

NATIONAL UNIVERSITY OF SINGAPORE
Department of Mathematics

2017/2018

QF4102 Financial Modelling

Semester I

QF4102 Assignment 3

A3.1 Consider the **transformed** Black-Scholes PDE model:

$$\begin{cases} \frac{\partial u}{\partial t} + \frac{\sigma^2}{2} \frac{\partial^2 u}{\partial x^2} + \left(r - q - \frac{\sigma^2}{2}\right) \frac{\partial u}{\partial x} - ru = 0, & x \in (-\infty, \infty), \quad t \in [0, T), \\ u(x, T) = \varphi(x). \end{cases}$$

- (i) Derive the $O[\Delta t + (\Delta x)^2]$ fully implicit scheme for a European vanilla call option. Assume that $x \in (-\infty, \infty)$ is truncated to $x \in [-x_{max}, x_{max}]$, with $x_{max} = I dx$ for a sufficiently large integer I . Write down the system of linear equations and its matrix form, as well as the boundary conditions.
- (ii) Implement a Matlab function for the fully implicit scheme derived in (i). Use the following function heading in your m-file:

```
function OptVal=FD_ids_call_trans(S0, X, r, q, T, sigma, I, N, xmax)
```

Test your implementation with a European call option which has a strike price of \$5 and time to maturity of 1 year. The underlier has a current price of \$5.25, volatility of 30% and dividend yield of 10%. The risk free rate is 3%. Use a grid with values of x lying in the truncated domain $[-5, 5]$. For $N = 1500$ and I taking values from 100 to 1500 in increments of 100, obtain option value estimates (with linear interpolation) for the given option. Plot these estimates versus I . Comment.

- (iii) Write another Matlab function to implement the same fully-implicit scheme but for pricing an American vanilla call option. Use the following function heading in your m-file:

```
function OptVal=FD_ids_Acall_trans(S0, X, r, q, T, sigma, I, N, xmax, omega, eps)
```

In your code, use the **PSOR** algorithm with over-relaxation parameter $\omega = 1.3$ and convergence tolerance $\text{eps} = 1 \times 10^{-6}$. Test your implementation with an American vanilla call option **with the same option and grid parameters as in (ii)**. For $N = 1500$ and I taking values from 100 to 1500 in increments of 100, obtain option value estimates (with linear interpolation) for the given option. Plot these estimates versus I . Comment.

Important: Besides the two .m files for the required Matlab functions, also prepare a Matlab script file (named **A3p1.m**) to contain all Matlab statements used to generate results in (ii) and (iii). This file will be executed during the grading of the assignment. If you use separate m-file(s) to implement the PSOR algorithm, then such file(s) have to be submitted as well.

A3.2 A digital (or binary) call option on an asset has the payoff function at expiry T given by

$$\varphi(S_T) = \begin{cases} 1 & S_T > X, \\ 0 & S_T \leq X \end{cases}$$

where S_T is the terminal asset price and X the strike price. The value of this European-styled option at time 0 has the closed form formula:

$$\exp(-rT)N(x)$$

where r is the risk-free rate, and

$$x = \frac{\ln(S_0/X) + (r - q - \sigma^2/2)T}{\sigma\sqrt{T}}.$$

- (i) Use a Matlab m-file to implement a function which returns the value of the above digital call option. Use the following function heading in your m-file:

```
function OptVal=BS_DigitalCall(S0, X, r, q, T, sigma)
```

Consider a 3-asset digital call option with the payoff function

$$\varphi(S_{1T}, S_{2T}, S_{3T}) = \begin{cases} 1 & \max(S_{1T}, S_{2T}, S_{3T}) > X, \\ 0 & \max(S_{1T}, S_{2T}, S_{3T}) \leq X \end{cases}$$

where S_{1T} , S_{2T} and S_{3T} are the terminal prices of three correlated assets S_1 , S_2 and S_3 , with $T = 0.75$ year. You are given the following parameters for the three assets:

i	$S_{i,0}$	volatility σ_i	dividend q_i
1	\$9.5	0.35	0.01
2	\$10.2	0.21	0.04
3	\$8.8	0.18	0

The risk free rate is 0.05, and assets are correlated with the following correlation coefficients:

	Asset 1	Asset 2	Asset 3
Asset 1	1	0.88	0.17
Asset 2	0.88	1	0.34
Asset 3	0.17	0.34	1

- (ii) Use a Matlab m-file to implement a function which returns the Monte-Carlo estimate to the value of this 3-asset digital call option. Use the following function heading in your m-file:

```
function MC_noCV=MC_3AssetMD(S0, X, sigma, C, r, q, T, no_samples)
```

where $S0$, σ , q are vectors (each with 3 elements) for initial prices, volatilities and dividend yields of the 3 assets respectively, and C is the 3×3 correlation matrix.

- (iii) Use the matlab function in (ii) to obtain option value estimates and standard errors for $X = \$8.5$, $\$9.5$, $\$10.5$ respectively. For each X , use a total of 30 simulation runs each with 100 price-path bundles.

- (iv) Repeat (iii) with 1,000, 10,000 and 100,000 price-path bundles. Tabulate your results and comment.
- (v) Write another Matlab function by modifying the above function to incorporate a control variate (a basket of three single-asset digital calls) for the purpose of variance reduction. Demonstrate numerically the effectiveness of this control variate method for the same set of option/grid parameter considered in (iii) and (iv). Tabulate your results and comment.

Important: Besides the three .m files for the required Matlab functions, also prepare a Matlab script file (named **A3p2.m**) to contain all Matlab statements used to generate results in (iii), (iv) and (v). This file will be executed during the grading of the assignment.

Due date, requirement, guidelines and regulations

- (i) The due date/time for the Matlab programmes and brief report is **2359hr on 19 November, 2017**. No late submission will be accepted.
- (ii) Work on the assignment problems should commence soonest possible as programming and debugging can be time consuming.
- (iii) **Use Matlab for all programming tasks.**
Please add suitable amount of comments to your codes and test your codes thoroughly. **The first line of each Matlab m-file should have a comment line containing the names of the group members.**
- (v) **Prepare your report in the Windows Word format or the PDF format** with a description of your work done plus supporting figures and tables etc, as well as all necessary analysis and comments.
- (vi) The .doc/.pdf and all .m files should all be archived in a **single** Zip/Rar file. **Name your .zip/.rar file with your group index (such as Gxx_Assignment.zip or Gxx_Assignment.rar where Gxx is your assigned group index), and submit it online to the IVLE workbin set up for this purpose. Only one such archive file** from each group will be used in the grading process.
- (vi) This assignment counts 10% towards the final assessment score of this module.
- (vii) Plagiarism (copying work from fellow students, groups or others) **would not be tolerated and all parties involved would be penalized severely.**
Please refer to <http://emodule.nus.edu.sg/ac/> for more information on NUS's disciplinary process on plagiarism.

✂✂ The End ✂✂