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Bilateral Control of the Degree of Connectivity in Multiple Mobile-Robot Teleoperation

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Abstract— **This paper presents a novel bilateral controller that allows to stably teleoperate the degree of connectivity in the mutual interaction between a remote group of mobile robots considered as the slave-side. A distributed leader-follower scheme allows the human operator to command the overall group motion. The group autonomously maintains the connectivity of the interaction graph by using a decentralized gradient descent approach applied to the Fiedler eigenvalue of a properly weighted Laplacian matrix. The degree of connectivity, and then the flexibility, of the interaction graph can be finely tuned by the human operator through an additional bilateral teleoperation channel. Passivity of the overall system is theoretically proven and extensive human/hardware in-the-loop simulations are presented to empirically validate the theoretical analysis.**

I. INTRODUCTION

The use of a group of simple robots rather than a single complex robot has proven to be very effective in several applications like surveillance of large perimeters, search and rescue in disaster regions and exploration of wide areas. As a consequence, over the last years a considerable amount of research efforts has been devoted to the problem of coordinating a group of agents (see, e.g. [1] for a survey). Nevertheless, when the tasks become extremely complex and high-level cognitive-based decisions are required online, complete autonomy is still far from being reached. In this context, teleoperation systems, where a human operator commands a remote robot through a local interface, can be used to exploit human's intelligence.

The problem of teleoperating a group of agents is receiving a lot of attention in the robotics community. In [2] a centralized control strategy for teleoperating multiple fixed slaves from multiple masters has been proposed, and in [3] a different approach for controlling multiple wheeled robots can be found. Finally, in [4] a bilateral control strategy for controlling a group of multiple mobile agents is shown. The main drawback of these strategies relies in the "rigidity" of the group of agents that are not allowed, e.g., to actively reshape when facing a particularly cluttered environment.

In [5], [6] a bilateral teleoperation system for controlling a remote group fo mobile robots in a flexible and decentralized way has been proposed. Nevertheless, connectivity maintenance was not considered, and during the task a part of the group could possibly detach from the component driven by the user. A passivity-based bilateral architecture for teleoperating a group of mobile robots in a flexible and decentralized way while preserving the connectivity of the group has instead been developed in [7]. The group interaction is modeled as a weighted graph and connectivity is maintained by keeping the second smallest eigenvalue λ_2 of the graph Laplacian, also known as algebraic connectivity, greater than a predefined minimum threshold $\lambda_2^{\min} > 0$. This is achieved by means of a gradient descent control action paired with a distributed estimation strategy for recovering the global quantity λ_2, by in particular employing the methodology introduced in [8], [9]. In this way the group is not constrained to maintain a (given) fixed connected topology, but it can freely switch among any of the connected ones. Furthermore, collision avoidance and formation control are both embedded in the definition of connectivity, and are enforced by the sole connectivity maintenance action. Finally, the overall teleoperation system can be shown to be passive, i.e., a sufficient condition for guaranteeing a stable behavior of the system.

The main drawback of the approach proposed in [7] is the fact that the minimum connectivity threshold $\lambda_2^{\min}$ is required to stay constant over time. This negatively affects the flexibility of the system. In fact $\lambda_2^{\min}$ represents a measure of the *minimum degree of connectivity* of the group. The lower $\lambda_2^{\min}$, the more dispersed the group and, vice-versa, the higher $\lambda_2^{\min}$ the more compact the group. Thus, even if, by using the approach proposed in [7], connectivity is always maintained, the possibility of changing online the value of $\lambda_2^{\min}$ would provide the user with an even better control of the behavior of the group. For example the group could be constrained to either move with a compact formation in free space, hence ensuring higher responsiveness, or to be more dispersed when navigating through cluttered environments, thus possessing higher adaptability. In other words, controlling the degree of connectivity would provide an additional degree of maneuverability to the group.

The goal of this paper is to extend the teleoperation architecture proposed in [7] in order to allow the user to bilaterally teleoperate the group and the parameter $\lambda_2^{\min}$ in a decentralized way. Besides specifying the desired value of $\lambda_2^{\min}$, the user will also be able to feel a force feedback informative of the discrepancy between the desired $\lambda_2^{\min}$ and the value actually implemented by the connectivity control action. At the best of our knowledge, this is the first time

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that a quantity linked to the connectivity is used to control the swarming behavior of the group. We believe that this is a very powerful way of controlling a group of robots since it can be implemented in a decentralized way and it does not require the typical stringent conditions on the group topology needed by standard controllability approaches (see e.g. [10]).

The paper is organized as follows: in Sec. II a short background on the approach proposed in [7] is given and in Sec. III a passive way of tuning the parameter $\lambda_2^{\min}$ is illustrated. In Sec. IV the overall teleoperation architecture is shown and in Sec. V the theoretical results of the paper are validated by means of human-hardware-in-the-loop simulations. Finally, in Sec. VI some conclusions and future work are discussed.

II. THE FRAMEWORK

In this section the passivity based decentralized control architecture proposed in [7] is briefly summarized. Each mobile robot is modeled as a floating mass in $\mathbb{R}^3$ and the slave side consists of a group of N agents among which a leader is chosen. The motion of each robot depends on the motion of the surrounding vehicles, while the motion of the leader depends also on its coupling with the master. In order to consider the difference between the bounded workspace of the master and the unbounded workspace of a robot, the position of the master is treated as a velocity setpoint for the leader at the slave side and the difference between the position of the master and the velocity of the leader generates a force at the master side for feeding back to the user an information about the motion status of the group.

A. The Master Side

The master is modeled as a generic fully actuated mechanical system described by the following Euler-Lagrange equations:

$$M_M(x_M)\ddot{x}_M + C_M(x_M, \dot{x}_M)\dot{x}_M + B_M\dot{x}_M = F_M \quad (1)$$

where $M_M(x_M)$ represents the inertia matrix, $C(x_M, \dot{x}_M)\dot{x}_M$ is a term representing the centrifugal and Coriolis effects, and B_M is a matrix representing both the viscous friction present in the system and any additional damping injection via local control actions. Gravity is assumed to be locally compensated. The variables x_M and $v_M := \dot{x}_M$ represent the position and the velocity of the end-effector. In order to passively couple the position of the master with the velocity of the leader, in [5], [11] we proposed a local control loop that renders the master passive with respect to the pair (F_M, r_M) and with a storage function $\mathcal{R}_M = \frac{1}{2}r_M^T r_M$, where

$$r_M = \rho v_M + \rho\sigma x_M, \qquad \rho > 0,\ \sigma > 0. \quad (2)$$

By properly choosing the design parameters ρ and σ, it is possible to reduce the contribution of v_M and to make the second term equal to Kx_M, where K is a desired scaling factor. Thus, the r_M variable approximates the (scaled) position of the master and, during fast motions, the contribution of the velocity term ρv_M can be rendered as small as desired.

B. The Slave Side

The robots are assumed to be endowed with a Cartesian trajectory tracking controller (as, for instance, in the case of a UAV, the one proposed in [12]) ensuring a closed loop behavior close enough to that of a fully actuated floating mass in $\mathbb{R}^3$. We then model each agent, and its local control structure, as:

$$\begin{cases} \dot{p}_i = F_i^\lambda + F_i^e - w_i x_{t_i} - B_i M_i^{-1} p_i \\ \dot{x}_{t_i} = \alpha_i \frac{1}{x_{t_i}} D_i + w_i^T v_i \\ y_i = \left(v_i^T \ x_{t_i}\right)^T \end{cases} \quad (3)$$

where $p_i \in \mathbb{R}^3$ and $M_i \in \mathbb{R}^{3\times 3}$ are the momentum and the inertia matrix of the i^{th} agent, $\mathcal{K}_i = \frac{1}{2}p_i^T M_i^{-1} p_i$ is the kinetic energy stored by the agent and B_i is a positive definite matrix representing the dissipation in the system, either naturally present or artificially added for stabilization purposes. Forces $F_i^e \in \mathbb{R}^3$ and $F_i^\lambda \in \mathbb{R}^3$ model the interaction with the external world and with the other agents respectively. The power dissipated by the i-th agent

$$D_i := p_i^T {M_i^{-1}}^T B_i M_i^{-1} p_i, \quad (4)$$

is monitored and stored into an energy storing element called *tank*, characterized by an energy function $T_i = \frac{1}{2}x_{t_i}^2$. The quantity $\alpha_i \in \{0, 1\}$ in (3) is a control parameter used to disable/enable the storage of D_i into the tank. In fact, because of the reasons reported in [13], the energy stored in the tanks needs to be upper bounded in order to avoid the implementation of practically unstable behaviors. Thus, α_i is set to 0 if the energy stored in the tank reaches an upper bound $\bar{T}_i$ to be selected depending on the particular application. We also set a small threshold $\epsilon > 0$ below which energy extraction from the tank is prevented, in order to avoid singularities in (3) (i.e., preventing $x_{t_i} \to 0$).

Vector $w_i \in \mathbb{R}^3$ is an additional input that can be used to realize an exchange of energy between the tank and the agent. Finally, the outputs of the overall system (3) are the velocity of the agent $v_i = M_i^{-1} p_i$ and the tank state x_{t_i}. The energy stored in the tank represents a sort of "passivity margin", namely the quantity of energy available for implementing non-passive actions without violating the passivity of the overall system.

Two agents are able to communicate, to measure their relative position, and ultimately to *interact* if and only if they are *neighbors* according to the following definition:

Definition 1: Agents i and j are *neighbors* if and only if: *i)* their relative distance d_{ij} is less than $D \in \mathbb{R}^+$ (the *sensing range*) and larger than $d_{\min} \in [0, D)$ (the *safety range*), *ii)* their line-of-sight is not occluded by an obstacle, and *iii)* neither i nor j are closer than $d_{\min}$ to any other agent.

Let $x_{ij} := x_i - x_j$ be the relative position of agent i wrt agent j and group all the relative positions in $x_{\mathrm{R}} = (x_{12}^T \ \ldots \ x_{1N}^T \ x_{23}^T \ \ldots \ x_{2N}^T \ \ldots \ x_{N-1N}^T)^T \in \mathbb{R}^{\frac{3N(N-1)}{2}}$. Similarly, given M obstacle points in the environment, vector x_{O} represents the relative position of the line of sights of the agents and the obstacles (see [7] for a more detailed

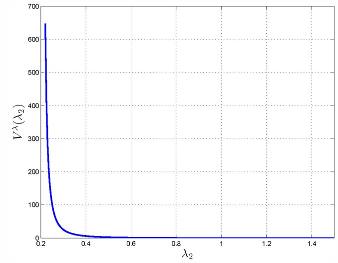


Fig. 1. An example of connectivity potential. $\lambda_2^{\min} = 0.2$ and $\Delta = 1$. This function can be obtained by joining an hyperbolic function to 0 using a smooth bump function.

definition). The interagent coupling is then implemented through a lower bounded connectivity potential function $V^\lambda(\cdot) \in \mathbb{R}^+$. This elastic coupling takes into account connectivity maintenance, collision avoidance among the agents and obstacle avoidance.

Let $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ be the graph formed by the agents together with their neighboring relationship, where $\mathcal{V} = \{1, \ldots, N\}$ is the set of agents and $(i, j) \in \mathcal{E}$ iff $j \in \mathcal{N}_i$. Let A_{ij} be the (i, j)-th entry of the Adjacency matrix $A \in \mathbb{R}^{N \times N}$ associated to the graph $\mathcal{G}$. For each $(i, j) \in \mathcal{E}$, a state dependent weight A_{ij} is computed: A_{ij} encodes information about the relative distance between the agents i and j, possible line-of-sight occlusions and possible collisions with agents or obstacles. Furthermore, it can be evaluated using only local and 1-hop information by agents i and j. In [7] it is shown that, by implementing the unique action of preserving the connectivity of graph $\mathcal{G}$, defined in terms of the weights A_{ij}, one obtains multiple results: (i) the 'physical connectivity' of the group (due to actual sensing/communication limitations) is maintained, (ii) a formation control keeping interdistances is implemented, and (iii) collisions among the agents and with obstacles are mandatorily avoided.

Let $\mathcal{L}$ be the Laplacian of $\mathcal{G}$ and λ_2 be the second smallest eigenvalue of $\mathcal{L}$. The graph $\mathcal{G}$ is connected iff $\lambda_2 > 0$ [8]. The connectivity potential is then defined as a smooth non-negative function of λ_2, $V^\lambda : (\lambda_2^{\min}, \infty) \to \mathbb{R}^+$, that grows unbounded as $\lambda_2 \to \lambda_2^{\min} > 0$ and vanishes (with vanishing derivative) for $\lambda_2 = \lambda_2^{\min} + \Delta$, $\Delta > 0$. An example of a possible V^λ is shown in Figure 1. Recalling that λ_2 is a function of the weights and that the weights depend on the state, connectivity is maintained by implementing a gradient descent of V^λ that can be written as (see, again [7]):

$$F_i^\lambda = \sum_{j \in \mathcal{N}_i} \left(\frac{\partial V^\lambda}{\partial x_{o_{ij}}} + \frac{\partial V^\lambda}{\partial x_{ij}} \right) = \frac{\partial V^\lambda}{\partial \lambda_2} \sum_{j \in \mathcal{N}_i} \left(\frac{\partial \lambda_2}{\partial x_{ij}} + \frac{\partial \lambda_2}{\partial x_{o_{ij}}} \right). \tag{5}$$

By implementing (5) one obtains that λ_2 will never become smaller than $\lambda_2^{\min} > 0$, and F_i^λ will vanish when the group beomes "connected enough", namely when $\lambda_2 > \lambda_2^{\min} + \Delta$.

The eigenvalue λ_2 and its associated eigenvector ν_2 are global quantities but, exploiting the strategy reported in [8], [9], it is possible to obtain a decentralized estimation of their values which, in turn, allows to implement (5) in fully distributed way. The use of the estimate of λ_2 for implementing F_i^λ can potentially lead to a loss of passivity. This problem is tackled by exploiting the energy stored in the tanks. Thus, the overall dynamics of the slave side can be modeled as:

$$\begin{cases} \begin{pmatrix} \dot{p} \\ \dot{x}_R \\ \dot{x}_O \\ \dot{x}_t \end{pmatrix} = \left[\begin{pmatrix} 0 & \mathcal{I} & -\mathbb{I} & \Upsilon \\ -\mathcal{I}^T & 0 & 0 & 0 \\ \mathbb{I}^T & 0 & 0 & 0 \\ -\Upsilon^T & 0 & 0 & 0 \end{pmatrix} - \begin{pmatrix} B & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -\alpha P B & 0 & 0 & 0 \end{pmatrix} \right] \nabla \mathcal{H} \\ +GF^{\mathrm{e}} = [J - R] \nabla \mathcal{H} + GF^{\mathrm{e}} \\ v = G^T \nabla \mathcal{H} \end{cases} \tag{6}$$

where $p = (p_1^T, \ldots, p_N^T)^T$ and $x_t = (x_{t_1}, \ldots, x_{t_N})^T$. Moreover, $\mathcal{I} = \mathcal{I}_{\mathcal{G}} \otimes I_3$, $\mathbb{I} = I_N \otimes \mathbf{1}_{N-1}^T \otimes I_3$, and $G = \begin{pmatrix} I_N \otimes I_3 & 0 & 0 \end{pmatrix}^T$, with $\mathcal{I}_{\mathcal{G}}$ being the incidence matrix of the graph $\mathcal{G}$ whose edge numbering is induced by the entries of the vector x_R, I_3 and I_N being the identity matrices of order 3 and N respectively, $\mathbf{1}_{N-1}$ a column vector of all ones of dimension $N-1$, 0 representing a null matrix of proper dimensions, and $\otimes$ denoting the Krönecker product. Finally, the system exchanges energy through the port (F^e, v) with the external world, where $F^e = (F_1^{e^T}, \ldots, F_N^{e^T})^T$ and $v = (v_1^T, \ldots, v_N^T)^T$. $B = diag(B_1, \ldots, B_N)$, $\Upsilon = diag(-w_i)$, $P = diag(\frac{1}{x_{t_i}} p_i^T M_i^{-T})$. Details on the design of Υ can be found in [7]. The overall system is passive with respect to the total energy function

$$\mathcal{H} = \sum_{i=1}^{N} (\mathcal{K}_i + T_i) + V^\lambda. \tag{7}$$

C. Master-Slave Coupling

Let the agent l be the leader. It is possible to decompose $F_l^{\mathrm{e}} = F_s + F_l^{\mathrm{env}}$, where F_l^{env} is the component of the force due to the interaction with the external environment (obstacles) and F_s is the component due to the interaction with the master side. Similarly, we can decompose F_M in (1) as $F_M = F_m + F_h$, where F_h is the component due to the interaction with the user and F_m is the force acting on the master because of the interaction with the slave.

For achieving the desired teleoperation behavior, master and slave sides are joined using the following interconnection:

$$\begin{cases} F_s = -b_T(v_l - r_M) \\ F_m = b_T(v_l - r_M) \end{cases} \tag{8}$$

where $b_T > 0$ is a design parameter. This is equivalent to joining the master and the leader using a damper which generates a force proportional to the difference of the two velocity-like variables of the master and the leader. Since r_M is "almost" the position of the master, the force fed back to the master and the control action sent to the leader are the desired ones. The complete teleoperation system, consists then of the interconnection of a passive master side, a passive interconnection and a passive slave side. Recalling that the interconnection of passive systems is again passive [14], we can conclude that the teleoperation system is passive w.r.t. the pair $((F_h^T, F^{env^T})^T, (r_M^T, v^T))^T$ where

$$F^{env} = \begin{pmatrix} F_1^{e^T} & \ldots & F_l^{env^T} & \ldots & F_N^{e^T} \end{pmatrix}^T. \tag{9}$$

III. PASSIVITY–PRESERVING TUNING OF $\lambda_2^{\min}$

The value of the parameter $\lambda_2^{\min}$ defining the location of the vertical asymptote of the connectivity potential V^λ (see Fig. 1) represents the *minimum degree of connectivity* allowed for the group. A large $\lambda_2^{\min}$ forces the group to evolve with a high connectivity level and, therefore, the agents to stay very close to each other to form a compact swarm. On the other hand, a small $\lambda_2^{\min}$ allows the group to evolve with a low connectivity level, making it possible for the agents to spread in the environment like a more "fluid body".

Therefore, changing the value of $\lambda_2^{\min}$ online would represent an important feature, as it would allow the user to gain control over the "degree of fluidity" of the group as the task evolves. Unfortunately, from a theoretical standpoint, freely changing a parameter of a potential energy function, such as V^λ, can threaten passivity and, consequently, lead to a potentially unstable teleoperation system [15]. The goal of this section is then to devise a passivity-preserving mechanism for allowing presence of a time-varying $\lambda_2^{\min}(t)$ while guaranteeing closed-loop passivity of the slave-side (the group of agents).

To this end, let $\lambda_m > 0$ be a *constant* parameter specified by the user at the beginning of the task. This value represents a fixed and pre-defined lower-bound for the minimum degree of connectivity $\lambda_2^{\min}(t)$ (i.e., the maximum degree of dispersion) that is allowed for the group of agents at time t. Due to the shape of the potential V^λ, $\lambda_2^{\min}(t)$ is also necessarily upper bounded by the current value $\lambda_2(t)$: in fact, V^λ is not defined for $\lambda_2 = \lambda_2^{\min}$, and no connectivity action is implemented if $\lambda_2 < \lambda_2^{\min}$. Summarizing, at any time t the following inequalities must hold:

$$\lambda_m \leq \lambda_2^{\min}(t) < \lambda_2(t). \tag{10}$$

In order to allow for changes in $\lambda_2^{\min}(t)$ while complying with these bounds, we define $\lambda_2^{\min}(t)$ as a function of a real parameter $\mu(t)$, called *increment*, in the following way:

$$\lambda_2^{\min}(\mu(t)) = \lambda_m + \mathrm{sat}_{[0,\lambda_2(t)-\lambda_m-\eta]}(\mu(t)) =: \lambda(\mu(t)), \tag{11}$$

where $\mathrm{sat}_{[0,\lambda_2(t)-\lambda_m-\eta]}$ is a linear saturation with 0 and $\lambda_2(t) - \lambda_m - \eta$ being its lower and upper limits, where $\eta > 0$ is an arbitrarily small positive number such that $\lambda_2(t) - \lambda_m - \eta > 0$. The introduction of η is necessary for coping with the strict inequality in the rhs. of (10).

We then denote with $V^{\lambda(\mu)}$ and with $F_i^{\lambda(\mu)}$ the connectivity potential and the connectivity control action evaluated using $\lambda_2^{\min}(\mu)$ as given in (11). Therefore, for example, $V^{\lambda(0)}$ will grow unbounded as $\lambda_2 \to \lambda_2^{\min}(0) = \lambda_m$, and similarly for other values of μ. We will now show how to use the energy stored in the tanks for passively modifying the connectivity control action in presence of a time-varying $\lambda_2^{\min}(t)$, i.e., when $\mu(t)$ changes over time.

First of all, we augment the dynamics of each agent with an extra variable $\mu_i \in \mathbb{R}$

$$\begin{cases} \dot{p}_i = F_i^{\lambda(0)} + F_i^e - w_i x_{t_i} - B_i M_i^{-1} p_i \\ \dot{x}_{ti} = \alpha_i \frac{1}{x_{t_i}} D_i + w_i^T v_i \\ \dot{\mu}_i = u_i \\ y_i = \begin{pmatrix} v_i^T & x_{t_i} & \mu_i \end{pmatrix}^T \end{cases}, \tag{12}$$

and add a 'baseline' interaction force $F_i^{\lambda(0)}$ in (12) to the agent dynamics: this guarantees maintenance of the minimum degree of connectivity represented by $\lambda_2^{\min} = \lambda_m \equiv const$. On the other hand, the state μ_i represents the i-th agent estimation of the current value μ, i.e., the desired increment with respect to λ_m.

Let $u_{lH} \in \mathbb{R}$ be an external input for the leader agent in charge of regulating the value of $\mu(t)$ over time. Input u_{lH} can either represent the action of an external higher-level planner influencing the leader, or it can represent, as in our case, an additional degree of freedom for the human operator teleoperating the swarm. In order to propagate the effect of input u_{lH} from the user to *all* the agents in a decentralized way, we design a consensus law over the states μ_i, $i = 1 \dots N$:

$$\begin{cases} u_i = -\sum_{j\in\mathcal{N}_i}(\mu_i - \mu_j) \quad i = 1, \dots, N \; i \neq L \\ u_l = -\sum_{j\in\mathcal{N}_l}(\mu_i - \mu_j) + u_{lH} \end{cases}. \tag{13}$$

At this point, the i-th agent can passively implement the desired connectivity control action $F_i^{\lambda(\mu_i)}$ evaluated upon the current (and locally available) μ_i, by exploiting the energy stored in its tank by setting

$$w_i = \begin{cases} \frac{1}{x_{ti}}(-F_i^{\lambda(0)} + F_i^{\lambda(\mu_i)}) & \text{if} \quad T(x_{t_i}) > \epsilon \\ 0 & \text{if} \quad T(x_{t_i}) \leq \epsilon \end{cases}. \tag{14}$$

Plugging (14) into (12) yields the following result: if the tank has enough energy for implementing the desired control action $F_i^{\lambda(\mu_i)}$ (i.e., if $T(x_{t_i}) > \epsilon$), then $F_i^{\lambda(0)}$ is replaced by $F_i^{\lambda(\mu_i)}$ in (12). If the tank is depleted (i.e., if $T(x_{t_i}) \leq \epsilon$), then the 'baseline' force $F_i^{\lambda(0)}$ is implemented in (12) and $F_i^{\lambda(\mu_i)}$ discarded.

Considering the augmented dynamics (12), the overall slave side is now represented by:

$$\begin{cases} \begin{pmatrix} \dot{p} \\ \dot{x}_{\mathcal{R}} \\ \dot{x}_{\bullet} \\ \dot{x}_t \\ \dot{\mu} \end{pmatrix} = \left[\begin{pmatrix} 0 & \mathcal{I} & -\mathbb{I} & \Upsilon & 0 \\ -\mathcal{I}^T & 0 & 0 & 0 & 0 \\ \mathbb{I}^T & 0 & 0 & 0 & 0 \\ -\Upsilon^T & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} - \begin{pmatrix} B & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -\alpha PB & 0 & 0 & 0 \\ 0 & 0 & 0 & \mathcal{L} \end{pmatrix} \right] \\ \nabla \mathcal{H}_a + G_a \begin{pmatrix} F^e \\ u_{lH} \end{pmatrix} \\ \begin{pmatrix} v \\ \mu_l \end{pmatrix} = G_a^T \nabla \mathcal{H}_a \end{cases} \tag{15}$$

where $\mu = (\mu_1, \dots \mu_N)^T$, $G_a = diag(G\ g)$, where $g \in \mathbb{R}^N$ $g(i) = 1$ if $i = l$ and $g(i) = 0$ otherwise, and matrix $\mathcal{L}$ is the Laplacian of $\mathcal{G}$. Furthermore:

$$\mathcal{H}_a = \mathcal{H} + \frac{1}{2}\mu^T\mu = \mathcal{H} + \mathcal{U} \tag{16}$$

and all the other quantities are the same as defined in (6).

Proposition 1: The augmented slave side represented in (15) is passive with respect to the pair

$((F^{e^T}, u_{lH})^T, (v^T, \mu_l)^T)$ with the the positive storage function $\mathcal{H}_a$.

Proof: Let $x = (p^T, x_R^T, x_O^T, x_t^T)^T$ be a vector grouping the state variables of the non augmented slave side. Using the notation introduced in (6), (15) can be rewritten as:

$$\begin{cases} \dot{x} = [J - R]\,\nabla\mathcal{H} + GF^{\mathrm{e}} \\ \dot{\mu} = -\mathcal{L}\nabla\mathcal{U} + gu_{lH} \\ v = G^T\nabla\mathcal{H} \\ \mu_l = g^T\nabla U \end{cases} \tag{17}$$

The dynamics of (6) is passive with respect to $\mathcal{H}$ and the pair (F^e, v) and, following the proof given in [7], we have that:

$$\dot{\mathcal{H}} \leq v^T F^e \tag{18}$$

Furthermore, we have that:

$$\dot{\mathcal{U}} = -\mu^T\mathcal{L}\mu + \mu^T g u_l \leq \mu_l u_{lH} \tag{19}$$

where the inequality follows form the positive semi-definiteness of $\mathcal{L}$. By combining (18) and (19) we obtain:

$$\dot{\mathcal{H}} + \dot{\mathcal{U}} \leq v^T F^e + \mu_l u_{lH} \tag{20}$$

thus proving the statement. ■

Passivity guarantees a stable behavior of the group. Thanks to the structure of (12) it is also possible to guarantee the connectivity of the slave side:

Proposition 2: Consider the slave side represented by (15) and assume that

$$\int_0^t v^T F^e d\tau < E, \quad E > 0, \tag{21}$$

namely that only a finite amount of energy can be injected in the system. The following statements hold:

(i) If the group of agents is initially connected, i.e. $\lambda_2(0) > \lambda_m$, then $\lambda_2(t) > \lambda_m$ $\forall t \geq 0$.

(ii) If:

(a) $\mu_1 = \mu_2 = \cdots = \mu_N = \bar{\mu}$ for $t \geq t_1$

(b) $\lambda_2(t_1) > \bar{\mu} + \lambda_m$ for $t_1 > 0$

(c) While implementing (14), $T_i(x_{t_i})(t) > \epsilon$ $\forall t > t_1$ $i = 1, \ldots, N$

then $\lambda_2(t) > \bar{\mu} + \lambda_m$ $\forall t \geq t_1$.

Proof: From (18), we have that $\dot{\mathcal{H}} \leq v^T F^e$ which implies that

$$\mathcal{H}(t) \leq \mathcal{H}(0) + \int_0^t v^T F^e d\tau \tag{22}$$

From (7), since $T(t) \geq 0$ and $\mathcal{K}(t) \geq 0$ $\forall t > 0$, we can write

$$0 \leq V^{\lambda(0)}(t) \leq H(0) + \int_0^t v^T F^e d\tau \tag{23}$$

Since only a finite amount of energy can be injected in the system, we can rewrite (23) as

$$0 \leq V^{\lambda(0)}(t) \leq H(0) + E \tag{24}$$

where the first inequality follows from the definition of $V^{\lambda(0)}$. Thus, because of the bound reported in (24), if the system starts from a configuration such that $\lambda_2 > \lambda_m$ then its evolution if characterized by a value of $\lambda_2(t)$ such that $V^{\lambda(0)} \leq H(0) + E$. Thus, since $V^{\lambda(0)}$ grows unbounded as $\lambda_2 \to \lambda_m$, we have that $\lambda_2(t) > \lambda_m > 0$ which implies that connectivity is always maintained.

Consider now the second statement. Because of assumption (c), the tanks always contain a sufficient amount of energy for implementing (14). Thus:

$$V^{\lambda(0)}(t) + T(t) \geq V^{\lambda(\bar{\mu})}(t) \forall t \geq t_1 \tag{25}$$

where the inequality follows from the fact that only part of the energy in the tanks is sufficient for implementing the new connectivity potential. From (25) and (24) it follows that

$$V^{\lambda(\bar{\mu})}(t) \leq H(0) + E \tag{26}$$

Exploiting the same argument used for the first statement and considering that $V^{\lambda(\bar{\mu})}$ grows unbounded when $\lambda_2 \to \bar{\mu} + \lambda_m$, we can conclude that if $\lambda_2(t_1) > \bar{\mu} + \lambda_m$ then $\lambda_2(t) > \bar{\mu} + \lambda_m$ $\forall t \geq t_1$. ■

The assumption reported in (21) is naturally satisfied. When teleoperating the group of agents, at steady state all the tanks will be full and no energy will be stored anymore. In this situation, all the energy injected into the system is dissipated by the local dampers, and the injected energy does not grow any more. Assumption (a) requires that a steady state is reached among the agents. In Sec. IV we will show that this actually happens and that $\bar{\mu}$ corresponds to the desired value of μ requested by the user. Assumption (b) is necessary for avoiding that the group starts with a degree of connectivity lower than the desired one. Finally, assumption (c) simply requires presence of enough energy in the tanks for always implementing the desired connectivity action. If this is not the case, $F^{\lambda(\bar{\mu})}$ cannot be passively implemented.

When a new desired value of the increment is transmitted from the user to the leader agent, the consensus protocol needs some time in order to propagate this value to all the other agents. During this transient phase, assumption (b) is not satisfied but, thanks to the term $F_i^{\lambda(0)}$ the minimum degree of connectivity ((i.e., $\lambda_2(t) > \lambda_m$) is always maintained.

If assumptions (a), (b) and (c) are satisfied, despite the fact that $V^{\lambda(\bar{\mu})}$ grows unbounded as $\lambda_2 \to \bar{\mu}$, an infinite amount of energy is not necessary for implementing the new connectivity control action. In fact, thanks to (26), the needed control action is always bounded. If more energy than that currently stored in the tank is necessary for implementing $F_i^{\lambda(\mu_i)}$, energy can be harvested increasing the local damping on the agent as shown in [7].

IV. THE TELEOPERATION SYSTEM

The desired increment μ can be set by the user in a bilateral teleoperation setting by using a 1-dof device at the master side called *connectivity master* and modeled as:

$$m_\lambda \ddot{x}_\lambda + b_\lambda \dot{x}_\lambda = \bar{F}_M^\lambda \tag{27}$$

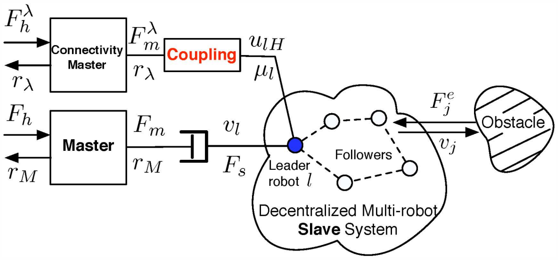


Fig. 2. The overall teleoperation system. Two master devices are interconnected in a passive way to a passive remote side. The coupling block represents the interconnection (30)

where $m_\lambda \in \mathbb{R}^+$, $b_\lambda \in \mathbb{R}^+$ and $x_\lambda \in \mathbb{R}$ represent the inertia, the damping and the position of the device respectively. In order to deal with the difference between the infinite range of values that can be assumed by μ and the finite range of x_λ, we propose to couple the position of the master with the input u_{lH} influencing $\dot{\mu}_l$.

As reported in Sec. II-A it is possible to implement a local control loop that transforms (27) into

$$\dot{r}_\lambda = F_M^\lambda \tag{28}$$

where $r_\lambda = \rho_\lambda \dot{x}_\lambda + \rho_\lambda \sigma_\lambda x_\lambda$. By properly choosing $\rho_\lambda, \sigma_\lambda > 0$, one has $r_\lambda \approx K_\lambda x_\lambda$, where $K_\lambda > 0$ is a desired scaling factor. We decompose F_M^λ into the sum of two contributions $F_M^\lambda = F_h^\lambda + F_m^\lambda$, where F_h^λ represents the force applied by the user to the device and F_m^λ is the force due to the coupling with the slave side. The system in (28) is passive with respect to the pair $(r_\lambda, F_m^\lambda + F_h^\lambda)$ using the positive storage function $\mathcal{R}_\lambda = \frac{1}{2} r_\lambda^2$, In fact we have that:

$$\dot{\mathcal{R}}_\lambda = r_\lambda \dot{r} \leq (F_h^\lambda + F_m^\lambda) r_\lambda \tag{29}$$

In order to allow the user to teleoperate the desired value of $\lambda_2^{\min}$ by acting on μ, and to feel a force feedback related to the tracking error of the desired increment, we couple (28) with the increment dynamics of the leader in the following way:

$$\begin{cases} F_m^\lambda = -b_\lambda (r_\lambda - \mathrm{sat}_{[0,\lambda_2(t)-\lambda_m-\eta]}(\mu_l)) \\ u_{lH} = b_\lambda (r_\lambda - \mu_l) \end{cases} \tag{30}$$

where $b_\lambda > 0$ is a design parameter.

The presence of the saturation function in F_m^λ is necessary to provide a proper force feedback to the user. The user can set any desired value for the increment but it will receive a feedback proportional to the difference between what s/he desire and what is actually implemented at the slave side due to the bounds in (10). The overall teleoperation system is represented in Fig. 2.

Using (30) the overall teleoperation system is passive and the user can assign to all the agents a desired increment using the connectivity master. In fact, the following result holds:

Proposition 3: If the connectivity master reported in (27) is coupled to the slave side represented in (15) through (30), then the overall teleoperation system is passive. Furthermore, if $r_\lambda(t) = \mu \in \mathbb{R}$, then $\mu_i \to \mu$, for $i = 1, \ldots, N$.

Proof: Considering the vector of external forces in (9) and the dissipative coupling (8) between the master and the the slave side we have that:

$$\dot{\mathcal{H}} + \dot{\mathcal{U}} + \dot{\mathcal{R}}_M \leq v^T F^{env} + u_{lH}\mu_l + r_M^T F_h \tag{31}$$

Summing (29) to (31) we obtain:

$$\dot{\mathcal{H}} + \dot{\mathcal{U}} + \dot{\mathcal{R}}_M + \dot{\mathcal{R}}_\lambda \leq v^T F^{ext} + u_{lH}\mu_l + r_M^T F_h + r_\lambda F_h^\lambda + r_\lambda F_m^\lambda \tag{32}$$

Using (30), we can write

$$\begin{aligned} &u_{lH}\mu_l + r_\lambda F_m^\lambda = \\ &= -b_\lambda(\mu_l^2 + r_\lambda^2 - \mu_l r_\lambda - r_\lambda \mathrm{sat}_{[0,\lambda_2(t)-\lambda_m]}(\mu_l)) \leq 0 \end{aligned} \tag{33}$$

The inequality follows from simple algebra. If $0 \leq \mu_l \leq \lambda_2(t) - \lambda_m - \eta$, then $\mathrm{sat}_{[0,\lambda_2(t)-\lambda_m-\eta]}(\mu_l) = \mu_l$ and (33) can be rewritten as $-b_\lambda(\mu_l - r_\lambda)^2 \leq 0$. In the other cases, we can rewrite (33) as

$$-b_\lambda \left[\begin{pmatrix} \mu_l & r_\lambda \end{pmatrix} \begin{pmatrix} 1 & -1 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \mu_l \\ r_\lambda \end{pmatrix} - r_\lambda \mathrm{sat}_{[0,\lambda_2(t)-\lambda_m-\eta]}(\mu_l) \right] \tag{34}$$

The first term within the brackets is always positive since it is a quadratic form associated to a positive definite matrix. If $\mu_l \leq 0$ then $r_\lambda \mathrm{sat}_{[0,\lambda_2(t)-\lambda_m]}(\mu_l) = 0$ and therefore the inequality in (33) holds. If $\mu_l \geq (\lambda_2(t) - \lambda_m - \eta)$ then $r_\lambda \mathrm{sat}_{[0,\lambda_2(t)-\lambda_m]}(\mu_l) = r_\lambda(\lambda_2(t) - \lambda_m - \eta)$. Since $\lambda_2(t) - \lambda_m - \eta > 0$, if $r_\lambda \leq 0$ then $-r_\lambda \mathrm{sat}_{[0,\lambda_2(t)]}(\mu_l) \geq 0$ and (33) holds. Finally, if $r_\lambda > 0$ then $\mu_l^2 + r_\lambda^2 - \mu_l r_\lambda - r_\lambda \mathrm{sat}_{[0,\lambda_2(t)-\lambda_m]}(\mu_l) \geq (\mu_l - r_\lambda)^2 \geq 0$ and, therefore, (33) holds. Thus, using (33) in (32), we have:

$$\dot{\mathcal{H}} + \dot{\mathcal{U}} + \dot{\mathcal{R}}_M + \dot{\mathcal{R}}_\lambda \leq v^T F^{env} + r_M^T F_h + r_\lambda F_h^\lambda \tag{35}$$

which proves the passivity of the overall teleoperation system.

Both the passified connectivity master in (28) and the increment dynamics in (12) are integrators. The graph formed by these integrators is always connected since the graph of the agents at the slave side is connected because of Proposition 2 and since we assume that a coupling between master and leader always exists. Because of the expression of u_{lH} in (30), if $r_\lambda(t) = \mu \in \mathbb{R}$ the system composed by the connectivity master and by the increment dynamics behaves as a set of integrators connected by a consensus protocol where an integrator stays constant. Thus, since a connected undirected graph always admits a spanning tree, if $r_\lambda(t) = \mu$, as shown in [16], then $\mu_i \to \mu$ for all $i = 1, \ldots, N$. ■

If the user takes the connectivity master at a position $r_\lambda = \mu$ corresponding to an increment outside of the saturation bounds, the desired setpoint will be propagated to all the agents which, nevertheless, using the tanks will implement the increment $\mathrm{sat}_{[0,\lambda_2(t)-\lambda_m-\eta]}(\mu_l) > 0$. In this case, the user will feel a force feedback on the connectivity master informing her/him that the requested increment cannot be implemented.

Finally, presence of communication delays between master and slave can also be taken into account by extending the strategy proposed in [6].

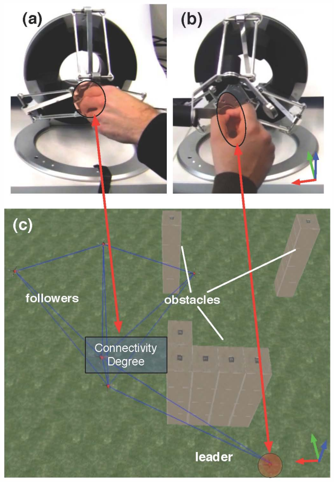


Fig. 3. A screenshot of the simulation environment with 6 UAVs (bottom) and the two haptic devices used as master side (top)

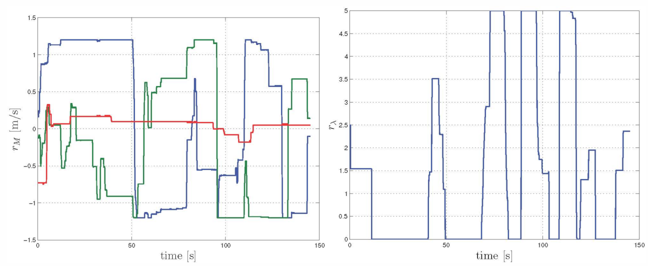
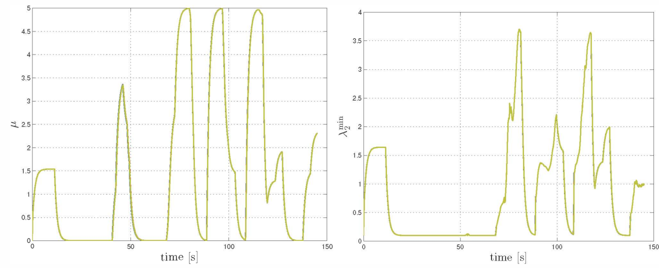
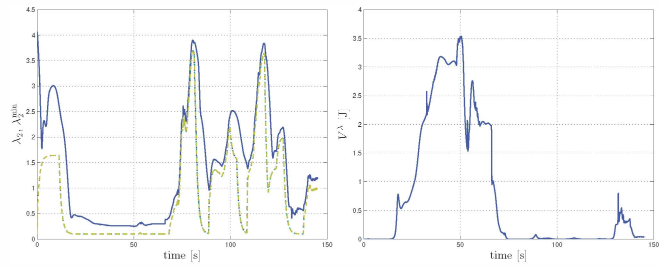


Fig. 4. Three components of the velocity-like command r_M for the leader robot (left) and speed-like command r_λ controlling the degree of connectivity (right).



(a) (b)

Fig. 5. (a) The desired increments $\mu_1 \dots \mu_N$ computed by every robot by implementing the consensus law (13). The N values are almost perfectly superimposed because of the very fast convergence rate of the consensus dynamics. (b) The actual $\lambda_{2,i}^{\min}$ used in the connectivity potentials.



(a) (b)

Fig. 6. (a) The actual $\lambda_{2,i}^{\min}$ used in the connectivity potentials (dashed lines) vs. the current value of λ_2 (solid line). (b) The behavior of $V^{\lambda(0)}(t)$ during the task execution which keeps always bounded, thus confirming that group connectivity was always preserved

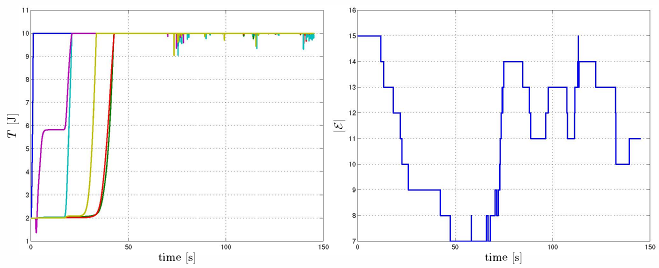
V. HARDWARE IN THE LOOP SIMULATIONS

We report here the results of a human/hardware-in-the-loop simulation (HHIL) obtained on a group of $N = 6$ simulated quadrotor UAVs (the slave side) and two real 3D haptic interfaces (the master side). The UAVs were simulated in a physically realistic 3D environment based on the Ogre3D engine and the PhysX libraries for simulating the interaction between the UAVs and the environment. A set of obstacles was placed in the scene in order to trigger line-of-sight occlusions and the collision avoidance capabilities of the connectivity maintenance force F^λ. The two haptic devices are a Omega.6 and Omega.3[1]: the Omega.6 device has 3 actuated translational dofs and 3 passive rotational dofs, and was used to represent the 3-dof master (1). The Omega.3 device has 3 actuated translational dofs, of which 2 were kept fixed via software so as to render it as a 1-dof device to play the role of the connectivity master (27). A picture illustrating the setup is given in Fig. 3.

Figure 4 shows the 3 components of the velocity command r_M and the scalar command r_λ, respectively, during the task execution. As clear from the plots, the leader UAV was teleoperated among the obstacles with 'random' commands in order to continuously stress the ability of the connectivity force $F^{\lambda(\mu)}$ to preserve physical connectivity and avoid collisions with other agents and obstacles. At the same time, the human operator acted upon r_λ in order to vary the minimum degree of connectivity $\lambda_2^{\min}$ over time (Fig. 4). In this respect, Fig. 5(a) reports the behavior of the 6 components of vector $\mu(t)$ over time, i.e., the result of having implemented the consensus law (13) for tracking r_λ via u_{lH} in (30). Because of the fast convergence rate of the consensus law, the components of $\mu(t)$ in Fig. 5(a) are almost perfectly superimposed, thus forming a unique thick line. Figure 5(b) reports the 6 values $\lambda_{2,i}^{\min}$ evaluated upon the different μ_i via (11): as expected these also result being almost perfectly coincident. It is then interesting to look at Fig. 6(a) where the 6 values of $\lambda_{2,i}^{\min}$ are compared against the current value of λ_2, the degree of connectivity of the group. Note how the values of $\lambda_{2,i}^{\min}$ remain *below* λ_2 at all times, as required by constraint (10). Note also how λ_2 tends to always increase/decrease whenever any of the $\lambda_{2,i}^{\min}$ increases/decreases (because, ultimately, of the issued command r_λ). Therefore: by acting upon r_λ, the human operator is able to make λ_2 changing almost at will, but while always coping with constraint (10). As a result, connectivity is never lost during the task: this is also evident from Fig. 6(b) where the shown connectivity potential $V^{\lambda(0)}(t)$ remains bounded over time.

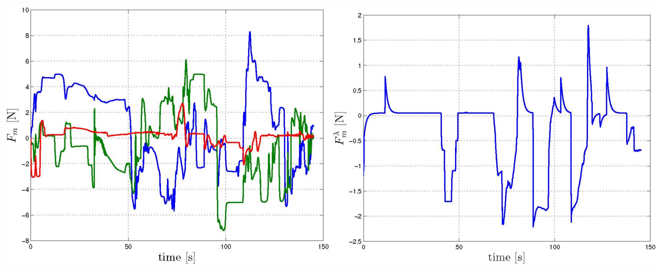
Additionally, in Fig. 7(a) we report the evolution of the 6 tank energies over time: as expected, the tanks get refilled

[1] http://www.forcedimension.com



(a) (b)

Fig. 7. (a) The tank energy values. (b) The time-varying number of edges of the interaction graph.



(a) (b)

Fig. 8. (a) Force F_m applied to the master controlling the leader robot. (b) Force F_m^λ applied to the master controlling the degree of connectivity.

during the motion and never deplete. It is possible to see some isolated phases of small discharges in the tank energies, e.g., at about $t = 75$ s and $t = 140$ s: these are due to the temporary non-passive behavior of $F^{\lambda(\mu)}$ because of the too erratic variation of $\lambda_2^{\min}$. However, these effects are completely within the passivity margin of the system as clear from the plot. Finally, in Figs 8(a)–8(b) we show the force cues F_m and F_m^λ displayed to the human operator during the task. Peaks in the force F_m^λ correspond to phases in which a connectivity change command r_λ was not executed by the group mainly because of constraint (10). As a last plot, Fig.7(b) reports the total number of edges $|\mathcal{E}|$ over time: this is meant to show the *time-varying* nature of the interaction graph $\mathcal{G}$ in which edges are allowed to be created or destroyed at anytime as long as the overall graph stays connected.

Finally, the interested reader can also appreciate these HHIL simulation results in the videoclip attached to the paper.

VI. CONCLUSIONS AND FUTURE WORK

In this paper we presented a novel bilateral controller that allows to stably teleoperate the degree of connectivity in the mutual interaction between a remote group mobile robots. Passivity of this new bilateral interaction channel is theoretically proven and ensured in a fully distributed way. The feasibility and applicability of the system has been the demonstrated by means of human/hardware-in-the loop simulations.

In future work we want to implement this framework with a group of real UAVs, and evaluate the benefit of the new control dimension provided to the operator by means of psychophysical experiments. Furthermore, we aim at defining an algorithm for tuning the upper limit of the tanks of the agents depending on the topology and on the interaction with the environment.

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