Image Processing and Imaging
Edge Detection

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Outline

- Edges are pixels, in which the image intensity function changes its magnitude
- Crack edges are a virtual edge entity between pixels

The are three different types of gradient operators:

1. Operators approximating the derivative of the image intensity function by differences: $\frac{\partial f}{\partial x} = f(x+1) - f(x)$
2. Operators approximating the zero-crossings of the second derivative of the image intensity function: $\frac{\partial^2 f}{\partial^2 x} = f(x+1) + f(x-1) - 2f(x)$
3. Operators mapping the image intensity function to a parameterised edge model

a b
c

FIGURE 10.2
(a) Image.
(b) Horizontal intensity profile that includes the isolated point indicated by the arrow.
(c) Subsampled profile; the dashes were added for clarity. The numbers in the boxes are the intensity values of the dots shown in the profile. The derivatives were obtained using Eqs. (10-4) for the first derivative and Eq. (10-7) for the second.

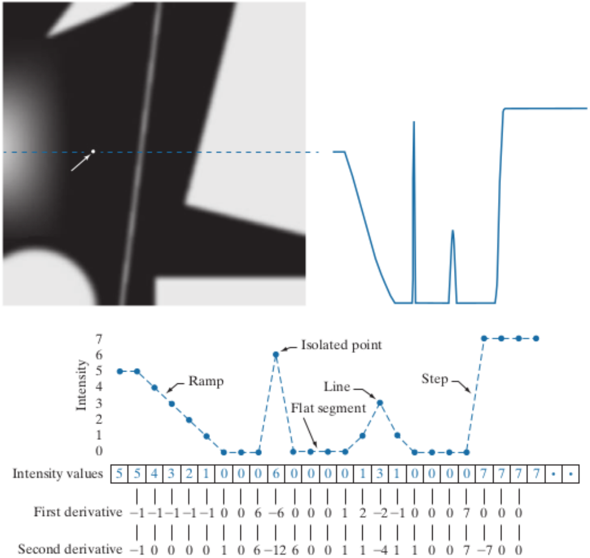


Figure: Numerics: 1st vs. 2nd order derivative

Outline

The Roberts operator uses a 2×2 neighbourhood with two convolution masks, thereby not considering the orientation of the edges

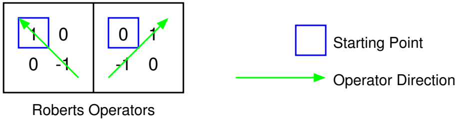


Figure: Roberts Operator

Disadvantage:
- High sensitivity against noise, since only a small number of pixels is used in the approximation
- Not symmetric around the center

Prewitt and Sobel Operator

A better approximation of the 1st order derivative can be achieved using a **Prewitt** or **Sobel operator**.

-1	-1	-1
0	0	0
1	1	1

-1	0	1
-1	0	1
-1	0	1

Figure: Prewitt operator

-1	-2	-1
0	0	0
1	2	1

-1	0	1
-2	0	2
-1	0	1

Figure: Sobel operator

Note: It can be demonstrated that using a 2 in the center location of the Sobel kernels provides image smoothing (better noise suppression).

The tool of choice for finding **edge strength** and **direction** at an arbitary location (x, y) of an image f is the gradient vector ∇f.

$$\nabla f(x,y) = \begin{bmatrix} g_x(x,y) \\ g_y(x,y) \end{bmatrix} = \begin{bmatrix} \frac{\partial f(x,y)}{\partial x} \\ \frac{\partial f(x,y)}{\partial y} \end{bmatrix}$$

We can determine the magnitude $M(x, y)$ of the gradient vector and the direction $\alpha(x, y)$ at point (x, y) as follows:

$$M(x,y) = ||\nabla f(x,y)|| = \sqrt{g_x^2(x,y) + g_y^2(x,y)}$$

$$\alpha(x,y) = tan^{-1}\left(\frac{g_y(x,y)}{g_x(x,y)}\right)$$

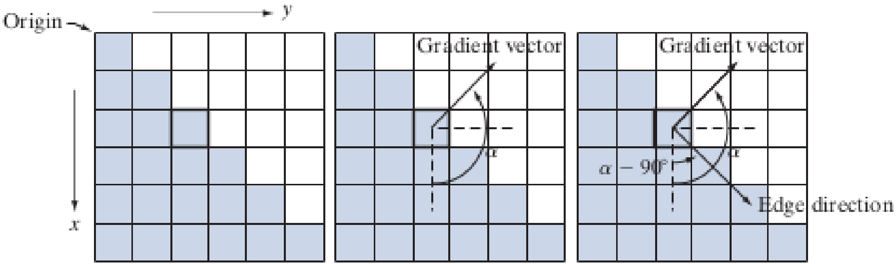
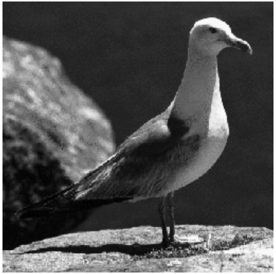


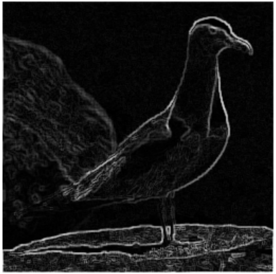
Figure: Using the gradient to determine edge strength and direction at a point. The edge direction is perpendicular to the direction of the gradient vector at the point where the gradient is computed.

Examples

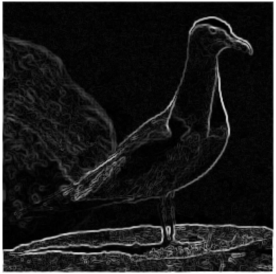
Source



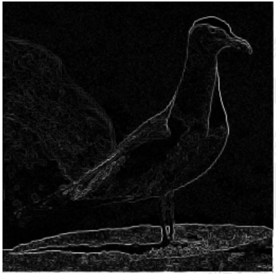
Prewitt operator



Sobel operator



Roberts operator



Compass Operators (1)

Compass Edge Detection is an alternative approach to the differential gradient edge detection. The used kernels are designed to detect the edge magnitude **AND** direction in different directions (a.k.a. compass directions). The are two well-known compass operators — Robinson and Kirsch operator

Approach: Convolve the image with a set (in general 8) convolution kernels. For each pixel, the local edge gradient magnitude is estimated with the maximum response of a kernel at certain pixel location. There are 8 possible edge directions for every location.

-1	0	1	0	1	2	1	2	1	2	1	0
-2	0	2	-1	0	1	0	0	0	1	0	-1
-1	0	1	-2	-1	0	-1	-2	-1	0	-1	-2
	r_0			r_1			r_2			r_3	

1	0	-1	0	-1	-2	1	2	1	-2	-1	0
2	0	-2	1	0	-1	0	0	0	-1	0	1
1	0	-1	2	1	0	-1	-2	-1	0	1	2
	r_4			r_5			r_6			r_7	

Figure: Robinson operator

−3	−3	5	−3	5	5	5	5	5	5	5	−3
−3	0	5	−3	0	5	−3	0	−3	5	0	−3
−3	−3	5	−3	−3	−3	−3	−3	−3	−3	−3	−3
	N			NW			W			SW	

5	−3	−3	−3	−3	−3	−3	−3	−3	−3	−3	−3
5	0	−3	5	0	−3	−3	0	−3	−3	0	5
5	−3	−3	5	5	−3	5	5	5	−3	5	5
	S			SE			E			NE	

Figure: Kirsch operator

Disadvantages of the previous approaches

The previous approaches / kernels are ...

- Sensitive to noise
- Sensitive to the size of the object and scale / type of edge (step edge vs. ramp edge).
- In many cases it is easier to locate zero-crossings than maxima or minima

$\rightarrow$ More advanced techniques are required that taking into account factors such as image noise and the nature of edges

Outline

The **Laplace operator** is a good choice if ...

- An application only requires the magnitude of the gradient regardless of its orientation
- Rotation invariance is needed. The Laplace Operator is **isotropic** (rotation-invariant).

$$\nabla^2(x,y) = \frac{\partial^2 f(x,y)}{\partial x^2} + \frac{\partial^2 f(x,y)}{\partial y^2} \tag{1}$$

Recall: It can be computed efficiently in the Fourier domain

In most cases, it is approximated using 3×3 masks for 4- und 8-neighbourhoods:

$$h_4 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & -4 & 1 \\ 0 & 1 & 0 \end{pmatrix} \qquad h_8 = \begin{pmatrix} 1 & 1 & 1 \\ 1 & -8 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$

Mexican Hat Operator (1)

Mexican Hat Operator (also denoted as Marr-Hildreth Operator) uses a two-dimensional Gauss filter as a smoothing operator:

$$G(x,y) = e^{-\frac{x^2+y^2}{2\sigma^2}}$$

Standard deviation σ is proportional to the size of the neighbourhood the filter is operating on. To compute the second derivative (in order to identify zero-crossings) the Laplace operator is applied to the smoothed image:

$$\nabla^2 \left(G(x,y,\sigma) * f(x,y)\right) \overset{\text{linear}}{\rightarrow} \left(\nabla^2 G(x,y,\sigma)\right) * f(x,y)$$

$\nabla^2 G$ is image independent and thus can be computed in advance.

$$r^2 = x^2 + y^2$$

$$G(r) = e^{-\frac{r^2}{2\sigma^2}}$$

$$G'(r) = -\frac{r}{\sigma^2} e^{-\frac{r^2}{2\sigma^2}}$$

$$G''(r) = \frac{1}{\sigma^2}\left(\frac{r^2}{\sigma^2} - 1\right) e^{-\frac{r^2}{2\sigma^2}}$$

Replacing r^2 by $x^2 + y^2$

- We obtain the *Laplacian of Gaussian* (LoG)
- Resembles the shape of a sombrero, i.e. Mexican Hat:

$$h(x,y) = \frac{1}{\sigma^2}\left(\frac{x^2+y^2}{\sigma^2} - 1\right) e^{-\frac{x^2+y^2}{2\sigma^2}} \quad (2)$$

Mexican Hat Operator (2) - Graphical Representation

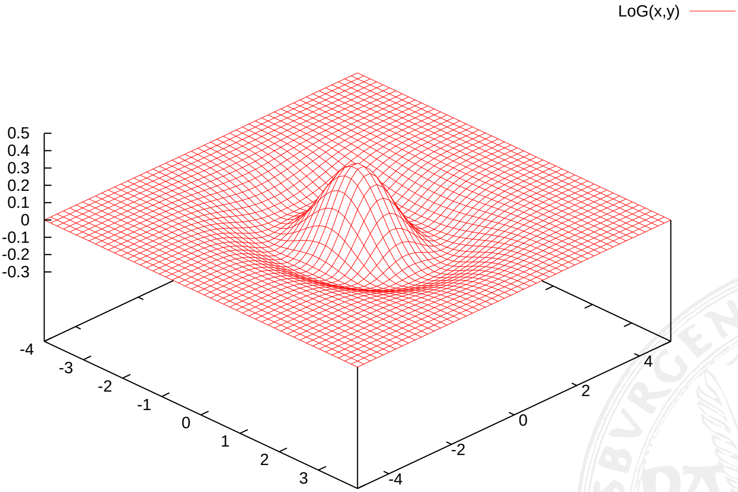


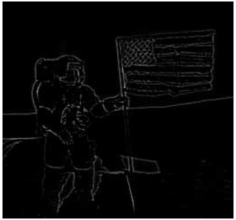
Figure: Mexican Hat

Mexican Hat Operator (3)

Examples for increasing values of σ:
The larger those values, the more responses we get from coarse edges (large σ in the Gauss mask leads to a strong smoothing effect).

Original

small σ



increasing σ

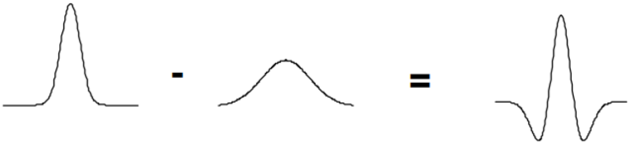
large σ

Mexican Hat Operator (4)

Remark:

$\bigtriangledown^2 G$ can be approximated efficiently by the difference of two Gauss-masks with different sigma.

This is called the *Difference of Gaussian* (DoG).



Mexican Hat Operator (5)

Advantage:

- By selecting σ, it is possible to set the scale with respect to which the edge property should be determined.

Disadvantages:

- significant smoothing
- trend to form closed curves (*plate of sphagetti*)

Outline

Canny Edge Detector - Introduction

- Has been developed in 1986
- Has been theoretically proven to be optimal with respect to the following criteria for noisy *step edges*:

Detection no important edges are missed and no false edges are listed (low value for false negative and false positive edges)

Localisation distance among actual edge position and computed edge position is minimal

One response multiple responses to one edge are minimised

Canny Edge Detector - Techniques

Canny Edge Detector achieves these results by applying the following techniques:

Thresholding with hysteresis improves detection performance in the presence of noise.

Edge pixels have to satisfy the following conditions:

- edge-magnitude $>$ *high threshold*
- edge-magnitude $>$ *low threshold* and is connected to an edge pixel $>$ *high threshold*

Non-maximal suppression ensures that only local maxima of the edge image are processed

Feature synthesis

- All edges with respect to a small scale (i.e. fine detail edges, small σ) are marked
- A predictor for larger σ (see Mexican Hat) is used to predict edge pixels for a larger σ:
  - Smoothing of the small scale edges
  - The predicted values are then compared to the actual ones and only those actual values are kept as additional values, which are significantly larger than the prediced ones
  - This procedure is conducted for several values of σ

Canny Edge Detector - Algorithm

1. Convolve the image with a Gaussian with σ
2. Compute the gradient and orientation image by calculating the gradient magnitude and direction for each pixel (see Slide 9).
3. Apply non-maximal suppression to the gradient image to thin wide ridges
4. Apply hysteresis thresholding
5. Feature synthesis: Repeat (1) to (4) for increasing values of σ. Combine the results.

Canny Edge Detector - Non-Maximal Suppression (1)

Goal: Get rid of thick ridge lines by thinning. We only keep the strongest value for each ridge.

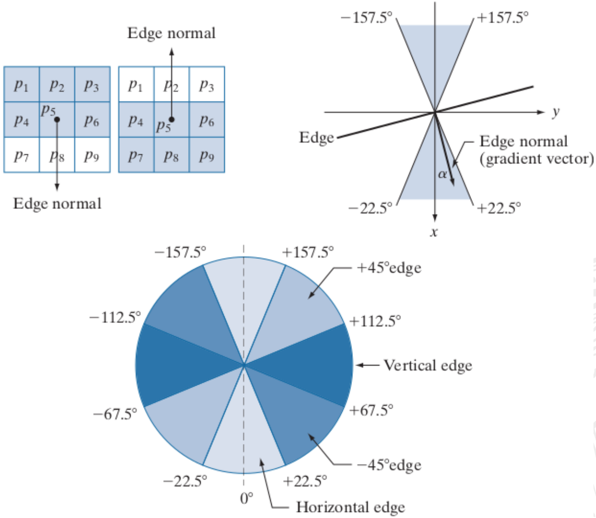
We refer to $M_s(x, y)$ as the gradient image and $\alpha(x, y)$ as the orientation image. Furthermore, we specify a number of discrete orientations d_k of the gradient vector (e.g., horizontal, vertical, 45°, -45°)

Algorithm:

1. Find the direction d_k that is closest to $\alpha(x, y)$
2. Let K denote the value of $M_s(x, y)$. If K is less than the value of M_s at one or both of the neighbors of point (x, y) along d_k, let $g_N(x, y) = 0$ (suppression). Otherwise, let $g_N(x, y) = K$.

a b
c

FIGURE 10.24
(a) Two possible orientations of a horizontal edge (shaded) in a 3×3 neighborhood. (b) Range of values (shaded) of α, the direction angle of the edge normal for a horizontal edge. (c) The angle ranges of the edge normals for the four types of edge directions in a 3×3 neighborhood. Each edge direction has two ranges, shown in corresponding shades.



Canny Edge Detector - Hysteresis Thresholding (1)

Goal: Reduce the number of false edge points.

The final operation is to threshold the non-maximal suppressed gradient image $g_N(x, y)$.

We can visualize the thresholding operation as creating two additional images:

$$g_{NH} = g_N(x, y) \geq T_H$$

$$g_{NL} = g_N(x, y) \geq T_L$$
$$g_{NL}(x, y) = g_{NL}(x, y) - g_{NH}(x, y)$$

where T_H is a high threshold and T_L is a low threshold.

Canny Edge Detector - Hysteresis Thresholding (2)

The nonzero pixels in $g_{NH}(x, y)$ and $g_{NL}(x, y)$ may be viewed as being "strong" and "weak" edge pixels, respectively. After the thresholding operations, all strong pixels in $g_{NH}(x, y)$ are assumed to be valid edge pixels, and are so kept immediately.

Depending on the value of T_H , the edges in $g_{NH}(x, y)$ typically have gaps.

Longer edges are formed using the following procedure:

1. Locate the next unvisited edge pixel, p, in $g_{NH}(x, y)$.
2. Mark as valid edge pixels all the weak pixels in $g_{NL}(x, y)$ that are connected to p.
3. If all nonzero pixels in $g_{NH}(x, y)$ have been visited go to Step (4). Else, return to Step (1).
4. Set to zero all pixels in $g_{NL}(x, y)$ that were not marked as valid edge pixels.

Figure: Canny Edge Detector: different σ

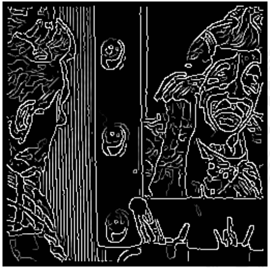
Canny Edge Detector - Examples Thresholding

Original

t1=255, t2=1

t1=255, t2=220

t1=128, t2=1



Outline

1. Introduction
2. Techniques Using the 1st Derivative
3. Techniques Using the 2nd Derivative
4. Canny Edge Detector
5. Line Finding Algorithms
   - Simple Kernels
   - Hough Transformation

Simple Kernels for line detection (1)

A **line** is a curve that does not bend sharply. In case a line has a width of one or two pixels, a line can be detected using the following kernels:

−1	−1	−1
2	2	2
−1	−1	−1

Horizontal

2	−1	−1
−1	2	−1
−1	−1	2

+45°

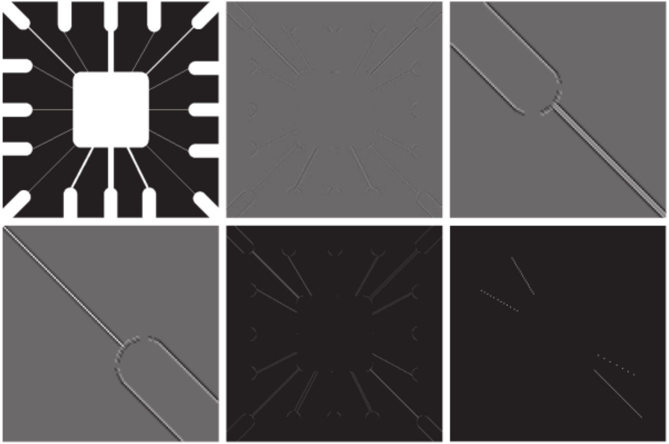
−1	2	−1
−1	2	−1
−1	2	−1

Vertical

−1	−1	2
−1	2	−1
2	−1	−1

−45°

Figure: Line detection kernels



a b c
d e f

FIGURE 10.7 (a) Image of a wire-bond template. (b) Result of processing with the +45° line detector kernel in F 10.6. (c) Zoomed view of the top left region of (b). (d) Zoomed view of the bottom right region of (b). (e) The ima in (b) with all negative values set to zero. (f) All points (in white) whose values satisfied the condition $g > T$, whe g is the image in (e) and $T = 254$ (the maximum pixel value in the image minus 1). (The points in (f) were enlarg to make them easier to see.)

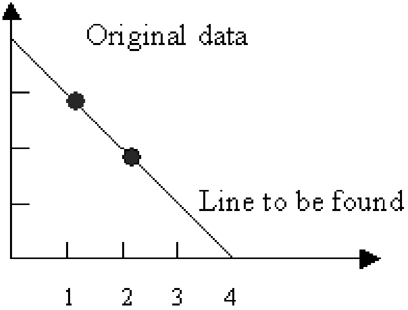
Outline

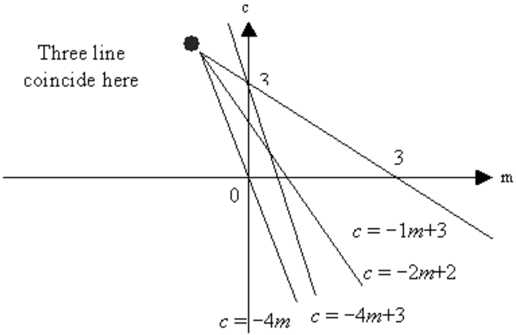
Hough Transformation (1)

- Maps edge pixels to a parametric model of a curve of a certain type
- To be applied, we have to know which curves we are looking for:

Straight line:

- Is defined by two points $A = (x_1, y_1)$ and $B = (x_2, y_2)$
- All lines passing through A are given by $y_1 = mx_1 + c$, for arbitrary m and c (this equation is associated with the parameter space m, c)
- In this representation, all lines through A are given by $c = -x_1 m + y_1$
- Lines through B are given by $c = -x_2 m + y_2$
- Only common point of those two lines in the m, c parameter space is the point representing the line connecting A and B
- Each line the the image is represented by a single point in the m, c parameter space





The points (1,3), (2,2) and (4,0) are situated on the line we are looking for, while (4,3) is not.

y	x	Gives	Transposed
3	1	3 = m 1 + c	c = -1 m + 3
2	2	2 = m 2 + c	c = -2 m + 2
0	4	0 = m 4 + c	c = -4 m
3	4	3 = m 4 + c	c = -4m + 3

Hough Transformation (3)

Based on the line definition as discussed so far, we obtain the following procedure:

1. determine all potential line pixels (edge detection)
2. determine all lines passing through these line pixels
3. transform these lines into (m, c)-parameter space
4. determine point (a, b) in parameter space, which is the result of the Hough transform of the line $y = ax + b$ ((a, b) is the only tupel that occurs several times)

Remark: Only a limited number of lines is considered passing through the line pixels. Collecting the parameters of all admissible lines results in an *accumulator array*, the elements of which are denoted *accumulator cells*.

For each line pixel, potential lines which point into an admissible direction are determined, the parameters m and c are recorded and the value of the corresponding accumulator cell $A(m, c)$ is incremented. Lines in the image are found by taking local maxima of the accumulator array.

Advantage:

- Robust against noise

Advantage and disadvantage:

- Missing line parts are interpolated

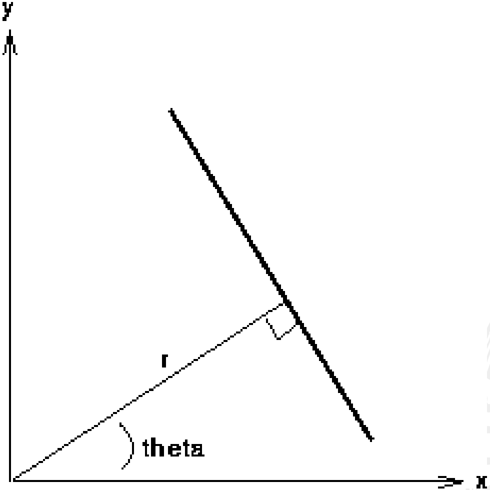
Disadvantages:

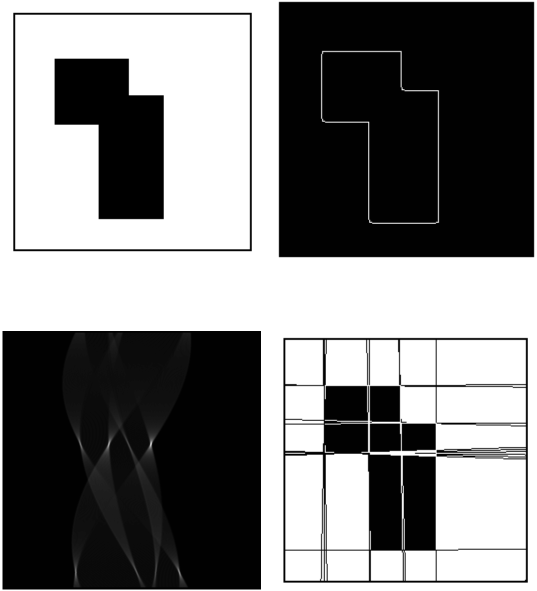
- Brute force approach with high complexity: $\mathcal{O}(A^{m-2})$ with A ... size of the image space and m ... number of parameters (e.g. 2 in case of a straight line)
- High memory consumption. Was improved for several cases in the literature
- Instead of one target line, several similar lines are found. Plateaus instead of peaks in parameter space. These plateaus need to be combined to a single peak

Check out the visualisation:
`http://liquiddandruff.github.io/hough-transform-visualizer/`

In the general setting the curve equation $f(x, a) = 0$ with a being the vector of curve parameters is used.

In actual implementations the usage of the common line equation $y = mx + c$ for $m \to \infty$ is sub-optimal in case of large slope. The line equation $r = x\cos(\theta) + y\sin(\theta)$ is better in such cases.





Hough Transformation - Examples 2

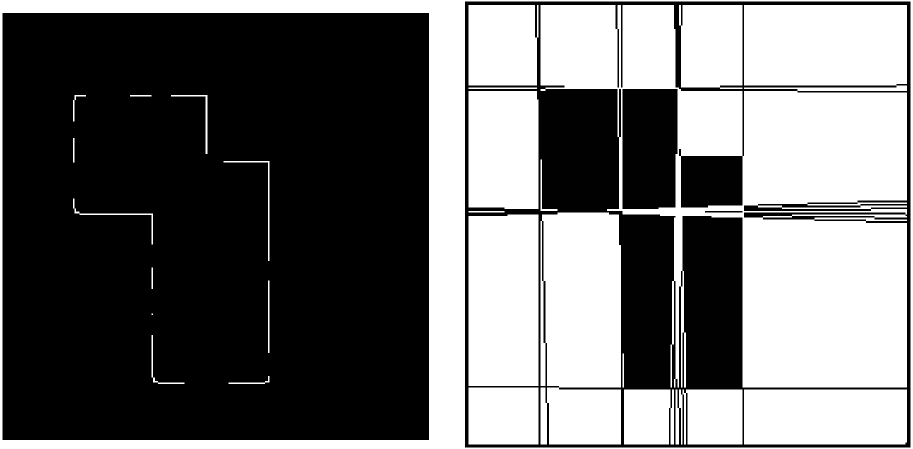


Figure: Example for Hough transform

Hough Transformation - Algorithm

1. Quantisation of the parameter space, dimension n of this space is the number of parameters in a
2. Generate the n-dimensional accumulator array $A(a)$ and set $A(b) = 0$
3. $\forall(x_1, y_1)$ in the appropriately thresholded gradient image increase accumulator cell $A(a)$ in case of $f(x, a) = 0$
4. Local maxima in the accumulator array are the curves we have been looking for

- Circles: $(x_1 - a)^2 + (y_1 - b)^2 = r^2$ three parameters and a corresponding three dimensional accumulator array are required
- By analogy: for more complicated curves, high dimensional accumulator cells are used
- Due to the high complexity, often local variants of the Hough transform are used