QF4102 Financial Modelling and Computation: Assignment 2 Report

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Abstract

This document is the report for QF4102 Financial Modelling and Computation, assignment 2. In this report, we will show all the numerical results we got, and comment on them with respect to the results and questions respectively. In general, in this assignment, we examined the Forward Shooting Grid method (FSGM) for American put arithmetic and look back options. We also looked at explicit numerical scheme and its convergence problem both for American and European vanilla call options.

1 A2.1(i) American fixed-strike arithmetic-average put option

1.1 Description of work done

This is the first question of assignment 2. We considered firstly a not newly issued American style fixed-strike arithmetic-average put option using Forward shooting grid method. We firstly wrote a Matlab function by using two state $FSGM$ algorithm with a slight modification that we firstly calculate a current running average and create the average grid based on this value. After writing out the function, we tried it out with different values of ρ and different values of number of time periods. We not only recorded the numerical value we got, we also recorded the time amount taken for each set of calculations.

1.2 Numerical results we get:

ρ, N	50	100	200	400
1	5.3942	5.4301	5.4552	5.4685
0.5	5.36	5.4181	5.4487	5.4639
0.2	5.3533	5.412	5.4446	5.4616

Table 1: Option values with different values of ρ and N

ρ, N	50	100	200	400
1	0.081859	0.556570	4.596088	35.672926
0.5	0.174664	1.131893	8.512614	67.209527
0.2	0.360371	2.708468	22.005674	193.933429

Table 2: Calculation time spent (in seconds) with different values of ρ and N

1.3 Comment on your numerical results and computation times taken.

When keeping ρ as a constant, as N increases, the estimated option value increases at a decreasing rate. Similarly, when we keep the number of periods N as a constant, as ρ used becomes smaller, the estimated option value will decrease at a decreasing rate. This implies that as N increases and ρ decreases, the estimated value will converge to its true value. When N is equal to 400, the three option values are all around 5.46, which is likely to be the true option value. Under this assumption, we can see using number of periods 100 and 200 still can hold two significant figures true, while using N=500 can only make it to one significant figures.

As for computation time, keeping N constant, when ρ is reduced to half, meaning the grid number doubles, the running times also doubles. Therefore, the running time is proportional to the number of grid, which is consistent with the theoretical complexity of $O((n+1)(2mn+1))$. On the other hand, keeping ρ constant, when N doubles, running times increases for around $7\,to\,9$ times, indicating a complexity of around $O(n^3)$. Even though it is larger than the theoretical $O(n^2)$ complexity, the saving time will still be tremendous when N is large, as compared to a normal BTM of $O(2^n)$ time complexity. As a side note, we can improve the calculation time spent by vectorization in the algorithm and loop.

2 A2.1(ii) American fixed-strike lookback put option

2.1 Description of work done

This is the second question of assignment 2. We considered a not newly issued American style fixed-strike lookback put option using Forward shooting grid method. We again wrote a Matlab function by using two state $FSGM$ algorithm. Compared to the last question, we also changed the algorithm for running minimum calculation. Because of the running minimum will surely be one of the price states of the underlier, we can choose $\rho = 1$. After writing the Matlab function, we tried it out with different number of time periods in the lattice being 50 to 500 in increments of 50. The first set of value is done by taking running minimum equal to 0.97 and for the second round, we choose the running minimum from the previous time periods equal to 0.57.

2.2 Taking previous running minimum as 0.97

2.2.1 Numerical results

table	50	100	150	200	250
value	0.086453	0.081535	0.08706	0.090432	0.092765
time	0.757948	6.335255	21.562594	49.020456	97.223307

Table 3: Option value and time spent with different values of N from 50 to 250

table	300	350	400	450	500
value	0.087203	0.089026	0.090541	0.091819	0.092904
time	160.402135	268.440798	373.777399	597.046388	812.959572

Table 4: Option value and time spent with different values of N from 300 to 500

2.2.2 Comment on your numerical results and computation times taken.

The payoff for immediate exercise of this American option is 0, which means it is not optimal to exercise this option now. The option estimated value is oscillating around the value between 0.086 and 0.093. The current estimations' oscillation does not show clear convergence, so we may need larger number of time periods to get the true value of options.

As for computation time, when N doubles, running times increases for around 8 times consistently, indicating a complexity of around $O(n^3)$. Similar to the arithmetic American option case, it is larger than the theoretical $O(n^2)$ complexity, the saving time will still be tremendous when N is large, as compared to a normal BTM of $O(2^n)$ time complexity.

2.3 Taking previous running minimum as 0.57

2.3.1 Numerical results

table	50	100	150	200	250
value	0.38	0.38	0.38	0.38	0.38
time	0.882802	6.878774	24.343880	49.430236	103.749121

Table 5: Option value and time spent with different values of N from 50 to 250

table	300	350	400	450	500
value	0.38	0.38	0.38	0.38	0.38
time	155.599389	246.965933	408.966377	587.768502	829.394949

Table 6: Option value and time spent with different values of N from 300 to 500

2.3.2 Comment on your numerical results and computation times taken.

The payoff for immediate exercise of this American option is $0.95 - 0.37 = 0.38$, which is what we have gotten here in the option value estimations. This demonstrates that, for American put lookback options, if the current running minimum is significantly lower than the strike price, It is always to the interest of the holder to exercise the option immediately.

Comparing the running time, we find that the time is still quite similar to the last case with running minimum equal to 0.97. Because of our implementation, we still go through the whole forward shooting grid before making up the decision and compare with the current immediate payoff. We can be quite sure that the true option value is just 0.38.

3 A2.2 Explicit difference scheme III

3.1 Description of work done

This is the second part of assignment 2. In this question, we examined the third explicit difference scheme (III) discussed in lecture. We firstly wrote a Matlab function using explicit scheme III for pricing European vanilla call option. We have identified the coefficients in the finite difference equations that are not positive. We also compared it against the exact value we should get. Next, we tried to derive a a lower bound for $N = \dfrac{T}{\Delta t}$ such that all coefficients in the finite difference equations are positive. rerunning the algorithm with this bound and a slightly lower value of N, we determined the cut-off value where the option estimate loses all its significant figures. In addition, we wrote another Matlab function for pricing American vanilla call options using explicit scheme III algorithm. Using this new function, we redid the previous experiments.

3.2 (i) Compare the estimate against the exact value of option.

At $S_0 = 9.8$, from the output, we get the exact value which is equal to 0.82551 but the option value estimate we get from this explicit scheme is $7.055274186430886 \times 10^{21}$. This estimation clearly loses all the significant figures and it's not acceptable as an answer for option value. Checking the conditions violation, we get the following:

Coeff a, Of 719 elements, 0 violated the positivity condition.
Coeff b, Of 719 elements, 653 violated the positivity condition.
Coeff c, Of 719 elements, 0 violated the positivity condition.

Clearly, the coefficient b has positivity condition violation thus making the scheme not convergent. As we can see, the estimated value we got is too far away from the true value. This is because the N we use is too small and the explicit scheme becomes not convergent.

3.3 (ii) A lower bound for N such to have all positive coefficients.

As we can see from the output as well as the explicit scheme formulation, only the parameter b can be negative sometimes which may affect the convergence of this scheme, which may thus in turn affect the precision of the final estimate for the option value. We can obtain the lower bound for N such to have all positive coefficients by getting the lowest N to make b positive for every i.

$$1 - \sigma^2 \times i^2 \times \Delta t - (r - q) \times i \times \Delta t > 0 \tag{1}$$

Substituting the parameters given into equation 1 we can get:

$$0.0225 \times i^2 \times \Delta t - 0.009 \times i \times \Delta t < 1 \tag{2}$$

$$\Delta t = \frac{T}{N} \tag{3}$$

From equation 2 and 3 we can have:

$$N > 0.0225 \times i^2 \times 0.25 - 0.009 \times i \times 0.25 \tag{4}$$

As we have the biggest i given as follows:

$$i_{max} = \frac{4 \times X}{\Delta S} \tag{5}$$

Substituting equation 5 into equation 4, we can have $N_{min} = 2914.38 \approx 2915$.

3.4 (iii) Obtain another estimate to the option value using this N.

At $S_0 = 9.8$, the exact value of the option is equal to 0.82551 and the estimate of this option value we get from explicit scheme is 0.82529. which is very close to the true value. Checking the condition violations we have:

Coeff a, Of 719 elements, 0 violated the positivity condition.
Coeff b, Of 719 elements, 0 violated the positivity condition.
Coeff c, Of 719 elements, 0 violated the positivity condition.

We can see no coefficients is violating positivity conditions. The estimated value we get is very close to the real option value calculated by Black-Scholes formula.

3.5 (iv) Determine the cut-off value where the option estimate loses all its significant figures.

This is the output from our function

—–question iv—– Finding cut-off value Euro
Calculating...
At N = 2459, option value is 0.82686
At N = 2458, option value is 0.82405
At N = 2457, option value is 0.82172
At N = 2456, option value is 0.81252
At N = 2455, option value is 0.83409
At N = 2454, option value is 0.81025
At N = 2453, option value is 0.91772
At N = 2453, option value is 0.91772, it loses all significant figures.

As we can see, the cut-off value is 2453 for N, where we have estimation of the option value equal to 0.91772, which clearly loses all the significant figures. This N is a little bit smaller than the lower bound 2915 we have previously got, which demonstrates the lower bound is a sufficient but not necessary bound for getting estimation at least one significant figures correct.

3.6 (v) Repeat iii for American vanilla call option pricing function.

At $S_0 = 9.8$, we have the exact value for European call vanilla option is equal to 0.82551, but the estimation for American vanilla call option is equal to 0.83189, which is slightly bigger than the European vanilla call option. This observation is normal as American option generally has a higher value than the corresponding European option for its ability to be exercised any time.