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Article · September 2002

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GREY PREDICTION GM(1,1) MODEL FOR FORECASTING DEMAND OF PLANNED SPARE PARTS IN NAVY OF TAIWAN

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Keywords: planning material, grey prediction model, reliability, spare parts

Summary: *The inventory management of maintenance spare parts plays an important role on their logistic policy. However, the ground personnel perceive difficulties in forecasting through the periodic nature of demand for aircraft repair parts. In addition, because of insufficient data or uncertain demand of maintenance requirement that we have, the traditional prediction method is generally hard to predict the optimal quantity of spare parts fitting the required quantity. In this study, we introduce Grey Prediction Model (GPM) to coping with such problem. After taking fourteen periodic items of planned material from 1999 to 2002, we then apply GM(1,1) model to predict the planning requirement of spare parts of 2003. In order to verify the performance of our forecasting model, we also compare the results with the observed data which are calculated by the rule of technical manual of equipments. Through this study, we demonstrate the GPM can conduct accurate prediction of spare parts especially in situations of insufficient data or resources within highly uncertain, that accurate prediction should reduce the operation cost and improve the reliability of maintenance equipment.*

1. Introduction

Inventory control of spare parts plays an increasingly important role in modern operations management. The trade-off is clearly, a large number of spare parts ties up a large amount of capital, while too little inventory may result in poor customer service or extremely costly emergency actions. In addition, demand forecasting is one of the most crucial issues of inventory management. Forecasts, which form the basis for the planning of inventory levels, are probably the biggest challenge in the regular repair and overhaul industry, as the one common problem facing logistic divisions is the need to know the short-term demand forecast with the highest possible degree of accuracy. From our empirical investigation, most overhaul

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materials manager deal with intermittent demand, it will enable them to select the appropriate forecasting method to meet their cyclical demand for spare parts.

Deng originally presented grey theory in 1982, which theory especially dealt with extreme insufficient data problems. Since there are many abstractive systems that cannot be specifically described in realistic world, furthermore, the fact of incomplete information and uncertain relation in the system, it is rather difficult to analyze it with ordinary statistical method. This present paper introduce grey prediction model GM(1,1) for forecasting the critical spare parts of aircraft in navy of Taiwan.

The organization of this paper is that, section 2 summarizes related research of forecasting on intermittent spare parts demand. We introduce grey prediction model for forecasting the intermittent or critical spare parts in section 3. We then apply the grey prediction model to forecast the critical spare parts of aircraft of navy in Taiwan in section 4. Finally, conclusions presented in section 5.

2. Forecasting methods for intermittent parts demand

In this section, we firstly briefly summarize frequently used forecasting methods for intermittent demand, we also review the alternative forecasting method, grey prediction model, for systems forecasting.

2.1 Traditional forecasting methods for intermittent demand

In order to determine suitable spare part inventory levels, one must know about maintenance schedules and parts forecasting that feed into the MRP system. Due to the nature of variation on demand pattern, periodic demand sometimes produces a series of random values that appear at random intervals, leaving many time periods with no demand. Watson (1987) found that the increase in average annual inventory cost resulted from the fluctuations in the forecast demand parameters of several lumpy demand patterns. The single exponential smoothing and the Croston methods are the most frequently used methods for forecasting low and intermittent demands (Croston,1972; Willemain et al., 1994) Nevertheless, Johnston and Boylan (1996) observed an improvement in forecast performance using the Croston method when compared with the straight Holt (EWMA) method in their experimental study.

On the other hand, Bartezzaghi et al. (1999) in their experimental simulation found that EWMA appears applicable only with low levels of lumpiness. Willemain et al. (1994) concluded that the Croston method is significantly superior to exponential smoothing under intermittent demand conditions. Both methods were shown to produce poor and unreliable forecasting results after being tested on the current research data and for that reason neither is included in the study. Zhao and Lee (1993) concluded in their study that forecasting errors significantly increases total costs and reduce the service level within MRP systems. They argued that the selection of the forecasting methods has a significant impact on system performance. Their results showed that forecasting errors increase as variations in the demand increase.

According to Ghobbar and Friend's survey (1996), Aircraft availability has to be maximized at these peaks and the maintenance fitted into a time slot when the planes are not required for commercial activities. As it is general within the aviation industry that the usage patterns for most parts are unpredictable, and the forecasting of future demand was made by considering available maintenance contract information and looking at scheduled maintenance plans. Forecasts are generally based on past usage patterns such as flying hours or parts demand. On the other hand, the annual budgets for all departments in the technical division are taken into account, along with the number of forecasted flight hours/cycles, the number and type of checks planned for every aircraft, and the fleet size. With this data, the purchasing department tries to determine the quantity of stock necessary for the particular period.

Ghobbar & Friend (2003) further examined and clarified thirteen traditional forecasting techniques through statistical analysis for intermittent demand of aircraft; they proposed a new approach to forecast the intermittent repairable parts of Fokker, BAe and ATR aircraft. In their study employed general linear model approach to explain the variation attributable to different experimental factors and their interactions.

2.2 Grey prediction model for systems forecasting

Grey theory, originally developed by Deng (1982), focuses on model uncertainty and information insufficiency in analyzing and understanding systems via research on conditional analysis, prediction and decision-making. In the field of information research, deep or light colors represent information that is clear or ambiguous, respectively. The Grey method has numerous application, related models have already been developed and extended to Multiple Criteria Decision Making (MCDM) problems. Similar to fuzzy set theory, grey theory is a feasible mathematical means to deal with systems analysis characterized by poor information is lacking. Research and its application covered by grey theory include systems analysis, data processing, modeling, prediction, as well as decision-making and control (Deng, 1986, 1989; Tzeng and Tsaur, 1994). The grey system puts each stochastic variable as a grey quantity that changes within a given range. It does differ from statistical analysis method to deal with the grey quantity. It deals directly with the original data and searches the intrinsic regularity of the data (Huang and Huang, 1996). The grey system theory include the following fields: (a) grey generating, (b) grey relational analysis, (c) grey forecasting, (d) grey decision making, and (e) grey control.

Grey forecasting differs from other statistical regression models. With a basis in probability theory, conventional regression requires amount of data for establishing forecast model. Grey forecasting is based on the grey generating function, GM(1,1) model, which uses the variation within the system to find the relations between sequential data and establish then the prediction model.

Considering the cycle of intermittent spars parts requirement sometimes are not regular, as well as the situation of have short life cycle in high-tech industry product, forecasting results are influenced by shifts in operational demand cycles, and traditional forecasting methods cannot explain the real situation. This work attempts to solve the above problems and develop a new forecasting method that only requires short-term, current and limited data.

3. Grey prediction model for forecasting of spare parts

In this section we focus on the grey prediction model, GM(1,1), which has been applied in many aspects of social and natural science, including decision-making, finance, economics, engineering and meteorology. The Grey forecasting model GM(1,1) is a time series prediction model encompassing a group of differential equations adapted for parameter variance as well as a first-order differential equation (Tseng, 2001).

The procedure of GM (1, 1) grey prediction model can be summarized as follows.

Step 1. Establish the initial sequence from observed data

$$\boldsymbol{x}^{(0)} = (x^{(0)}(1), x^{(0)}(2), \ldots, x^{(0)}(n)) \tag{1}$$

where $x^{(0)}(i)$ represents the base line (state = 0)data with respect to time i.

Step 2. Generate the first-order accumulated generating operation (AGO) sequence $\boldsymbol{x}^{(1)}$ based on the initial sequence $\boldsymbol{x}^{(0)}$

$$\boldsymbol{x}^{(1)} = (x^{(1)}(1), x^{(1)}(2), \ldots, x^{(1)}(n)) \tag{2}$$

where $x^{(1)}(k)$ is derived as following formula:

$$x^{(1)}(k) = \sum_{i=1}^{k} x^{(0)}(i) \tag{3}$$

Step 3. Compute the mean value of the first-order AGO sequence:

$$z^{(1)}(k) = 0.5x^{(1)}(k) + 0.5x^{(1)}(k-1) \tag{4}$$

Step 4. Define the first-order differential equation of sequence $\boldsymbol{x}^{(1)}$ as:

$$\frac{dx^{(1)}(k)}{dk}+ax^{(1)}(k)=b \tag{5}$$

where a and b express the estimated parameters of grey forecasting model.

Step 5. Utilize the least squares estimation, we can derive the estimated first-order AGO sequence $\hat{x}^{(1)}(k+1)$ and the estimated inversed AGO sequence $\hat{x}^{(0)}(k+1)$ as follows,

$$\hat{x}^{(1)}(k+1)=\left[x^{(0)}(1)-\frac{b}{a}\right]e^{-ak}+\frac{b}{a} \tag{6}$$

$$\hat{x}^{(0)}(k+1)=\hat{x}^{(1)}(k+1)-\hat{x}^{(1)}(k) \tag{7}$$

where parameter a and b can be conducted by following equations:

$$\begin{bmatrix} a \\ b \end{bmatrix}=(\boldsymbol{B}^T\boldsymbol{B})^{-1}\boldsymbol{B}^T\boldsymbol{y} \tag{8}$$

$$\boldsymbol{B}=\begin{bmatrix} -\frac{1}{2}(x^{(1)}(1)+x^{1}(2) & 1 \\ -\frac{1}{2}(x^{(1)}(2)+x^{1}(3) & 1 \\ \vdots & \vdots \\ -\frac{1}{2}(x^{(1)}(n-1)+x^{1}(n) & 1 \end{bmatrix} \tag{9}$$

$$\boldsymbol{y}=\left[x^{(0)}(2),x^{(0)}(3),\ldots,x^{(0)}(n)\right]^T \tag{10}$$

Furthermore, in order to categorize the performance of grey forecasting model, Deng (1986) introduced two indicators, accuracy and ratio of post-error test, to evaluate the accuracy of the grey forecasting. The forecasting errors of initial state at time stage k was defined as $e^{(0)}(k)$:

$$e^{(0)}(k)=x^{(0)}(k)-\hat{x}^{(0)}(k),\ \text{for}\ \ k=1,2,\ldots,n \tag{11}$$

The relative forecasting errors of initial state at time stage k was defined as $d^{(0)}(k)$:

$$d^{(0)}(k)=\left(\frac{x^{(0)}(k)-\hat{x}^{(0)}(k)}{x^{(0)}(k)}\right)\times 100\%,\ \text{for}\ \ k=2,3,\ldots,n \tag{12}$$

where $d^{(0)}(k)>0$ if the observed data larger than estimated data, i.e. $x^{(0)}(k)>\hat{x}^{(0)}(k)$, and vice versa.

Furthermore, the precision of forecasting model was defined as parameter p,

$$p=\frac{\sum_{k=2}^{n}\left(1-\left|d^{(0)}(k)\right|\right)}{n-1} \tag{13}$$

In addition, the mean and the root mean square error of the estimated data were defined as x and S_1:

$$x=\frac{\sum_{k=1}^{n}e^{(0)}(k)}{n} \tag{14}$$

$$S_1=\sqrt{\frac{\sum_{k=1}^{n}(e^{(0)}(k)-x)^2}{n}} \tag{15}$$

The mean and the root mean square error of the observed data were defined as m and S_2:

$$m = \frac{\sum_{k=1}^{n} x^{(0)}(k)}{n} \tag{16}$$

$$S_2 = \sqrt{\frac{\sum_{k=1}^{n} (x^{(0)}(k) - m)^2}{n}} \tag{17}$$

The parameter of post-error ratio C is derived then by divided S_1 by S_2, $C = S_1 / S_2$. The lower value the post-error ratio is, the better performance the model is. The pairs of the forecasting indicators p and C as shown in Table 1.

TABLE 1 The categorization of grey forecasting accuracy

Classification	Parameters	
	p	C
Good	> 0.95	< 0.35
Qualified	> 0.80	< 0.50
Just	> 0.70	< 0.65
Unqualified	≤ 0.70	≥ 0.65

4. Illustrative case

There are two fundamental types of maintenance – scheduled or preventive maintenance, and unplanned repair. For preventive, or scheduled, maintenance, the demand for spare parts is predictable. For such maintenance, it may be possible to order parts to arrive just in time for use, and it may not be necessary to stockpile repair parts at all. For unplanned repair, the consequences of stockouts often include production loss with significant costs, and some kind of safety stock policy is necessary. If safety stock is necessary, the amount depends upon general management policies, obsolescence, the base-depot-echelon structure, and upon circumstances unique to each application.

In this study we search fourteen intermittent spare parts of on kind weapon system from inventory statistics, because the requirement are irregular and the historical data are insufficient with imprecise, we only collect four year data from 1999 to 2002 shown as Table 2. We then employ the grey prediction model to forecast the planning requirement of spare parts as mentioned in section 3 step by step, and the results shown as Table 3. We conduct the precision of forecasting model and the ratio of post-error test are 79.85% and 0.6673, respectively.

TABLE 2 Observed data of spare parts of weapon system

Item	1	2	3	4	5	6	7	8	9	10	11	12	13	14
1999	1	0	10	21	14	21	612	33	0	0	0	0	0	0
2000	2	29	7	11	15	28	485	12	33	71	91	35	32	15
2001	1	38	5	118	19	72	698	54	152	367	309	325	397	14
2002	1	26	2	10	2	19	326	15	87	162	110	162	99	8

TABLE 3 Results of performance of grey prediction model

Item No.	1	2	3	4	5	6	7

Precision (%)	83.96	85.29	85.00	174.53	110.09	36.36	73.13
Post-error Ratio (C)	0.372	0.304	0.159	0.970	0.761	0.919	0.867
Item No.	8	9	10	11	12	13	14
Precision (%)	26.88	50.26	50.45	38.23	102.10	113.05	88.64
Post-error Ratio (C)	0.987	0.673	0.753	0.756	0.751	0.867	0.203

5. Conclusion

Forecasting trends in intermittent spare parts using empirical methods is very difficult, because the intermittent demand is strongly fluctuating affected by irregular requirement. Consequently, the issue of how to obtain an accurate forecast is very important. The grey prediction model not only requires minimal data but also is the best among all existing models at short-term predictions. Essentially, the grey prediction model obtains higher quality short-term predictions than do the time series and exponential smoothing approaches, while the time series approach obtains more accurate predictions in the medium- and long-term.

This work only demonstrates grey forecasting models to determine the short-term predictions, especially in situation of insufficient historical data.

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