Grey system: theory, methods, applications and challenges

Dr. Meng Lu
Chair, IBEC
Member, Board of Governors, IEEE ITSS
Email: wklm@xs4all.nl

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Grey system theory

- Initially proposed by Julong Deng in 1982
- Multidisciplinary character
- Aims to cope with the uncertainty of a system (so-called "grey system", especially for multi-variable, discrete and incomplete data)
- Grey methods
  - grey relational analysis (GRA)
  - grey clustering
  - grey statistics
  - operational research
  - system control
  - system modelling
  - system forecast
- Application
  - agriculture
  - environment
  - earthquakes
  - marketing

Biography of grey system theory / applications

- List of journals in English
  - Applied Mathematics and Computation
  - Biosystems Engineering
  - Building and Environment
  - EJOR?
  - Electric Power Systems Research
  - Energy
  - Energy Conversion and Management
  - Environment Pollution and Control
  - Fuel Processing Technology
  - International Journal of Advanced Manufacturing Technology
  - International Journal of Machine Tools & Manufacture
  - International Journal of Reliability, Quality and Safety Engineering
  - IEEE Transactions on Automatic Control
  - IEEE Transactions on Cybernetics
  - IEEE Transactions on Power Systems
  - Journal of Grey System (published by CGSA)
  - Journal of the Franklin Institute
  - Measurement Science and Technology
  - Smart Material Structure
  - Systems and Control Letters
  - System Engineering
  - The Journal of Grey System
  - Wear

Evaluation method for decision making

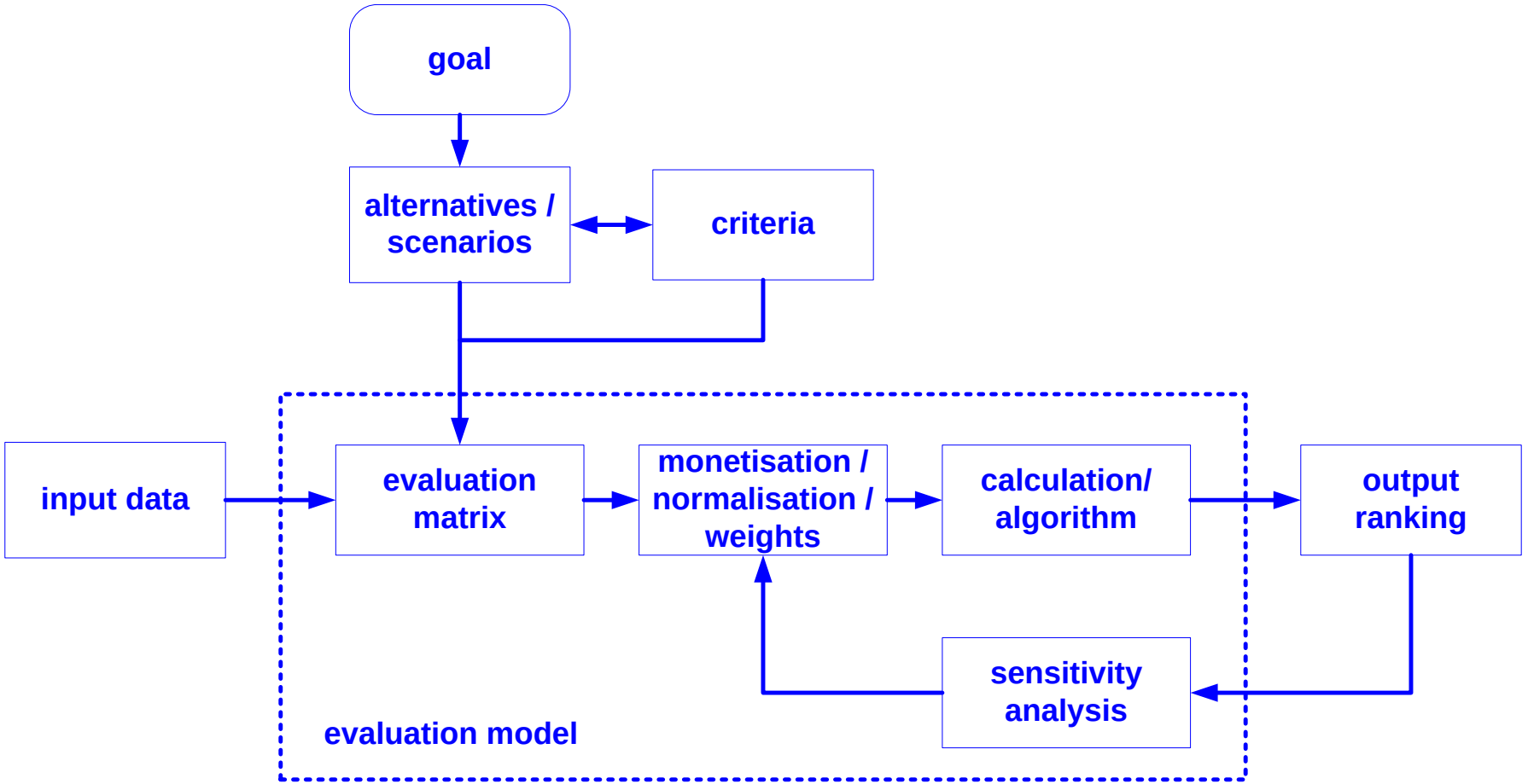
- Economics based methods
  - cost-benefit analysis (CBA)
  - cost-effectiveness analysis (CEA)
  - planning balance sheet (PBS)
  - goals-achievement matrix (GAM)
  - cost-utility analysis (CUA)
- Normalisation based methods
  - analytical hierarchy process (AHP)
  - simple additive weighting (SAW)
  - Technique for Order Preference by Similarity to Ideal Solution (TOPSIS)
  - ÉLimination Et Choix Traduisant la RÉalité (ELECTRE)
  - Preference Ranking Organization METHod for Enrichment Evaluations (PROMETHEE)
  - fuzzy measures for decision making
  - grey relational analysis (GRA)

Evaluation matrix

Evaluation - provide an analysis and ranking of different available alternatives to achieve a certain goal or objective

	alternative 1	alternative 2	…	alternative *i*
(sub-)criterion 1				
(sub-)criterion 2				
…				
(sub-)criterion *k*				

Evaluation process



Algorithm of GRA

- Create matrix of i alternatives and k attributes with proper data
- Grey normalisation
- Grey relational coefficient of attribute k for alternative i

$$\gamma(x_0(k), x_i(k)) = \frac{\min_{i,k} |x_0(k) - x_i(k)| + \zeta \max_{i,k} |x_0(k) - x_i(k)|}{|x_0(k) - x_i(k)| + \zeta \max_{i,k} |x_0(k) - x_i(k)|}$$

$x_0(k)$ - referential series for attribute k

$x_i(k)$ - compared series for attribute k

$\zeta \in (0,1)$ - identification or distinguishing coefficient

- Grey relational grade

$$\Gamma_{0i} = \sum_{k=1}^{n} w_k \gamma_{0i}(k)$$

w_k - k-th weight of $\gamma(x_0(k), x_i(k))$

Prominent forecasting techniques

- Time-series methods
  - exponential smoothing
  - decomposition
  - autoregressive/moving average (ARMA)
- Causal forecasting methods
  - regression analysis
  - econometric models
  - input-output models
  - simulation modelling
- Markov process
- Grey (system) model
- Qualitative methods
  - product life-cycle analogy
  - expert knowledge
  - Delphi

Essential principle of grey model

- Purport of grey model
  - deal with the problem of system modelling with limited data (a minimum of four)
- GM(n,h)
  - n-th order pseudo differential equation of h variables
  - grey differential equation represents discrete time sequences
- Assumption of grey model
  - pattern of discrete data series to be processed is exponential
  - or can be transformed to an exponential pattern by pre-processing
- Aims of grey model
  - describe this exponential pattern between each series and the discrete elements in the series
- Fundamental schema
  - to deal with the causality between the different elements of the data set, given or represented as an exponential discrete series
- Unique and distinguished characteristic of grey model
  - to provide a tool for dynamic modelling small-sample discrete data series

Prerequisite of grey model

- Exponential pattern of the data series: data pre-processing
  - original series is denoted as:

$x^{(0)} = (x^{(0)}(1), x^{(0)}(2), \ldots, x^{(0)}(k)), k = 1,2, \ldots, n$

  - accumulated generating operation (AGO)

$$x^{(1)} = (x^{(1)}(1), x^{(1)}(2), \ldots, x^{(1)}(k)) = \left(\sum_{k=1}^{1} x^{(0)}(k), \sum_{k=1}^{2} x^{(0)}(k), \cdots, \sum_{k=1}^{n} x^{(0)}(k) \right)$$

- Series satisfies that successive terms $\sigma(k) = \frac{x(k-1)}{x(k)}$ close to a constant c, $c \in \left[e^{-2/(n+1)}, e^{2/(n+1)} \right]$

- Result of data pre-processing
  - to decrease randomness and noise of the original data series
  - (exponential) regularity and smoothness are increased

Grey model GM(1,1) algorithm

- Grey differential equation $\frac{dx^{(1)}}{dt} + ax^{(1)} = u$
  assume $x^{(0)}(k+1) + az = u$
- Mean value generating operation (MGO) $z = \frac{1}{2}\left(x(k+1) + x(k)\right)$
- optimised solution of the parameters a, u is derived by the least-square method as:

$$\mathbf{\Theta} = \begin{bmatrix} a \\ u \end{bmatrix} = (\mathbf{B}^{\mathrm{T}}\mathbf{B})^{-1}\mathbf{B}^{\mathrm{T}}\mathbf{Y} \qquad \mathbf{B} = \begin{bmatrix} -z^{(1)}(2) & 1 \\ -z^{(1)}(3) & 1 \\ \vdots & \vdots \\ -z^{(1)}(n) & 1 \end{bmatrix} \qquad \mathbf{Y} = \begin{bmatrix} x^{(0)}(2) \\ x^{(0)}(3) \\ \vdots \\ x^{(0)}(n) \end{bmatrix}$$

- Solution of the GM (1,1) differential equation (discrete time response series) $\hat{x}^{(1)}(k+1) = \left(x^{(0)}(1) - \frac{u}{a}\right)e^{-ak} + \frac{u}{a}$
- To restore original series by IAGO (inverse AGO)

$$\hat{x}^{(0)}(1) = x^{(0)}(1), \quad \hat{x}^{(0)}(k) = \hat{x}^{(1)}(k) - \hat{x}^{(1)}(k-1), \quad k = 1, 2, ..., n$$

- Grey forecasting by extrapolating the modelled series

$$\hat{x}^{(1)}(n+1), \hat{x}^{(1)}(n+2), \cdots, \hat{x}^{(1)}(n+m)$$

General comments on grey system

- Grey theory does not have solid mathematical foundations yet like probability theory, fuzzy theory (based on Credibility Measure) and others
  - Prof. Deng pioneered a unique theory for modelling, and decision when the data is sparse. His contribution greatly appreciated.
  - Appreciation does not means that we accept everything proposed by him or his school.
  - Why essentially none of the professors works on grey theory No solid mathematical foundation was established yet is probably the most rooting cause. The grey theory so far developed is too mixed.

- The is inevitably being lost.
  - An example with the authors' data would be most convincing.
  - Are the least-squares estimators unbiased, consistent and efficient?

Comments received on GM(1,1) (1/2)

- The grey modelling assume that the data is exponential must be explained further.
  - What are the consequences of data that is not exponential?
  - Why not consider other distributions or perhaps a non-parametric approach?
  - Presumably issues like this have come up before in other grey modelling applications
- The data pre-processing seems like it could be potentially problematic since information is inevitably being lost.
  - What are the potential down-side consequences of this?
  - How does the grey approach compare to more traditional econometric methods?
  - Are the least-squares estimators unbiased, consistent and efficient?

Comments received on GM(1,1) (2/2)

- Regarding the consecutive terms, so-called class ratio sigma(k)=x(k-1)/x(k), k=2,3,...,n that should satisfy: exp(- 2/(n+1))<=sigma(k)<=exp(2/(n+1)), k=2,3,...,n,
  - AGO does create a monotone-increasing sequence but do not guarantee a exponentially increasing sequence.
- According to Prof. Deng, the "weight" equals to 0.5. What are the fundamental reasons to take the average? z(1)(k)=1/2[x(1)(k)+x(1)(k-1)]
  - Why not the optimal weight w for z^{1} (k)=w x^{1}(k)+(1-w)x^{1}(k-1)]?
  - The weight choice depends upon the criterion to optimise the information availability.
  - The optimal weight for minimize the model relative error, in most of the cases, is not 0.5.

Comments received on GRA

- I don't see that the authors have demonstrated that this technique will achieve a better result than any alternative decision making methods.
- GRA is a recipe for prioritisation. Is there one "true" ranking?
- How much proportion can GRA catch up?
- Could GRA be formulated in different ways? If so, which version of GRA formulation is better?
- Is any counterpart of autocorrelation concept in GRA analysis?
- How to produce predictable results?

Challenges for researchers of grey system

- Quality of English and writing style
- Limited readers and audience
- Limitation of the grey system theory application
- Lack of an axiomatic foundation for the grey theory
  - rigorous fundamental theory in a mathematical sense (not philosophical sense)
  - grey model
    - why to obtain an exponential pattern of the data series AGO is the unique way? and could it be possible to rigorously proof the exponential law of data pre-processing?
    - why it should be MGO? if not, could it be proofed in mathematical way instead of by empirical study?
    - could it be possible to proof the reliability of GM(1,1) by theoretical research into the foundations of the model, e.g. clarification of constrains and prerequisites of the model, instead of currently used error analysis for model accuracy verification
    - what are the relationships between multidisciplinary methods
  - fundamental concept of the essential elements
    - grey number
    - haze set

Potential (further) applications in logistics

- Transport and logistics – big data (in general)
- Limited data for decision making
- Full of uncertainty
- No general accepted evaluation method
- Systems and control – why do we need grey?

Conclusion

- Aims
  - to cope with the uncertainty of a system
  - especially for multi-variable, discrete and incomplete data
    - ✓ *relational analysis, operational research, system control, system modelling and system forecasting*
- For any new theory, the progress from its birth, growth to maturity requires a long time and considerable research efforts
- Most distinguished methods that are in use are GRA and GM(1,1)
- Purpose of GRA is not to provide a perfect evaluation method, but a practical one especially for solving some specific questions
- GM (1,1) provides a powerful tool for modelling / forecasting of discrete series with few data, based on determination of an exponential pattern
- Various grey system methods are theoretically very appealing
- Language obstacles and ambiguous assumptions, prerequisites and restrictions may hinder their widespread adoption
- Further development of the methods and their foundations is required

Recommendations

- To publish more papers on theoretical foundations of grey system and applications in "white" journals
  - International Journal of Forecasting
  - OR
  - MCA
- To improve readability of publications by improving the quality of English and writing style
- To promote the understanding of the background and foundations of grey system theory, and of its applications
  - by e.g. organising seminars on grey system theory in different universities outside China
  - by enriching the contents of the web site, and timely provision of up-to-date information