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Abstract

Purpose – The purpose of this paper is to introduce the elementary concepts and fundamental principles of grey systems and the main components of grey systems theory. Also to discuss the astonishing progress that grey systems theory has made in the world of learning and its wide-ranging applications in the entire spectrum of science.

Design/methodology/approach – The characteristics of unascertained systems including incomplete information and inaccuracies in data are analysed and four uncertain theories: probability statistics, fuzzy mathematics, grey system and rough set theory are compared. The scientific principle of simplicity and how precise models suffer from inaccuracies are also shown.

Findings – The four uncertain theories, probability statistics, fuzzy mathematics, grey system and rough set theory are examined with different research objects, different basic sets, different methods and procedures, different data requirements, different emphasis, different objectives and different characteristics.

Practical implications – The scientific principle of simplicity and how precise models suffer from inaccuracies are shown. So, precise models are not necessarily an effective means to deal with complex matters, especially in the case that the available information is incomplete and the collected data inaccurate.

Originality/value – The elementary concepts and fundamental principles of grey systems and the main components of grey systems theory are introduced briefly. The reader is given a general picture of grey systems theory as a new method for studying problems where partial information is known, partial information is unknown; especially for uncertain systems with few data points and poor information.

Keywords Grey systems theory, Mathematics, Appearance, Development, Uncertain systems

Paper type Research paper

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1. The scientific background for appearance of grey systems theory

On the basis of dividing the spectrum of scientific and technological endeavors into finer sections, the overall development of modern science has shown the tendency of synthesis at a higher level. This higher level synthesis has caused the appearance of the various studies of systems science with their specific methodological and epistemological significance. Systems science reveals the much deeper and more intrinsic connections and interlockings of objects and events and has greatly enriched the overall progress of science and technology. Many of the historically difficult problems in different scientific fields have been resolved successfully along with the appearance of systems science and its specific branches. And because of the emergence of various new areas in systems science, our understanding of nature and the laws that govern objective evolutions has been gradually deepened. At the end of the 1940s, there appeared general systems theory, information theory, cybernetics and operations research. During the 1950s and the 1960s, systems dynamics and the theory of dissipative structures have been put forwarded. During the 1970s, there appeared one by one such new transfield and interfiled theories of systems science as the synergetics, catastrophe theory, hypercycle theory, genetic algorithms, chaos theory and fractal theory, etc.

When people investigate systems, due to both the existence of internal and external disturbances and the limitation of our understanding, the available information tends to contain various kinds of uncertainty and noises. Along with the development of science and technology and the progress of the mankind, our understanding of uncertainties of systems has been gradually deepened and the research of uncertain systems has reached at a new height. During the second half of the twentieth century, in the areas of systems science and systems engineering, the seemingly non-stoppable emergence of various theories and methodologies of unascertained systems has been a great scene. For instance, Zadeh (1965) established fuzzy mathematics in the 1960s, Julong Deng developed grey systems theory and Pawlak (1982) advanced rough set theory in the 1980s, etc. All these works represent some of the most important efforts in the research of uncertain systems of this time period. From different angles, these works provide the theories and methodologies for describing and dealing with uncertain information.

The grey systems theory, established by Julong Deng in 1982, is a new methodology that focuses on the study of problems involving small samples and poor information (Deng, 1982). It deals with uncertain systems with partially known information through generating, excavating and extracting useful information from what is available. So, systems' operational behaviors and their laws of evolution can be correctly described and effectively monitored. In the natural world, uncertain systems with small samples and poor information exist commonly. That fact determines the wide range of applicability of grey systems theory.

2. The development history and current state of grey systems theory

In 1982, *Systems & Control Letters*, an international journal by North-Holland, published the first paper in grey systems theory, "The control problems of grey systems," by Julong Deng. In the same year, the *Journal of Huazhong University of Science and Technology* published the first paper, also by Julong Deng, on grey systems theory in Chinese language. The publication of these papers signaled the official appearance of the cross disciplinary grey systems theory. As soon as these works appeared, they immediately caught the attention of many scholars and scientific practitioners from across the world. Numerous well-known scientists strongly supported the validity and livelihood of such research. Many young scholars actively participated in the investigation of grey systems theory. With great enthusiasm these young men and women carried the theoretical aspects of the theory to new heights and employed their exciting results to various fields of application. In particular, successful applications in great many fields have won the attention of the international world of learning. Currently, a great number of scholars from China, the USA, England, Romania, South Africa, Germany, Japan, Australia, Canada, Poland, Spain, Cuba, Korea, Russia, Turkey, The Netherlands, Iran and others, have been involved in the research and application of grey systems theory. In 1989, the British journal, *The Journal of Grey System*, was launched. Currently, this publication is indexed by INSPEC (formerly Science Abstracts) of England, *Mathematical Review* of the USA, Science Citation Index and other important indexing agencies from around the world. In 1997, a Chinese publication, named *Journal of Grey System*, is launched in Taiwan. It is later in 2004 that this publication becomes all English. In 2011, a new journal, named *Grey Systems: Theory and Application,* edited by the faculty of Institute for Grey Systems Studies, Nanjing University of Aeronautics and Astronautics (NUAA), launched by Emerald of England. There are currently thousands of different professional journals in the world that have accepted and published papers in grey systems theory. As of this writing, an journal of the Association for Computing Machinery (USA), *Communications in Fuzzy Mathematics* (Taiwan), *Kybernetes: The International Journal of Systems & Cybernetics,* also sponsored by Emerald, have published several special issues on grey systems theory.

Many universities around the world offer courses in grey systems theory. For instance, NUAA not only offers such courses to PhD level and master level students, but also provides a service course on grey systems to all undergraduate students in different majors.

Huazhong University of Science and Technology, NUAA, Wuhan University of Technology, Fuzhou University, De Montfort University, Bogazici University, University of Cape Town, Bucharest University of Economics, Kanagawa University and several universities in Taiwan recruit and produce PhD students focusing on the research in grey systems. It is estimated that thousands upon thousands of graduate students from around the world employ the thinking logic and methodology of grey systems in their research and the writing of their dissertations.

Many publishers from around the world, such as Science Press, Press of National Defense Industry, Huazhong University of Science and Technology Press, Jiangsu Press of Science and Technology, Shangdong People's Press, Literature Press of Science and Technology, Taiwan Quanhua Science and Technology Books, Taiwan Guaoli Books Limited, Science and Engineering Press of Japan, the IIGSS Academic Publisher and Taylor & Francis Group (USA), Springer-Verlag (Germany), etc. have published over 100 different kinds of monographs in grey systems.

A whole array of brand new hybrid branches of study, such as grey hydrology, grey geology, grey theory and methods of breeding, grey medicine, grey systems analysis of regional economics, etc. have appeared along with the opportunity presented by grey systems theory. Agencies at national, provincial and local governments all actively sponsored research works in grey systems. Each year a very good number of theoretical and applied works on grey systems are financially supported by various foundations.

It is estimated that throughout China more than 200 research outcomes of grey systems were recognized officially by national, provincial, or ministerial agencies. Since 2002, several Chinese scholars, Professor Julong Deng *et al.*, received recognition from the World Organization of Cybernetics and Systems and IEEE Systems, Man and Cybernetics Society.

It is found that such internationally recognized indexing sources as SCI, EI, ISTP, SA, MR, MA and others, have collected research works in grey systems theory more than 20,000 times. From 1982 to 2010, more than 50,000 papers on grey system were retrieved from Chinese academic periodical database in China National Knowledge Infrastructure. Along with them, more than 10,000 papers were retrieved by subject “grey system” only (Table I).

From Table I, it is easy to find that the papers with subject “grey system” increased rapidly from 1982 to 2010.

In 1993, the *Blue Book of Chinese Science and Technology* (No. 8), edited and published by Chinese Ministry of Science and Technology, recognized grey systems theory as a new methodology of soft science established by Chinese scholars. In 2008, the Reports of Scientific Developments and Researches (2007-2008), edited and published by Chinese Association of Science, spent a good amount of space on introducing grey systems theory as one of the major innovative achievements in management science and engineering disciplines.

In 2006, the conference of grey systems theory and applications was held successfully in Beijing with financial sponsorship provided by the Chinese Center for Advanced Science and Technology, headed by Li Zhengdao (a Nobel Laureate), Zhou Guangzao and Lu Yongxian (both of Academicians and former President of Chinese Academy of Sciences). In 2008 and 2010, the 16th and 19th National Conference of Grey Systems were financially supported by Chinese Center for Advanced Science and Technology, respectively.

Many important international conferences, such as the Conference on Uncertain Systems Modeling, Systems Prediction and Control, Workshops of the International Institute for General Systems Studies, Congress of World Organization of Cybernetics and Systems, IEEE Conferences on Systems, Man and Control, International Conferences of Computers and Industrial Engineering, the IEEE Conference on Networks, Sensing and Control, etc. have listed grey systems theory as a special topic of interest or arranged special topic sessions for grey systems research. Grey systems theory has caught the attention of many important international conferences and become a center of discussion at many international events, which no doubt will play an important and active role for the world of systems researchers to get better acquainted with grey systems theory.

During November 18-20, 2007, the inaugural IEEE International Conference on Grey Systems and Intelligent Services (IEEE GSIS) was successfully held in Nanjing, China. Nearly 300 scholars from around the world participated in this event. The IEEE

Table I.
Retrieved results from Chinese academic periodical database

Year	1982	1983	1984	1985	1986	1987	1988	1989	1990	1991
Numbers	6	4	6	11	42	82	125	126	149	181
Year	1992	1993	1994	1995	1996	1997	1998	1999	2000	2001
Numbers	195	203	517	477	481	483	448	456	418	435
Year	2002	2003	2004	2005	2006	2007	2008	2009	2010	Total
Numbers	512	556	551	576	724	767	599	789	791	13,693

headquarter approved the organization of this event and National Natural Science Foundation (China), NUAA and Grey Systems Society of China jointly sponsored the actual details of the conference. The Institute for Grey Systems Studies and College of Economics and Management of the NUAA hosted the event. Scholars from China, the USA, England, Japan, South Africa, Russia, Turkey, Malaysia, Iran, Taiwan, Hong Kong and other countries or geographic regions submitted 1,019 papers for possible presentation at this event. Eventually, 332 of the submissions were accepted.

As decided by the conference organizing committee, the IEEE GSIS will be organized biannually. At the end of 2007, The Technical Committee of IEEE on Grey Systems was approved by System, Man and Cybernetics Society. On the basis of such an international platform as IEEE, grey systems theory will surely be widely recognized and studied by many scholars from around the world in the years to come.

During November 10-12, 2009 and September 15-18, 2011, the second and third International Conference on Grey Systems and Intelligent Services (IEEE GSIS) were also successfully held in Nanjing. The fourth international Conference on Grey Systems and Intelligent Services (IEEE GSIS) will held in Macao in 2013. Scholars from the USA, England, South Africa and Romania, expressed their interest of hosting such events in the future.

3. Characteristics of unascertained systems

The fundamental characteristic of uncertain systems is the incompleteness and inadequacy in their information. Due to the dynamics of system evolutions, the biological limitations of the human sensing organs and the constraints of relevant economic conditions and technological availabilities, uncertain systems exist commonly.

3.1 Incomplete information

Incompleteness in information is one of the fundamental characteristics of uncertain systems. The situation involving incomplete system information can have the following four cases:

(1) the information about the elements (parameters) is incomplete;

(2) the information about the structure of the system is incomplete;

(3) the information about the boundary of the system is incomplete; and

(4) the information on the system's behaviors is incomplete.

The situation of incomplete information is often seen in our social, economic and scientific research activities. For instance, in agricultural productions, even if we know all the exact information regarding the areas of plantation, seeds, fertilizers, irrigations, due to the uncertainties in areas like labor quality, natural environments, weather conditions, the commodity markets, etc. it is still extremely difficult to precisely predict the production output and the consequent economic values. For biological prevention systems, even if we clearly know the relationship between insects and their natural enemies, it is still very difficult for us to achieve the expected prevention effects due to our ignorance with the knowledge on the relationships between the insects and the baits, their natural enemies and the baits and a specific kind of natural enemy with another kind of natural enemy. As for the adjustment and reform of the price system, it is often difficult for the policy makers to take actions because of the lack of the information about how much psychological pressure the consumers could bear and how price change on a certain commodity would affect the prices of other commodities. On the security markets, even the brightest market analysts cannot be assured of winning constantly due to their inability to correctly predict economic policy and interest rate changes, management changes at various companies, the direction of the political winds, investors' behavioral changes in the international markets and price changes in one block of commodities on other blocks. For the general social-economic system, because there are not clear relationships between the "inside" and the "outside" and between the system itself and its environment and because the boundary between the inside of the system and the outside is difficult to define, it is hard to analyze the effect of input on the output.

Incompleteness in available information is absolute, while completeness in information is relative. Man employs his limited cognitive ability to observe the infinite universe so that it is impossible for him to obtain the so-called complete information. The concept of large samples in statistics in fact represents the degree of tolerance man has to give to incompleteness. In theory, when a sample contains at least 30 objects, it is considered "large." However, for some situations, even when the sample contains thousands or several tens of thousands of objects, the true statistical laws still cannot be successfully uncovered.

3.2 Inaccuracies in data

Another fundamental characteristic of uncertain systems is the inaccuracy naturally existing in the available data. The meanings of uncertain and inaccurate are roughly the same. They both stand for errors or deviations from the actual data values. From the essence of how uncertainties are caused, they can be categorized into three types: the conceptual, level and prediction types:

(1) *The conceptual type.* Inaccuracies of the conceptual type item from the expression about a certain event, object, concept, or wish. For instance, all such frequently used concepts as "large," "small," "many," "few," "high," "low," "fat," "thin," "good," "bad," "young," "beautiful,", etc. are inaccurate due to the lack of clear definition. It is very difficult to use exact quantities to express these concepts. As a second example, suppose that a job seeker with an MBA degree wishes to get an offer of an annual salary of no less than \$150,000. A manufacturing firm plans to control its rate of deficient products to be less than 0.01 per cent. These are all cases of inaccurate wishes.

(2) *The level type.* This kind of inaccuracy of data is caused by a change in the level of research or observation. The available data, when seen on the level of the system of concern, that is the macroscopic level, or on the level of the whole, or in broad outline level of cognitive, might be accurate. However, when they are seen on a lower level, that is a microscopic level or a partial localized level of the system, they generally become inaccurate. For example, the height of a person can be measured accurately to the unit of centimeters or millimeters. However, if the measurement has to be accurate to the level of one ten-thousandth micron, the earlier accurate reading will become extremely inaccurate.

(3) *The prediction type* (*the estimation type*). Because it is difficult to completely understand the laws of evolution, prediction of the future tends to be inaccurate. For instance, it is estimated that two years from now, the GDP of a certain specified area will surpass \$10 billion; it is estimated that a certain bank will attract as much savings from individual residents in an amount between \$70 and \$90 billion for the year in 2015; it is predicted that in the coming years the temperature in Nanjing during the month of October will not go beyond 30°C;, etc. All these examples provide the uncertain numbers of the prediction type. In statistics, it is often the case that samples are collected to estimate the whole. So, many statistical data are inaccurate. As a matter of fact, no matter what method is used, it is very difficult for anyone to obtain the absolutely accurate (estimated) value. When we draw out plans for the future and make decisions about what course of action to take, we in general have to reply on not completely accurate predictions and estimates.

4. The scientific principle of simplicity

In the development history of science, achieving simplicity has been a common goal of almost all scientists. As early as the sixth century BC, natural philosophers had a common wish in their efforts of understanding the material nature: build the knowledge of the material world on the basis of a few common, simple elements. The ancient Pythagoras of Greece introduced the theory of four elements (earth, water, fire and gas) at around 500 BC. They believed that all material matters in the universe were composed of these four simple elements. Around the same time, the ancient Chinese also had the theory of five elements of water, fire, wood, gold and earth. These are the most primitive and elementary thoughts about simplicity.

The scientific principle of simplicity originates from the simplicity of thinking employed in the process of understanding nature. As the natural science matures over time, simplicity becomes the foundation for man to fathom the world and becomes a guiding principle of scientific research.

Newtonian laws of motion unify the macroscopic phenomena of objective movements in their form of extreme simplicity. In his *Mathematical Principles of Natural Philosophy*, Newton pointed out that nature does not do useless work; because nature is fond of simplicity, it does not like to employ extra reasons to flaunt itself. During the era of relativity, Einstein (1976) introduced two criteria for testing a theory: external confirmation and internal completeness, that is, the logical simplicity. He believed that from the angle of employing scientific theories to reflect the harmony and orderliness of nature, a true scientific theory must comply with the principle of simplicity. In the 1870s, Ampere, Weber, Maxwell and others one after another established theories to explain the phenomenon of electromagnetism based on their different assumptions. Because Maxwell's theory is the one that best complies with the principle of simplicity, it becomes well accepted. Another example is the well-known Kepler's third law of planetary motion: $T^2 = D^3$, in form it looks very simple.

According to the slaving principle of synergetics, one can transform the original high-dimensional equation into a low-dimensional evolution equation of order-parameters by eliminating the fast-relaxing variables in the high-dimensional nonlinear equation that describes the evolution process of a system. Because the order-parameters dominate the dynamic characteristics of the system near the boundary points, through solving the evolution equation of order-parameters, one can obtain the system's time structure, space structure or time-space structure so that he can materialize efficient control over the system's behavior.

The simplicity of scientific models is actualized by employing simple expressions and by ignoring unimportant factors of the system of concern. In the area of economics, the methods of using Gini coefficient to describe differences among consumers' incomes and of employing Cobb-Dauglas production function to measure the contribution of advancing technology in the economic growth are all introduced on the basis of simplifying realistic systems. The model F. Modigliani uses to describe the average propensity to consume:

$$\frac{C_t}{y_t} = a + b\frac{y_0}{y_t}, \ \ a > 0, \ \ b > 0$$

the curve Alban W. Phillips employs to describe the relationship between the rate of inflation $\Delta p/p$ and the unemployment rate x:

$$\frac{\Delta p}{p} = a + b\frac{1}{x}$$

and the well-known capital asset pricing model:

$$E[r_i] = r_f + \beta_i(E[r_m] - r_f)$$

can all be essentially reduced to the simplest linear regression model with some straightforward transformations.

5. Precise models suffer from inaccuracies

When the available information is incomplete and the collected data inaccurate, any pursuit of chasing after precise models in general becomes meaningless. This fact has been well described by Lao Tzu more than 2,000 years ago. The mutually antagonistic principle proposed by L.A. Zadeh, the founder of fuzzy mathematics, also clearly addresses this end: when the complexity of a system increases, our ability to precisely and meaningfully describe the characteristics of the system decreases accordingly until such a threshold that as soon as it is surpassed, the preciseness and meaningfulness become two mutually excluding characteristics. This mutually antagonistic principle reveals that single minded pursuit after preciseness could reduce the operationality and meaningfulness of the cognitive outcome. So, precise models are not necessarily an effective means to deal with complex matters.

In 1994, Jiangping Qiu and Xisheng Hua, respectively, established a theoretically delicate statistical regression model and relatively coarse grey model based on the deformation data and leakage data of a certain large-scale hydraulic dam. Their work shows that their grey model provided better fit than the statistical regression model. When comparing the errors between the predictions of the two models with the actual observations, it is found that the prediction accuracy of the grey model is generally better than that of the regression model (Table II).

In 2001, Dr Haiqing Guo, Academician Zhongru Wu *et al.*, respectively, established a statistical regression model and a grey time series combined model using the observational data of displacement in the vertical direction of a certain large clay-rock filled dam of inclined walls. They compared the data fitting and predictions of the two models and the actual observations and found that the data fitting of the grey combined model is much more superior than that of the statistical regression model.

Xiaobing Li, Haiyan Sun *et al.* employed fuzzy prediction functions to dynamically trace and precisely control the fuel oil feeding temperature for anode baking. The control effect was clearly better than that obtained by utilizing the traditional PID control method.

Academician Caixin Sun and his research group made use of grey incidence analysis, grey clustering and various new types of grey prediction models to diagnose and predict insulation related accidents of electric transformers. Their huge amount of results indicates that these relatively coarse methods and models are more operational and provide more efficient results (Sun *et al.*, 2002).

6. Comparison of several uncertain systems models

Probability and statistics, grey systems theory, rough set theory and fuzzy mathematics are four main research methods employed for the investigation of uncertain systems. Their research objects all contain certain kinds of uncertainty, which represents their commonality. It is exactly the differences among the uncertainties in the research objects that these four theories of uncertainty are different from each other with their respective characteristics.

Probability and statistics study the phenomena of stochastic uncertainty with emphasis placed on revealing the historical statistical laws. They investigate the chance for each possible outcome of the stochastic uncertain phenomenon to occur. Their starting point is the availability of large samples that are required to satisfy a certain typical form of distribution.

Fuzzy mathematics emphasizes on the investigation of problems with cognitive uncertainty, where the research objects possess the characteristic of clear intension and unclear extension. For instance, "young man" is a fuzzy concept, because each person knows the intension of "young man." However, if you are going to determine the exact range within which everybody is young and outside which each person is not young, then you will find yourself in a great difficulty. That is because the concept of the young man does not have a clear extension. For this kind of problem of cognitive uncertainty with clear intension and unclear extension, the situation is dealt with in fuzzy mathematics by making use of experience and the so-called membership function.

The focus of grey systems theory is on the uncertainty problems of small samples and poor information that are difficult for probability to handle. It explores and uncovers the realistic laws of evolution and motion of events and materials through information

Order no.	Type	Average error Statistical model	Grey model
1	Horizontal displacement	0.862	0.809
2	Horizontal displacement	0.446	0.232
3	Vertical displacement	1.024	1.029
4	Vertical displacement	0.465	0.449
5	Water level of pressure measurement hole	6.297	3.842
6	Water level of pressure measurement hole	0.204	0.023

Table II. Comparison between the prediction errors of a statistical model and a grey model coverage and through the works of sequence operators. One of its characteristics is construct models with small amounts of data. What is clearly different with fuzzy mathematics is that grey systems theory emphasizes the investigation of such objects that process clear extension and unclear intension. For example, by the year of 2050, China will control its total population within the range of 1.5-1.5 billion people. This range from 1.5 to 1.6 billion is a grey concept. Its extension is definite and clear. However, if one inquires further regarding exactly which specific number within the said range it will be, then he will not be able to obtain any meaningful and definite answer.

Rough set theory deals with rough non-overlapping class and rough concepts, which signify the indiscernibility between objects. The object is approximated by both the lower approximation and the upper approximation. The redundancy can be reduced by algorithm of attribute reduction, which make pattern discovery possible from the data which may be blurred by too much detail.

Based on what is discussed above, we summarize the differences among these four most studies subject matters in Table III.

The research of uncertain (stochastics, fuzzy, grey and rough) systems can be categorized into the following three aspects:

(1) the mathematical foundation of the uncertain systems theories;
(2) the modeling of uncertain systems and computational schemes, including various uncertain system modeling, modeling combined with other relevant methods and related computational methods; and
(3) the wide-range applications of uncertain systems theories in natural and social sciences.

Currently, the theoretical studies of uncertain (stochastics, fuzzy, grey, rough) systems have been widely applied in all areas of natural science, social science and engineering, including aviation, spaceflight, civil aviation, information, metallurgy, machinery, petroleum, chemical industry, electrical power, electronics, light industries, energy resources, transportation, medicine, health, agriculture, forestry, geography, hydrology, seismology, meteorology, environment protection, architecture, behavioral science,

Object	Prob. statistics	Fuzzy math	Grey systems	Rough sets
Research objects	Stochastics	Cognitive uncertainty	Poor information	Indiscernibility
Basic sets	Cantor sets	Fuzzy sets	Grey sets	Approximation sets
Methods	Mapping	Mapping	Information coverage	Partition
Procedures	Frequency distribution	Cut set	Sequence operator	Lower and upper approximation
Data requirement	Typical distribution	Known membership	Any distribution	Equivalent relations
Emphasis	Intension	Extension	Intension	Intention
Objective	Historical laws	Cognitive expression	Laws of reality	Concept approximation
Characteristics	Large sample	Experience	Small sample	Information systems (tables)

Table III.
Comparison of the different uncertainty models

management science, law, education, military science, etc. These practical applications have brought forward definite and noticeable social and economic benefits.

Both the theoretical and applied research of uncertain (stochastics, fuzzy, grey, rough) systems has been extremely active. However, all these works have shown the emphasis on applications without enough effort placed on further developing the theory and establishing innovative methods. In particular, not enough attention is given to investigate the differences and commonalities between the various available uncertain systems theories so that not enough works on melting together the traditional theories and methods with the newly appearing uncertain systems theories and methods were published. This fact more or less affected the development of uncertain (stochastics, fuzzy, grey, rough) systems theories.

As a matter of fact, each traditional or newly emerging uncertain systems theory and method complements each other and cannot be clearly separated from each other. When facing various kinds of uncertainty problems different uncertain systems theory, different uncertain system theories and methods have their strengths, which complement and supplement each other instead of repelling each other. Many complex, dynamic uncertainty problems are really way beyond the scope and capacity of any single uncertain systems theory and method. It requires the researcher to combine various kinds of classical theories with the newly developed uncertain systems theories and methods. It is a must for this kind of interaction, exchange and combination of the relevant theories and methods to occur for the further, healthy development of science.

7. Elementary concepts and fundamental principles of grey systems

7.1 Elementary concepts of grey systems

Many social, economic, agricultural, industrial, ecological, biological, etc. systems are named by considering the features of classes of the research objects, while grey systems are labeled using the color of the systems of concern.

In the theory of control, people often make use of colors to describe the degree of clearness of the available information. For instance, Ashby refers the objects with unknown internal information to as black boxes. This terminology has been widely accepted in the scientific community. As another example, as a society moves toward democracy, the citizens gradually demand more information regarding the formation of policies and more in depth meaning of the policies. That is, the citizens want to have an increased degree of transparency. We use "black" to indicate unknown information, "white" the completely known information and "grey" the partially known and partially unknown information. Accordingly, the systems with completely known information will be regarded as white, those systems with completely unknown information black and the systems with partially known information and partially unknown information will be seen as grey.

At this junction, we need to pay attention to the difference between "systems" and "boxes." Usually, "boxes" are used when one does not pay much attention or does not attempt to utilize the information regarding the interior while focusing on the external characteristics. In this case, the researcher generally investigates the properties and characteristics of the object through analyzing the input-output relation. On the other hand, "systems" are employed to indicate the study of the object's structure and functions through analyzing the existing organic connections between the object, relevant factors and its environment and the related laws of change.

The research objects of grey systems theory consist of such uncertain systems that they are known only partially with small samples and poor information. The theory focuses on the generation and excavation of the partially known information to materialize the accurate description and understanding of the material world.

Incompleteness in information is the fundamental meaning of being "grey." From different angles and in varied situations, the meaning of "grey" can be expanded or stretched. For this end, see the details in Table IV.

7.2 Fundamental principles of grey systems

In the process of establishing the grey systems theory, Professor Julong Deng discovered and extracted the following fundamental principles of grey systems. It is readily for the reader to see that these principles contain intrinsic philosophical intensions:

- *Axiom 1 (principle of informational differences)*. "Difference" implies the existence of information. Each piece of information must carry some kind of "difference".
- *Axiom 2 (principle of non-uniqueness)*. The solution to any problem with incomplete and indeterminate information is not unique.
- *Axiom 3 (principle of minimal information)*. One characteristic of grey systems theory is that it makes the most and best use of the available "minimal amount of information".
- *Axiom 4 (principle of recognition base)*. Information is the foundation on which people recognize and understand (nature).
- *Axiom 5 (principle of new information priority)*. The function of new pieces of information is greater than that of old pieces of information.
- *Axiom 6 (principle of absolute greyness)*. "Incompleteness" of information is absolute.

8. Main components of grey systems theory

Through nearly 30 years of development, grey systems theory has been built up as a newly emerging scientific discipline with its very own theoretical structure consisting of systems analysis, evaluation, modeling, prediction, decision making, control and techniques of optimization. Its main contents contain:

- Grey numbers and algebraic system, grey matrices, grey equations, etc. constitute the foundation of grey systems theory. In terms of the theoretical beauty and completeness of the theory, there are still a lot of problems left open in this area.

	Situation		
Concept	Black	Grey	White
From information	Unknown	Incomplete	Completely known
From appearance	Dark	Blurred	Clear
From processes	New	Changing	Old
From properties	Chaotic	Multivariate	Order
From methods	Negation	Change for better	Confirmation
From attitude	Letting go	Tolerant	Rigorous
From the outcomes	No solution	Multi-solutions	Unique solution

Table IV.
Extensions of the concept of "grey"

- The sequence operators mainly include buffer operators (weakening buffer operators, strengthening operators), mean generation operators, stepwise ratio generators, accumulating generators, inverse accumulating generators, etc.
- Grey incidence analysis includes such materials as grey incidence axioms, degree of grey incidence, generalized degree of grey incidence (absolute degree, relative degree, synthetic degree), the degrees of grey incidence based on either similar visual angles or nearness visual angles, grey incidence order, superiority analysis and others.
- Grey cluster evaluation includes such contents as grey variable weight clustering, grey fixed weight clustering, cluster evaluations based on (center-point or end-point) triangular whitenization weight functions and other related materials.
- Through grey generations or the effect of sequence operators to weaken the randomness, grey prediction models are designed to excavate the hidden laws; and through the interchange between difference equations and differential equations, a practical jump of using discrete data sequences to establish continuous dynamic differential equations is materialized. Here, GM(1,1) is the central model that has been most widely employed; and discrete grey models are a class of new models initially developed by Dr Naiming Xie.
- In terms of grey predictions, they produce quantitative forecasts on the basis of the GM model. Based on their functions and characteristics, grey predictions can be grouped into sequence predictions, interval predictions, disaster predictions, seasonal disaster predictions, stock-market-like predictions and system predictions, etc.
- The grey combined models include grey econometric models (G-E), grey Cobb-Douglass models (G-C-D), grey Markov models (G-M), grey-rough mixed models, etc.
- Grey decision making includes multi-attribute weighted grey target models, grey incidence decision making, grey cluster decision making, grey situation decisions, grey stratified decisions, etc.
- The main contents of grey control include the control problems of essential grey systems, the controls composed of grey systems methods, such as grey incidence control, GM(1,1) prediction control, etc.

For the convenience of practical applications, Dr Zeng Bo have designed a computer software on most of grey systems modeling, which can be downloaded from the web site of Institute for Grey Systems studies of NUAA (http:// ieeegsss.nuaa.edu.cn/ institute) for free.

References

Deng, J.L. (1982), "Control problems of grey systems", *Systems & Control Letters*, Vol. 1 No. 5, pp. 288-94.

Einstein, A. (1976), *Collection of Einstein*, compiled by Xu Liangying, Vol. 1, The Commercial Press, Beijing, pp. 216-17.

Pawlak, Z. (1982), "Rough sets", *International Journal of Computer & Information Sciences*, Vol. 11 No. 5, pp. 341-56.

Sun, C., Li, J. and Zheng, H. (2002), "A new method of faulty insulation diagnosis in power transformer based on degree of area incidence analysis", *Power System Technology*, Vol. 26 No. 7, pp. 24-9.

Zadeh, L.A. (1965), "Fuzzy sets", *Information and Control*, Vol. 8, pp. 338-53.

Further reading

Andrew, A.M. (2011), "Why the world is grey", *Grey Systems: Theory and Application*, Vol. 1 No. 2, pp. 112-16.

Deng, J.L. (1985), *Grey Control Systems*, Press of Huazhong University of Science and Technology, Wuhan (in Chinese).

Deng, J.L. (1990), *A Course in Grey Systems Theory*, Press of Huazhong University of Science and Technology, Wuhan (in Chinese).

Deng, J.L. (1996), "Several problems on the research and development of grey systems", in Liu, S.F. and Xu, Z.X. (Eds), *New Developments in Grey Systems Research*, Press of Huangzhong University of Science and Technology, Wuhan, pp. 1-12.

Fang, Z.G. and Liu, S.F. (2003), "Study on linear programming model based on grey matrix game", *Journal of Automobile Management Institute*, Vol. 1 No. 1, pp. 61-7 (in Chinese).

Gu, J.F. and Xu, G.Z. (1999), *Preface for Grey Systems Theory and Its Application*, 2nd ed., Science Press, Beijing (in Chinese).

Haken, H. (1978), *Synegetics*, 2nd ed., Springer, Berlin.

Hsu, Y.T. and Yeh, J. (2000), "A novel image compression using grey models on a dynamic window", *International Journal of Systems Science*, Vol. 31 No. 9, pp. 1125-41.

Huang, Y.P. and Wang, S.F. (1997), "Identification of fuzzy grey prediction system by genetic algorithms", *International Journal of Systems Science*, Vol. 28 No. 1, pp. 15-24.

Jian, L., Caixin, S. and Weigen, C. (2002), "Study on fault diagnosis of insulation of oil-immersed transformer based on grey cluster theory", *Transactions of China Electrotechnical Society*, Vol. 17 No. 4, pp. 80-3.

Kuo, H.C. and Chang, H.K. (2003), "A real-time shipboard fire-detection system based on grey-fuzzy algorithms", *Fire Safety Journal*, Vol. 38 No. 4, pp. 341-63.

Lin, C.R. and Deng, J.L. (1996), "On grey prediction of gas pool", in Liu, S.F. and Xu, Z.X. (Eds), *New Developments in Grey Systems Research*, HUST Press, Wuhan, pp. 25-31 (in Chinese).

Lin, Y. and Liu, S.F. (1999), "Several programming models with unascertained parameters and their application", *Journal of Multi-Criteria Decision Analysis*, Vol. 8 No. 1, pp. 206-20.

Lin, Y. and Liu, S.F. (2000a), "A systemic analysis with data (I)", *International Journal of General Systems*, Vol. 29 No. 6, pp. 989-99.

Lin, Y. and Liu, S.F. (2000b), "A systemic analysis with data (II)", *International Journal of General Systems*, Vol. 29 No. 6, pp. 1001-13.

Lin, Y. and Liu, S.F. (2000c), "Law of exponentiality and exponential curve fitting", *Systems Analysis Modelling Simulation* (*SAMS*), Vol. 38 No. 1, pp. 621-36.

Liu, B., Liu, S.F., Zhai, Z.J. and Dang, Y.G. (2003), "Optimum time response sequence for GM (1,1)", *Chinese Journal of Management Science*, Vol. 11 No. 4, pp. 54-7 (in Chinese).

Liu, S.F. (1991), "The three axioms of buffer operator and their application", *The Journal of Grey System*, Vol. 3 No. 1, pp. 39-48.

Liu, S.F. (1996), "Axiom on grey degree", *The Journal of Grey System*, Vol. 8 No. 4, pp. 397-400.

Liu, S.F. (2006a), "On index system and mathematical model for evaluation of scientific and technical strength", *Kybernetes: The International Journal of Systems & Cybernetics*, Vol. 35 Nos 7/8, pp. 1256-64.

Liu, S.F. (2006b), "On measures of information content of grey numbers", *Kybernetes: The International Journal of Systems & Cybernetics*, Vol. 35 No. 6, pp. 899-904.

Liu, S.F. and Lin, Y. (1998), *An Introduction to Grey Systems Theory*, IIGSS Academic Publisher, Grove City, PA.

Liu, S.F. and Lin, Y. (2006), *Grey Information: Theory and Practical Applications*, Springer, London.

Liu, S.F. and Lin, Y. (2011), *Grey Systems: Theory and Applications*, Springer, Berlin.

Liu, S.F. and Zhu, Y.D. (1996), "Grey-econometrics combined model", *The Journal of Grey System*, Vol. 8 No. 1, pp. 103-10.

Liu, S.F., Fang, Z.G. and Lin, Y. (2006), "Study on a new definition of degree of grey incidence", *Journal of Grey System*, Vol. 9 No. 2, pp. 115-22.

Liu, S., Xie, N. and Forrest, J. (2011), "Novel models of grey relational analysis based on visual angle of similarity and nearness", *Grey Systems: Theory and Application*, Vol. 1 No. 1, pp. 8-18.

Liu, S.F., Dang, Y.G., Fang, Z.G. and Xie, N.M. (2010), *Grey System Theory and Application*, 5th ed., Science Press, Beijing.

Luo, D., Liu, S.F. and Dang, Y.G. (2003), "Study on optimization of grey model GM (1,1)", *Engineering Science of China*, Vol. 5 No. 8, pp. 50-3.

Morita, H., Kase, T., Tamuray, Y. and Iwamoto, S. (1996), "Interval prediction of annual maximum demand using grey dynamic model", *International Journal of Electrical Power and Energy System*, Vol. 18 No. 7, pp. 409-13.

OuYang, S.C., McNeil, H.D. and Lin, Y. (2002), "Walking into non-regular", *Weather press*.

Scarlat, E. and Delcea, C. (2011), "Complete analysis of bankruptcy syndrome using grey systems theory", *Grey Systems: Theory and Application*, Vol. 1 No. 1, pp. 19-32.

Song, Z.M. and Zhang, S.H. (2002), "Separated modeling method in grey system", *Systems Engineering – Theory & Practice*, Vol. 22 No. 5, pp. 103-7.

Vallee, R. (2002), "Cybernetics and systems, from past to future", *Proceedings of the Joint Conference of WOSC and IIGSS, Pittsburgh, PA*.

Vallee, R. (2008), "Book reviews: grey information: theory and practical applications", *Kybernetes*, Vol. 37 No. 1, p. 89.

Wang, Z.L. and Liu, S.F. (2004), "Trend testing of grey dynamic models", *IEEE International Conference on Systems, Man and Cybernetics*, pp. 2438-42.

Wei, Y. (2007), "Two effective methods of solving approximately the parameter of GM(1,1)", *Mathematics in Practice and Theory*, Vol. 37 No. 7, pp. 108-12.

Wen, K.L., Chang, T.C. and You, M.L. (1998), "Grey entropy and its application in weighting analysis", *Proceedings of the IEEE International Conference on Systems, Man and Cybernetics*, pp. 1842-4.

Wu, G.P. and Xu, Z.X. (2001), "Grey ART dynamic clustering evaluation for oil and gas information in oxido-reducing potential", *Geophysical Prospecting for Petroleum*, Vol. 40 No. 3, pp. 107-11 (in Chinese).

Xiao, X.P. and Peng, K.K. (2011), "Research on generalized non-equidistance GM(1,1) model based on matrix analysis", *Grey Systems: Theory and Application*, Vol. 1 No. 1, pp. 87-96.

Xiao, X.P., Xie, L.C. and Huang, D.R. (1995), "A modified computation method of grey correlation degree and its application", *Application of Statistics and Management*, Vol. 14 No. 5, pp. 27-30.

Yang, Y. and Liu, S.-F. (2011), "Reliability of operations of grey numbers using kernels", *Grey Systems: Theory and Application*, Vol. 1 No. 1, pp. 57-71.

Yue, J. (1994), "An grey regression model and accuracy analysis", *Dam & Safety*, Vol. 2, pp. 23-8.

Zhang, Q.S. (2003), "Genetic algorithm-based optimization of the distinguishing coefficient in relational analysis", *Chinese Journal of Management Science*, Vol. 11 No. 5, pp. 76-9.

Zhao, Z. and Zhu, Y. (2004), "The study of the rules of simplicity of science theories", *Studies in Dialectics of Nature*, Vol. 20 No. 4, pp. 8-12.

Zheng, Y. (1993), "On the optimization concept of grey systems", *Applied Mathematical Modelling*, Vol. 17 No. 7, pp. 388-92.

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3. Professor Naiming Xie, Dr Yingjie Yang and Dr Chuanmin Mi, Yow-jyy Joyce Lee, Lawrence W. Lan. 2014. Application of grey relational analysis to expose individual student's cognitive difficulties in English public speaking. *Grey Systems: Theory and Application* **4**:2, 232-249. [Abstract] [Full Text] [PDF]
4. Jun Liu, Jian-Zhong Qiao. 2014. A grey rough set model for evaluation and selection of software cost estimation methods. *Grey Systems: Theory and Application* **4**:1, 3-12. [Abstract] [Full Text] [PDF]
5. Camelia Delcea, Emil Scarlat, Liviu-Adrian Cotfas. 2013. Companies' quality characteristics vs their performance. *Grey Systems: Theory and Application* **3**:2, 129-141. [Abstract] [Full Text] [PDF]
6. Chen Hongzhuan, Fan Kaifeng, Fang Zhigeng. 2013. Research on complex product cost estimation based on the N-GM (0, N) model. *Grey Systems: Theory and Application* **3**:1, 46-59. [Abstract] [Full Text] [PDF]