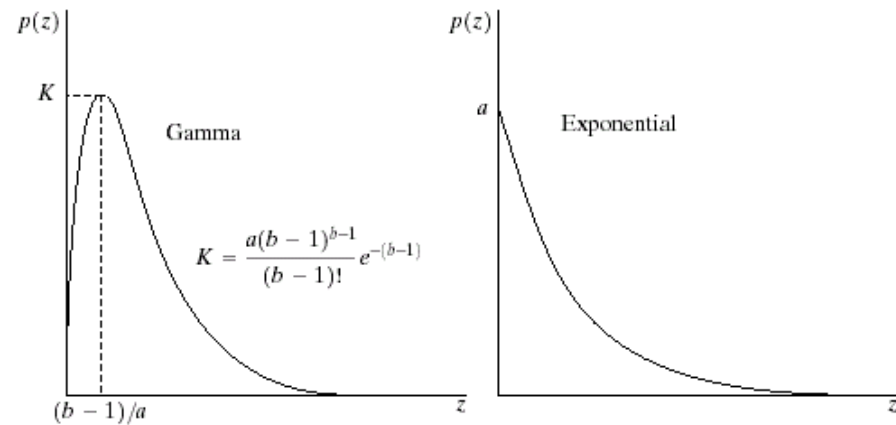
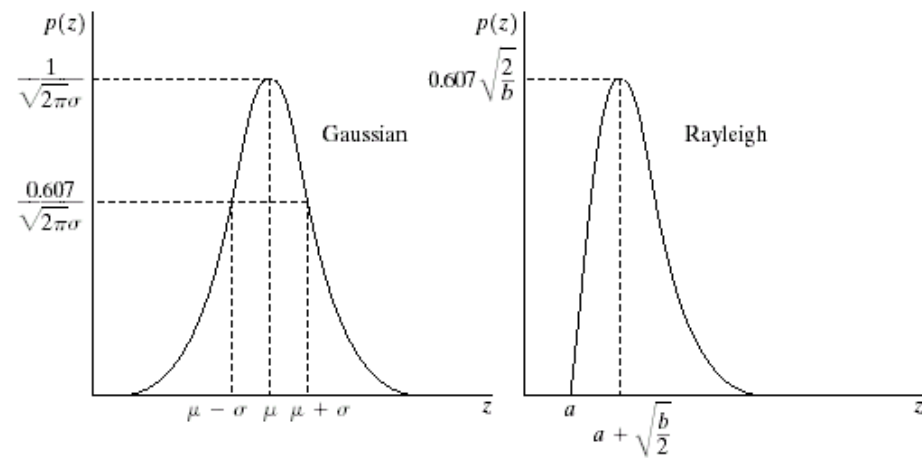
$$p(z) = \begin{cases} \frac{2}{b} (z-a) e^{-(z-a)^2/b} & \text{for } z \geq a \\ 0 & \text{for } z < a \end{cases} \quad (\text{瑞利噪声})$$

$$p(z) = \begin{cases} \frac{a^b z^{b-1}}{(b-1)!} e^{-az} & \text{for } z \geq 0 \\ 0 & \text{for } z < 0 \end{cases} \quad (\text{厄兰噪声})$$

$$p(z) = \begin{cases} a e^{-az} & \text{for } z \geq 0 \\ 0 & \text{for } z < 0 \end{cases} \quad (\text{指数噪声})$$

$$p(z) = \begin{cases} \frac{1}{b-a} & \text{if } a \leq z \leq b \\ 0 & \text{otherwise} \end{cases} \quad (\text{均匀噪声})$$

$$p(z) = \begin{cases} p_a & \text{for } z = a \\ p_b & \text{for } z = b \\ 0 & \text{otherwise} \end{cases} \quad (\text{椒盐噪声})$$



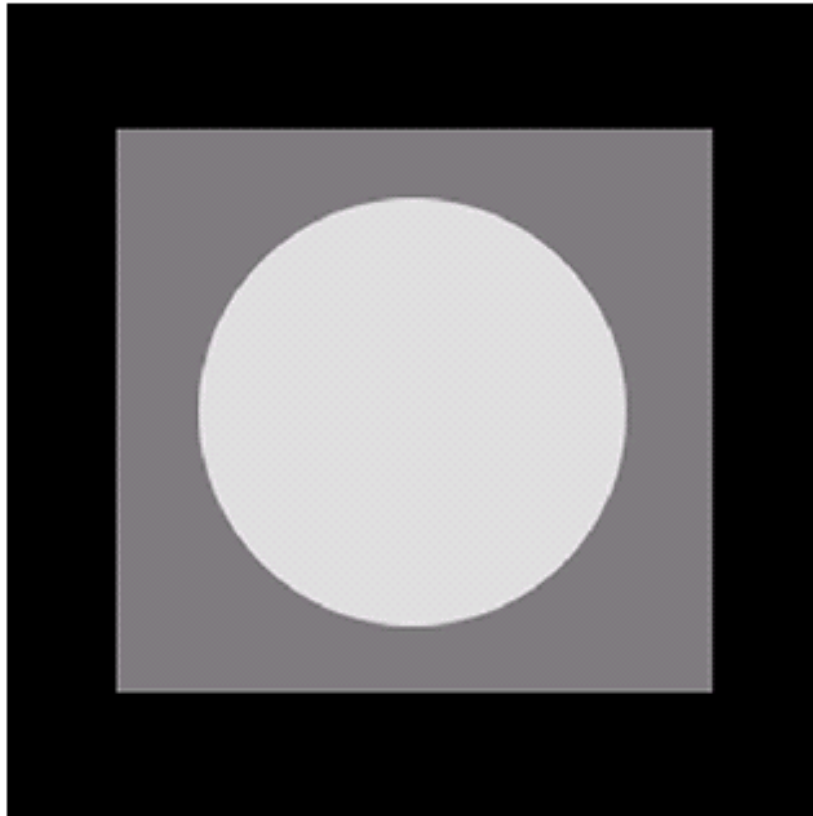
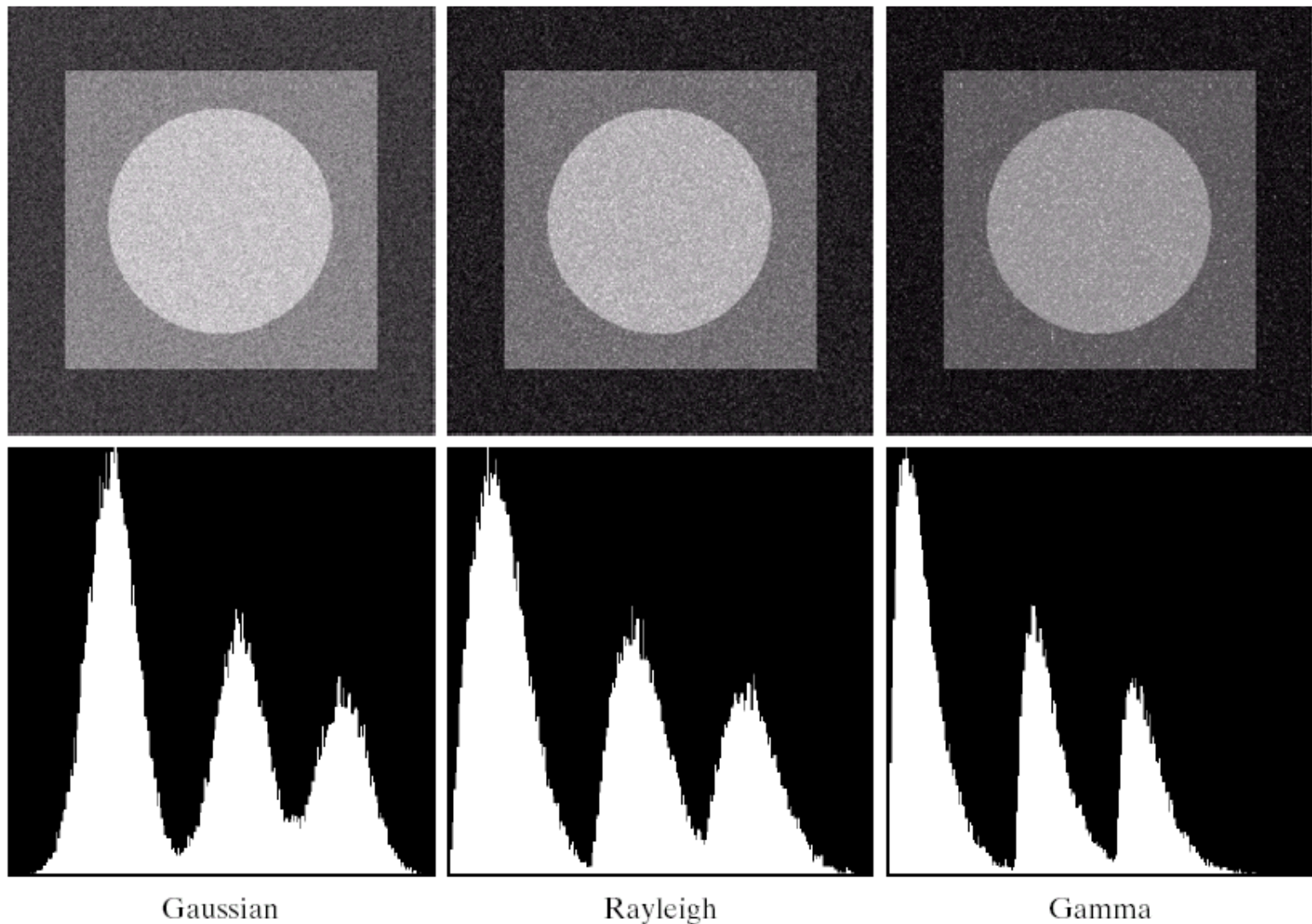
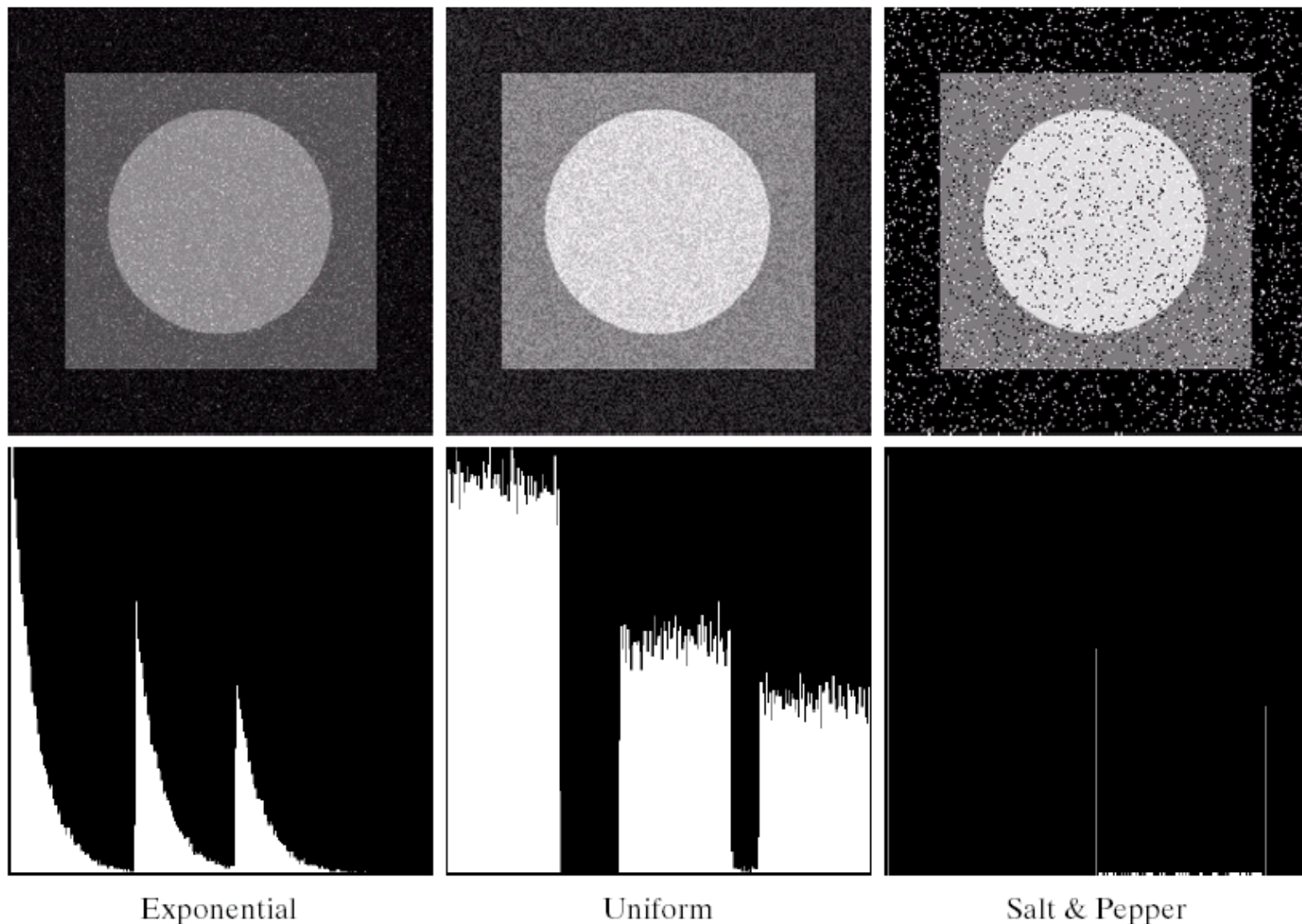


FIGURE 5.3 Test pattern used to illustrate the characteristics of the noise PDFs shown in Fig. 5.2.



a	b	c
d	e	f

FIGURE 5.4 Images and histograms resulting from adding Gaussian, Rayleigh, and gamma noise to the image in Fig. 5.3.



g h i
j k l

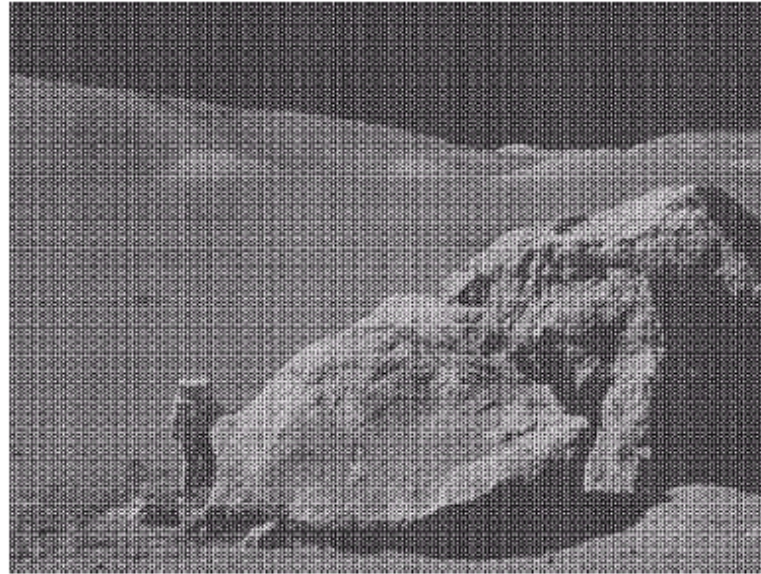
FIGURE 5.4 (Continued) Images and histograms resulting from adding exponential, uniform, and impulse noise to the image in Fig. 5.3.

周期噪声(Periodic Noise)

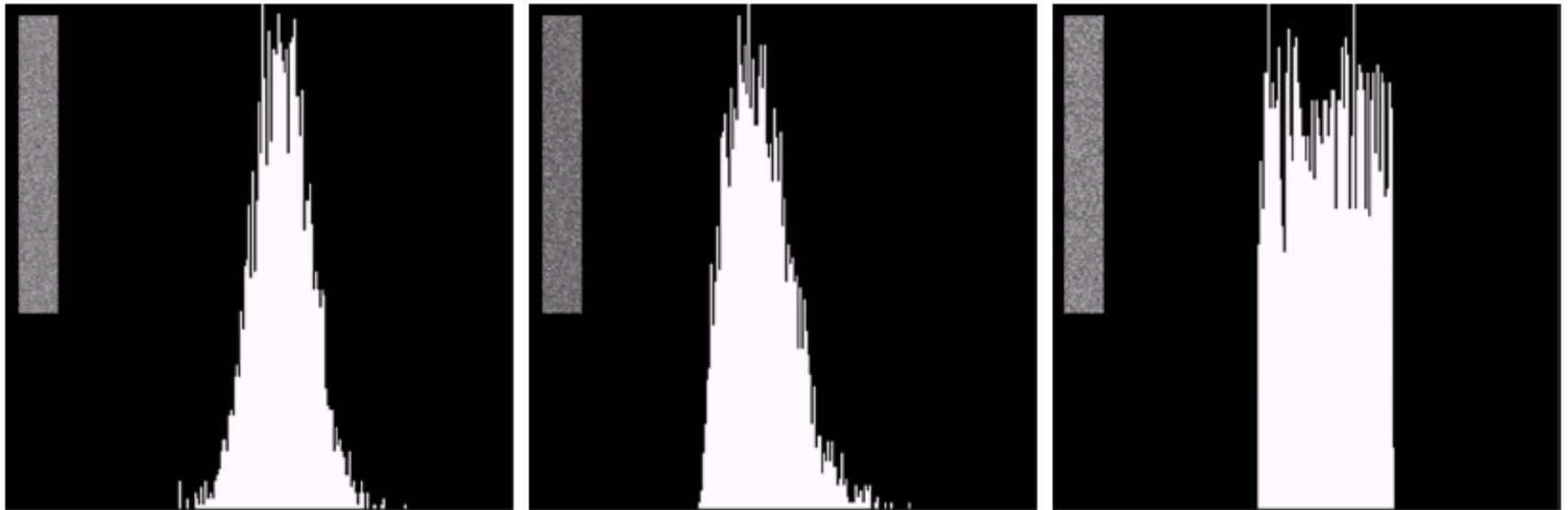
a
b

FIGURE 5.5

(a) Image corrupted by sinusoidal noise.
(b) Spectrum (each pair of conjugate impulses corresponds to one sine wave).
(Original image courtesy of NASA.)



估计噪声参数



a b c

FIGURE 5.6 Histograms computed using small strips (shown as inserts) from (a) the Gaussian, (b) the Rayleigh, and (c) the uniform noisy images in Fig. 5.4.



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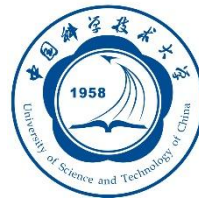
5.4 用频率域消除周期噪声

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5.6 估计退化函数

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5.8 最小均方误差（维纳）滤波



5.3 只存在噪声的复原——空间滤波

- 基于空域滤波的方法仅针对加性噪声
- 与空域增强原理相同
- 几种常用滤波器
 - 均值滤波器 (Mean Filters)
 - 次序统计滤波器 (Order Statistic Filters)
 - 自适应滤波器 (Adaptive Filters)

几种常用滤波器

- 算术平均滤波器(Arithmetic mean filters)

$$\hat{f}(x, y) = \frac{1}{mn} \sum_{(s,t) \in S_{xy}} g(s, t)$$

- 几何平均滤波器(Geometric mean filter)

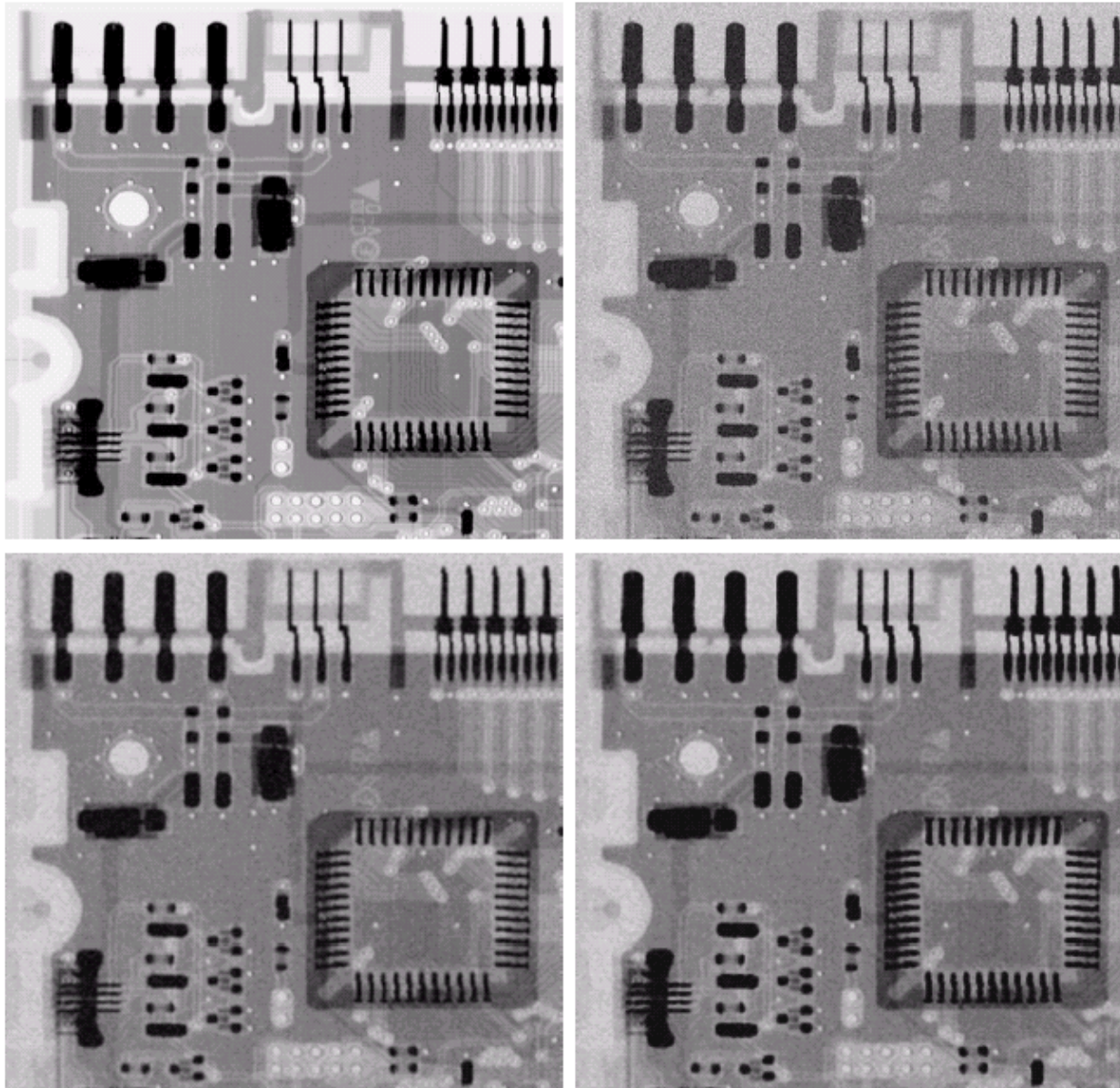
$$\hat{f}(x, y) = \left[\prod_{(s,t) \in S_{xy}} g(s, t) \right]^{\frac{1}{mn}}$$

- 调和平均滤波器(Harmonic mean filter)

$$\hat{f}(x, y) = \frac{mn}{\sum_{(s,t) \in S_{xy}} \frac{1}{g(s, t)}}$$

- 反调和平均滤波器(Contraharmonic filter)

$$\hat{f}(x, y) = \frac{\sum_{(s,t) \in S_{xy}} g(s, t)^{Q+1}}{\sum_{(s,t) \in S_{xy}} g(s, t)^Q}$$



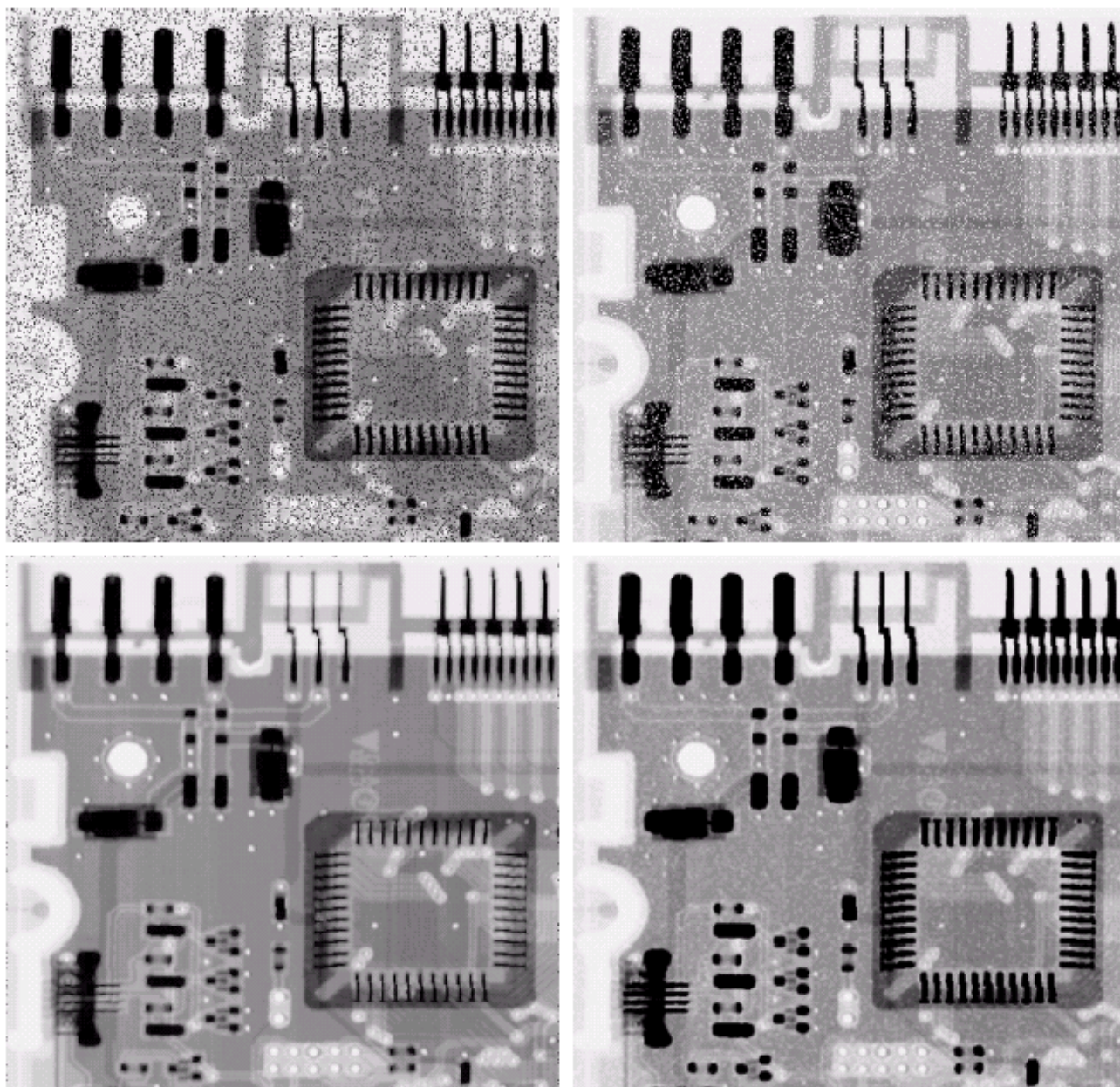
a	b
c	d

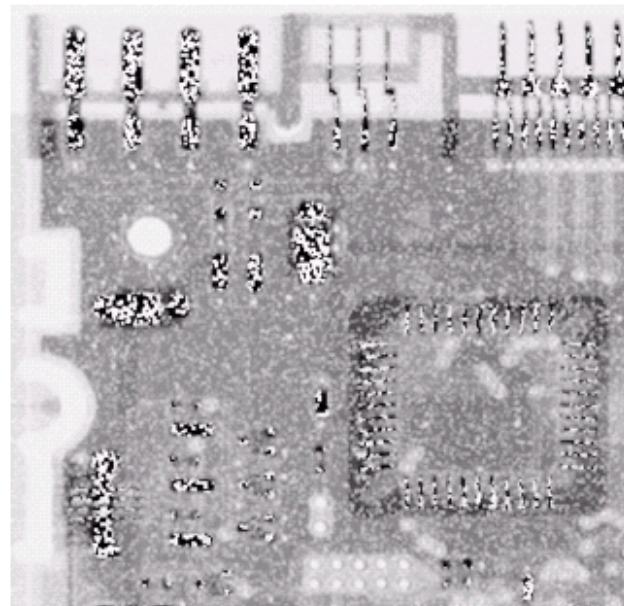
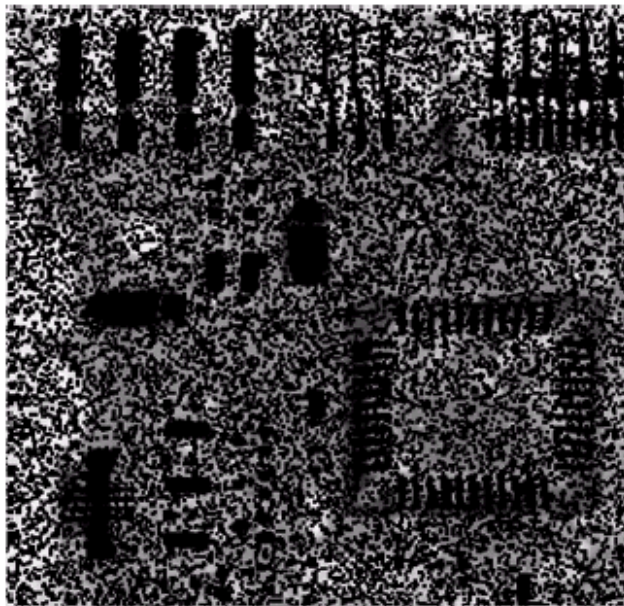
FIGURE 5.7 (a) X-ray image. (b) Image corrupted by additive Gaussian noise. (c) Result of filtering with an arithmetic mean filter of size 3×3 . (d) Result of filtering with a geometric mean filter of the same size. (Original image courtesy of Mr. Joseph E. Pascente, Lixi, Inc.)

a	b
c	d

FIGURE 5.8

(a) Image corrupted by pepper noise with a probability of 0.1. (b) Image corrupted by salt noise with the same probability. (c) Result of filtering (a) with a 3×3 contraharmonic filter of order 1.5. (d) Result of filtering (b) with $Q = -1.5$.





a b

FIGURE 5.9 Results of selecting the wrong sign in contraharmonic filtering. (a) Result of filtering Fig. 5.8(a) with a contraharmonic filter of size 3×3 and $Q = -1.5$. (b) Result of filtering 5.8(b) with $Q = 1.5$.

次序统计滤波器

中值滤波器(Median filter)

$$\hat{f}(x, y) = \underset{(s,t) \in S_{xy}}{\text{median}} \{g(s, t)\}$$

最大最小滤波器(Max and min filters)

$$\begin{cases} \hat{f}(x, y) = \max_{(s,t) \in S_{xy}} \{g(s, t)\} \\ \hat{f}(x, y) = \min_{(s,t) \in S_{xy}} \{g(s, t)\} \end{cases}$$

中点滤波器(Midpoint filter)

$$\hat{f}(x, y) = \frac{1}{2} \left[\max_{(s,t) \in S_{xy}} \{g(s, t)\} + \min_{(s,t) \in S_{xy}} \{g(s, t)\} \right]$$

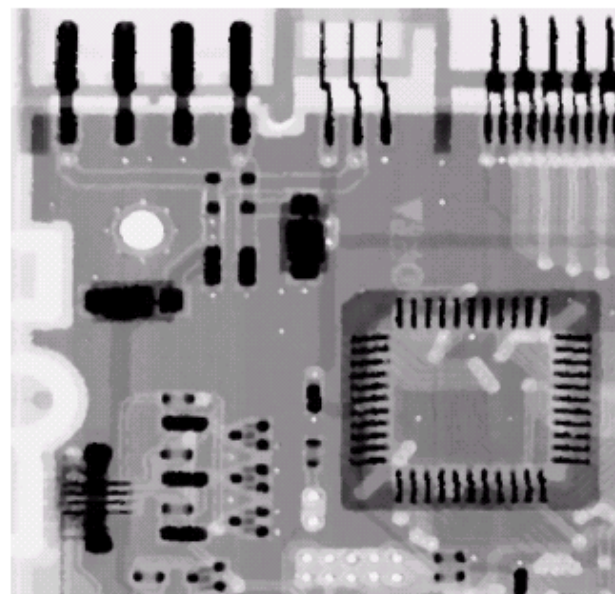
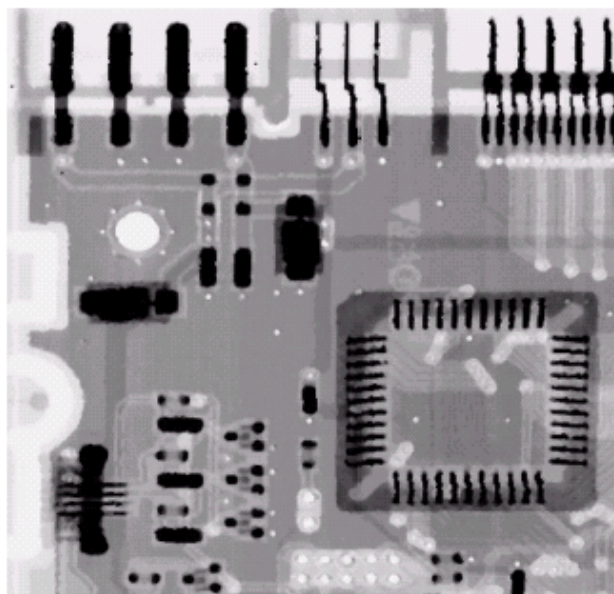
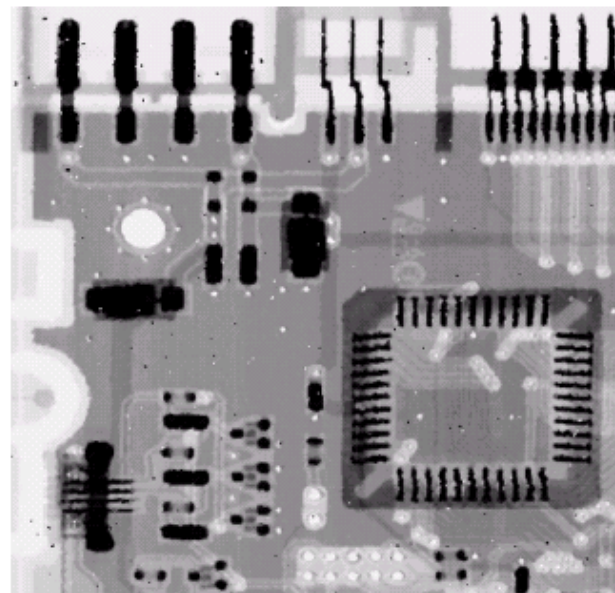
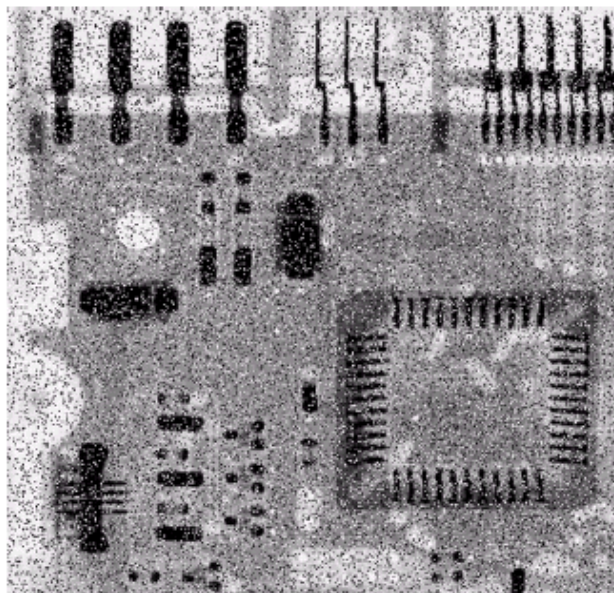
Alpha截取中值滤波器(Alpha-trimmed mean filter)

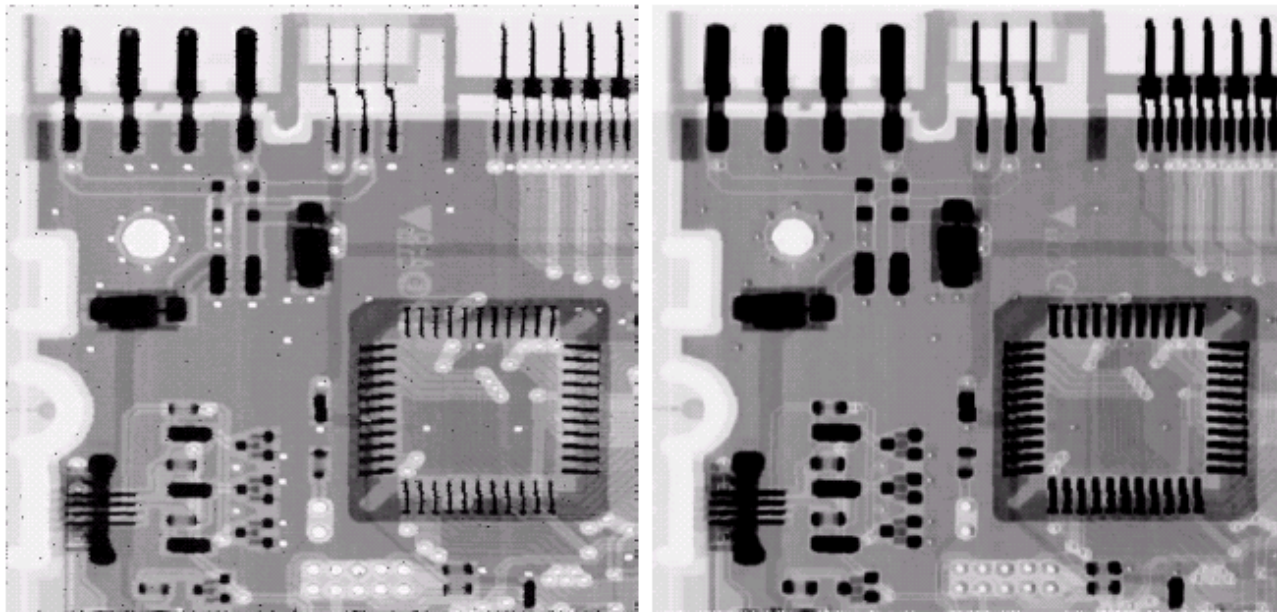
$$\hat{f}(x, y) = \frac{1}{mn - d} \sum_{(s,t) \in S_{xy}} g_r(s, t)$$

a	b
c	d

FIGURE 5.10

- (a) Image corrupted by salt-and-pepper noise with probabilities $P_a = P_b = 0.1$.
 (b) Result of one pass with a median filter of size 3×3 .
 (c) Result of processing (b) with this filter.
 (d) Result of processing (c) with the same filter.



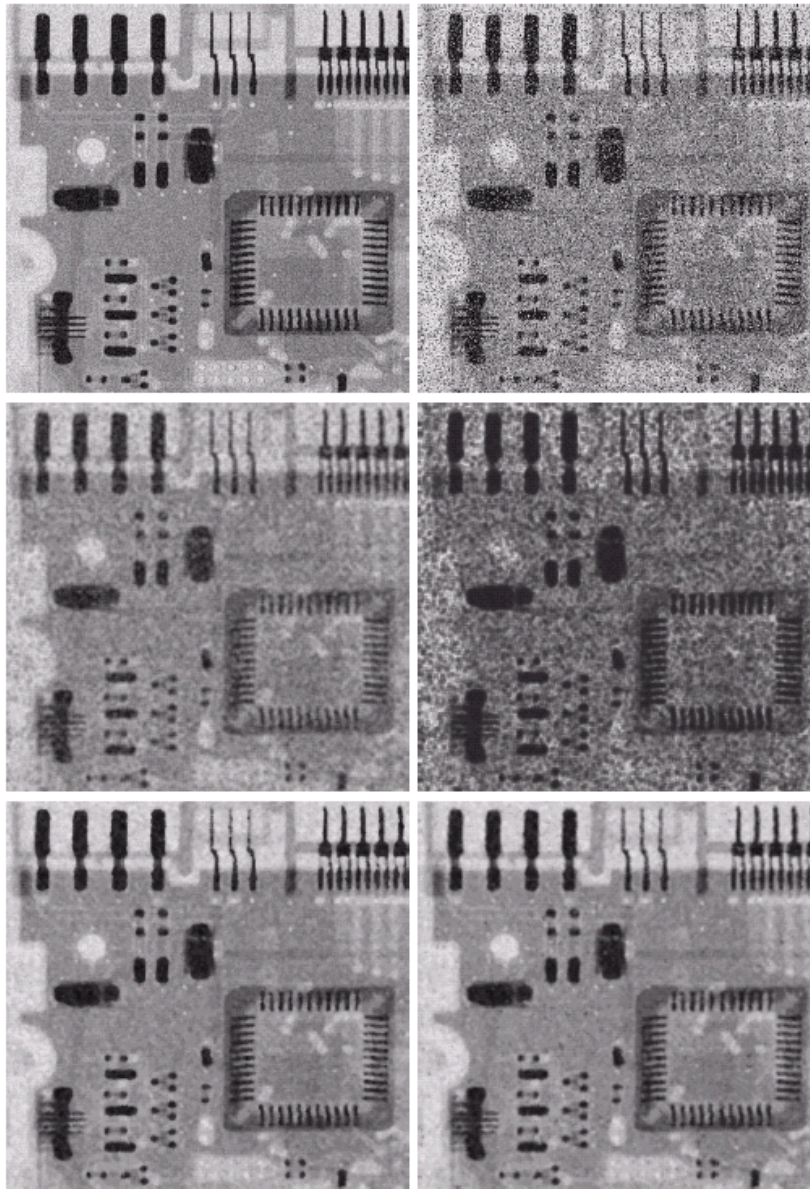


a b

FIGURE 5.11

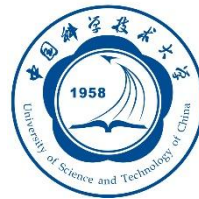
(a) Result of filtering

Fig. 5.8(a) with a max filter of size 3×3 . (b) Result of filtering 5.8(b) with a min filter of the same size.



a b
c d
e f

FIGURE 5.12 (a) Image corrupted by additive uniform noise. (b) Image additionally corrupted by additive salt-and-pepper noise. Image in (b) filtered with a 5×5 : (c) arithmetic mean filter; (d) geometric mean filter; (e) median filter; and (f) alpha-trimmed mean filter with $d = 5$.



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5.4 用频率域消除周期噪声

Periodic Noise Reduction by Frequency Domain Filtering

- 带阻滤波器(Bandreject Filters)
- 带通滤波器(Bandpass Filters)
- 槽口滤波器(Notch Filters)

带阻滤波器 (Bandreject Filters)

理想带阻滤波器(Ideal Bandreject Filters)

$$H(u, v) = \begin{cases} 1 & \text{if } D(u, v) < D_0 - \frac{W}{2} \\ 0 & \text{if } D_0 - \frac{W}{2} \leq D(u, v) \leq D_0 + \frac{W}{2} \\ 1 & \text{if } D(u, v) > D_0 + \frac{W}{2} \end{cases}$$

Butterworth带阻滤波器(Butterworth Bandreject Filters)

$$H(u, v) = \frac{1}{1 + \left[\frac{D(u, v)W}{D^2(u, v) + D_0^2} \right]^{2n}}$$

高斯带阻滤波器(Gaussian Bandreject Filters)

$$H(u, v) = 1 - e^{-\frac{1}{2} \left[\frac{D^2(u, v) + D_0^2}{D(u, v)W} \right]^2}$$

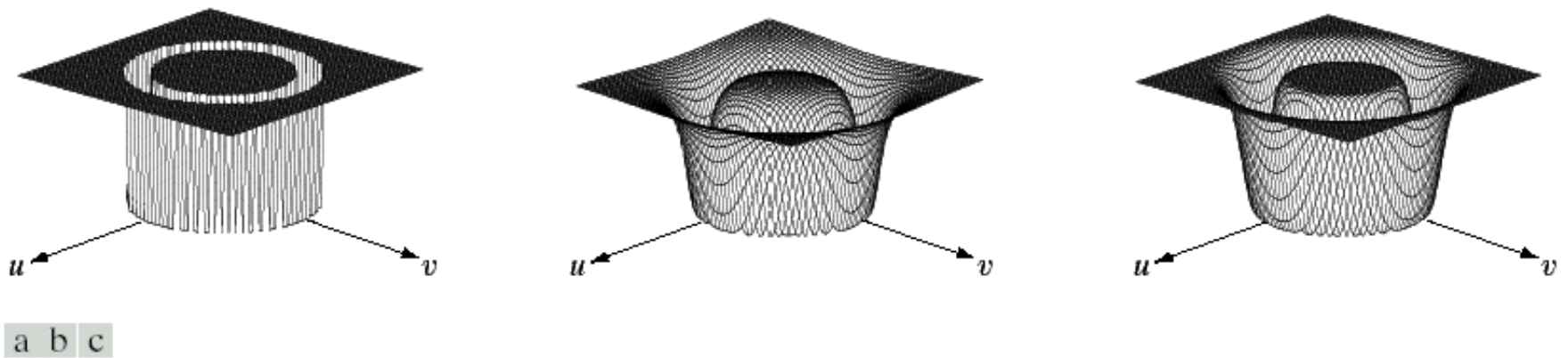
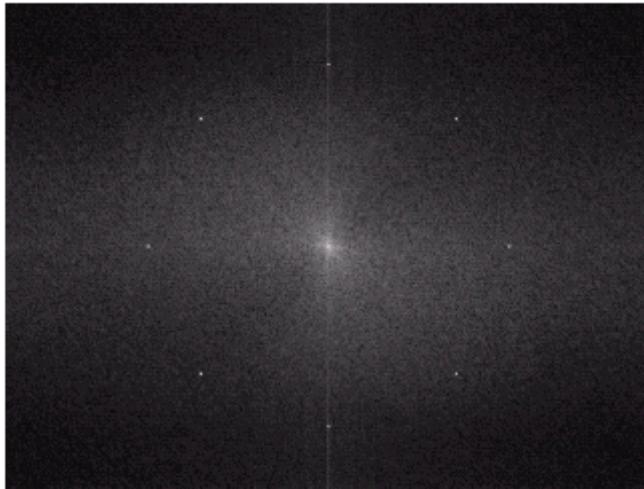
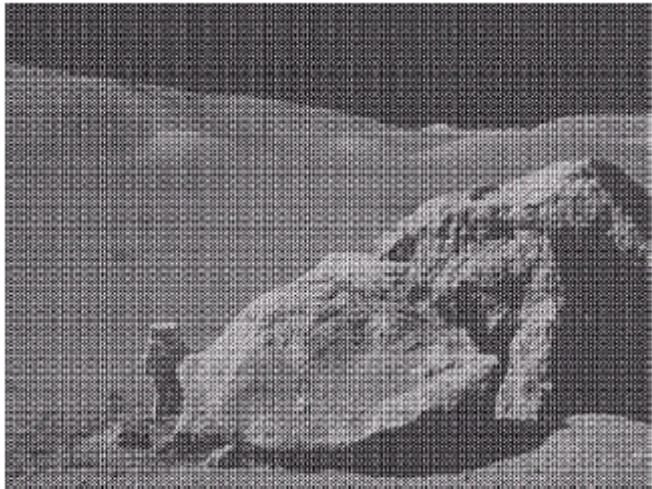


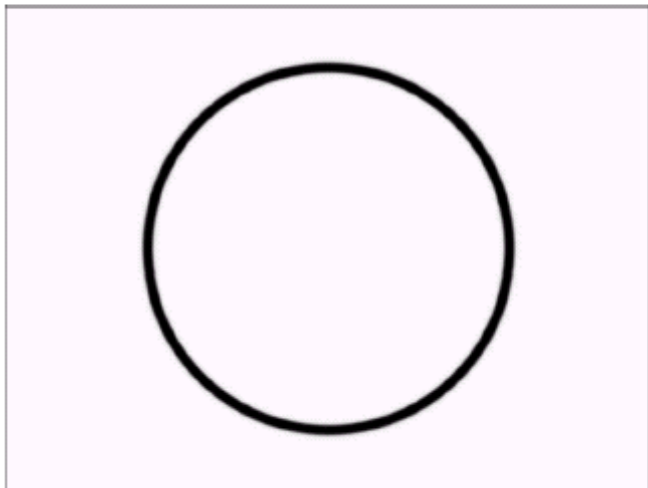
FIGURE 5.15 From left to right, perspective plots of ideal, Butterworth (of order 1), and Gaussian bandreject filters.



a	b
c	d

FIGURE 5.16

(a) Image corrupted by sinusoidal noise. (b) Spectrum of (a). (c) Butterworth bandreject filter (white represents 1). (d) Result of filtering. (Original image courtesy of NASA.)

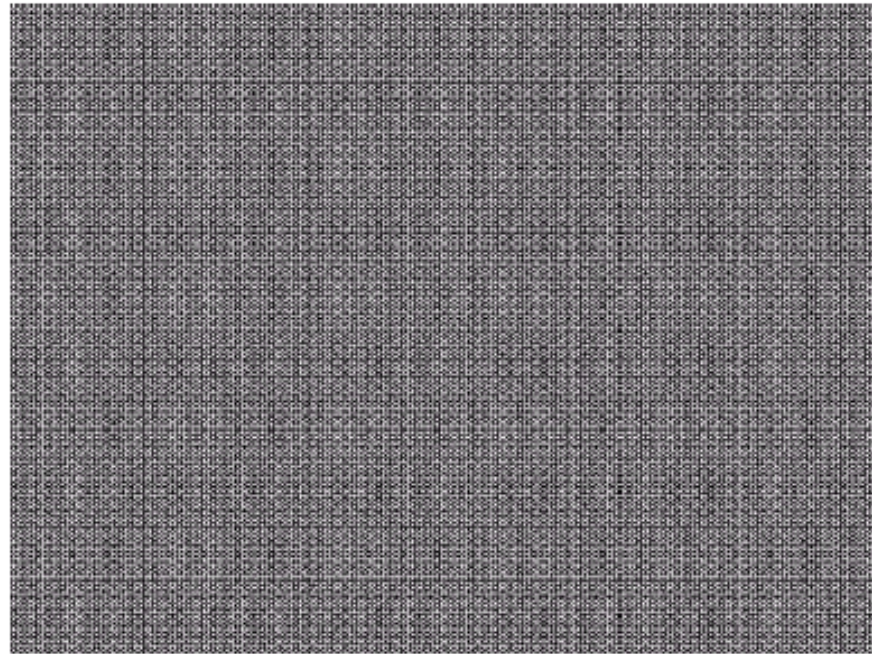


带通滤波器(Bandpass Filters)

带通滤波器与带阻滤波器功能相反

$$H_{bp}(u, v) = 1 - H_{br}(u, v)$$

FIGURE 5.17
Noise pattern of
the image in
Fig. 5.16(a)
obtained by
bandpass filtering.



槽口滤波器

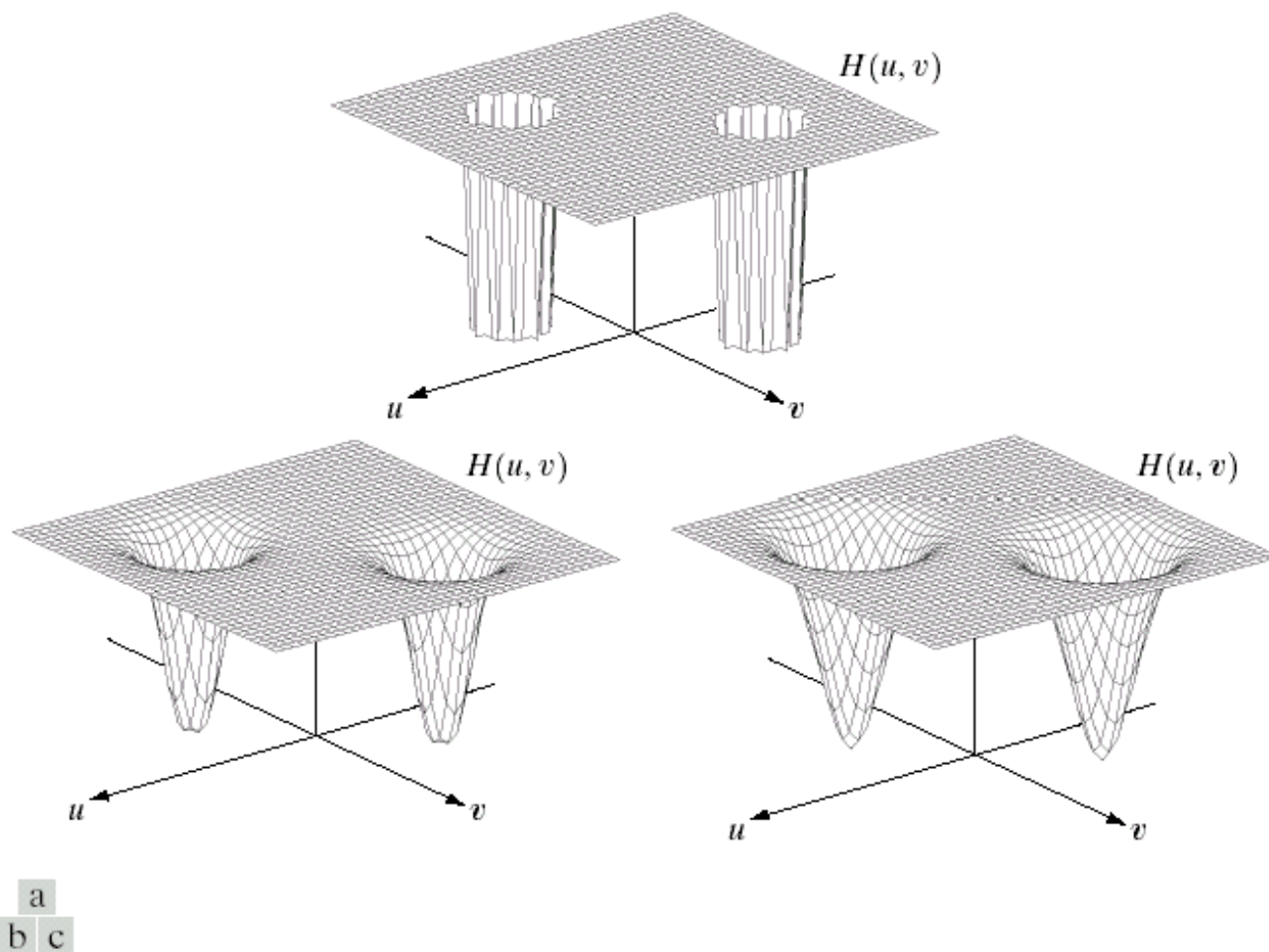
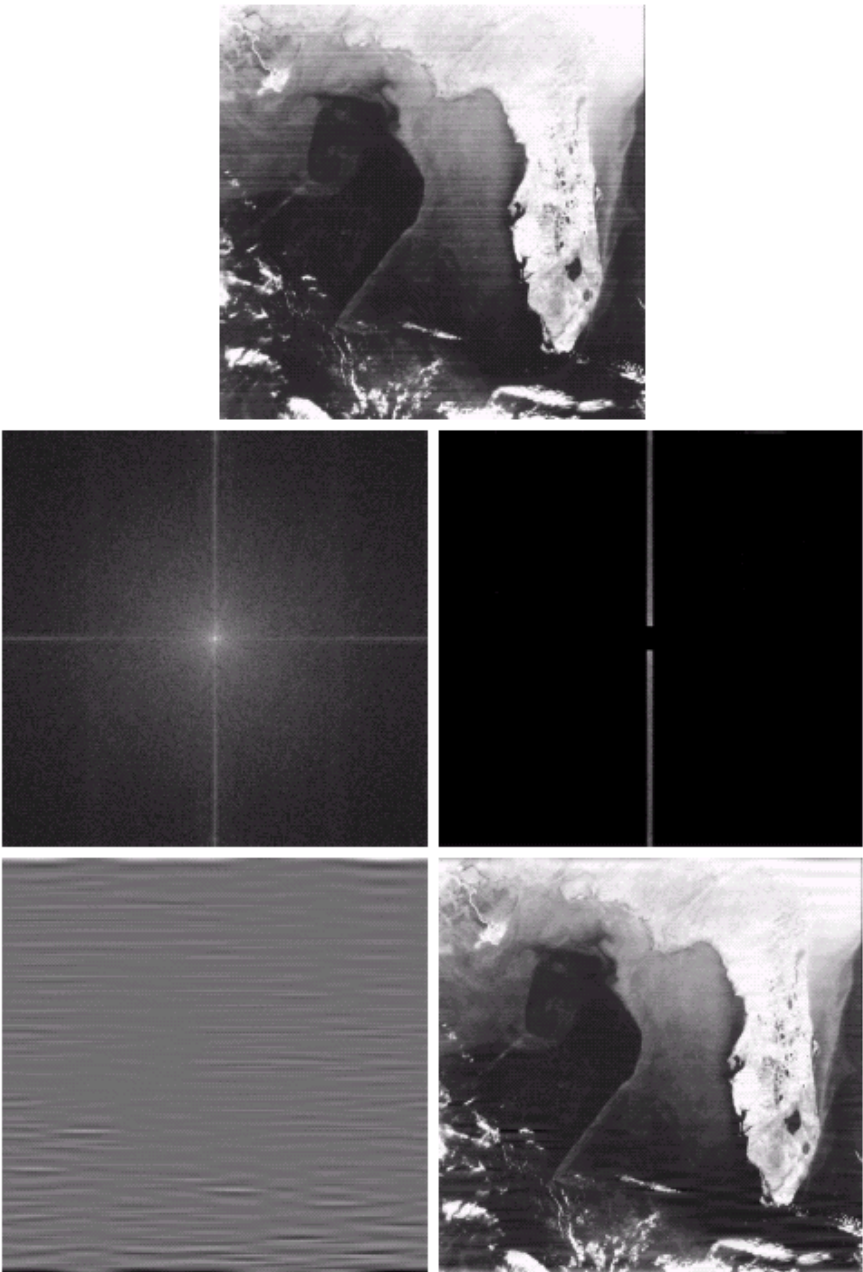
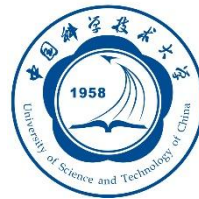


FIGURE 5.18 Perspective plots of (a) ideal, (b) Butterworth (of order 2), and (c) Gaussian notch (reject) filters.

a
b c
d e

FIGURE 5.19 (a) Satellite image of Florida and the Gulf of Mexico (note horizontal sensor scan lines). (b) Spectrum of (a). (c) Notch pass filter shown superimposed on (b). (d) Inverse Fourier transform of filtered image, showing noise pattern in the spatial domain. (e) Result of notch reject filtering. (Original image courtesy of NOAA.)





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5.5 线性、位置不变的退化

降质过程:

$$g(x, y) = H[f(x, y)] + \eta(x, y)$$

线性系统:

$$H[af_1(x, y) + bf_2(x, y)] = aH[f_1(x, y)] + bH[f_2(x, y)]$$

位置不变系统:

$$H[f(x - \alpha, y - \beta)] = g(x - \alpha, y - \beta)$$

线性位置不变降质模型

$$g(x, y) = h(x, y) * f(x, y) + \eta(x, y)$$

$$G(u, v) = H(u, v)F(u, v) + N(u, v)$$



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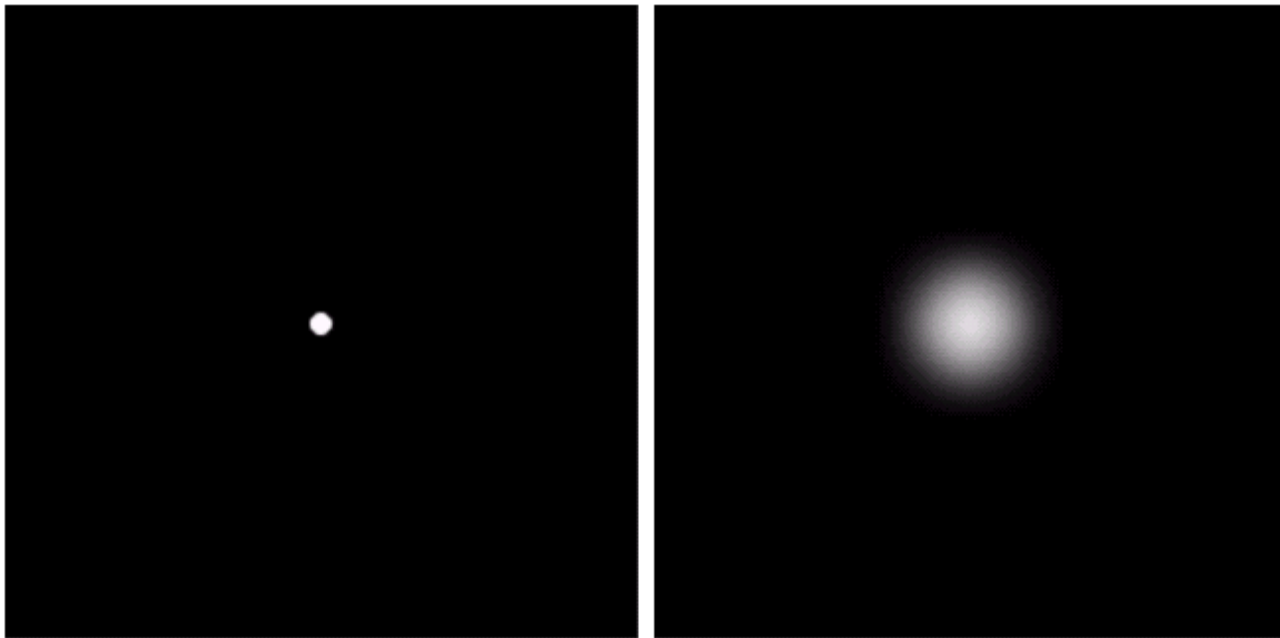
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5.6 估计退化函数

Estimating the Degradation Function

- 通过观测图像估计(Estimation by Image Observation)
- 通过实验估计(Estimation by Experimentation)
- 通过数学建模估计(Estimation by Modeling)



a b

FIGURE 5.24

Degradation estimation by impulse characterization.
(a) An impulse of light (shown magnified).
(b) Imaged (degraded) impulse.

a b
c d

FIGURE 5.25

Illustration of the
atmospheric
turbulence model.

(a) Negligible
turbulence.

(b) Severe
turbulence,
 $k = 0.0025$.

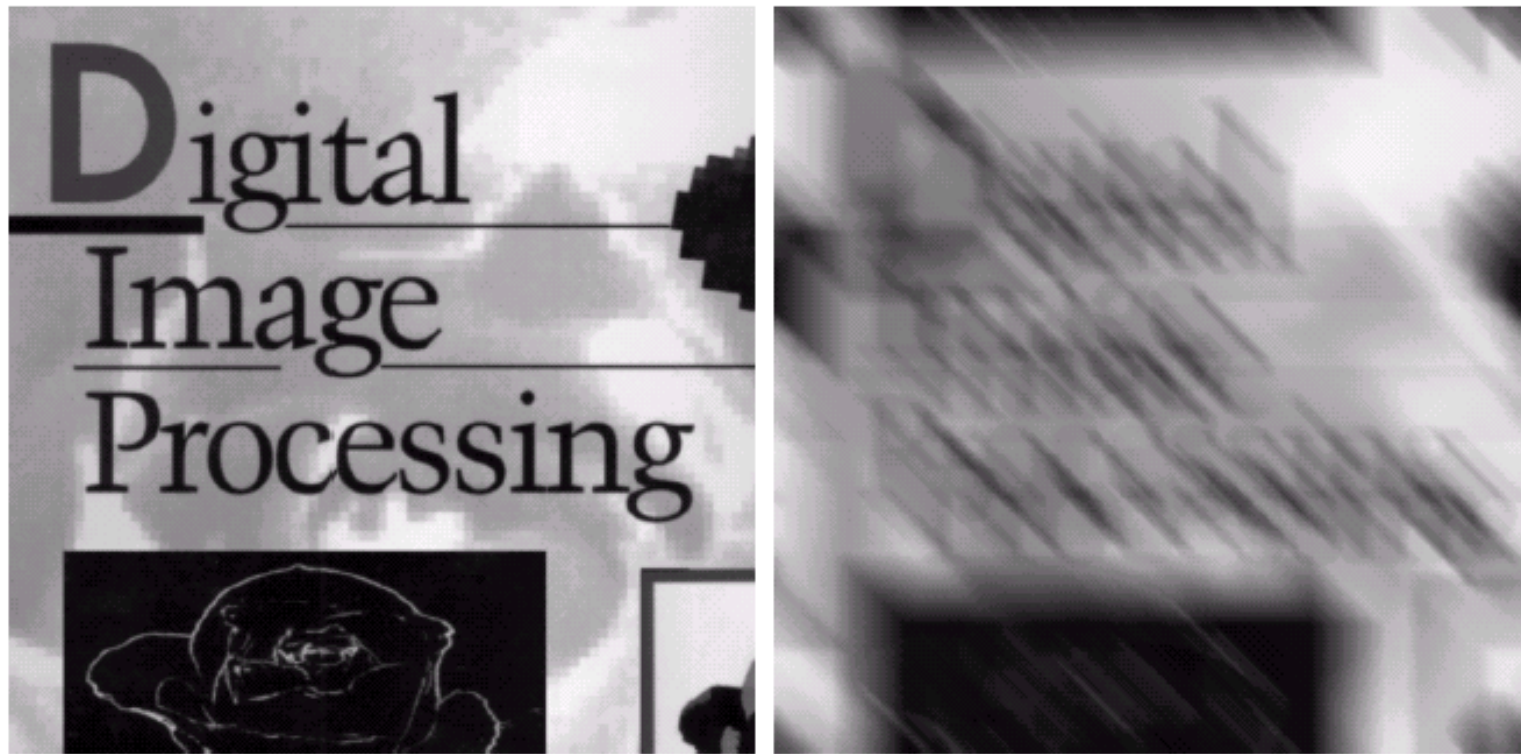
(c) Mild
turbulence,
 $k = 0.001$.

(d) Low
turbulence,
 $k = 0.00025$.

(Original image
courtesy of
NASA.)



$$H(u, v) = e^{-k(u^2 + v^2)^{5/6}}$$

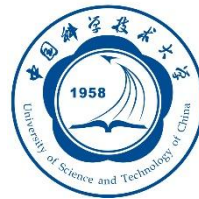


a b

FIGURE 5.26 (a) Original image. (b) Result of blurring using the function in Eq. (5.6-11) with $a = b = 0.1$ and $T = 1$.

$$H(u, v) = \frac{T}{\pi(ua + vb)} \sin[\pi(ua + vb)] e^{-j\pi(ua + vb)}$$

$$x(t) = at / T \quad y(t) = bt / T$$



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5.7 逆滤波

- 最简单且粗糙的恢复方法
- 降质图像频谱直接除以降质函数：

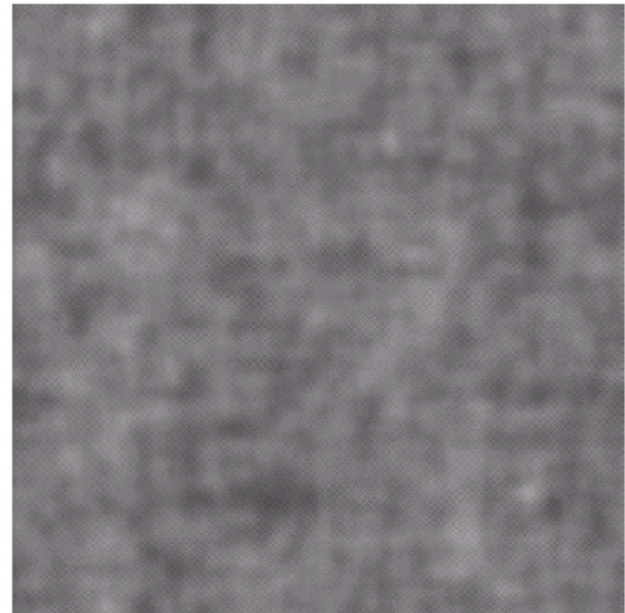
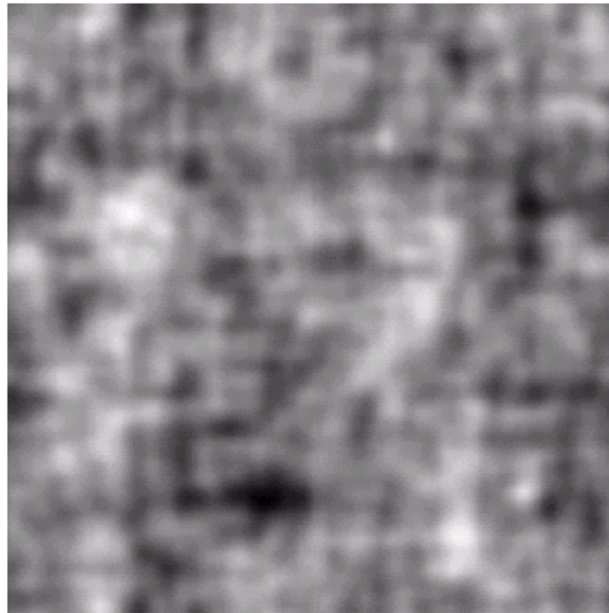
$$\hat{F}(u, v) = \frac{G(u, v)}{H(u, v)} \quad \hat{F}(u, v) = F(u, v) + \frac{N(u, v)}{H(u, v)}$$

- 即使得到降质函数，难以精确重建
- 解决方法：限制频谱范围

a	b
c	d

FIGURE 5.27

Restoring
Fig. 5.25(b) with
Eq. (5.7-1).
(a) Result of
using the full
filter. (b) Result
with H cut off
outside a radius of
40; (c) outside a
radius of 70; and
(d) outside a
radius of 85.





第5章 图像复原与重建

- 5.1 图像退化/复原过程的模型
- 5.2 噪声模型
- 5.3 只存在噪声的复原——空间滤波
- 5.4 用频率域消除周期噪声
- 5.5 线性、位置不变的退化
- 5.6 估计退化函数
- 5.7 逆滤波
- 5.8 最小均方误差（维纳）滤波



5.8 最小均方误差（维纳）滤波

最小均方差准则(Minimum Mean Square Error)

$$e^2 = E\{(f - \hat{f})^2\}$$

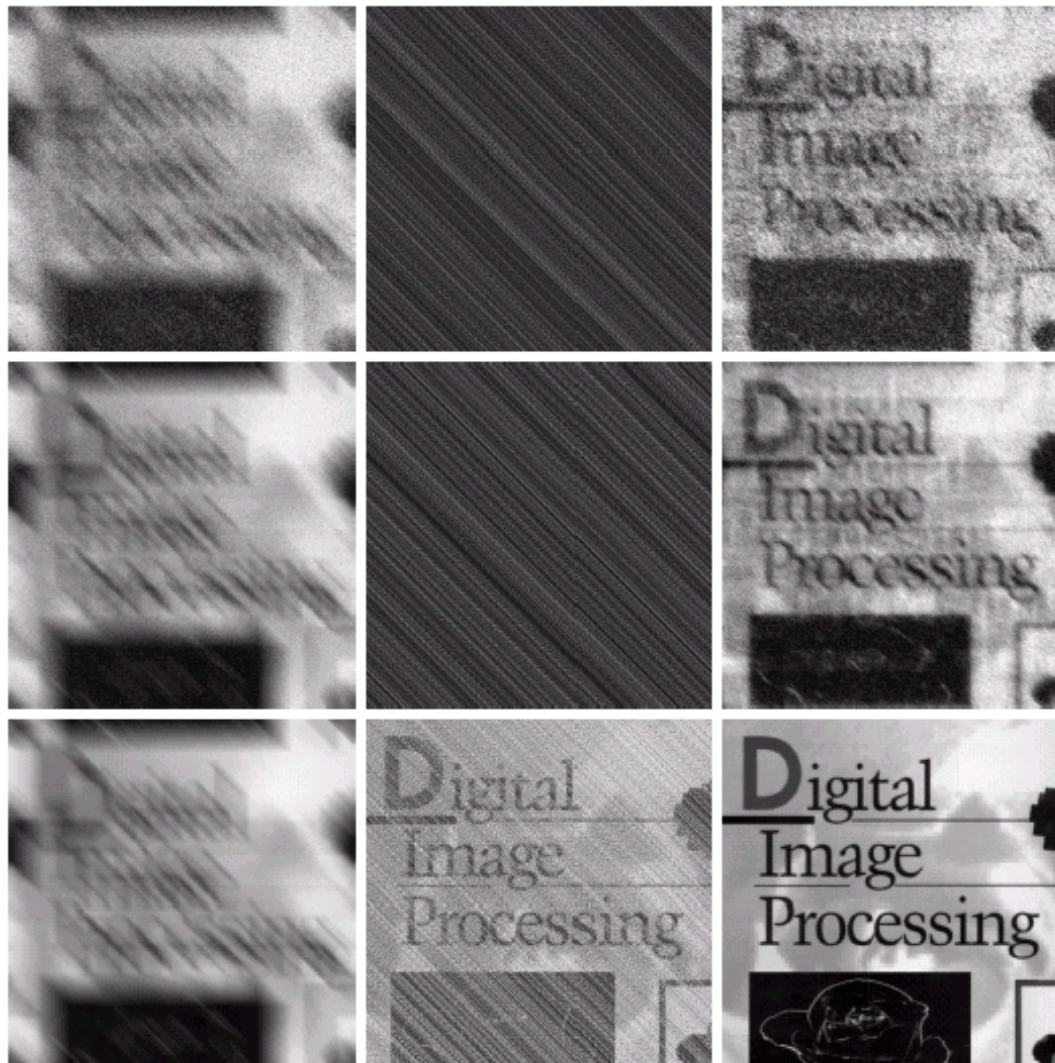
$$F(u, v) = \left[\frac{1}{H(u, v)} \frac{|H(u, v)|^2}{|H(u, v)|^2 + S_\eta(u, v) / S_f(u, v)} \right] G(u, v)$$

$$F(u, v) = \left[\frac{1}{H(u, v)} \frac{|H(u, v)|^2}{|H(u, v)|^2 + K} \right] G(u, v)$$



a b c

FIGURE 5.28 Comparison of inverse- and Wiener filtering. (a) Result of full inverse filtering of Fig. 5.25(b). (b) Radially limited inverse filter result. (c) Wiener filter result.



a	b	c
d	e	f
g	h	i

FIGURE 5.29 (a) Image corrupted by motion blur and additive noise. (b) Result of inverse filtering. (c) Result of Wiener filtering. (d)–(f) Same sequence, but with noise variance one order of magnitude less. (g)–(i) Same sequence, but noise variance reduced by five orders of magnitude from (a). Note in (h) how the deblurred image is quite visible through a “curtain” of noise.



第5章 图像复原与重建

Thanks