

Image Processing and Imaging

Image Segmentation

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2 Thresholding-based segmentation

- Thresholding Variations
- Threshold Selection
- Optimal Thresholding Techniques

3 Region-based segmentation

- Region Growing
- Region Merging and Splitting
- K-Means Clustering Segmentation



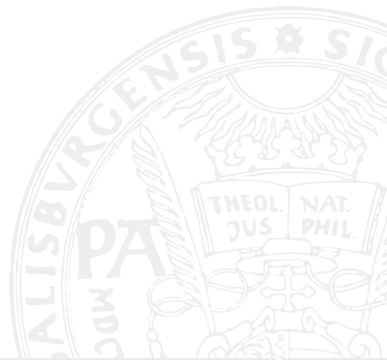
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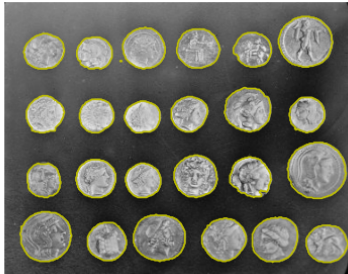
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What is segmentation?



- Image segmentation is the division of an image into **regions** or **objects**
- Spatially close regions very often can be combined to an **object**
- Objects are typically what we are interested in (depends on the application). The rest of the image is background.



Complete Segmentation

- Divides an image into non-overlapping regions that match to the real world objects
- Nowadays this is very often tackled with deep learning (DL) based techniques (see **semantic segmentation** or **instance segmentation**)
- Simple tasks can be approached with simple (non-DL based) techniques
 - Text with letters and numbers
 - Objects in front of uniform background or high contrast

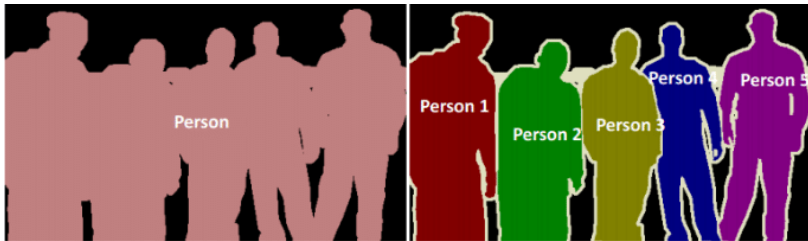


Figure: Semantic segmentation (left) vs. Instance segmentation (right)

Partial Segmentation:

- Regions which are homogeneous with respect to some criterion are identified
- These regions do NOT necessarily correspond to real-world objects
- In many cases, over-segmentation occurs, i.e. too many regions are found

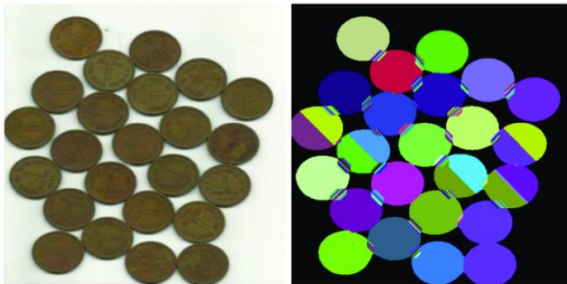
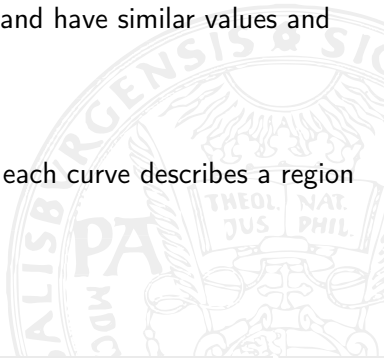


Figure: Example of oversegmentation

Segmentation techniques can be grouped into three categories:

- 1 **Thresholding-based:** Use global information of the image or parts of it (e.g. the histogram).
 - 2 **Edge-based:** An edge filter is applied to the image. Closed edge-chains are generated as object borders
 - 3 **Region-based:** Group together pixels which are neighbours and have similar values and splitting groups of pixels which are dissimilar in value
- The latter two techniques solve a dual problem:
Each region can be described by its closed border curve and each curve describes a region which is enclosed by the curve



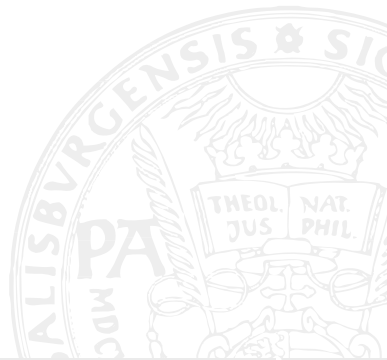
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- Great due to its **simplicity** and **computational speed**
- A threshold T is determined to separate objects and the background
- Input image $f(x, y)$ is transformed into a binary segmented output image $g(i, j)$ as follows:

$$g(i, j) = \begin{cases} 1 & f(i, j) \geq T \\ 0 & f(i, j) < T \end{cases}$$

- If T is constant over the entire image, we call refer to it as **global thresholding**
- Thresholding is a suited approach if objects are not joined and the object's gray-scale values are different from background's gray-scale values

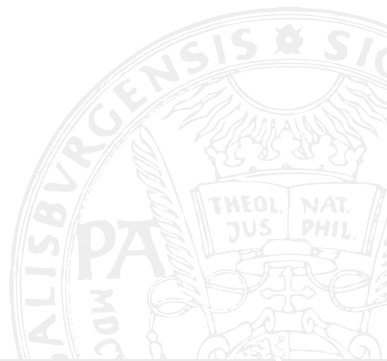
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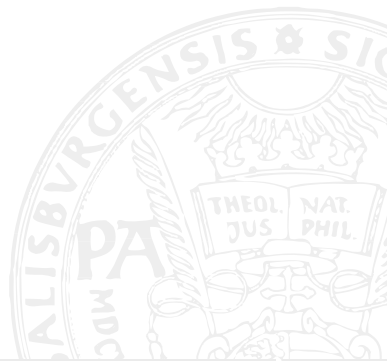
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- Using a one fixed threshold is successful in rare cases only
- **Variable thresholding** is often more successful. The value of T is changed dependent on a local image region's characteristics.



Thresholding Variations (2)

Using a single threshold T typically works well if the distribution of the is bimodal. If the histogram has, for example, three dominant modes (i.e., cause by two types of light objects on a dark background) we require **multiple thresholds**.

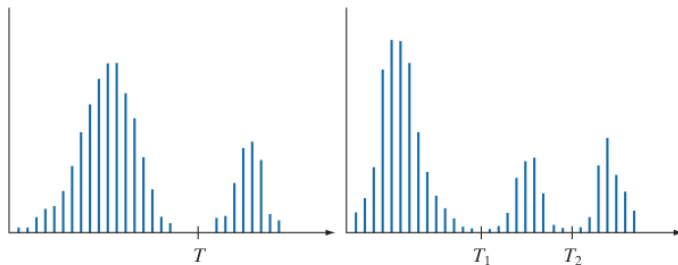


Figure: Intensity histograms that can be partitioned by a single threshold (left) and by dual thresholds (right)

Band Thresholding:

- Grayscale values in a specific grayscale range are determined to be object pixels
- Example application: CT imaging — Different tissue types cause different grayscale values

$$g(i,j) = \begin{cases} 1 & f(i,j) \in D \\ 0 & \text{otherwise} \end{cases}$$

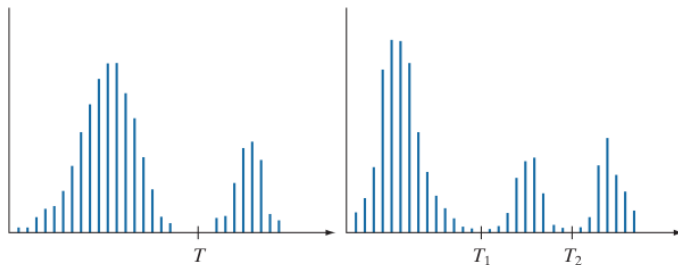


Figure: Intensity histograms that can be partitioned by a single threshold (left) and by dual thresholds (right)

Multi Thresholding: Similar to band thresholding but the output image is not binary

$$g(i,j) = \begin{cases} 1 & \text{for } f(i,j) \in D_1 \\ 2 & \text{for } f(i,j) \in D_2 \\ 3 & \text{for } f(i,j) \in D_3 \\ \dots & \end{cases}$$

Semi Thresholding: Removes the background but keeps the grayscale information in the objects

$$g(i,j) = \begin{cases} f(i,j) & \text{for } f(i,j) \geq T \\ 0 & \text{otherwise} \end{cases}$$

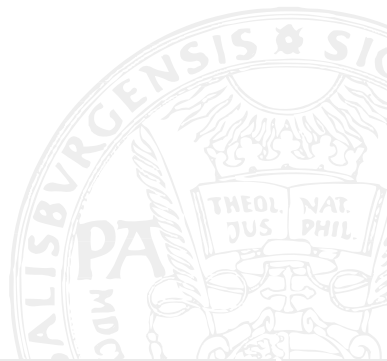
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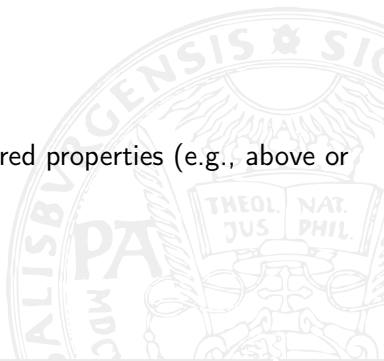


The crucial question — How to choose a threshold?

In general we can say, the more **priori information** the better.

Illustrative examples:

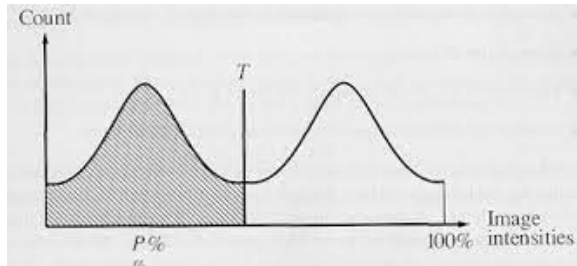
- Shape of the object region
- Position or orientation of the object
- Known initial and final point of the boundary
- Relation of the region considered to other regions with required properties (e.g., above or inside)



■ Assumption:

We have dark objects against a light background and can estimate the area/size of the objects present in the image (e.g., written letters on a sheet)

- If p % of the image is occupied by the objects of interest, choose the threshold T so that the p % darkest pixels are classified as object pixels.
- **How?** Compute the commulative histogram distribution. Select T such that $1/p$ of the image is $\leq T$.



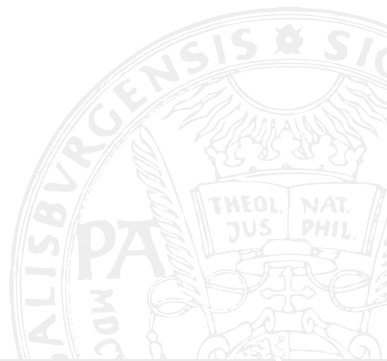
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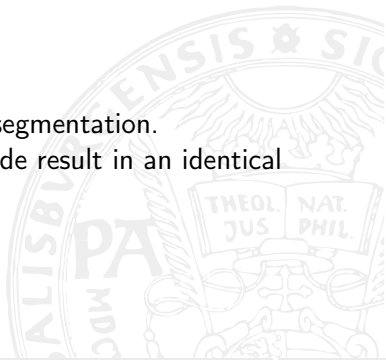
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- Optimal thresholding selects a threshold value that is statistically optimal
- If the histogram is **bimodal**, the threshold is selected to be the minimal value between the two extrema
- If the histogram is **multimodal**, the thresholds can be selected between two corresponding maxima.

Attention: A bimodal histogram does not guarantee a correct segmentation.

Example: 50% B/W pixel mixed or concentrated on one image side result in an identical bimodal histogram.



Concept of Optimal Thresholding

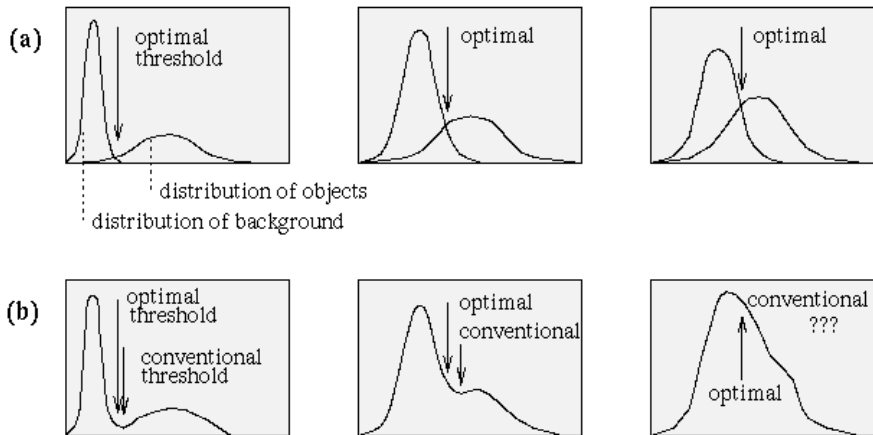


Figure: Concept of optimal thresholding, first row represents the individual distribution, second row is the combined distribution.

Optimal global thresholding using Iterative Threshold Selection (1)

Assumption: Existence of regions with two dominant grayscale values.

Algorithm:

- 1 Select an initial estimate for the global threshold T^0 .
- 2 Produce two regions by global thresholding with T^0 : Foreground and object region
- 3 Compute the mean intensity value μ_B^t of the background and object region μ_O^t .

$$\mu_B^t = \frac{\sum_B f(i,j)}{\text{number of background pixel}}$$

$$\mu_O^t = \frac{\sum_O f(i,j)}{\text{number of object pixel}}$$

- 4 Compute a new threshold value midway between μ_B and μ_O .

$$T^{t+1} = \frac{\mu_B^t + \mu_O^t}{2}$$

- 5 Repeat Steps 2 through 4 until the difference between values of T^{t+1} in successive iterations is smaller than a predefined value ΔT .

How to choose the initial threshold value?

- Assuming that the corner pixels contain background values → Choose the average corner pixel value
- Choose the global average pixel value
- **Note:** The initial intensity value must be greater than the minimum and less than the maximum intensity value.

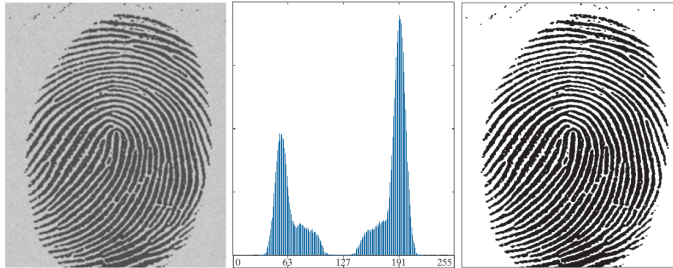


Figure: Example of segmentation using the preceding iterative algorithm. Application of the algorithm resulted in the threshold $T=125.4$ after three iterations, starting with T equal to the average intensity of the image, and using $\Delta T = 0$.

Optimal thresholding using Otsu's method (1)

Idea: View thresholding as statistical-decision theory problem → Minimize the average error incurred in assigning pixels to two or more regions (also called classes).

How?

By choosing a threshold T that **minimizes the intra-class variance** σ_{intra}^2 , defined as a **weighted sum of variances** of the two classes.

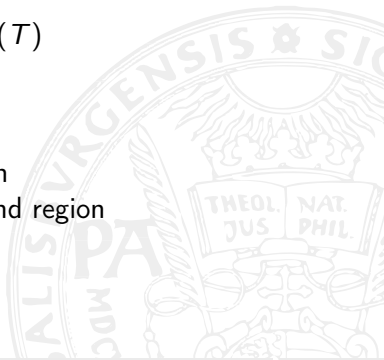
$$\sigma_{intra}^2(T) = q_O(T)\sigma_O^2(T) + q_B(T)\sigma_B^2(T)$$

where

$\sigma_O^2(T)$... Threshold-dependent variance the object region

$\sigma_B^2(T)$... Threshold-dependent variance of the background region

$q_O(T)$, $q_B(T)$... Weight terms



Optimal thresholding using Otsu's method (2)

The **weight terms** are chosen based on the **probability of the class** occurring.

Assuming that $P(i)$ denotes the normalized histogram, $q_O(T)$ and $q_B(T)$ can simply be computed as follows:

$$q_O(T) = \sum_{i=0}^{t-1} P(i) \quad \text{and} \quad q_B(T) = \sum_{i=t}^{L-1} P(i)$$

Calculating the class variance $\sigma_O^2(T)$ where $P_{O,T}(i)$ is the probability distribution of class O :

$$\begin{aligned} \sigma_O^2(T) &= \sum_{i=1}^T (i - \mu_O(T))^2 P_{O,T}(i) \\ &= \sum_{i=1}^T (i - \mu_O(T))^2 P(i|O) && [\text{Apply Bayes rule}] \\ &= \sum_{i=1}^T (i - \mu_O(T))^2 P(O|i)P(i)/P(O) \end{aligned}$$

Finally we obtain for $\sigma_O^2(T)$:

$$\sigma_O^2(T) = \sum_{i=1}^T (i - \mu_O(T))^2 \frac{P(i)}{q_O(t)}$$

Similary, we can derive the **formula for the object mean** $\mu_O(T)$:

$$\mu_O(t) = \sum_{i=1}^t i \frac{P(i)}{q_O(t)}$$

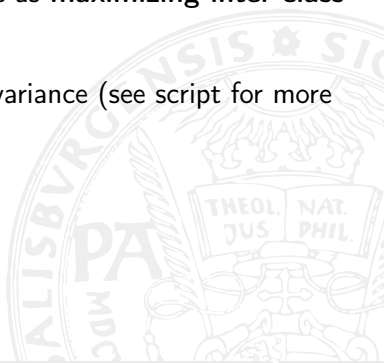
(background mean and variance by analogy)

Last but not least, all we need to do is just **run through the full range of T values and pick the value that minimizes σ_{intra}^2** .

Final note: Having to do this computation for the entire range of pixel values is not very efficient computationally. However, there also exists a **fast recursive approach**.

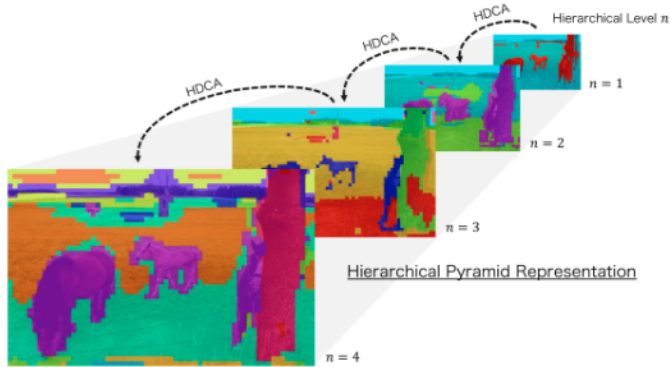
The recursive approach, exploits the relationship between the intra-class σ_{intra}^2 and inter-class σ_{inter}^2 variances \rightarrow Minimizing the intra-class variance is the same as **maximizing inter-class variance**.

There exists a fast recursive algorithm to maximizing inter-class variance (see script for more details).



Further variants - Hierarchical Data Structures (1)

Basic idea: View the image as a pyramidal data structure. Identify regions in the low resolution image and refine them in the higher resolutions



Advantage: Robustness against noise, since the initial segmentations start with a significantly smoothed image.

Variants:

- Segment the low-resolution image. In the next higher resolution pixels close to the region border are re-assigned to object(s) or background (depending on a similarity criterion) → Repeat until the highest resolution
- Apply a *significant pixel detector* (*) in the lowest resolution which identifies pixels different to their neighbourhood.

The corresponding image part in the full-resolution image is thresholded with T being between the gray-scale of the significant pixel and the average of the other 8-neighbours (this is done locally with different thresholds for different regions).

(*) e.g, a 3x3 masks which indicates if the central pixel is different from its neighbors

Basic idea: Adapt the threshold for every point in the image based properties of the local neighborhood (e.g., based on its mean and standard deviation).

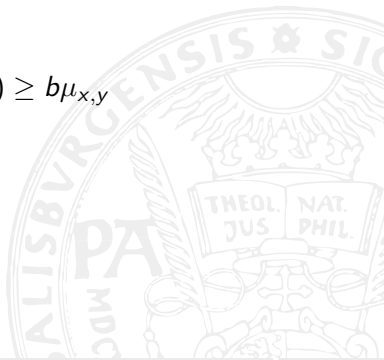
Illustrative approach:

Let m_{xy} and σ_{xy} denote the mean and standard deviation of a pixel (x, y) 's local neighborhood.

For instance, we can compute the segmentation mask $g(x, y)$:

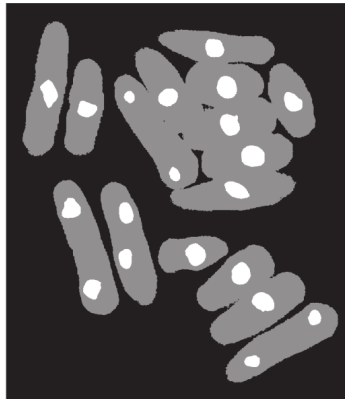
$$g(x, y) = \begin{cases} 1 & f(x, y) \geq a\sigma_{x,y} \text{ AND } f(x, y) \geq b\mu_{x,y} \\ 0 & \text{otherwise} \end{cases}$$

where a and b are nonnegative constants.

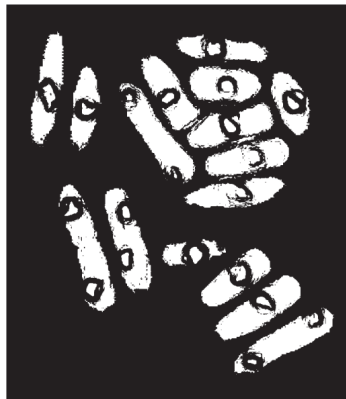




(a)



(b)



(c)

Figure: Segmentation of a yeast image. (a) Original yeast image. (b) Yeast image segmented using a global dual thresholding approach. The mid-gray regions on the right side of the image were not separated properly. (c) Yeast image segmented using variable thresholding ($a = 30$ and $b = 1.5$). All regions are nicely separated.

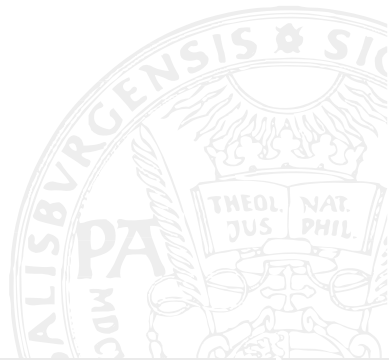
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- **Idea:** Partition the image into regions of maximal homogeneity
- Primary applied to noisy images or in circumstances where the difference between objects is not "just" the luminance (e.g. they exhibit a different structures — texture)

Homogeneity:

- We need a metric for measuring the similarity between pixels/regions
- For example: average gray-value, shape of a local histogram, texture properties
- Target regions should exhibit the following property (apart from their zero intersection):

$$\begin{array}{ll} H(R_i) = \text{True} & i = 1, \dots, S \\ H(R_i \cup R_j) = \text{False} & i \neq j \text{ AND } R_i \text{ adjacent to } R_j \end{array}$$

S ... number of regions, $H(R_i)$ is a binary evaluation of homogeneity of R_i

- This means that regions are homogeneous and maximal (i.e. if they would be larger, homogeneity is lost)

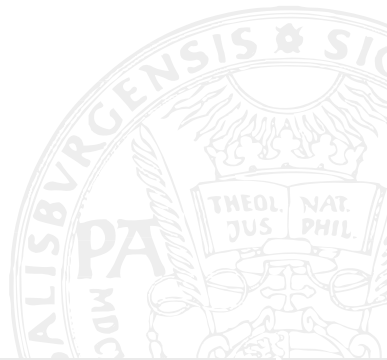
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Region Growing (1)

Idea: Group pixels or subregions into larger regions while the homogeneity criterion is satisfied.

Compare one pixel with its neighbors. If the homogeneity criterion is satisfied, the pixel belongs to the same region.

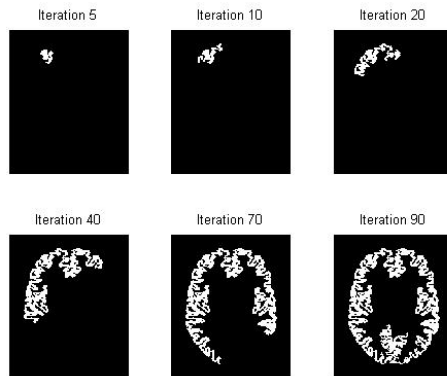
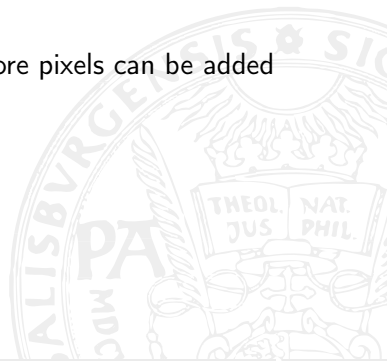


Figure: Illustrative example of region growing

Region Growing Algorithm:

- 1 Choose seed pixel(s)
- 2 Check neighbouring pixels following a chosen strategy and add them to the region if they are “similar” to the seed pixel
- 3 Repeat (2) and (3) for the newly added pixels; stop if no more pixels can be added



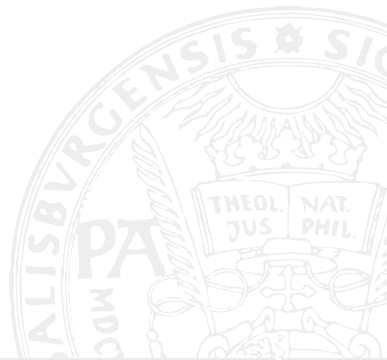
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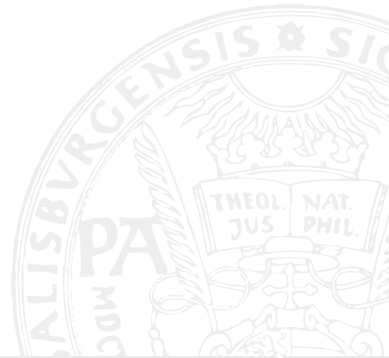
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Region Growing requires seed points.

Instead, **Region Merging and Splitting** initially subdivides the image into a set of disjoint regions and then merges/splits those regions.



Region Splitting Algorithm:

- 1 Start with the whole image where the homogeneity requirement is usually not satisfied. If the homogeneity criterion is not satisfied, split the image into four disjoint quadrants.
- 2 Keep subdividing the quadrants until the homogeneity is satisfied for all of them.

The splitting technique has a convenient representation in the form of structure called **quadtree**.

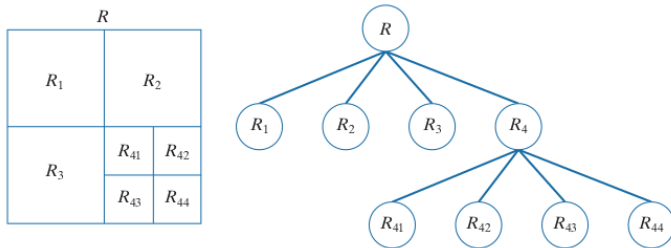


Figure: Left - Partitioned image. Right - Corresponding quadtree

Note that if only splitting is used, the final partition normally contains adjacent regions with identical properties. Hence, we need to merge those regions.

Region Merging Algorithm:

- 1 Start with a set of regions which all satisfy the homogeneity requirement.
- 2 Keep merging regions until the homogeneity requirement is no longer satisfied.

Techniques differ in terms of their initial segmentations and the different criteria for homogeneity.

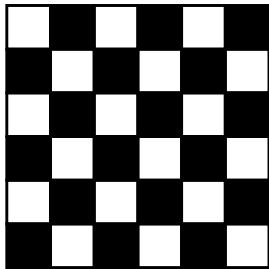
Improvement: Additional employment of edge information

- Neighbouring regions are merged if a significant share of their common border consists of weak edges
- Result of edge relaxation can be used to determine if an edge is weak or not

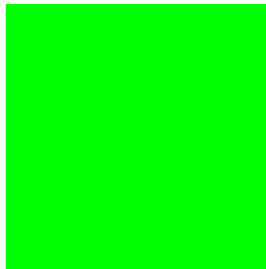
Region Splitting and Merging Comparison

Remark:

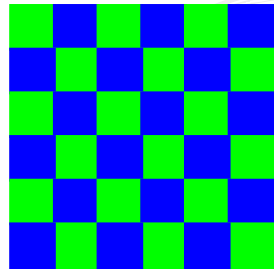
- Merging is not dual to splitting even if the same homogeneity criterion is used
- Figure: Splitting and merging applied to a chess board. A region is homogeneous, if the average gray-value is 0.5.
- Splitting: The mean value of the overall image is 0.5. Hence the initial region is already homogeneous.
- Merging: The gray value of a each individual field is either 0 or 1. Hence, the homogeneity criterion is never satisfied.



source



splitting



merging

Split-and-Merge Algorithm:

In many cases a combination of splitting and subsequent merging is applied
Data structure employed is often a quad-tree (*split and merge*):

- 1 Define an initial segmentation into regions, a criterion for homogeneity and a pyramidal data structure
- 2 In case a region in the data structure is not homogeneous, it is split into its four children regions; In case four regions corresponding to the same parent can be merged, they are merged. If no further region can be processed, GOTO 3)
- 3 In case two neighbouring regions (either in different levels of the pyramid or with different parent nodes) can be merged according the criterion for homogeneity they are merged
- 4 Regions being too small are merged with the most similar neighbouring region

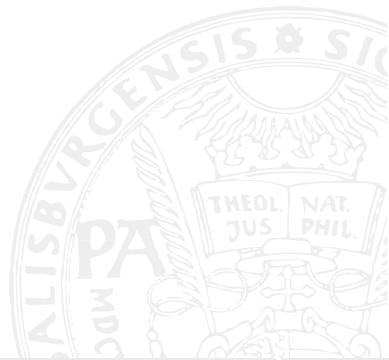
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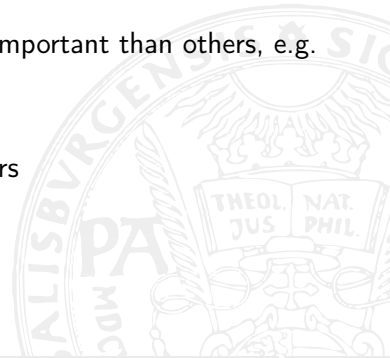
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Idea: Image segmentation can be viewed as a clustering task. “Similar” pixels should be clustered into one region

Procedure:

- 1 Represent each pixel in the image with a vector (e.g. intensity, colour and location, texture descriptors, etc.)
- 2 Choose distance weights (which vector component is more important than others, e.g. color vs. location)
- 3 Apply k-means clustering
- 4 Pixels belong to the segment corresponding to cluster centers



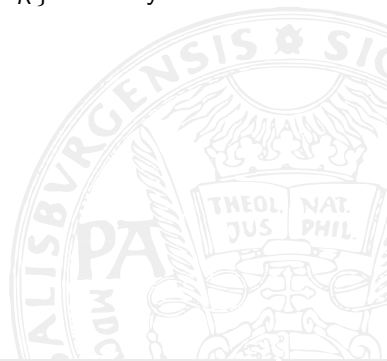
K-means Clustering Algorithm:

Let $\{x_1, \dots, x_n\}$ be a set of pixels (observations), where each observation x is a vector and has the form $x = [x_1, \dots, x_D]^T$. For instance, in case of a grayscale image, x is a scalar.

The aim is to partition the n pixels into K sets $S = \{S_1, S_2, \dots, S_K\}$ in a way that the **within-cluster sum of squares (WCSS)** is minimized.

$$\arg \min_S \sum_{i=1}^k \sum_{x_j \in S_i} \|x_j - \mu_i\|^2$$

where μ_i the mean or centroid of pixels in cluster S_i



Lloyd Algorithm

- 1 Specify an initial set of means μ_i .
- 2 Assign samples to clusters: Each sample should be assigned to the cluster set whose mean is the closest.

$$S_i^{(t)} = \{x_j : \|x_j - m_i^{(t)}\|^2 \leq \|x_j - m_{\hat{i}}^{(t)}\|^2 \text{ for all } \hat{i} = 1, \dots, k\}$$

- 3 Update the cluster centers (means)

$$m_i^{(t+1)} = \frac{1}{|S_i^{(t)}|} \sum_{x_j \in S_i^{(t)}} x_j$$

- 4 Repeat (2) and (3) until the WCSS is less than certain threshold T .



Figure: Left - Original image. Right - K-Means clustered image with $k = 3$. Clustering was done solely based on the intensity.

- Finding the optimal solution is an **NP-hard problem**. When $T = 0$, the algorithm is known to converge in a finite number of iterations to a local minimum. It is not guaranteed to yield the global minimum.
- In practical applications, heuristic approximations are used
- "Stability" Test: Run Lloyd's algorithm multiple times using different random initializations for the cluster means each time.
- Lloyd's algorithm has a complexity of $\mathcal{O}(knT)$ with k is the number of clusters, n is the number of data points and T is the number of iterations

