

QF4102 Financial Modelling and Computation: Assignment 1 Report

G04: Dong Shaocong, He Xinyi

October 8, 2017

Abstract

This document is the report for QF4102 Financial Modelling and Computation, assignment 1. In this report, we will show all the output diagrams we got, as well as the relevant comments we want to add. In general, we examined two types of the options, the first one being European down-and-out call option, and the second one being European floating strike look back put option.

1 Question A1.1 European down-and-out call option

1.1 Description of work done

This is the first question of assignment 1. In this question, we work on implementing European down-and-out call option. We firstly adopted the exact solution formula and a direct calculation for the exact option price. Afterwards, we implement a binomial tree method for this European down-and-out option. We make sure the underlier price at time 0 is fixed at 8 and the barrier is fixed at 6. By using the previously-implemented exact option value function, we compared the result and plot the error as we use different values of step sizes. By inspecting the plot, we find two values of number of steps yielding local minimum errors. Using the equation from lecture notes, we calculated the corresponding value i . The plots, comments and workings are given in the subsequent sections.

1.2 Sub Question (iii) Comment on the relationship between the three sets of option prices.

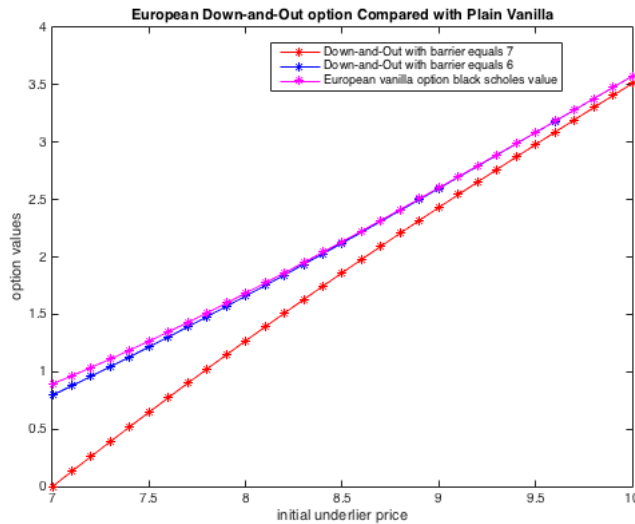


Figure 1: European Down-and-Out option compared with plain vanilla option

In general, the vanilla option value is higher than the barrier option values with the same underlier and maturity. This comes from the fact that with the barrier, some of the branches of the price path will be down and out, which produces zero option values, so the discounted price (value) of which will be cheaper.

Furthermore, the lower the barrier price, the higher the option value. Meanwhile, within the same barrier price, the higher the initial underlier price, the gap between the barrier option value and vanilla call value also reduces. This is because, with lower barrier value, the less likely that the price of the underlier will fall below the barrier and the branch becomes zero, therefore the closer to the price of the vanilla call. Similarly, the higher the initial price, the less likely the underlier price will go down below the barrier, thus having less zero values price path, which increases its value to nearly as much as the vanilla option. In this graph, the option value with $barrier = 6$ is especially close to vanilla call, indicating that for $S_0 = 7..10$, it is very unlikely for the price to go down below 6.

1.3 Sub Question (iv) Comments on the error range plot with different step sizes

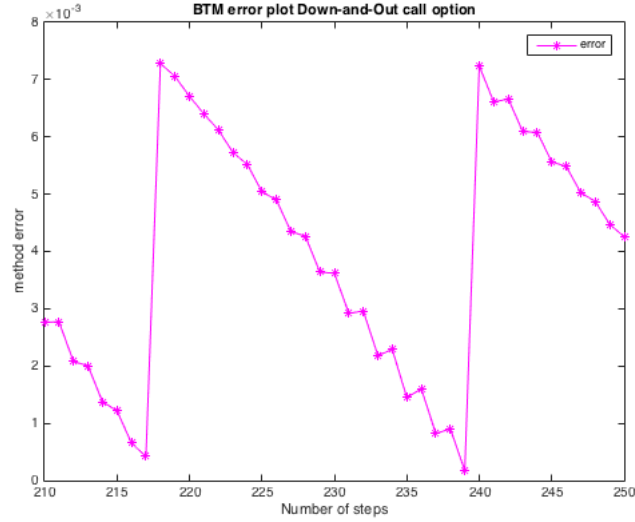


Figure 2: Prices of European down-and-out call against number of time intervals

The error pattern is highly cyclical. After the error gradually decreases to a local minimum value, it will increase sharply at the next N value, producing the new cycle. This is because, during each cycle, the artificial barrier will gradually move closer to the actual barrier as N increases. After it reaches the closest position, the next N will result in the artificial barrier being furthest away from the actual barrier, and the cycle continues.

1.4 Sub Question (v) Value i corresponding to the locally minimum error

Two N values 217 and 239 (with 239 achieving slightly lower minimum error) We find the corresponding i value as follows (20 and 21 respectively, -21,-20 also possible):

The working is as follows:
$$\frac{\sqrt{217} \times (-\ln(\frac{6}{8}))}{0.3 \times \sqrt{0.5}} = 20; \quad \frac{\sqrt{239} \times (-\ln(\frac{6}{8}))}{0.3 \times \sqrt{0.5}} = 21$$

2 European floating strike look back put option using BTM with single state variable

2.1 Description of work done

This is the second question of assignment 1. In this question, we examined the single state variable binomial tree method on the European floating strike look back put options. Firstly, we implemented the algorithm (version 1 in lecture notes) for a newly issued European floating strike look back put option, and we plotted the option values versus the number of steps we use. Secondly, we implemented the algorithm (version 2 in lecture notes) for a European floating strike look back put option which is not newly issued with a running max value greater than the current underlier price. Again, we plotted the option values versus the number of steps we use. The number of steps for both of the two sub questions ranges from 200 to 20000 with a step size of 100.

2.2 Sub Question (i) Comments on the newly issued options

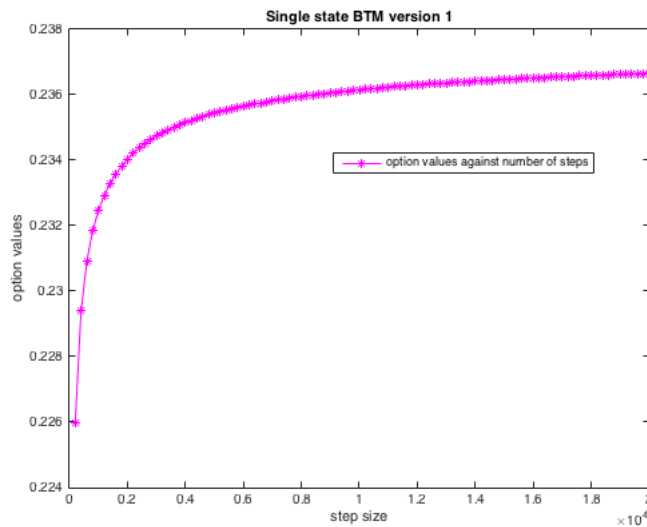


Figure 3: Single State BTM version 1 (newly issued)

The option value increases with decreasing rate, and it converges quickly to a stable value of around 0.2365 which means the actual value should be close to it as well. As step size increase, the accuracy of the estimation also increases. As step size becomes larger (larger than 6000), the change in value becomes very small thus becoming insignificant. From the lecture on option value convergence, we know the price will converge asymptotically to the real value, therefore, we can estimate this look back put option price to be the estimated value we obtained using steps larger than 6000.

2.3 Sub Question (ii) Comments on not newly issued options

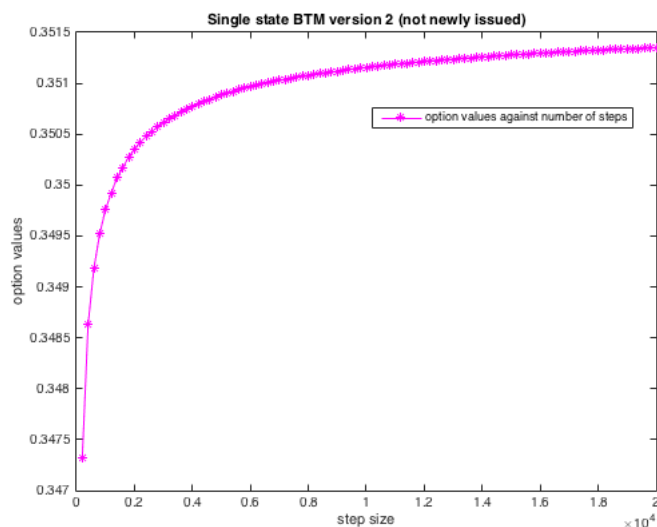


Figure 4: Single State BTM version 2 (not newly issued)

This not newly issued option expresses a similar trend in its value with the increasing number of steps. The value converges to around 0.3514. The option value is higher than the options that are newly issued when at time 0, the current running max is bigger than the underlier price. This arises from a higher probability of getting a higher payoff as the payoff function is composed of these two factors by this form $(RunningMax - S_T)^+$.