Assignment Report of ADSP Part 2

Liu Xiangyu G1802061L

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Q1.a) Given,

$$x[n] = s[n] + w[n] \tag{1}$$

$$s[n] = 0.8s[n-1] + v[n] \tag{2}$$

from eq.(2), the power spectral density of s[n],

$$\begin{aligned}\Gamma_{ss}(z) &= \sigma_v^2 H(z)H(z^{-1}) \\ &= \sigma_v^2 \tfrac{1}{1-0.8z^{-1}} \tfrac{1}{1-0.8z}\end{aligned} \tag{3}$$

By taking the inverse Z-transform, the autocorrelation functions

$$\gamma_{ss}[k] = 0.8^{|k|} \tag{4}$$

$$\gamma_{xx}[k] = \gamma_{ss}[k] + \gamma_{ww}[k] = 0.8^{|k|} + \delta[k] \tag{5}$$

b) Solve the following normal equations:

$$\begin{bmatrix} \gamma_{xx}[0] & \gamma_{xx}[-1] \\ \gamma_{xx}[1] & \gamma_{xx}[0] \end{bmatrix} \begin{bmatrix} h[0] \\ h[1] \end{bmatrix} = \begin{bmatrix} \gamma_{ss}[0] \\ \gamma_{ss}[1] \end{bmatrix} \tag{6}$$

$$\begin{bmatrix} 2 & 0.8 \\ 0.8 & 2 \end{bmatrix} \begin{bmatrix} h[0] \\ h[1] \end{bmatrix} = \begin{bmatrix} 1 \\ 0.8 \end{bmatrix} \tag{7}$$

Therefore,

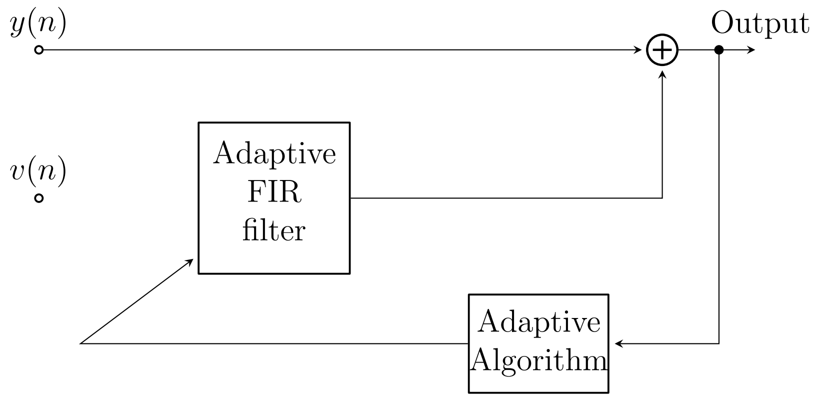
$$h_{opt}[0] = 0.405, h_{opt}[1] = 0.238 \tag{8}$$

c) Hence,

$$\varepsilon_2^h = \min \varepsilon_2^h = \gamma_{ss}[0] - \begin{bmatrix} \gamma_{ss}[0] & \gamma_{ss}[1] \end{bmatrix} \begin{bmatrix} h[0] \\ h[1] \end{bmatrix} = 0.405 \tag{9}$$

Q2.a) A linear adaptive filtering algorithm involves two basic processes: (1) a filtering process designed to produce an output in response to a sequence of input data, and (2) an adaptive process, the purpose of which is to provide a mechanism for the adaptive control of an adjustable set of parameters used in the filtering process

b)



The LMS algorithm is given by,

$$\boldsymbol{h}_M(n+1) = \boldsymbol{h}_M(n) + \mu e(n)\boldsymbol{X}_M^*(n), \quad n = 0, 1, 2, ... \tag{10}$$

where,

1. $\mathbf{h}_M(n)$ is the matrices of estimates of the true coefficients
2. μ is a fixed step size
3. $e(n) = d(n) - \hat{d}(n)$
4. $\boldsymbol{X}_M(n)$ is the set of M signal samples in the filter at the n^{th} iteration.

Therefore, the adaptive FIR filter to estimate the noise $\hat{w}_2(n)$ works as,

$$\hat{w}_2(n) = \sum_{m=0}^{M} h(m)v(n-m) \tag{11}$$

c) The convergence of the mean of the coefficient vector in the LMS algorithm depends on the range of μ,

$$0 < \mu < \frac{2}{\lambda_{max}} \tag{12}$$

where $lambda_{max}$ is the largest eigenvalue of Γ_M,

$$\lambda_{max} < \sum_{k=0}^{M-1} \lambda_k = trace\Gamma_M = M\gamma_{xx}[0] \tag{13}$$

Given the input white noise signal has zero mean and variance w^2,

$$\lambda_{max} < M\gamma_{xx}[0] = M\sigma^2\delta[0] = M\sigma^2 \tag{14}$$

Therefore, the condition for convergence is when the selected fixed step size μ is less than the upper bound,

$$\mu < \frac{2}{M\sigma^2} \tag{15}$$

Q3. Given $x(n) = s(n) + w(n)$,

$$\gamma_{xx}[m] = \gamma_{ss}[m] + \gamma_{ww}[m] = \gamma_{ss}[m] + \sigma_w^2 \delta[m] \tag{16}$$

$$\Gamma_{xx}(z) = \Gamma_{ss}(z) + \Gamma_{ww}(z) = \frac{\sigma_s^2}{|A(z)|^2} + \sigma_w^2 = \frac{\sigma_s^2 + \sigma_w^2 |A(z)|^2}{|A(z)|^2} \tag{17}$$

To model $x(n)$ as ARMA(2,2) process, the power density spectrum of $x(n)$ needs to have the formation:

$$\hat{\Gamma}_{xx}(z) = \hat{\sigma}_x \frac{\hat{B}(z)\hat{B}(z^{-1})}{\hat{A}(z)\hat{A}(z^{-1})} \tag{18}$$

where,

$$\begin{aligned} \hat{A}(z) &= a_0 + a_1 z^{-1} + a_2 z^{-2} \\ \hat{B}(z) &= b_0 + b_1 z^{-1} + b_2 z^{-2} \end{aligned} \tag{19}$$

In this case, since $s(n)$ is given as an AR(2) process, we only need to find $B(z)$ so that,

$$B(z)B(z^{-1}) = \sigma_s^2 + \sigma_w^2 |A(z)|^2 \tag{20}$$

Therefore, let

$$B(z) = c + \sigma_w A(z) \tag{21}$$

where c is a constant to determine. If there exists a value of c that makes $\hat{B}(z)\hat{B}(z^{-1})$ equal to $\sigma_s^2 + \sigma_w^2 |A(z)|^2$, x(n) can be modelled as an ARMA(2,2) process. Based on eq.(21),

$$\begin{aligned} B(z)B(z^{-1}) &= c^2 + c\sigma_w (A(z) + A(z^{-1})) + \sigma_w^2 |A(z)|^2 \\ &= c^2 + 2c\sigma_w Re\{A(z)\} + \sigma_w^2 |A(z)|^2 \end{aligned} \tag{22}$$

Considering eq.(20) equal to eq.(22), the proper c can be obtained,

$$\hat{c} = \sqrt{\sigma_s^2 + \sigma_w^2 Re^2\{A(z)\}} - \sigma_w Re\{A(z)\} \tag{23}$$

With c selected as above, the condition in eq.(20) is satisfied, and x(n) is an ARMA(2,2) process with A(z) given and B(z) as,

$$B(z) = \hat{c} + \sigma_w A(z) \tag{24}$$