Closure of a Set of FDs (F^+)

- $F = \{ A \rightarrow B, B \rightarrow C \}$

➢ If $X \rightarrow Y \in F$ then $X \rightarrow Y \in F^+$

$A \rightarrow B \in F^+$ $B \rightarrow C \in F^+$

➢ By **Inclusion Rule**，the follow FDs are element of F^+

$A \rightarrow A$	$B \rightarrow B$	$C \rightarrow C$
$AB \rightarrow A$	$AB \rightarrow B$	$AB \rightarrow AB$
$AC \rightarrow A$	$AC \rightarrow C$	$AC \rightarrow AC$
$BC \rightarrow B$	$BC \rightarrow C$	$BC \rightarrow BC$
$ABC \rightarrow A$	$ABC \rightarrow B$	$ABC \rightarrow C$
$ABC \rightarrow AB$	$ABC \rightarrow AC$	$ABC \rightarrow BC$
$ABC \rightarrow ABC$		

- By $A \rightarrow B$ and $B \rightarrow C$ and **Transitivity Rule** we have:

  $A \rightarrow C \in F^+$

- By $A \rightarrow B$ and $A \rightarrow C$ and **Union Rule** we have:

  $A \rightarrow BC \in F^+$

- By $A \rightarrow B$ and **Augmentation rule** we have:

  $A \rightarrow AB \in F^+$, $AC \rightarrow BC \in F^+$, $AC \rightarrow ABC \in F^+$

- By $B \rightarrow C$ and **Augmentation rule** we have:

  $AB \rightarrow AC \in F^+$, $B \rightarrow BC \in F^+$, $AB \rightarrow ABC \in F^+$

- By $A \rightarrow C$ and **Augmentation rule** we have:

  $A \rightarrow AC \in F^+$, $AB \rightarrow BC \in F^+$

- By $A \rightarrow BC$ and **Augmentation rule** we have:

  $A \rightarrow ABC \in F^+$

- By AB→B and B→C and Transitivity Rule we have:

  $AB \rightarrow C \in F^+$

- By AC→A and A→B and Transitivity Rule we have:

  $AC \rightarrow B \in F^+$

- By AC→B and Augmentation rule we have:

  $AC \rightarrow AB \in F^+$

- $F = \{ A \rightarrow B, B \rightarrow C \}$

  $F^+ = \{$

  $A \rightarrow A,\ A \rightarrow B,\ A \rightarrow C,\ A \rightarrow AB,\ A \rightarrow BC,$

  $A \rightarrow AC,\ A \rightarrow ABC,$

  $B \rightarrow B,\ B \rightarrow C,\ B \rightarrow BC,$

  $C \rightarrow C,$

  $AB \rightarrow A,\ AB \rightarrow B,\ AB \rightarrow C,\ AB \rightarrow AB,\ AB \rightarrow BC,$

  $AB \rightarrow AC,\ AB \rightarrow ABC$

  $AC \rightarrow A,\ AC \rightarrow B,\ AC \rightarrow C,\ AC \rightarrow AB,\ AC \rightarrow BC,$

  $AC \rightarrow AC,\ AC \rightarrow ABC$

  $BC \rightarrow B,\ BC \rightarrow C,\ BC \rightarrow BC,$

  $ABC \rightarrow A,\ ABC \rightarrow B,\ ABC \rightarrow C,\ ABC \rightarrow AB,$

  $ABC \rightarrow BC,\ ABC \rightarrow AC,\ ABC \rightarrow ABC$

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