Assignment Report of ADSP Part 2

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Q1.a) Given,

$$x[n] = s[n] + w[n] \tag{1}$$

$$s[n] = 0.8s[n-1] + v[n] \tag{2}$$

from eq.(2), the power spectral density of s[n],

$$\begin{aligned} \Gamma_{ss}(z) &= \sigma_v^2 H(z)H(z^{-1}) \\ &= \sigma_v^2 \tfrac{1}{1-0.8z^{-1}} \tfrac{1}{1-0.8z} \end{aligned} \tag{3}$$

By taking the inverse Z-transform, the autocorrelation functions

$$\gamma_{ss}[k] = 0.8^{|k|} \tag{4}$$

$$\gamma_{xx}[k] = \gamma_{ss}[k] + \gamma_{ww}[k] = 0.8^{|k|} + \delta[k] \tag{5}$$

b) Solve the following normal equations:

$$\begin{bmatrix} \gamma_{xx}[0] & \gamma_{xx}[-1] \\ \gamma_{xx}[1] & \gamma_{xx}[0] \end{bmatrix} \begin{bmatrix} h[0] \\ h[1] \end{bmatrix} = \begin{bmatrix} \gamma_{ss}[0] \\ \gamma_{ss}[1] \end{bmatrix} \tag{6}$$

$$\begin{bmatrix} 2 & 0.8 \\ 0.8 & 2 \end{bmatrix} \begin{bmatrix} h[0] \\ h[1] \end{bmatrix} = \begin{bmatrix} 1 \\ 0.8 \end{bmatrix} \tag{7}$$

Therefore,

$$h_{opt}[0] = 0.405, h_{opt}[1] = 0.238 \tag{8}$$

c) Hence,

$$\varepsilon_2^h = \min \varepsilon_2^h = \gamma_{ss}[0] - \begin{bmatrix} \gamma_{ss}[0] & \gamma_{ss}[1] \end{bmatrix} \begin{bmatrix} h[0] \\ h[1] \end{bmatrix} = 0.405 \tag{9}$$

Q2.a) A linear adaptive filtering algorithm involves two basic processes: (1) a filtering process designed to produce an output in response to a sequence of input data, and (2) an adaptive process, the purpose of which is to provide a mechanism for the adaptive control of an adjustable set of parameters used in the filtering process.
b) The implementation of the adaptive noise canceller is demonstrated in fq.(1).

The LMS algorithm is given by,

$$\boldsymbol{h}_M(n+1) = \boldsymbol{h}_M(n) + \mu e(n) \boldsymbol{X}_M^*(n), \quad n = 0, 1, 2, ... \tag{10}$$

where,

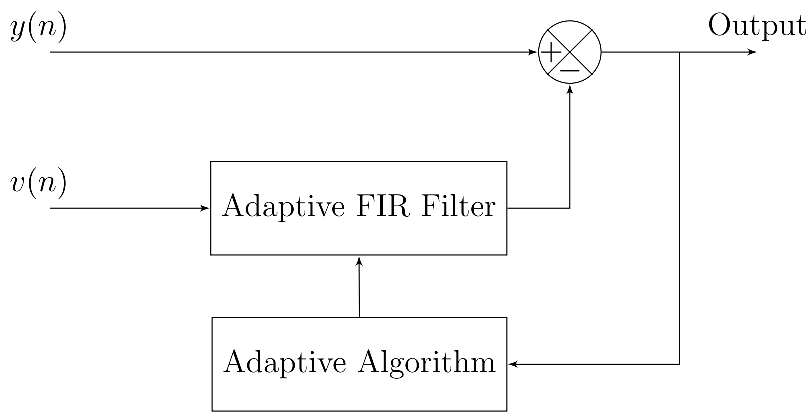

Figure 1: Adaptive Noise Canceller

1. $\mathbf{h}_M(n)$ is the matrices of estimates of the true coefficients,
2. μ is a fixed step size,
3. $e(n) = d(n) - \hat{d}(n)$,
4. $\boldsymbol{X}_M(n)$ is the set of M signal samples in the filter at the n^{th} iteration.

Therefore, the adaptive FIR filter to estimate the noise $\hat{w}_2(n)$ works as,

$$\hat{w}_2(n) = \sum_{m=0}^{M} h_M(m)v(n-m) \tag{11}$$

c) The convergence of the mean of the coefficient vector in the LMS algorithm depends on the range of μ,

$$0 < \mu < \frac{2}{\lambda_{max}} \tag{12}$$

where $lambda_{max}$ is the largest eigenvalue of Γ_M,

$$\lambda_{max} < \sum_{k=0}^{M-1} \lambda_k = trace\Gamma_M = M\gamma_{xx}[0] \tag{13}$$

Given the input white noise signal has zero mean and variance w^2,

$$\lambda_{max} < M\gamma_{xx}[0] = M\sigma^2\delta[0] = M\sigma^2 \tag{14}$$

Therefore, the condition for convergence is when the selected fixed step size μ is less than the upper bound,

$$\mu < \frac{2}{M\sigma^2} \tag{15}$$

Q3. Given $x(n) = s(n) + w(n)$,

$$\gamma_{xx}[m] = \gamma_{ss}[m] + \gamma_{ww}[m] = \gamma_{ss}[m] + \sigma_w^2\delta[m] \tag{16}$$

$$\Gamma_{xx}(z) = \Gamma_{ss}(z) + \Gamma_{ww}(z) = \frac{\sigma_s^2}{|A(z)|^2} + \sigma_w^2 = \frac{\sigma_s^2 + \sigma_w^2 |A(z)|^2}{|A(z)|^2} \tag{17}$$

To model $x(n)$ as ARMA(2,2) process, the power density spectrum of $x(n)$ needs to have the formation:

$$\hat{\Gamma}_{xx}(z) = \hat{\sigma}_x \frac{\hat{B}(z)\hat{B}(z^{-1})}{\hat{A}(z)\hat{A}(z^{-1})} \tag{18}$$

where,

$$\begin{aligned} \hat{A}(z) &= a_0 + a_1 z^{-1} + a_2 z^{-2} \\ \hat{B}(z) &= b_0 + b_1 z^{-1} + b_2 z^{-2} \end{aligned} \tag{19}$$

In this case, since $s(n)$ is given as an AR(2) process, we only need to find $B(z)$ so that,

$$B(z)B(z^{-1}) = \sigma_s^2 + \sigma_w^2 |A(z)|^2 \tag{20}$$

Therefore, let

$$B(z) = c + \sigma_w A(z) \tag{21}$$

where c is a constant to determine. If there exists a value of c that makes $\hat{B}(z)\hat{B}(z^{-1})$ equal to $\sigma_s^2 + \sigma_w^2|A(z)|^2$, x(n) can be modelled as an ARMA(2,2) process. Based on eq.(21),

$$\begin{aligned} B(z)B(z^{-1}) &= c^2 + c\sigma_w(A(z) + A(z^{-1})) + \sigma_w^2|A(z)|^2 \\ &= c^2 + 2c\sigma_w Re\{A(z)\} + \sigma_w^2|A(z)|^2 \end{aligned} \tag{22}$$

Considering eq.(20) equal to eq.(22), the proper c can be obtained,

$$\hat{c} = \sqrt{\sigma_s^2 + \sigma_w^2 Re^2\{A(z)\}} - \sigma_w Re\{A(z)\} \tag{23}$$

With c selected as above, the condition in eq.(20) is satisfied, and x(n) is an ARMA(2,2) process with A(z) given and B(z) as,

$$B(z) = \hat{c} + \sigma_w A(z) \tag{24}$$