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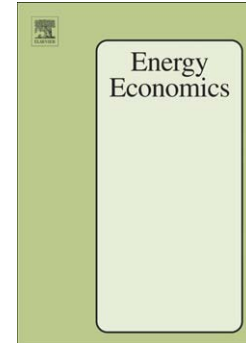
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# Value-at-Risk estimation of energy commodities: a long-memory GARCH-EVT approach

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**ABSTRACT**

In this paper, we evaluate Value-at-Risk (VaR) and Expected Shortfall (ES) for crude oil and gasoline market. We adopt three long-memory - models including, FIGARCH, HYGARCH and FIAPARCH to forecast energy commodity volatility by capturing some volatility stylized fact such as long-range memory, heteroscedasticity, asymmetry and fat-tails. Then we consider Extreme Value Theory which concentrates on the tail distribution rather than the entire distribution. EVT is considered as a potentially framework for the separate treatment of tails of distributions which allows for asymmetry. Our results show that the FIAPARCH model with extreme value theory performs better in predicting the one-day-ahead VaR. Using the fitted long-memory GARCH-model and a simulation approach to estimate VaR for horizons over than one day, backtesting results show that our approach still performs for lower estimations frequencies. Overall, our findings confirm that taking into account long-range memory, asymmetry and fat tails in the behavior of energy commodity prices returns combined with filtering process such as EVT are important in improving risk management assessments and hedging strategies in the high volatile energy market.

**Key words:** Extreme Value Theory, long- range- memory, Value-at-Risk, expected shortfall oil price and energy commodities volatility.

## Value-at-Risk estimation of energy commodities: a long-memory GARCH-EVT approach

### 1. Introduction

The world crude oil prices have raised dramatically during the past decade, consequently the oil market have become very volatile and risky. Moreover, volatile oil prices may lead to price variability of other energy commodities and can have wide-spread impacts on the international economy. Therefore, forecasts of oil price volatility are important for both academicians and market participants. Many forecasting approaches and risk measurement tools have been proposed in the existing literature, in order to provide financial institutions, risk managers and market participant with appropriate and easy technical approaches to measure financial and energy markets risk. In this paper we are interested on commodity assets, especially on forecasting oil and gasoline prices risk. In fact, a large body of empirical studies shows that oil price fluctuations have considerable effects on economic activity. Hamilton (1983) argues that oil price shocks are responsible, at least partly, for every U.S recession after the second world-war. Sadorsky (1999) find that oil price volatility shocks have asymmetric effects on the economy and find evidence of the importance of oil price movements in explaining movements in stock returns. Consequently, it's important to model these oil price fluctuations and implement an accurate tool for energy price risk management. In this context, Value-at-Risk (VaR), originally proposed by J.P. Morgan in 1994, has become a popular risk measures in the financial industry (Duffie and Pan (1997) and Engle and Manganelli (2004)). The concept of VaR is defined as a potential amount of loss on a portfolio with a given probability over a certain fixed time horizon. In fact, VaR reduces the risk associated with any portfolio to just one number, the loss occurred given a certain probability. Since the diffusion of the Risk Metrics system (RM), an academic debate has been emerged among academicians and practitioners about the appropriate approach to calculate VaR. Different approaches have been proposed in the existing literature and may be classified into three families. First, the non-parametric historical simulation (HS) approaches. Second, the parametric models approaches based on an econometric model for volatility dynamics under the normality assumption of the returns distribution. Third, the extreme value theory approach which models only the tails of the return distribution. Since VaR estimations are only related to the tails of a probability distribution, techniques from EVT may be particularly effective. Extreme value theory has been applied in different area where extreme losses may appear, in hydrology: (Davidson and Smith, 1990; Katz and al., 2002). In insurance: (McNeil, 1997; RootzNen and Tajvidi, 1997) and in finance: (Longin, 1996; Danielsson and De varies, 1997; McNeil, 1998; Embrechts and al., 1999, McNeil and Frey, 2000; Gençay and Selçuk, 2004...). However, none of the previous study has reflected the current volatility background. In order to overcome the drawbacks of these methods, McNeil and Frey (2000) proposed a combined approach that reflects two stylized facts exhibited by most financial returns series, namely stochastic volatility and fat-tailedness of conditional returns distribution. In this context, the use of extreme value theory in oil market to implement a risk measure constitutes an important issue. Different empirical studies have investigated the predictive performance of EVT approach to measure risk forecasts on the oil market despite the significant need and interest to manage energy price risks (Krehbiel and Adkins, 2006; Cabedo and Moya, 2003; Costello et al., 2008, Huang et al., 2008, Fan et al., 2008, Marimoutou and al., 2009). To the best of our knowledge, in the empirical literature, the above mentioned studies have not considered the eventual long-range memory in the crude oil volatility when using the theory of extremes to quantify market risks. They were

focusing on heteroscedasticity, fat tails and normality in the empirical distribution of returns time series. Degiannakis (2004) have analyzed the forecasting performance of different risk models in order to estimate the one-day ahead realized volatility and the daily VaR. He concludes that the Fractional Integrated APARCH (FIAPARCH) with skewed-student-t conditionally distributed innovations is more appropriate to take into account the major stylized facts of equity price behavior. Tang and Shieh (2006) have investigated the long-memory proprieties of three stock index futures markets. They conclude that the hyperbolic GARCH (HYGARCH) with skewed-student-t distribution performs better. Similar conclusions have been confirmed by Kang and Yoon (2007). These studies are concerned only with developed financial assets. There are only few studies focused on energy markets. Aloui and Mabrouk (2010) have estimated the Value-at-Risk for three long memory models, FIGARCH, HYGARCH and FIAPARCH with different error's distribution assumptions for some major crude oil and gas commodities. They conclude that models taking into account asymmetries in the volatility specifications and fractional integration in the volatility process perform better in the VaR's prediction. The focus of this paper is to further explore the usefulness of EVT in predicting extreme risks in oil market. To this end, we adopt some long-memory approach GARCH-type models to forecast energy prices volatility by capturing some volatility stylized fact such as long-range memory, heteroscedasticity, asymmetry and fat-tails. Then we consider EVT approach which concentrates on the tail distribution rather than the entire distribution. For his finality, conditional EVT approaches are used to forecast VaR. This paper differs from the existing literature in at least two points. Firstly, for our knowledge, the long-memory GARCH type models have not been implemented with EVT to forecast energy market risk. Secondly, compared with same previous studies, we have considered both the right and left tail to evaluate the VaR and ES accuracy for predicting commodities market risk. This paper is organized as follow. Section 2 represents a brief review of extreme value theory and exposes the forecasting models of long-range memory of energy market volatility. Data and preliminary analysis are provided in section 3. Section 4 provides our empirical findings and section 5 outlines some portfolio implication and section 6 concludes the paper.

## 2. Methodology

### 2.1. Measures of extreme risk

Consider  $(X_t, t \in \mathbb{Z})$  a strictly stationary time series representing daily observations of the negative log-return on a financial asset price. The dynamic of  $X_t$  is assumed to be governed by:

$$X_t = \mu_t + \sigma_t Z_t \quad (1)$$

Where the innovations  $Z_t$  are a strict white noise process, independent and identically distributed, with zero mean, unit variance and marginal distribution function  $F_z(z)$ . We assume that  $\mu_t$  and  $\sigma_t$  are measurable with respect to  $I_{t-1}$  the information about the return process available up to time  $t-1$ .

Let  $F_X(x)$  denote the marginal distribution of  $(X_t)$  and, for a horizon  $h \in \mathbb{N}$  let  $F_{X_{t+1}+\dots+X_{t+h}|I_t}(x)$  denote the predictive distribution of the return over the next  $h$  days, given knowledge of returns up to and including day  $t$ .

We are interested of estimating unconditional and conditional quantiles in the tails of negative log-returns in these distributions. We remind that for  $0 < q < 1$  the  $q$ th unconditional quantile is a quantile for the marginal distribution denoted by:

$$x_q = \inf \{x \in \mathcal{R} : F_X(x) \geq q\},$$

And a conditional quantile is a quantile of the predictive distribution for the return over the next  $h$  days denoted by

$$x_q^t(h) = \inf \{x \in \mathcal{R} : F_{X_{t+1}+\dots+X_{t+h}|I_t}(x) \geq q\},$$

We also consider the expected shortfall (ES), known to be a measure of risk for the tail of a distribution. The ES is a coherent measure of risk in the sense of Artzner, Delbaen, Elsner, and Heath (2000). The unconditional expected shortfall is defined as:

$$ES_q = E[X | X > x_p]$$

And the conditional shortfall to be

$$ES_q^t = \left[ \sum_{j=1}^h X_{t+j} \middle| \sum_{j=1}^h X_{t+j} > x_q^t(h), I_t \right]$$

We are principally interested in quantiles and expected shortfalls for the 1-step predictive distribution. Thus we denote the quantiles respectively by  $x_q^t$  and  $ES_q^t$ . Since

$$\begin{aligned} F_{X_{t+1}|I_t}(x) &= P\{\sigma_{t+1}Z_{t+1} + \mu_{t+1} \leq x | I_t\} \\ &= F_Z\left(\frac{x - \mu_{t+1}}{\sigma_{t+1}}\right) \end{aligned}$$

These measures simplify to:

$$x_q^t = \mu_{t+1} + \sigma_{t+1}z_q \tag{2}$$

$$ES_q^t = \mu_{t+1} + \sigma_{t+1}E[Z | Z > z_q] \tag{3}$$

Where  $z_q$  is the upper  $q$ th quantile of the marginal distribution of  $Z_t$  which by assumption does not depend on  $t$ .

To implement an estimation procedure for these measures we must choose a particular model for the dynamics of the conditional mean and volatility. Many different models for volatility dynamics has been proposed in the econometric literature including models from the ARCH/GARCH family (Bollerslev et al., 1992), HARCH process (Muller et al., 1997) and stochastic volatility models (Sheppard, 1996). In this paper, we use the parsimonious but effective GARCH (1.1) process for the volatility. Our paper differ from the others papers existing in the literature by introducing some long-memory GARCH-type models to forecast energy price volatility by capturing some volatility stylized facts such as asymmetry and fat tails in the energy price return innovations and to provide better VaR's computations.

## 2.2. Modeling oil price volatility

For predictive purpose we fix a constant memory  $n$  so that at the end of day  $t$  our data consist of the last  $n$  negative log-returns  $(x_{t-n+1}, \dots, x_{t-1}, x_t)$ . We consider these to be a realization from a GARCH (p, q) process. Hence, the conditional variance of the mean-adjusted series  $\epsilon_t = X_t - \mu_t$  is given by

$$\sigma_t^2 = \omega + \sum_{j=1}^p \beta_j \sigma_{t-j}^2 + \sum_{j=1}^q \alpha_j \epsilon_{t-j}^2$$

Where  $p > 0, q > 0, \omega > 0, \alpha_i \geq 0 \forall (i=1,2,\dots,p)$  and  $\sum_{j=1}^p \beta_j + \sum_{i=1}^q \alpha_i < 1$ .

### 2.2.1. The long memory concept

According to Ding and Granger (1996), a series is said to have a long-memory if it displays a slowly declining autocorrelation function (ACF) and an infinite spectrum at zero frequency. Specifically, the series  $\{y_t\}_{t=0}^{\infty}$  is said to be a stationary long-memory process if the ACF,  $\rho(k)$  behaves as,

$$\rho(k) \approx c|k|^{2d-1} \text{ as } |k| \rightarrow \infty \quad (4)$$

Where  $0 < d < 0.5$  and  $c$  is some positive constant. The left hand side and the right hand side in Eq. (4) tend to 1 as  $k \rightarrow \infty$ . The ACF in Eq (1) displays a very slow rate of decay to zero as  $k$  goes to infinity and  $\sum_{k=-\infty}^{\infty} |\rho(k)| = \infty$ . This slow rate of decay can be contrasted with ARMA processes which have an exponential rate of decay, and satisfy the following bound:  $|\rho(k)| \leq ba^k$ ,  $0 < b < \infty$ ,  $0 < a < 1$  and consequently  $\sum_{k=-\infty}^{\infty} |\rho(k)| < \infty$ . A process that satisfies this condition is termed short-memory. Equivalently, long-memory can be defined as a spectrum that goes to infinity at the origin. This is,  $f(w) \approx cw^{-2d}$  as  $w \rightarrow 0$ . A simple example of long-memory is the fractionally integrated noise process,  $I(d)$ , with  $0 < d < 1$ , which is  $(1-L)^d y_t = \mu_t$ , where  $L$  is the lag operator, and  $\mu_t \rightarrow i, i, d(0, \sigma^2)$ . This model includes the traditional extremes of a stationary process,  $I(0)$  and a non stationary process. The fractional difference operator  $(1-L)^d$  is well defined for a fractional  $d$  and the ACF of this process displays a hyperbolic decay consistent with Eq. (4). A model that incorporates the fractional differencing operator is a natural starting point to capture long-memory. This is the underlying idea of the ARFIMA and FIGARCH class of processes. In practice we must resort to estimating the ACF with usual sample quantities

$$\hat{\rho}(k) = \frac{\left(\frac{1}{T}\right) \sum_{k+1}^T (y_t - \bar{y})}{\left(\frac{1}{T}\right) \sum_{k+1}^T (y_t - \bar{y})^2} \quad (5)$$

A second approach to measure the degree of long-memory has been to use semi-parametric methods. This allows one to review the specific parametric form, which is miss-specified and

could lead to an inconsistent estimate of the long memory parameter. In this paper, we consider the most two frequently used estimators of long memory parameter  $d$ . The first is the Geweke and Porter- Hudak (GPH) (1983) estimator, based on a log-periodogram regression. Suppose  $\{y_t\}$ ,  $t = 0, 1, \dots, T-1$ , is the dataset and define the periodogram for the first  $m$  ordinates as

$$I_j = \frac{1}{2\pi T} \left| \sum_{t=0}^{T-1} y_t \exp(iw_j t) \right|^2 \quad (6)$$

Where  $w_j = \frac{(2\pi j)}{T}$ ,  $j = 1, 2, \dots, m$  and  $m$  is chosen positive integer. The estimate of  $\hat{d}$  can then be derived from linear regression of  $\log(I_j)$  on a constant and the variable  $X_j = \log \left| 2 \sin \frac{w_j}{2} \right|$  which gives

$$\hat{d} = \frac{\sum_{j=1}^m (x_j - \bar{x}) \log I_j}{\sum_{j=1}^m (x_j - \bar{x})} \quad (7)$$

Robinson (1995a) provides formal proofs of consistency and asymptotic normality for the Gauss case with  $-0.5 < d < 0.5$ . The asymptotic standard error is  $\pi\sqrt{24m}$ . The bandwidth parameter  $m$  must converge infinitely with the sample size, but at a slower rate than  $\sqrt{T}$ . Clearly, a larger choice of  $m$  reduces the asymptotic standard error, but the bias may increase. The bandwidth parameter was set to  $(T)$  in Geweke and Porter-Hudak (1983). While Hurvich et al. (1998) show the optimal rate to be  $O(T^{4/5})$ . Velasco (1999) has shown that consistency extends to  $0.5 < d < 1$  and asymptotic normality to  $0 < d < 0.75$ . The other popular semi-parametric estimator is due to Robinson (1995b). Essentially, this estimator is based on the log-periodogram and solves:

$$\hat{d} = \arg \min R(d)$$

where

$$R(d) = \log \left( \frac{1}{m} \sum_{j=1}^m \omega_j^{2d} I_j \right) - \frac{2d}{m} \sum_{j=1}^m \omega_j \quad (8)$$

The estimator is asymptotically more efficient than the GPH estimator and consistency and asymptotic normality of  $\hat{d}$  are available under weaker assumptions than for the Gaussian case.

### 2.2.2. The fractional integrated GARCH model

Baillie et al. (1996) have extended the traditional GARCH model by considering an eventual fractional integration model. They suggested the FIGARCH mode which is able to distinguish between short memory and long memory in the conditional variance behavior. Formally, the FIGARCH (p, d, q) process is defined as follows:

$$\left[ \phi(L)(1-L)^d \right] \varepsilon_t^2 = \omega + [1 - \beta(L)](\varepsilon_t^2 - \sigma_t^2) \quad (9)$$

And



$$\begin{aligned}\sigma_t^2 &= \omega + \beta(L)\sigma_t^2 + [1 - \beta(L)\varepsilon_t^2] - \phi(L)(1-L)^d \varepsilon_t^2 \\ &= \omega + [1 - \beta(L)]^{-1} + \{1 - [1 - \beta(L)]^{-1} \phi(L)(1-L)^d\} = \omega[1 - \beta(L)]^{-1} + \lambda(L)\varepsilon_t^2\end{aligned}$$

Where  $(L)$  is the lag operator.  $\lambda(L) = \sum_{i=1}^{\infty} \lambda_i L_i$ , and  $0 \leq d \leq 1$ .  $\lambda(L)$  is an infinite summation which, in practice, has to be truncated. According to Baillie et al. (1996),  $\lambda(L)$  should be truncated at 1000 lags.  $(1-L)^d$  is the fractional differencing operator. It can be defined as follows:

$$\begin{aligned}(1-L)^d &= \sum_{k=0}^{\infty} \frac{\Gamma(d+1)L^k}{\Gamma(k+1)\Gamma(d-k+1)} = 1 - dL - \frac{1}{2}d(1-d)L^2 - \frac{1}{6}d(1-d)(2-d)L^3 - \dots \\ &= 1 - \sum_{k=1}^{\infty} c_k(d)L^k\end{aligned}$$

$$\text{Where } c_1(d) = d, \quad c_2(d) = \frac{1}{2}d(1-d), \text{ ect.} \quad (10)$$

### 2.2.3. The fractional integrated asymmetric power ARCH model

Tse (1998) have extended the FIGARCH (p,d,q) model in order to take into account asymmetry and the long-memory feature in the process of the conditional variance. He has introduced the function  $(|\varepsilon_t| - \gamma\varepsilon_t)^\delta$  of the APARCH process. The FIAPARCH (p,d,q) can be written as follows;

$$\sigma_t^\delta = \omega[1 - \beta(L)]^{-1} + \{1 - [1 - \beta(L)]^{-1} \phi(L)(1-L)^d\} (|\varepsilon_t - \gamma\varepsilon_t|)^\delta \quad (11)$$

Where  $\gamma, \delta$  and  $\lambda$  are the model parameters. The FIAPARCH process can consider for some stylized facts on volatility of financial and commodity prices. More specifically, (1) if  $0 < d < 1$  then volatility exhibits the long-memory property; (2) if  $\gamma > 0$ : negative shocks have more impact on volatility than positive shocks and inversely; (3)  $\lambda$  is the power term in the volatility structure. It should be specified by the data; (4) the FIAPARCH process also nests the FIGARCH process when  $\gamma = 0$  and  $\delta = 2$ . Consequently, the FIAPARCH process is superior to the FIGARCH because it takes into account asymmetry and long memory in the conditional variance behavior.

### 2.2.4. The hyperbolic GARCH

The HYGARCH model (Davidson, 2004) is obtained by extending the conditional variance of the FIGARCH model by introducing weights in the difference operator. The conditional variance of the HYGARCH model is expressed as follows:

$$\sigma_t^2 = \omega[1 - \beta(L)]^{-1} + \{1 - [1 - \beta(L)]^{-1} \rho(L)[1 + \alpha(1-L)^d]\} \varepsilon_t^2 \quad (12)$$

The FIGARCH and stable GARCH cases correspond to  $\alpha = 1$  and 0, respectively. We should note that the main advantage of this model is to take jointly into account volatility clustering, long-range memory and leptokurtosis in the time series behavior. However, this model is unable to consider for asymmetry in the return distribution.

### 2.3. Extreme Value Theory and the peaks-over-threshold model

In this section, we briefly describe how we obtain the quantile  $z_q$  by applying EVT techniques to the distribution of GARCH-models filtered innovations. We fix a high threshold  $u$  and we assume that excess residuals over this threshold have a generalized Pareto distribution (GPD) with tail index  $\xi$ .

$$G_{\xi, \beta}(y) = \begin{cases} 1 - \left(1 + \frac{\xi y}{\beta}\right)^{-\frac{1}{\xi}} & \text{if } \xi \neq 0 \\ 1 - \exp\left(-\frac{y}{\beta}\right) & \text{if } \xi = 0 \end{cases} \quad (13)$$

Where  $\beta > 0$  and the support is  $y \geq 0$  when  $\xi \geq 0$  and  $0 \leq y \leq -\beta/\xi$  when  $\xi < 0$ . The choice of this distribution follows from a limit result in EVT. Consider a general distribution function  $F$  and the corresponding excess distribution above the threshold  $u$  defined by:

$$F_u(y) = P\{X - u \leq y | X > u\} = \frac{F(y+u) - F(u)}{1 - F(u)} \quad (14)$$

For  $0 \leq y < x_0 - u$ , where  $x_0$  is the (finite or infinite) right endpoint of  $F$ . Balkema and de Haan (1974) and Pickands (1975) showed for a large class of distributions  $F$  that is possible to find a positive measurable function  $\beta(u)$  such that

$$\lim_{u \rightarrow x_0} \sup_{0 \leq y < x_0 - u} |F_u(y) - G_{\xi, \beta(u)}(y)| = 0$$

This result was shown by Balkema and de Haan (1974) and Pickands (1975). This result holds for continuous distributions used in statistics.

In our case we assume that the tail of the underlying distribution begins at the threshold  $u$ , with  $N$  the random variables of exceeding observations. For a sample for total size  $n$  the random proportion of extremes is then  $N/n$ . If we assume that the  $N$  excesses over the threshold are i.i.d with exact GPD distribution, the parameters  $\xi$  and  $\beta$  are estimated by maximum likelihood. Smith (1987) has shown that maximum likelihood estimates  $\hat{\xi}$  and  $\hat{\beta}$  of the GPD parameters  $\xi$  and  $\beta$  are consistent and asymptotically normal as  $N \rightarrow \infty$ , provided  $\xi > -1/2$ . Even under an approximate GPD distribution, parameters estimates  $\hat{\xi}$  and  $\hat{\beta}$  are unbiased and asymptotically normal, provided a sufficient rate of convergence. Under the assumption of dependent data, the GPD-based tail estimator is still asymptotically valid, but provides much less stable results compared to the i.i.d case. Embrechts and al (1997) provides a related example involving an  $AR(1)$  process.

The following equality holds for points  $x > u$  in the tail of  $F$

$$1 - F(x) = (1 - F(u))(1 - F_u(x - u)) \quad (15)$$

If we estimates the first term,  $(1 - F(u))$ , using the random proportion of the data on the tail  $N/n$ , and if we estimate the term  $1 - F_u(x - u)$ , where  $F_u(x)$  is defined in Eq (13), by approximating the excess distribution with a GPD fitted by maximum likelihood, we get the tail estimator

$$\hat{F}(x) = 1 - \frac{N}{n} \left(1 + \hat{\xi} \frac{x - u}{\hat{\beta}}\right)^{-1/\hat{\xi}},$$

For  $x < u$ , let  $Z_{(1)} \geq Z_{(2)} \geq \dots \geq Z_{(n)}$  represent the ordered residuals. If we fix the number of the data in the tail to be  $N = k$ , this gives us a random threshold at the  $(k+1)th$  order statistic. The GPD with parameters  $\xi$  and  $\beta$  is fitted to the data  $Z_{(1)} - Z_{(k+1)}, \dots, Z_{(k)} - Z_{(k+1)}$ , the excess amounts over the threshold for all residuals exceeding the threshold. The form of the tail estimator for  $F_z(z)$  is then

$$\hat{F}_z(z) = 1 - \frac{k}{n} \left( 1 + \frac{\hat{\xi} (z - z_{(k+1)})}{\hat{\beta}} \right)^{-1/\hat{\xi}} \quad (16)$$

For  $q > 1 - k/n$  we can invert this tail formula to get

$$\hat{z}_q = \hat{z}_{q,k} = z_{k+1} + \frac{\hat{\beta}}{\hat{\xi}} \left( \left( \frac{1-q}{k/n} \right)^{-\hat{\xi}} - 1 \right) \quad \text{the } qth \text{ quantile of the data distribution} \quad (17)$$

In order to estimate risk measures, VaR for crude oil market, our main interest is on extreme value theory based models: we consider only the conditional GPD approach.

#### 2.4. The peak over threshold: conditional GPD approach

Different approaches have proposed in the literature to estimate risk measures. The unconditional GPD represent the advantage is that it focuses attention directly on the tail of the distribution. However it doesn't recognize the fact that returns are none-i.i.d. The econometric models of volatility dynamics such as GARCH-process under different innovation's distributions yield VaR estimates which reflects the current volatility background. The weakness of this GARCH modeling approach is that focuses in the modeling of the whole conditional return distribution as time-varying, and not only on the part we are interested in, the tail. This approach may fail to estimate accurately risk measures like VaR and ES. In order to overcome drawbacks of each of the above methods, McNeil and Frey (2000) proposed to combine ideas from these two approaches. The advantage of this GARCH-EVT combination lies in its ability to capture conditional heteroscedasticity in the time series through the GARCH framework. While, simultaneously, modeling the extreme tails shape through the EVT method. Bali and Neftci (2003) apply the GARCH-EVT model to U.S short-term interest rates and show that the models yields best estimates of VaR than that obtained from a student-t distributed with GARCH models. Bystrom (2004), find similar conclusions indicating that GARCH-EVT performs better than the parametric models in forecasting VaR different international stock markets.

In the context of energy market, Bystrom (2005) apply GARCH-EVT to NordPool hourly electricity returns. Krehbiel and Adkins (2005) employ the some approach to the NYMEX energy complex. Marimoutou and al., (2009) apply this methodology to estimate risk measures in oil market. They find similar results confirming that conditional EVT model produces more accurate estimates of extreme tails.

The combined approach, denoted conditional GPD, may be presented in the following three steps:

**-Step1:** fit a GARCH-type model to the return data by the quasi-maximum likelihood. Estimate  $\mu_{t+1}$  and  $\sigma_{t+1}$  from the fitted model and extract the residuals  $z_t$ .

**-Step 2:** consider the standardized residuals computed in step 1 to be a realization of a white noise process, and estimate the tail of the innovations using the extreme value theory and then compute the quantiles of the innovations.

**-step 3:** construct VaR from parameters estimated in step 1 and 2.

## 2.5. Statistical accuracy of model-based VaR estimations

In order to back-test the accuracy for the estimated VaRs, we computed the empirical failure rates. By definition, the failure rate is the number of times returns (in absolute values) exceed the forecasted VaR. If the model is correctly specified, the failure rate should be equal to the specified VaR's level. In this study, the backtesting VaR is based on the Kupiec's (1995) and Engel and Manganelli (2004) for unconditional and conditional coverage tests.

### 2.5.1. The Kupiec's (1995) ( $LR_{uc}$ ) test

The main idea of the Kupiec's (1995) is to estimate of the probability of observing a loss greater than the VaR's amount. In order to test the accuracy and to evaluate the performance of the model-based VaR estimates, Kupiec (1995) provided a likelihood ratio test ( $LR_{uc}$ ) for testing whether the failure rate of the model is statistically equal to the expected one (unconditional coverage).

Consider that  $N = \sum_{t=1}^T I_t$  is the number of exceptions in the sample size  $T$ . Then

$$I_{t+1} = \begin{cases} 1 & \text{if } r_{t+1} < -VaR_{t+1|\mathcal{I}_t}(\alpha) \\ 0 & \text{else} \end{cases} \quad (18)$$

Follows a binomial distribution,  $N \rightarrow B(T, \alpha)$ . If  $p = E(N/T)$  is the expected exception frequency (i.e. the expected ratio of violations), then the hypothesis for testing whether the failure rate of the model is equal to the expected one is expressed as follows:  $H_0 : \alpha = \alpha_0$ ,  $\alpha_0$  is the prescribed VaR level. Thus the appropriate likelihood ratio statistic in the presence of the null hypothesis is given by

$$LR_{uc} = -2 \log \left\{ \alpha_0^N (1 - \alpha_0)^{T-N} \right\} + 2 \log \left\{ \left( \frac{N}{T} \right)^N \left( 1 - \left( \frac{N}{T} \right)^{T-N} \right) \right\} \quad (19)$$

Under the null hypothesis,  $LR_{uc}$  has a  $\chi^2(1)$  as an asymptotical distribution. Consequently, a preferred model of VaR prediction should display the property that the unconditional coverage measured by  $p = E(N/T)$  should equals the desired coverage level  $p_0$ .

### 2.5.2. Dynamic quantile test

Engel and Manganelli (2004) proposed a conditional coverage test by using a linear regression model based on the process of hit function:

$$H_t = I_t - \alpha = \begin{cases} 1 - \alpha & \text{if } r_t < -VaR_t \\ -\alpha & \text{else} \end{cases}$$

Where  $\{H_t\}$  is a centered process in the target probability  $\alpha$ . The dynamic of the hit function is modeled as:

$$H_t = \beta_0 + \sum_{j=1}^p \beta_j H_{t-j} + \sum_{k=1}^K \gamma_k g_k(z_t) + \varepsilon_t \quad (20)$$

Where  $\varepsilon_t$  an i.i.d process with is mean of zero and  $g(\cdot)$  is a function of past exceedences and of variable  $z_t$ . Under the hypothesis that the VaR estimation can deliver accurate VaR estimates and also the occurrence of  $p$  consecutive exceedences is uncorrelated, the

regressors should have no explanatory power. Hence the dynamic quantile ( $DQ$ ) test is defined as:

$$H_0 : \Psi = (\beta_0, \beta_1, \dots, \beta_p, \gamma_1, \gamma_2, \dots, \gamma_k)^T = 0$$

It's easy to show that the dynamic quantile test statistic, in association with the Wald statistic, is:

$$DQ = \frac{\hat{\Psi}^T X^T \hat{\Psi}}{\alpha(1-\alpha)} \rightarrow \chi^2_{1+p+K} \quad (21)$$

Where  $X$  denotes the covariates matrix in equation (19), and  $g(z_t) = \hat{Va}R_t$ .

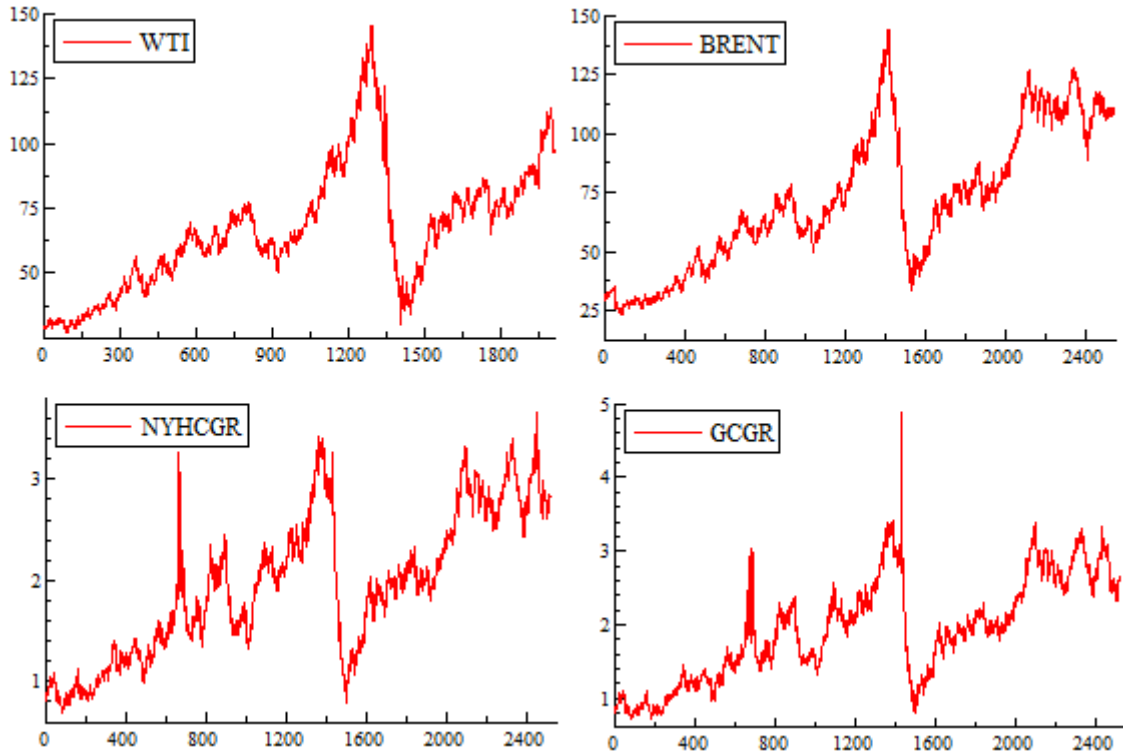


Fig.1. Daily energy commodities closing prices

### 3. Data description and preliminary analysis

#### 3.1. Data description and preliminary results

In this study we tend to estimate risk measures for energy commodities market. For this aim, we consider daily closing spot prices of four energy commodities: Cushing West Intermediate crude oil (WTI), Europe Brent crude oil (Brent), New York Harbor Conventional Regular gasoline (NYHCGR) and U.S. Gulf Coast Conventional Gasoline Regular (GCCGR). Closing prices of energy commodities are provided by the Energy Information Administration (EIA). For the selected series, the data covers the period (January, 2003-December 2012), totaling more than 2500 observations. The continuously compounded daily returns are computed as follows:

$$r_t = 100 \ln \left( \frac{p_t}{p_{t-1}} \right) \quad (22)$$

Where  $r_t$  and  $p_t$  are the return in percent and the energy commodity closing price on day  $t$ , respectively. This data set is challenging to model as is characterized by large energy commodities price increases and decreases that reflect a substantial rise in the volatility of real oil price (see Fattouh, 2005). We should mention that for the series, under study, the dataset is subdivided into two subsets. The last 1000 daily returns are reserved for the out- of-sample analysis. The first subset is used for the in-sample analysis.

Descriptive statistics, unit root, stationarity and long range memory tests are reported in Table 1. From the panel A, we can see that crude oil returns (WTI and Brent) returns are skewed toward the left, energy commodities are (NYHCGR and GCCGR) are right-skewed, and they did not correspond to the normal distribution assumption. According to the Jarque-Béra (1980) test statistic, we can surely reject the null hypothesis of Gaussian distribution for all returns. Using the Ljung-Box  $Q$  statistic of order 10 based on the squared returns, we can also reject the hypothesis of white noise and assert that time series are autocorrelated.

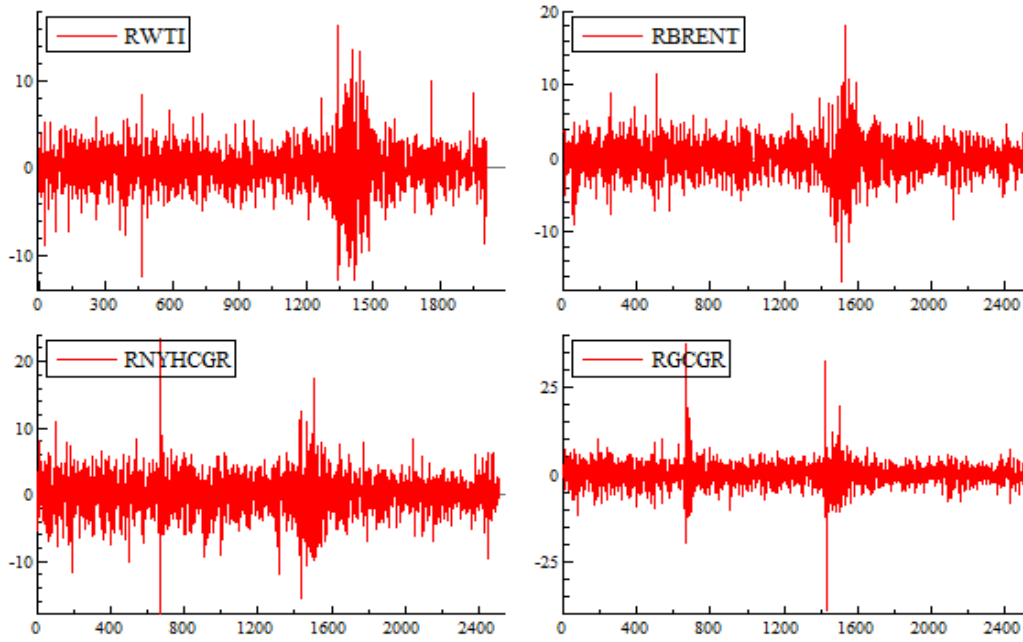


Fig.2. Daily energy commodities returns

Descriptive graphs (daily price levels, daily returns, normal probability plots of daily returns and histogram of daily returns against normal distribution) for each commodity are reported in figs 1-3. The graphs of daily returns show that all energy commodities were extremely volatile around the 2007-2008 periods, which led to a succession of extremely large positive and negative returns within a very short time horizon, indicating that risk management against market risk for oil related commodities is essential. Volatility clustering is manifestly apparent for energy commodities returns revealing the presence of heteroscedasticity. Moreover, the normal probability plots and the histograms against normal distribution graphs show that each returns distribution of data exhibit asymmetry and fat tails.

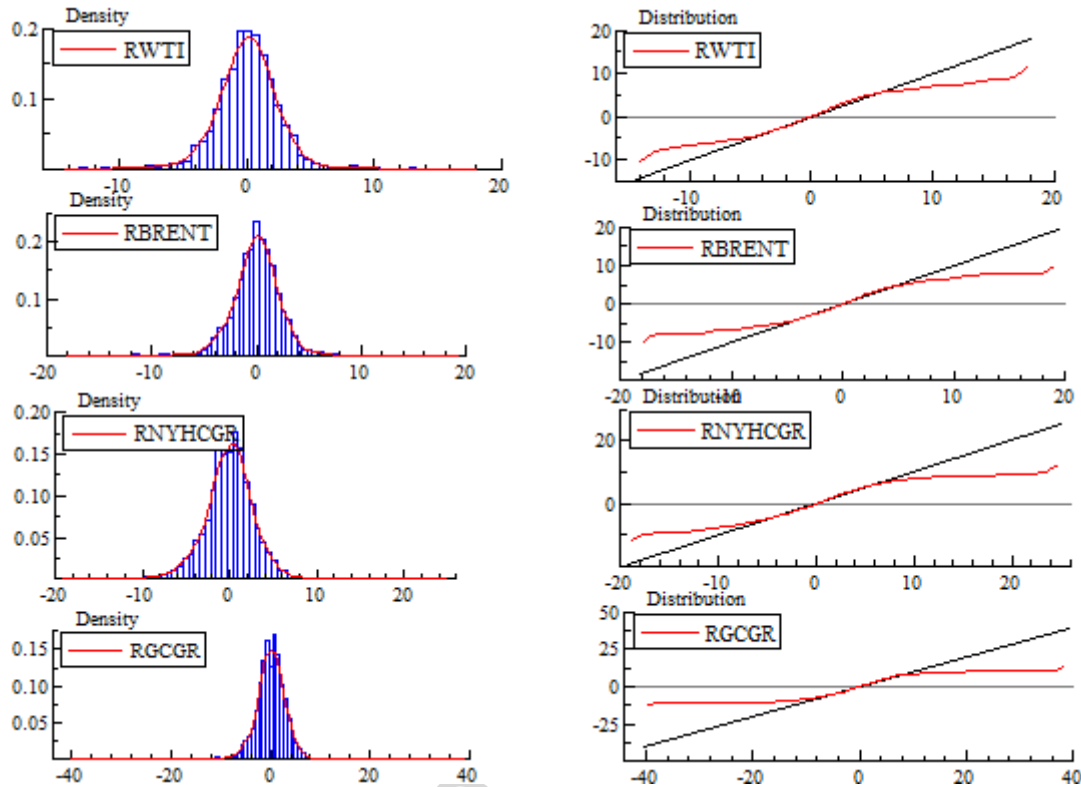


Fig.3. Normal probability plots and histogram X normal distribution for energy commodities daily returns.

Finally, before fitting series, we employ some tests in order to check for the presence of unit roots and to test stationarity. In panel *B* (table 1) we represent the results of the augmented Dickey-Fuller (1979) (ADF) and the Philips Peron (1988) (PP) unit roots tests and the Kwiatkowski, Phillips, Schmidt and Shin (1992) (KPSS) stationarity test. The ADF and PP tests undoubtedly reject hypothesis of unit root for the time series studied. So, we can conclude that energy commodities price returns are governed by an  $I(0)$  process which have no long-range memory. Furthermore, the KPSS test's results reveal that we cannot reject the stationarity null hypothesis at a 1% significant level for all the energy returns time series.

### 3.2. Testing long memory in crude oil volatility

In this paper, we consider two proxies of the daily volatility squared return and absolute returns. To test long memory we use two different long-memory tests: Lo's (1991) test and two semi-parametric estimators of long memory parameter, the log-periodogram regression (GPH) of Geweke and Porter-Hudak (1983) and the Gaussian semi-parametric (GSP) of Robinson (1995a). Empirical results are given in table 2. From these results reported in (Panel *C* table 1) we can conclude that energy commodities returns does not display long-rang memory in their mean's returns equation. Furthermore, Lo's test (R/S) does not reject the null hypothesis of no long-range memory.

Table 1: summary statistics, unit root, stationary test and long memory tests for daily log returns

	WTI	Brent	NYHCGR	GCCGR
<i>panel A: descriptive statistics</i>				
mean	0,0608	0.0472	0.03791	0.0493
maximum	16,4137	18.130	23.505	37.174
minimum	-12,8267	-16.832	-17.89	-38.675
S.D	2,5519	2.2687	2.8609	3.2313
skweness	-0.0076 (0.8879)	-0.0603 (0.2168)	0.1127 (0.0209)	0.2413 (0.0000)
kurtosis (excess)	4.3764** (0.0000)	4.9073** (0.0000)	5.2332** (0.0000)	21.097** (0.0000)
J-B test	1612,7058** (0.0000)	2522.0 ** (0.0000)	2870.7 ** (0.0000)	46589** (0.0000)
$Q^2(10)$	981,759** (0.0000)	365.982** (0.0000)	912.635** (0.0000)	498.549** (0.0000)
<i>panel B: unit root and stationary tests</i>				
ADF test	-24,4858** (0.0000)	-28.07** (0.0000)	-29.96** (0.0000)	-29.584** (0.0000)
PP test	-45,71223** (0.0000)	-77,43 (0.0000)	-66,28 (0.0000)	-55.65** (0.0000)
KPSS test	0,08229** (0.0463)	0.0662** (0.0347)	0.0321** (0.0423)	0.0242** (0.0453)
<i>Panel C: long-memory test statistics</i>				
$(r_t)$				
return				
Lo's R/S test	1,44791 (0.9476)	1.3541 (0.3324)	1.2728 (0.8779)	1.0871 (0.7493)
squared return $(r_t^2)$				
Lo's R/S test	4,9465 ** (0.0000)	4.0762** (0.0000)	2.7179** (0.0000)	2.3887** (0.0000)
absolute return $ r_t $				
Lo's R/S test	5,162** (0.0000)	4.5683** (0.0000)	4.2741** (0.0000)	4.6199** (0.0000)

**Note:** S.D. is the standard deviation of WTI returns, the descriptive statistics for cash daily returns are expressed on percentage, J-B test is the Jarque- Béra (1980) normality test statistic,  $Q^2(10)$  is the Ljung-Box Q-statistics of order 10 on the squared returns. ADF is the augmented Dickey Fuller (1979) unit-root test statistics, PP is the Phillip-Peron (1988) unit-root test statistics, KPSS is the Kwiatkowski, Phillips, Schmidt and Shin (1992) stationary test statistic. P-values are given into brackets. \*\*\*, \*\* denotes significance at 1% and 5% level respectively.



Table 2: GPH, GSP and R/V tests results

	WTI	Brent	NYHCGR	GCCGR
GPH (1983)				
Squared returns $(r_t^2)$				
$m = T^{0.5}$	0.6104	0.8133	0.2902	0.2588
$m = T^{0.6}$	0.4923	0.7409	0.2285	0.1918
$m = T^{0.8}$	0.2429	0.1831	0.2635	0.3294
Absolute returns $ r_t $				
$m = T^{0.5}$	0.6889	0.9255	0.7423	0.6715
$m = T^{0.6}$	0.5762	0.6228	0.4716	0.5408
$m = T^{0.8}$	0.2186	0.2497	0.2975	0.3991
GSP (1995a) Robinson				
Squared returns $(r_t^2)$				
$m = T/4$	0.2221	0.1813	0.2839	0.3335
$m = T/16$	0.3703	0.5907	0.2589	0.2514
$m = T/64$	0.7425	0.6733	0.4059	0.2991
Absolute returns $ r_t $				
$m = T/4$	0.2411	0.2552	0.3458	0.3869
$m = T/16$	0.3978	0.5631	0.4994	0.5266
$m = T/64$	0.7608	0.8592	0.7300	0.6550
Lo's (1991) test				
Squared returns $(r_t^2)$				
$m = 10$	3.0465	2.9191	1.8669	1.6609
$m = 40$	1.8241	1.7514	1.4694	1.2818
$m = 110$	1.2387	1.2293	1.4740	1.0680
Absolute returns $ r_t $				
$m = 10$	3.1924	3.0178	2.9545	2.9329
$m = 40$	1.8950	1.8616	2.0312	1.8932
$m = 110$	1.2783	1.2858	1.1932	1.3809

Notes:  $(r_t)$ ,  $(r_t^2)$  and  $|r_t|$  are, respectively, log return, squared log return and absolute log return.

$(m)$  denotes the bandwidth for the Geweke and Porter-Hudak's (1983) and the GSP Robinson (1995a) tests.

In fact we fail to reject the long-range memory in the return volatilities of energy commodities at the 5% significance level since the evaluated statistic are over the critical value. Moreover the GPH and GSP tests statistics confirm the existence of long-range memory in the squared and absolute returns series. Thus, energy market volatilities seem to be will described by a fractionally integrated process. Finally, we can suppose that clustering volatility, fat tails, asymmetry and long-range memory characteristics could be captured by an accurate model describing the dynamic behavior of energy commodities.

Table 3: Estimation results and diagnostic tests for FIGARCH, HYGARCH and FIAPARCH models

<i>Panel A: Estimation results</i>	FIGARCH				HYGARCH				FIAPARCH			
	WTI	Brent	NYHCGR	GCGR	WTI	Brent	NYHCGR	GCGR	WTI	Brent	NYHCGR	GCGR
$\mu$	0.1348*** (2.9930)	0.1036** (2.536)	0.1014** (2.073)	0.1389*** (2.829)	0.12193** (2.487)	0.1051** (2.578)	0.1003** (2.054)	0.1390*** (2.831)	0.0903* (1.8980)	0.0842** (2.137)	0.0912* (1.862)	0.1359*** (2.760)
$\omega$	0.1653*** (2.8150)	0.1461* (1.717)	0.3739** (2.199)	0.4540** (2.018)	0.0560** (2.072)	0.2415* (1.846)	0.0261 (0.0686)	0.4994 (1.496)	0.1909*** (3.8550)	0.1983** (2.261)	0.3611 (1.458)	0.4917*** (2.171)
$d$	0.7123*** (12.6600)	0.9476*** (68.58)	0.5303*** (4.574)	0.3220* (1.818)	0.9551*** (53.57)	0.6609*** (6.337)	0.4456** (0.0256)	0.3382* (1.743)	0.7081*** (14.12)	0.5773*** (6.882)	0.4814*** (3.384)	0.3712* (1.858)
$\beta$	0.4689*** (7.7830)	0.0415*** (4.094)	0.3571*** (2.976)	0.1646 (0.9625)	0.0200 (0.3341)	0.2607*** (4.084)	0.3546* (1.860)	0.1725 (0.9736)	0.4516*** (8.809)	0.3354*** (4.731)	0.3407** (2.391)	0.1891 (1.041)
$\alpha$	0.3836*** (6.1020)	0.3804*** (4.630)	0.2888*** (6.060)	0.2851*** (5.882)	1.0368*** (0.0000)	0.4697*** (3.702)	0.1464 (1.251)	0.2990*** (3.969)	0.4025*** (6.6400)	0.3021*** (4.915)	0.2524*** (3.753)	0.3108*** (5.883)
$Logalpha(HY)$					(-0.0107* (-1.786)	-0.0570 (-1.147)	0.3525 (0.7232)	-0.0202 (-0.2073)				
$\gamma_1$									0.3418*** (4.5920)	0.5013** (2.548)	0.0949 (1.141)	0.0256 (0.3651)
$\delta$									1.5346*** (8.484)	1.6092*** (7.105)	2.1066*** (6.862)	1.8724*** (10.44)
$Ln(1)$	-4489.9400	-5422.21	-5946.95	-6046.6	-4452.51	-5421.83	-5946.24	-6046.589	-4482.13	-5410.722	-5945.88	-6046.290
<i>Panel B: diagnostic tests</i>												
$Q^2(10)$	4.2178 (0.8370)	4.8058 (0.7781)	18.8162 (0.0158)	33.3417 (0.0151)	4.4910 (0.8103)	4.9902 (0.7586)	18.9248 (0.0152)	20.9966 (0.0071)	5.5851 (0.6936)	5.7066 (0.6800)	22.7289 (0.0037)	22.0881 (0.0047)
$ARCH(10)$	0.43279 (0.9311)	0.4676 (0.9115)	1.9811 (0.0316)	2.1730 (0.0169)	0.4442 (0.9250)	0.4859 (0.9002)	1.9986 (0.0298)	2.2412 (0.0478)	0.57115 (0.8386)	0.5553 (0.8511)	2.3796 (0.0084)	2.2717 (0.0121)
AIC	4.4770	4.2850	4.7430	4.8216	4.4427	4.2855	4.7425	4.8224	4.4383	4.2776	4.7422	4.8230
SIC	4.4910	4.3012	4.7639	4.8379	4.4650	4.3040	4.7610	4.8410	4.4634	4.2983	4.7585	4.8439

Notes:  $Ln(1)$  is the value of the maximized log-likelihood, t-values are reported in brackets.  $Q(10)$  and  $Q^2(10)$  are the box-Pierce statistics for remaining serial correlation for respectively standardized and squared standardized returns with p-values in brackets. AIC and SIC are the Akaike (1974) and Shibata information criterion respectively. \*, \*\*, \*\*\*denotes significantly at the 10%, 5% and 1% respectively.

## 4. Empirical results

### 4.1. Estimating GARCH-type models

Tables.3 represents the estimation results of the FIGARCH, HYGARCH and FIAPARCH models. Results in panel A (table 3) reveal that (FI) GARCH models are able to capture the long-range memory phenomenon for crude oil and gasoline returns volatilities. Therefore all the long-memory parameters ( $d$ ) strongly reject the GARCH null hypothesis ( $d$ )=0 at a 1% significance level. Turning to the goodness-of-fit tests, our results indicate that we cannot reject the null hypothesis of correct model specification since the Box-Pierce test statistics computed with 10 lags for both standardized residuals and squared standardized residuals show no serial correlation and no remaining ARCH effect.

Table (3) reports the estimation results of the HYGARCH model. The hyperbolic parameters  $\log(\hat{\alpha})(HY)$  are not significantly different from zero for both crude oil and gasoline price returns indicating that the GARCH components are covariance stationary. For the goodness-of-fit test (*panel B*), the diagnostic results reveal that the HYGARCH model is suitable to depict the heteroscedasticity exhibited in the time series since the Box-Pierce and the ARCH effect tests do not reject the null hypothesis of a correct model specification

Concerning the FIAPARCH estimation results (see table 3), energy commodities display strong evidence of volatility asymmetry since the ( $\delta$ ) parameters is statistically significant. The coefficient of asymmetric response of volatility to news ( $\gamma$ ) is significant and positive only for crude oil (WTI and Brent), providing strong evidence of leverage effect for negative oil prices returns in the conditional variance specification. Diagnostic tests reported in panel B including the Box-Pierce statistics for remaining serial correlation for squared standardized residuals and the ARCH effect test provide evidence of correct model specification and show the power of the FIAPARCH model to take into account the major stylized facts of time series prices behavior.

For practical forecasting purposes, the used information criteria including AIC and SIC as well as the likelihood ratio test came together to show that the FIAPARCH model outperforms the GARCH, FIGARCH and HYGARCH models and provides the best representation of the conditional mean and variance dynamics for energy commodities market except the GCGR. This model is more appropriate to capture long-range memory, asymmetry and clustering volatility in the time series behavior. On the other hand, it's shown that residual series is found to be free from autocorrelation. We can conclude that the filtering process successfully removes the time series dynamics from the return series and obtain an i.d.d series free from any time series dynamics. Therefore, we can apply successfully EVT methods to the i.d.d residual series. Obviously, in what follow we choose the FIAPARCH-EVT approach to compute the one-day-ahead VaRs for both crude oil and gasoline markets. Perceptibly, this result should be taken with caution. The forecast performance of this model should be evaluated for out-of-sample period and using more accurate performance criteria.

### 4.2. Forecasting performance analysis

In this section, we tend to test the forecasting performance of the selected long-memory GARCH-class models. Various criteria are employed in order to evaluate the in- sample one-day-ahead forecasting performance of the selected long-range memory models, namely, the Mincer-Zarnowitz (1969) regression, the mean squared error (MSE), the mean absolute

prediction error (MAPE) and the logarithmic loss function(LL). The in-sample one-day-ahead forecasting performance results are reported in table 4.

Table 4: The in sample one-day-ahead forecasting performance of the long-memory GARCH class models

	$\alpha$	$\beta$	$R^2$	MSE	MAPE	LL
<b>FIGARCH</b>						
WTI	0.0124	0.8295	0.1440	147.2	296.1	7.515
Brent	0.8782	0.9583	0.1760	96.45	575.9	8.921
NYHCGR	0.0441	0.0831	0.085	47.77	724	18.73
GCGR	0.4105	0.6572	0.0875	80.51	73.11	6.083
<b>HYGARCH</b>						
WTI	0.0340	0.8642	0.1419	146.7	287.1	7.434
Brent	0.9118	1.0135	0.1694	96.63	574.7	8.916
NYHCGR	0.1115	0.0910	0.0809	47.78	762.3	19.1
GCGR	0.0205	0.7145	0.0845	47.78	75.11	6.171
<b>FIAPARCH</b>						
WTI	0.7782	1.0060	0.1564	143.4	277.5	7.402
Brent	1.8450	1.1790	0.1640	97.73	588.6	8.829
NYHCGR	0.0583	0.0846	0.0852	48.05	727.6	18.81
GCGR	0.4239	0.6591	0.0886	80.38	72.72	6.069

Notes:  $\alpha$  and  $\beta$  are the estimated coefficients of the Mincer-Zarnowitz (1969) regression. While  $R^2$  is the determination coefficient of the Mincer-Zarnowitz (1969). MSE is the mean square error. MAPE is the mean absolute prediction error. LL is the logarithmic loss function.

The reported results indicate that the FIAPARCH model provides the best in-sample one-day-ahead forecasting for both energy and gasoline markets. Thus, the LL is minimized with the FIAPARCH model. With reference to Mincer- Zarnowitz (1969) regression's results we perceive that the FIAPARCH model outperforms the FIGARCH and HYGARCH models. In this regression, the  $R^2$  coefficient appreciates the predictive power for each model. The largest coefficients correspond to the FIAPARCH model indicating that the realized volatility is well explained by the FIAPARCH predicted volatility. This result is supported by the MSE and the MAPE prediction error criteria. From these results, we can attest that the FIAPARCH model is the best specification for considering for long-memory, asymmetry and fat-tails in energy commodities markets.

#### 4.3. Parameters analysis of the FIAPARCH-EVT model

The POT method deeply depends on the selection of threshold. The asymptotic theory will break down while the threshold is low enough. Inversely, the number of samples used to estimate the parameters in the excess distribution would be insufficient if the threshold value is too high. Table 5 reports the parameters calculated using the FIAPARCH-EVT model. To apply EVT, the threshold  $u$  is selected using mean excess function (MEF) and Hill plots. Table 5 reports the threshold selected in each market. In each case, the resulting exceedences  $Nu$  are between 5-10% of the sample which follows the recommendations from the simulation study in McNeil and Frey (2000). The tail index parameter ( $\xi$ ) and scaling parameter ( $\beta$ ) are determined by fitting the GPD to the standardized residuals obtained from the FIAPARCH model. For each energy commodity, the  $\xi$  estimates are positive and statistically significantly different from zero

suggesting that the right tail (results for left tail not reported but available upon request) of standardized residuals is characterized by heavy-tailed distributions.

Table 5: parameters estimates for the FIAPARCH-EVT model

		WTI	Brent	NYHCGR	GCCR
Total observation	N	2534	2534	2511	2511
number of exceedences	$Nu$	190	190	200	190
EVT thershold	$u$	2,9822	0,8034	1,4218	1,6765
% of exceedences	Nu/N	7,50%	7,50%	7,96%	7,57%
GPD Tail parameter	$\xi$	0,3511***	0,3184***	0,2448***	0,2861***
		3,1492	(-2,6885)	(-2,6647)	(-3,1056)
GPD scale parameter	$\beta$	1.6979***	0,3414***	0,7678***	0,8446***
		(8,8709)	(9,6038)	(7,0664)	(8,6448)

Notes:the table reports ML estimates of the GPD distribution for the FIAPARCH-EVT model. t-values are reported in parentheses. \*\*\* denotes significance at the 1% level.

#### 4.5. In-sample VaR estimations

In this sub-section, we estimate the one-day-ahead VaRs and ES via the FIAPARCH-EVT approach and GARCH-EVT approach as a benchmark model (for GARCH-EVT results are not reported but available upon request). As mentioned above, we compute the Kupiec's (1995) LR tests and the  $DQ$  test of Engle and Manganelli (2004) for VaR's levels ranging from 5% to 0.25% and for both short and long trading positions. Regarding the backtesting VaR's and the GARCH-EVT approach results, we perceive that the GARCH-EVT model provide poor forecasting for the energy and gasoline market risk. For this model p-values for the Kupiec's and Engle and Manganelli tests are less than 5% for major energy commodities and especially for high confidence levels (99% and 99,75%). Results are the same for both short and long trading positions.

As shown in table 6, the backtesting VaR results reveal that the FIAPARCH-EVT approach performs very well for the one day time horizon. More specifically, for the short trading position, the p-values associated to the Kupiec's (1995) show that we are unable to reject the null hypothesis for all the  $\alpha$  significance levels and for the energy and gasoline commodities. Concerning the  $DQ$  test, p-values reveal that the FIAPARCH-EVT specification does not perform well for the Brent and gasoline market especially for high confidence levels. Our empirical results show different findings for the long trading position. Results from (table 6) reveals that the FIAPARCH-EVT model performs very well for the one day- ahead forecasts for long trading position and for both crude oil and gasoline markets. More importantly, we observe that with more consecutive choice of the quantile level, the model is able to consider for the fat tails exhibited in the returns very well.

With reference to the GARCH-EVT as a benchmark approach we perceive that the FIAPARCH model combined with GPD distribution provides marginally better VaR forecasts. Over all, we can conclude that the FIAPARCH model combined with extreme value theory does very well in predicting critical loss for both crude oil and gasoline markets. Our findings reveals that models considering for some stylized facts such that volatility clustering, long range memory and leptokurtosis in the time series behavior enhances the VaR predicting for both high and low confidence levels and for the short and the long trading position.

Table 6: In-sample VaR and Expected Shortfalls (ES) estimations calculated by the FIAPARCH-EVT approach

<i>Quantile</i>	<i>WTI</i>			<i>BRENT</i>			<i>NYHCGR</i>			<i>GCGR</i>		
	$LR_{uc}$	$DQ$	ES	$LR_{uc}$	$DQ$	ES	$LR_{uc}$	$DQ$	ES	$LR_{uc}$	$DQ$	ES
Short position												
0.95	0.8945	0.8078	0.41	0.3706	0.6179	0.63	0.9671	0.7731	0.70	0.4851	0.4143	0.61
0.975	0.5364	0.2910	0.52	0.1363	0.3732	0.62	0.5364	0.3240	0.42	0.0906	0.1433	0.38
0.99	0.2003	0.8411	0.40	0.7872	0.0094	0.46	0.2003	0.0000	0.50	0.0840	0.0816	0.44
0.995	0.1672	0.8416	0.38	0.9262	0.0000	0.96	0.8743	0.0000	0.91	0.6530	0.0034	0.53
0.9975	0.3293	0.9699	0.48	0.3189	0.0000	0.75	0.1711	0.0002	0.23	0.5965	0.0000	0.51
long position												
0.05	0.6751	0.0989	0.53	0.7645	0.5903	0.66	0.4438	0.4204	0.95	0.6853	0.6980	0.51
0.025	0.7208	0.5242	0.48	0.7376	0.9544	0.51	0.4334	0.2750	0.85	0.9770	0.8844	0.42
0.01	0.7080	0.0668	0.42	0.6350	0.3172	0.21	0.5239	0.9618	0.97	0.8225	0.7310	0.49
0.005	0.9003	0.9991	0.93	0.9262	0.9991	0.68	0.5021	0.9901	0.48	0.8743	0.0139	0.41
0.0025	0.3072	0.9859	0.51	0.5246	0.9981	0.58	0.7768	0.9999	0.61	0.5092	0.9978	0.62

Notes: table reports p-values for the Kupiec's (1995) and Engle and Manganelli (2004) tests. ES represent p-value for a one-sided block-bootstrap test of the hypothesis that standardized exceedences of residuals in the GPD case have mean zero against the alternative that the mean is greater than zero.

Table 7: Out-of-sample VaR and Expected Shortfalls (ES) estimations calculated by the FIAPARCH-EVT approach

<i>Quantile</i>	<i>WTI</i>			<i>BRENT</i>			<i>NYHCGR</i>			<i>GCGR</i>		
	<i>DQ</i>		ES	<i>DQ</i>		ES	<i>DQ</i>		ES	<i>DQ</i>		ES
Short position												
0.95	0.0074	0.0493	0.65	0.0002	0.000	0.10	0.0187	0.1184	0.40	0.0286	0.1857	5.07
0.975	0.0012	0.0002	0.38	0.0030	0.0001	0.56	0.1912	0.0360	0.54	0.0067	0.0562	0.64
0.99	0.0284	0.1542	0.19	0.0747	0.5038	0.81	0.9797	0.1674	0.51	0.1624	0.8227	0.58
0.995	0.3243	0.9635	0.16	0.1219	0.5893	0.27	0.9857	0.9999	0.98	0.3243	0.9658	0.91
0.9975	0.7336	0.9999	0.79	0.2745	0.8882	0.13	0.3901	0.9962	0.70	0.7336	0.9999	0.42
long position												
0.05	0.4270	0.2417	0.91	0.0286	0.0102	0.97	0.3449	0.5135	0.39	0.3449	0.2176	0.44
0.025	0.7195	0.7686	0.88	0.0790	0.0097	0.78	0.1912	0.4792	0.65	0.1912	0.5817	0.38
0.01	0.7276	0.0744	0.22	0.3021	0.9510	0.30	0.3021	0.9510	0.69	0.3021	0.9510	0.41
0.005	0.1219	0.5893	0.56	0.3243	0.9635	0.90	0.9857	0.9999	0.17	0.6299	0.9991	0.24
0.0025	0.2745	0.8882	0.53	0.2745	0.8882	0.24	0.2745	0.8882	0.67	0.7688	0.9999	0.87

Notes: table reports p-values for the Kupiec's (1995) and Engle and Manganelli (2004) tests. ES represent p-value for a one-sided block-bootstrap test of the hypothesis that standardized exceedences of residuals in the GPD case have mean zero against the alternative that the mean is greater than zero.

#### 4.6. Out-of-sample VaR estimations

In this sub-section, we are interested to evaluate the performance of the selected long-memory GARCH model combined with extreme value theory by computing the out-of-sample VaR and ES forecasts. As in Giot (2003), Giot and Laurent (2003, 2004), Kang and Yoon (2007), Aloui and Mabrouk (2010), our predictions are based on a window updating the model parameters every 50 observations in the out-of-sample period. As in the in-sample VaR analysis, these out-of-sample VaRs are compared with observed returns and then both results are recorded for latter assessment via Kupiec's (1995) LR and DQ test statistics of Engle and Manganelli (2004). The out-of-sample VaR's forecasts for long and short trading positions are provided in table 7. The obtained results (see table 7) reveal that the FIAPARCH-EVT performs well for the out-of-sample forecasts for both energy and gasoline markets and under all the confidence levels. The reported results indicate that the out of sample VaR forecasts are noticeably better than their counterparts for the in-sample period. This result provides strong evidence that the long-memory GARCH-class models are able to capture major stylized facts on energy commodities market return and volatility dynamics. With reference to the GARCH-EVT approach as a benchmark model, we perceive that the FIAPARCH model with GPD distributions provide better forecasts for all the critical levels. Regarding the reported results, we can see that for the long trading positions, the out of sample VaR forecasts are marginally better than those for the short position. In fact, the LR Kupiec test and DQ test's p-values show that we cannot reject the null hypothesis of adequate model for all confidence levels and for all markets under study. In addition, we remark that the in-sample p-values are greater than those of the out-of-sample forecasts. Furthermore, the FIAPARCH-EVT method yields accurate VaR quantification which account for volatility dynamics. In facts, this model performs well for crude oil and gasoline markets and for various confidence levels considered. The current findings also show that conditional GPD model performs well, especially for markets where the distributions of returns exhibits extreme moments.

#### 4.7. ES and backtesting exercise

In this section we discuss the expected shortfall (ES) estimations in our models. To backtest the method we use the one sided test proposed by McNeil and Frey (2000). This test will compare standardized returns with the expected shortfall to establish criteria to investigate if expected shortfalls correctly describe actual average exceedences. We are interested in the size of the discrepancy between  $X_{t+1}$  and  $ES'_q$  in the event of quantile violation. We define residuals

$$R_{t+1} = \frac{X_{t+1} - ES'_q}{\sigma_{t+1}} = Z_{t+1} - E[Z|Z > z_q]$$

For the particular case where the estimated model is correctly specified, these residuals are i.i.d., and, conditional on  $\{X_{t+1} > x'_q\}$  or equivalently  $\{Z_{t+1} > z_q\}$ , they have an expected value of zero. To measure ES violations, we then consider residuals that correspond to days when returns actually exceed the VaR threshold. Thus,



$$r_{t+1} = \frac{X_{t+1} - \hat{ES}_q^t}{\hat{\sigma}_{t+1}} \text{ if } \{X_{t+1} > \hat{x}_q^t\}$$

Where  $\hat{ES}_q^t$  is an estimate of the expected shortfall. Under the null hypothesis that we correctly estimate the dynamics of the process ( $\mu_{t+1}$  and  $\sigma_{t+1}$ ) and the first moment of the truncated innovation distribution ( $E[Z|Z > z_q]$ ), this residuals should behave like an iid sample with mean zero. To test the hypothesis of mean zero we use a bootstrap test that makes no assumption about the underlying distribution of the residuals. We conduct a one-sided test against the alternative hypothesis that the residuals have mean greater than zero, or equivalently, that conditional expected shortfall is systematically underestimated. The test is based on the usual t-statistic that we obtain using the algorithm of Efron and Tibshirani (1993, p224) but in a dependent bootstrap setup. The p-values are deduced from the empirical distribution of the t-statistic that we obtain from our simulations.

In table 6 and 7 we report jointly to the in-sample and the out-of-sample VaR's backtesting the p-value corresponding to the test of ES violation applied for the GPD residuals for the energy and the gasoline commodities and for both long and short trading position for the in-sample and out-of-sample periods. For instance, results reveal that p-values relative to the test of ES violation are in the major case over 5% for energy and gasoline markets and under different confidence levels considered.

The overall picture that emerges from these tables is that the FIAPARCH-EVT method used in this paper does not fail the test on any occasion for both the in-sample and the out-of-sample results. However, we note that the GARCH-EVT approach, considered as a benchmark in this work, may perform as well as the FIAPARCH-EVT for this ES exercise.

#### 4.8. Multiple day horizons

In this section we are interested to VaR's estimates for multiple day horizons ( $h > 1$ ). In fact, according to the existent literature, the VaR forecasting performance is depending on the time-horizon. Explicit estimation of the risk measure for horizons longer than one day is difficult, and is tempting to use a sample scaling rule, such as the square root of time rule to turn one-day VaR into  $h$ -day VaR. Unfortunately, square root of time is designed for cases when returns are normally distributed and is not appropriate for the kind of stochastic volatility model driven by heavy-tailed noise.

To estimate dynamic risk measures for longer time horizons it's possible to adopt a simulation approach to obtain this estimate. Possible future paths for the stochastic volatility model of the returns are simulated and possible  $h$ -day losses are calculated.

To calculate a single future path on day  $t$  we could proceed as follow. The noise distribution is modeled with a composite model consisting of GPD tail estimates for both tails and a simple empirical (ie. Simulation) estimate based on the model residuals in the centre.  $h$  independent values ( $Z_{t+1}, \dots, Z_{t+h}$ ) are simulated from this model. Using this composite estimate of the noise distribution and the current estimated volatility from the fitted long-memory GARCH model we can simulate future paths ( $x_{t+1}, \dots, x_{t+h}$ ) and calculate the corresponding cumulative sums which are simulated iid observations of the  $h$ -day loss. This loss is taken as a realization from the conditional distribution of the  $h$ -day loss.

For horizons of  $h=5$  and  $h=20$  days the out-of-sample backtesting results are collected in table 8. We compare the simulation method proposed above with the first approach FIAPARCH-EVT firstly used to estimate the one-day ahead VaRs.

Table 8: backtesting results for the out-of-sample VaR's estimations for  $h=5$  and  $h=20$  using the FIAPARCH-EVT approach.

5-days ahead VaR estimation								
Quantile	WTI		BRENT		NYHCGR		GCCGR	
Short position	$LR_{uc}$	DQ	$LR_{uc}$	DQ	$LR_{uc}$	DQ	$LR_{uc}$	DQ
0.95	0,4133	0,4082	0,8421	0,3718	0,0232	0,0156	0,0023	0,0037
0.975	0,4879	0,8687	0,2318	0,5378	0,0015	0,0020	0,0229	0,1999
0.99	0,8357	0,9989	0,5542	0,9953	0,0577	0,3189	0,1528	0,7792
0.995	0,6764	0,9996	0,9935	0,9767	0,3130	0,9514	0,3130	0,9514
0.9975	0,9367	1,0000	0,9957	0,9995	0,4761	0,9921	0,9368	1,0000
long position								
0.05	0,1384	0,2425	0,6363	0,1099	0,8934	0,2128	0,7160	0,1402
0.025	0,6306	0,8173	0,4880	0,8836	0,4814	0,9510	0,6307	0,0103
0.01	0,5541	0,9953	0,5770	0,3189	0,8358	0,9989	0,3954	0,3122
0.005	0,9105	1,0000	0,6765	0,9997	0,9105	1,0000	0,1380	0,9372
0.0025	0,4567	0,9981	0,9368	1,0000	0,4568	0,9981	0,4568	0,9983
20-days ahead VaR estimation								
Short position								
0.95	0,3207	0,4217	0,3207	0,3982	0,0138	0,0057	0,0138	0,0085
0.975	0,1456	0,6780	0,2318	0,5378	0,0042	0,0168	0,0229	0,1999
0.99	0,5542	0,9953	0,1528	0,7708	0,1528	0,7708	0,1528	0,7792
0.995	0,3130	0,9514	0,9595	0,9557	0,3130	0,9514	0,3130	0,9514
0.9975	0,4761	0,9921	0,9964	0,9957	0,4761	0,9921	0,9368	1,0000
long position								
0.05	0,9734	0,6307	0,3207	0,3654	0,8934	0,3135	0,7160	0,7642
0.025	0,2318	0,5428	0,1456	0,6922	0,4814	0,6897	0,7996	0,4217
0.01	0,5542	0,9953	0,5770	0,3189	0,6093	0,9917	0,2390	0,3884
0.005	0,6765	0,9997	0,6765	0,9997	0,6765	0,9997	0,1380	0,9372
0.0025	0,9368	1,0000	0,9368	1,0000	0,4761	0,9921	0,9368	1,0000

Note: table reports p-values for the Kupiec's (1995) and Engle and Manganelli (2004) tests for the out-of-sample 5-days-ahead VaR's and 20-days-ahead VaR's estimation using the FIAPARCH-EVT approach.

Results reported in table 8 indicate that the FIAPARCH-EVT approach still perform well for time horizons over than one day. In fact, reported p-values relative to the unconditional coverage and the DQ tests are over than 5% for both short and long trading position and that for all energy markets and all confidence levels considered. Some cases are detected for both long and short trading position where we can't accept the null hypothesis of the tests for gasoline markets (NYHCGR and GCCGR) for low confidence levels (95% and 97,5%). Overall, we can conclude that the FIAPARCH-EVT provide accurate VaR's prediction for energy commodities markets even for low estimations frequencies.

## 5. Policy implications

The rise in commodity prices over the last ten years and their recent volatility has generated considerable interest on the part of investors, regulators and policy-makers. Attracted by the prospect of robust returns, diversification benefits, and potential for hedging inflation and macroeconomic risks, investors have increased their allocations to commodities over the period, primarily via passive investment into long-only commodity futures indices. However, unexpected price changes are fundamentally determined by supply and demand imbalances, storage limitations and seasonality effects. Equivalently, price volatility can stem from the behavior of same market participant who engage in short-term speculation. In this context, academic research has made it clear that investors should take long as well as short positions in commodity energy markets.

Our findings show that crude oil and gasoline prices present similar future prices and behave like a typical term structure and represent high volatility especially in period of commodity spikes of 2007-2008 and 2009-2011. Consequently investors should investigate the conditional volatility of energy commodities and their conditional correlations to reduce the dimension of the long-short commodity portfolio risk and consequently their exposure to unexpected losses. Indeed, the strategic decision to include commodities in a well-diversified portfolio does not solely depend on the risk premium of each commodity futures viewed as an asset class but is also driven by a desire for risk diversification and thus depends on how the returns of commodity investments correlate with the rest of the investor's portfolio over time.

## 6. Conclusion

As the volatility in the energy market increases, it's extremely important to implement an effective risk management system against market risk. In this context, VaR has become the most popular tool to measure risk for institutions and regulators. In addition extreme value theory has been successfully applied in many fields where extreme values can appear. In this paper we introduce an extension of the McNeil and Frey (2000) approach based on the conditional-GPD method. We extend the GARCH model to the fractional integrated GARCH models to take into account major stylized facts in to the price return volatilities of energy and gasoline markets. Our findings reveal that energy commodities markets are characterized by asymmetry, fat-tail and long range memory. In addition, oil and gasoline market's volatilities exhibit strong evidence of long-range memory, which can be well captured by the FIAPARCH model. In terms of predictive accuracy and referring to several statistical criteria, our results show that the FIAPARCH model outperforms the GARCH, FIGARCH and the HYGARCH models for the energy commodities markets. The FIAPARCH model combined with extreme value theory performs better in predicting the one-day ahead VaRs. This approach show strong ability to correctly forecast out-of-sample VaR at time horizon longer than one day. Since commodities prices over the long-run are fundamentally determined by the economic cycles and availability of resources, it's interesting to assess the performance of our model for 1-week or 1-month risk framework. Using a combination of bootstrap and GPD simulation we estimate 5-days-ahead and 20-days-ahead VaRs for all commodities market. The backtesting results show that our approach still performs well in predicting energy market risk for time horizons over than one day.

In this paper we advocate the expected shortfall as an alternative risk measure to the quantile based risk-measures such as VaR, which overcomes the deficiencies of the latter. Using a circular bootstrap method to backtest the performance prediction of our approach, we find that

we cannot reject the assumption that the forecasted expected shortfall measure captures actual shortfalls in a satisfactory manner for both in-sample and out-of-sample periods. Overall, our findings confirm that taking into account long-range memory clustering volatility and fat tails in the behavior of time series, combined with filtering process such as EVT is important in improving risk management assessments and hedging strategies.

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**Highlights**

- ✓ We evaluate Value-at-Risk (VaR) and Expected Shortfall (ES) for crude oil and gasoline commodities market for both long and short position.
- ✓ We adopt three (FI)GARCH models to forecast energy commodities volatility.
- ✓ Then we consider Extreme Value Theory (EVT) to model the tail distribution.
- ✓ The FIAPARCH-EVT model performs better in predicting VaR for energy commodities for different time horizons (1-day, 5-days and 20-days) and for both short and long trading position.