1 Settings

Consider the following MS-VAR model with M regimes on K-dimensional data: let $Y = (Y_1, Y_2, ..., Y_n)$ be the target time series where each Y_i ($1 \leq i \leq n$) is a subset of $\mathbb{R}^K$. Denoting the state of the series at time t as s_t, suppose that each observation is generated from the model

$$Y_t(s_t) = \boldsymbol{\mu}_{s_t} + \sum_{i=1}^{s} A_i(s_t) Y_{t-i} + \mathbf{u}_{s_t}$$

in state s_t. With exogeneous variables, the model can be also expressed as

$$Y_t(s_t) = \boldsymbol{\mu}_{s_t} + \sum_{i=1}^{s} A_i(s_t) Y_{t-i} + \Gamma_0 \mathbf{x}_0 + \Gamma_{s(t)} \mathbf{x}_1 + \mathbf{u}_{s_t}$$

where $\mathbf{x}_0$ is a column of length p_0 that has corresponding coefficients that are not switching, $\mathbf{x}_1$ is a column of length p_1 with coefficients that are switching. Parameters of each model include

$A_i(s_t)$ is a $K \times K$ matrix

$\boldsymbol{\mu}_{s_t}$ is a $K \times 1$ column

Γ_0 is a $K \times p_0$ matrix

Γ_{s_1} is a $K \times p_1$ matrix

$\mathbf{u}_{s_t}$ is a $K \times 1$ random vector that follows $\mathcal{N}(\mathbf{0}, \Sigma_{s_t})$ for some positive-definite square matrix Σ_{s_t}.