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Semiparametric Estimation of the Fractional Differencing Parameter of Measures of the U.K. Unemployment

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Abstract. The order of integration of different measures of the U.K. unemployment is investigated in this paper by means of semiparametric techniques. Several methods, such as the $R\backslash S$ statistic, along with others proposed by Robinson in a number of papers are applied in this article. The methods perform poorly when using the time domain approaches, however, when using the frequency domain procedures, the results are fairly conclusive, with the order of integration of the series fluctuating around 1.50.

Key words: fractional integration, unemployment, semiparametric estimation, long memory

1. Introduction

The aim of this paper is to obtain estimates of the fractional differencing parameter for different measures of the U.K. unemployment by means of semi-parametric techniques. It is a well known fact that the U.K. unemployment is a highly persistent variable, and the traditional views of modelling this series either as trend-deterministic $I(0)$ or as stochastic trends (or unit roots) $I(1)$ processes seem too restrictive compared with the wide scope of possibilities covered by the fractionally integrated $I(d)$ processes. These processes belong to a broader class called long memory, due to their ability to display significant dependence between distant observations in time. Given a discrete covariance stationary time series process, x_t, with autocovariance function γ_j, according to McLeod and Hipel (1978), the process is long memory if

$$\lim_{T\to\infty} \sum_{j=-T}^{j=T} |\gamma_j|$$

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is infinite. A second way of characterising these type of processes is in the frequency domain. For that purpose, suppose that x_t has absolute continuous spectral distribution, so that it has a spectral density function, $f(\lambda)$, defined as

$$\gamma_j = \int_{-\pi}^{\pi} f(\pi) \cos\, j \, d\lambda \qquad j = 0, \pm 1, \pm 2, \ldots .$$

Thus, we can say that x_t displays the property of long memory if the spectral density function has a pole at some frequency λ in the interval $[0, \pi]$. A popular technique to analyze fractionally integrated models is through the fractional difference ∇^d, where

$$\nabla^d = (1 - L)^d = \sum_{k=0}^{\infty} (-1)^k \binom{d}{k} L^k = 1 - d\, L + \frac{d(d-1)}{2} L^2 + \ldots ,$$

and L is the lag operator ($Lx_t = x_{t-1}$). To illustrate this in the case of a scalar time series x_t, $t = 1, 2, \ldots$, suppose that v_t is an unobservable covariance stationary sequence with spectral density that is bounded and bounded away from zero at any frequency, and

$$(1 - L)^d \, x_t = v_t, \qquad t = 1, 2, \ldots . \tag{1}$$

The process v_t could itself be a stationary and invertible ARMA sequence, when its autocovariances decay exponentially, however, they could decay much slower than exponentially. When $d = 0$ in (1), $x_t = v_t$ and thus x_t is 'weakly autocorrelated', also termed 'weakly dependent'. If $d < 0 < 1/2$, x_t is still stationary but its lag-j autocovariance γ_j decreases very slowly, like the power law j^{2d-1} as $j \to \infty$ and so the γ_j are non-summable. We say then that x_t has long memory given that its spectral density $f(\lambda)$ is unbounded at the origin. It may also be shown that these kind of processes satisfy

$$\gamma_j \sim c_1 \, j^{2d-1} \quad \text{as} \quad j \to \infty \quad \text{for} \quad |c_1| < \infty \tag{2}$$

and

$$f(\lambda) \sim c_2 \, \lambda^{-2d} \quad \text{as} \quad \lambda \to 0^+ \quad \text{for} \quad 0 < c_2 < \infty , \tag{3}$$

where the symbol $\sim$ means that the ratio of the left hand side and the right hand side tends to 1, as $j \to \infty$ in (2), and as $\lambda \to 0^+$ in (3). Finally, as d in (1) increases beyond 1/2 and through 1 (the unit root case), x_t can be viewed as becoming 'more nonstationary' in the sense, for example, that the variance of the partial sums increases in magnitude. This is also true for $d > 1$, so a large class of nonstationary processes may be described by (1) with $d \geq 1/2$. The distinction between $I(d)$ with different values of d is also important from an economic viewpoint: if $d < 1$, the process is mean-reverting, with shocks affecting the system returning to its original level sometime in the future. On the contrary, $d \geq 1$ means that the series is nonstationary and non mean-reverting. In view of the preceding remarks, there is some interest in estimating the fractional differencing parameter d. This is important, not only because it reflects the degree of strong dependence in a series, but also because rates of convergence of some statistics that are relevant for statistical inference depend on d.

Two main approaches can be used to estimate the parameter d. The first approach is parametric, i.e., the model is specified up to a finite number of parameters of which d is one. The second is semi-parametric and is based on the limiting relationships (2) and (3). Practically all the methods presented below require that d must belong to the stationary region, so that if the time series is nonstationary, then an appropriate number of differences have to be taken before proceeding to the estimation. Velasco (1999a, b) shows however that the fractional differencing parameter d can be consistently semiparametrically estimated even for nonstationary series by means of tapering.

Using parametric techniques, Sowell (1992) analysed in the time domain the exact maximum likelihood estimates of the parameters of a fractional ARIMA model, using recursive procedures that allow quick evaluation of the likelihood function. However, when estimating with parametric approaches, the correct choice of the model is important: if it is misspecified, the estimates of d are liable to be inconsistent. In fact, misspecification of the short run components of the series can invalidate the estimation of its long run behaviour. Thus, there might be some advantages in estimating d on the basis of semiparametric approaches. They are called semiparametric models because they parameterize only the long run characteristics of the series. There is a price to be paid in terms of efficiency in not using a correct parametric model, but when the sample size is large the greater robustness of semiparametric models-based procedures is relevant.

Before considering some semiparametric estimates, we should mention an estimate (Hurst, 1951) that is based on the so-called 'adjusted rescaled range', or $R\backslash S$ statistic, and defined as

$$R\backslash S = \frac{\max_{1\le j<T} \sum_{t=1}^{j}(x_t - \overline{x}) - \min\limits_{1\le j<T} \sum_{t=1}^{j}(x_t - \overline{x})}{\left(\frac{1}{T}\sum_{t=1}^{T}(x_t - \overline{x})^2\right)^{\frac{1}{2}}},$$

where $\overline{x}$ is the sample mean of the process x_t. The specific estimate of d (Mandelbrot and Wallis, 1968) is given by:

$$\overline{d}_1 = \frac{\log(R\backslash S)}{\log T} - \frac{1}{2}. \tag{4}$$

Its properties were analyzed in Mandelbrot and Wallis (1969), Mandelbrot (1972, 1975) and Mandelbrot and Taqqu (1979). Beran (1994) provides a neat explanation of how to implement the $R\backslash S$ procedure and Lo (1991) modified the $R\backslash S$ statistic to be robust to weak dependence. Problems related to its lack of robustness with respect to departures from stationarity, and the difficulty of obtaining an exact distribution for it, which depends on the actual distribution of the data generating process, may suggest the use of this estimate as a mere approximation for the value of d.

Several methods of estimating semiparametrically the fractional differencing parameter d were examined in a number of papers by Robinson (1994a, b; 1995a, b) which we are to describe. Using the time domain, Robinson (1994a) suggested the log autocovariance estimate, which is based on taking logs in expression (2),

$$\log \gamma_j \sim \log c_1 + (2d-1)\log j, \quad \text{as} \quad j \to \infty,$$

substituting

$$\overline{\gamma}_j = \frac{1}{T}\sum_{t=1}^{T-j}(x_t - \overline{x})(x_{t+j} - \overline{x}), \quad j = 0, 1, \ldots, T-1$$

for γ_j. The OLS regression of $\log \overline{\gamma_j}$ on $\log j$ then leads to the estimate

$$\overline{d}_2 = \frac{1}{2}\left(1 + \frac{\sum_{j=T-r}^{T-1} \log \overline{\gamma_j}(\log j - \overline{\log j})}{\sum_{j=T-r}^{T-1} (\log j - \overline{\log j})^2}\right), \tag{5}$$

where $\overline{\log j} = \frac{1}{r}\sum_{j=T-r}^{T-1} \log j$, and r is a large integer less than T. A disadvantage of this estimate is that even if the γ_j are all positive for large j, some $\overline{\gamma_j}$ can be negative, especially when γ_j is close to zero. An alternative procedure described in the same article is the minimum distance autocovariance estimate, which is implicitly defined by

$$(\overline{d}_3, \overline{c}_3) = \arg\min_{(d\,c)} \sum_{j=T-r}^{T-1} (\overline{\gamma_j} - c\, j^{2d-1})^2, \tag{6}$$

for $d \in (0, 1/2)$ and $c \in R$, and concentrating out c in (6), we have

$$\overline{d}_3 = \arg\max_{d} \frac{\left(\sum_{j=T-r}^{T-1} \overline{\gamma_j}\, j^{2d-1}\right)^2}{\sum_{j=T-r}^{T-1} j^{2(2d-1)}}. \tag{7}$$

The sets over which the optimization is carried out in (6) and (7) will be typically compact with respect to d, such as the interval $[\varepsilon, \varepsilon/2]$ for small ε, and though the asymptotic properties of $\overline{d}_3$ still have not been obtained, it is likely that $\overline{d}_3$ is consistent for d, under regularity conditions and for a suitable sequence r.

Semiparametric estimates based on the frequency domain are the log-periodogram regression estimate proposed by Geweke and Porter-Hudak (1983) and modified by Künsch (1986) and Robinson (1995a); the averaged periodogram estimate proposed by Robinson (1994b), and the quasi maximum likelihood estimate (Robinson, 1995b). The first of these estimates is based on the regression model

$$\log I(\lambda_j) = c - 2d \log \lambda_j + \varepsilon_j \,, \tag{8}$$

where

$$I(\lambda_j) = (2\pi T)^{-1/2} \left| \sum_{t=1}^{T} x_t e^{i\lambda_j t} \right|^2; \quad \lambda_j = \frac{2\pi j}{T}\,, \quad j = 1, \ldots, m, \quad \frac{m}{T} \to 0\,,$$

$$C \sim \log\left(\frac{\sigma^2}{2\pi} f(0)\right), \quad \varepsilon_j = \log\left(\frac{I(\lambda_j)}{f(\lambda_j)}\right),$$

and the estimate is just the OLS estimate of d in (8). Unfortunately, it has not been proved that this estimate is consistent for d, but Robinson (1995a) modifies the former regression introducing two alterations: the use of a pooled periodogram instead of the raw periodogram, and introducing a trimming number q, so that frequencies λ_j, $j = 1, 2, \ldots q$, are excluded from the regression, where q tends to infinity slower than J, so that q/J tends to zero. So the final regression model is

$$Y_K^{(J)} = C^{(J)} - 2d \log \lambda_K + U_K^{(J)}, \quad \text{with} \quad Y_K^{(J)} = \log\left(\sum_{j=1}^{J} I(\lambda_{k+j-J})\right)$$

with $k = q + J, q + 2J, \ldots, m$, where J controls the pooling and q controls the trimming. The estimate of d is

$$\overline{d}_4 = -\frac{1}{2} \frac{\sum_{j=q+1}^{J} \left(\log \lambda_j - J^{-1} \sum_{j=q+1}^{J} \log \lambda_j\right) \log I(\lambda_j)}{\sum_{j=q+1}^{J} \left(\log \lambda_j - J^{-1} \sum_{j=q+1}^{J} \log \lambda_j\right)^2}\,, \tag{9}$$

and assuming Gaussianity, he proves the consistency and asymptotic normality of $\overline{d}_4$ in a multivariate framework.

The averaged periodogram estimate of Robinson (1994b) is based on the limiting relationship (3). The estimate employs an average of the periodogram near zero frequency,

$$\overline{F(\lambda_m)} = \frac{2\pi}{T} \sum_{j=1}^{m} I(\lambda_j)\,,$$

suggesting the estimator

$$\overline{d}_5 = \frac{1}{2} - \log\left(\frac{\overline{F(q\lambda_m)}}{\overline{F(\lambda_m)}}\right) \Big/ 2\log q\,, \tag{10}$$

where $\lambda_m = \frac{2\pi m}{T}$, $\frac{m}{T} \to 0$, for any constant $q \in (0, 1)$. He proves the consistency of this estimate under very mild conditions, and Lobato and Robinson (1996) shows the asymptotic normality for $0 < d < 1/4$, and the non-normal limiting distribution for $1/4 < d < 1/2$.

Finally, the quasi maximum likelihood estimate in Robinson (1995b) is basically a 'local Whittle estimate' in the frequency domain, considering a band of frequencies that degenerates to zero. The estimate is implicitly defined by:

$$\overline{d}_6 = \arg\min_d \left(\log \overline{C(d)} - 2d\frac{1}{m}\sum_{j=1}^{m} \log \lambda_j\right) \tag{11}$$

for

$$d \in (-1/2, 1/2); \quad \overline{C(d)} = \frac{1}{m}\sum_{j=1}^{m} I(\lambda_j)\lambda_j^{2d}, \quad \lambda_j = \frac{2\pi j}{T}, \quad \frac{m}{T} \to 0.$$

Under finiteness of the fourth moment and other conditions, Robinson (1995b) proves the asymptotic normality of this estimate, which is more efficient than the former ones (Robinson, 1995a, 1994b). Multivariate extensions of these estimation procedures can be found in Lobato (1999). In the following section we apply these methods to different measures of the U.K. unemployment in order to decide which is the correct order of integration of the series. Section 3 contains some concluding remarks. A diskette containing the FORTRAN codes for the programs is available for the author upon request and it is also available from the online archives of the journal.

2. Application to the U.K. Unemployment

We initially take as a measure of unemployment the number of people claiming unemployment benefit. This measure, known as the Claimant Count (CC) is available monthly, and though it does not provide a measure corresponding to the International Labour Organization (ILO) definition, it moves approximately in the same way as other measures such as the one obtained from the Labour Force Survey (LFS). We will call this series U_t and its logarithmic transformation log U_t, but we will also look at the CC series as a percentage of the workforce, u_t, and its logistic transformation,

$$u_t^* = \log \frac{u_t}{(1 - u_t)} \, .$$

We report the results mostly for the latter series, u_t^*, though the same conclusions hold for the remaining ones.[1] The monthly observations start in January 1971 and end in August 1998. These series have been analyzed in Gil-Alana (1999a, b), estimating and testing their orders of integration by means of parametric techniques. He found in these papers strong evidence in favour of fractional roots of orders higher than 1 and thus, rejecting in practically all cases the hypothesis of a unit root. In this paper, we concentrate on estimating d semiparametrically, and the conclusions obtained here are completely in line with those in these previous works, finding evidence of something higher than a unit root. Plots of the four series, with their corresponding correlograms and periodograms are given in Figure 1.

We observe that all the series move in a very similar way, with the correlograms decaying very slowly and the periodograms showing a large peak around the zero frequency, both indicating clearly the nonstationary nature of these series. Figures 2 and 3 show similar pictures for the first and the second differences respectively. In Figure 2 we observe through the correlograms that the autocorrelations still decay slowly, with significant values even at large lags, which could be indicative of fractional integration. This is corroborated by looking at the periodograms, where we still observe large values at the zero frequency. Taking second differences, in Figure 3, we observe through the correlograms, large negative values at lag 1. In adittion, the periodograms show values around 0 at the zero frequency, and though the periodogram is not a consistent estimate of the spectral density function, both figures may suggest that the series are now overdifferenced.

Our first task is to calculate the estimate of d based on the $R\backslash S$ statistic, i.e., d_1 in (4), for the four original series and their first and second differences. Results are given in Table I. When looking at the undifferenced series, the estimated values of d are around 0.35 for the four series, however, since these series are clearly nonstationary, these estimates are not very reliable. Looking at the first differences, (in the second row of the table), $\overline{d}_1$ ranges between 0.217 and 0.236, indicating that the original series might be fractionally integrated with $d > 1$. Finally, the results for the second differenced series give results with d_1 ranging between –0.156 and –0.068, suggesting estimates for the series between 1.844 and 1.932. However, as we mentioned in the previous section, the lack of robustness of d_1 with respect to different departures from stationary suggests that this estimate should be taken with great care and, in any case, as a mere approximation about the degree of integration of the series. Using Lo's (1991) modified version of the R/S statistic, the results were very similar to those reported here, suggesting orders of integration much higher than one in all series.

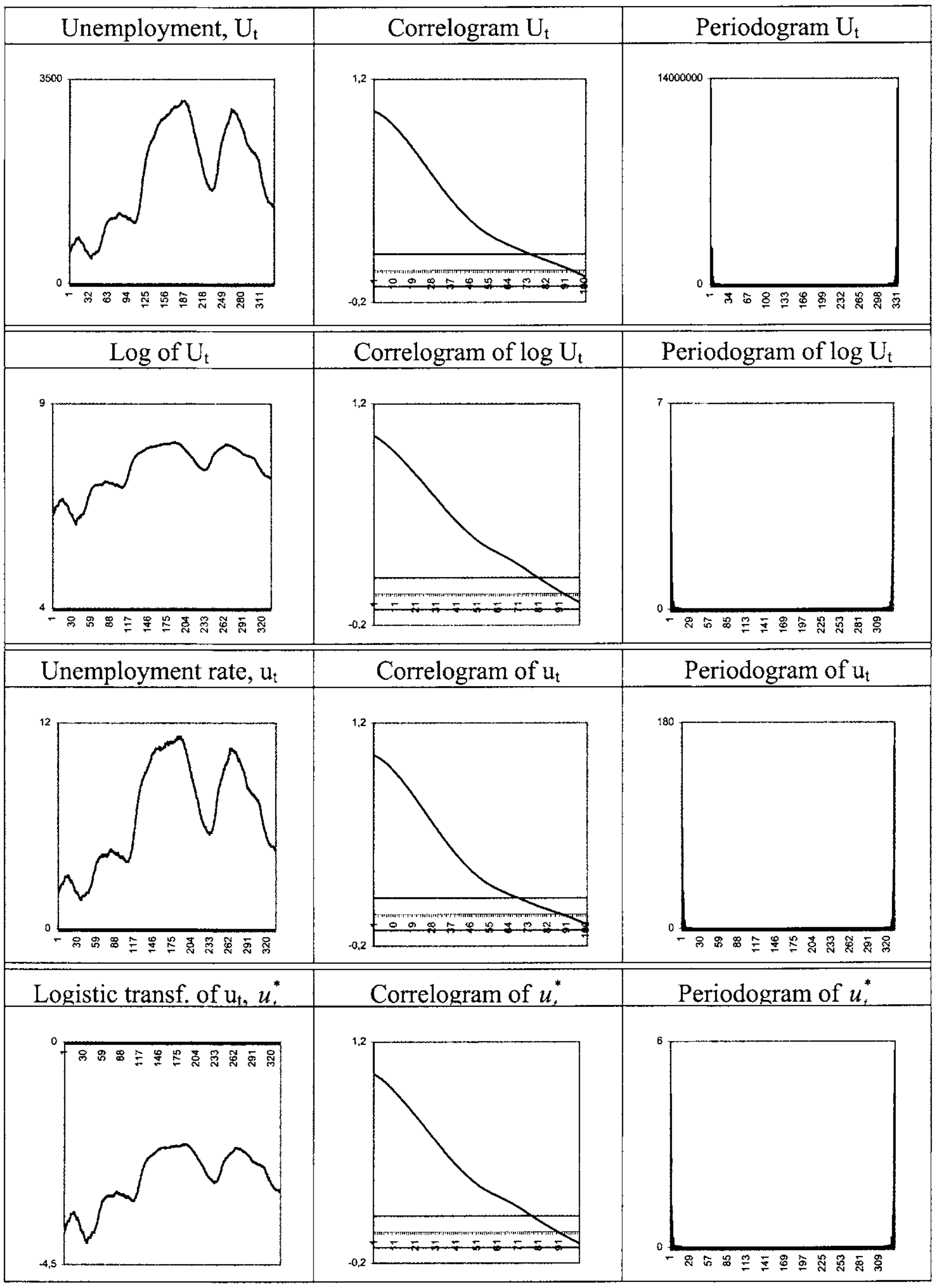

Figure 1. The large sample standard error under the null hypothesis of no autocorrelation is $T^{-1/2}$ or roughly 0.10 for series of length considered here.

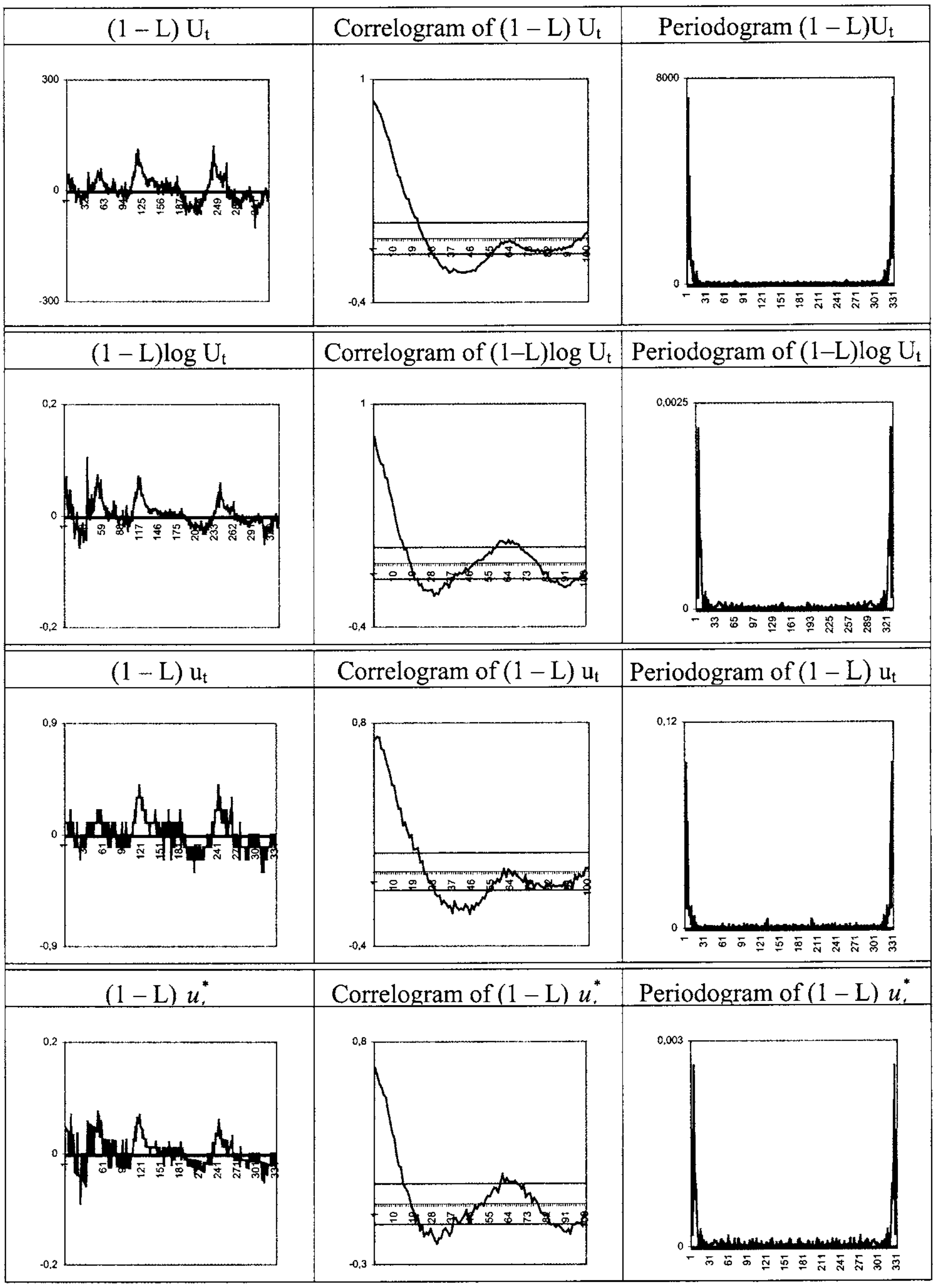

Figure 2. The large sample standard error under the null hypothesis of no autocorrelation is $T^{-1/2}$ or roughly 0.10 for series of length considered here.

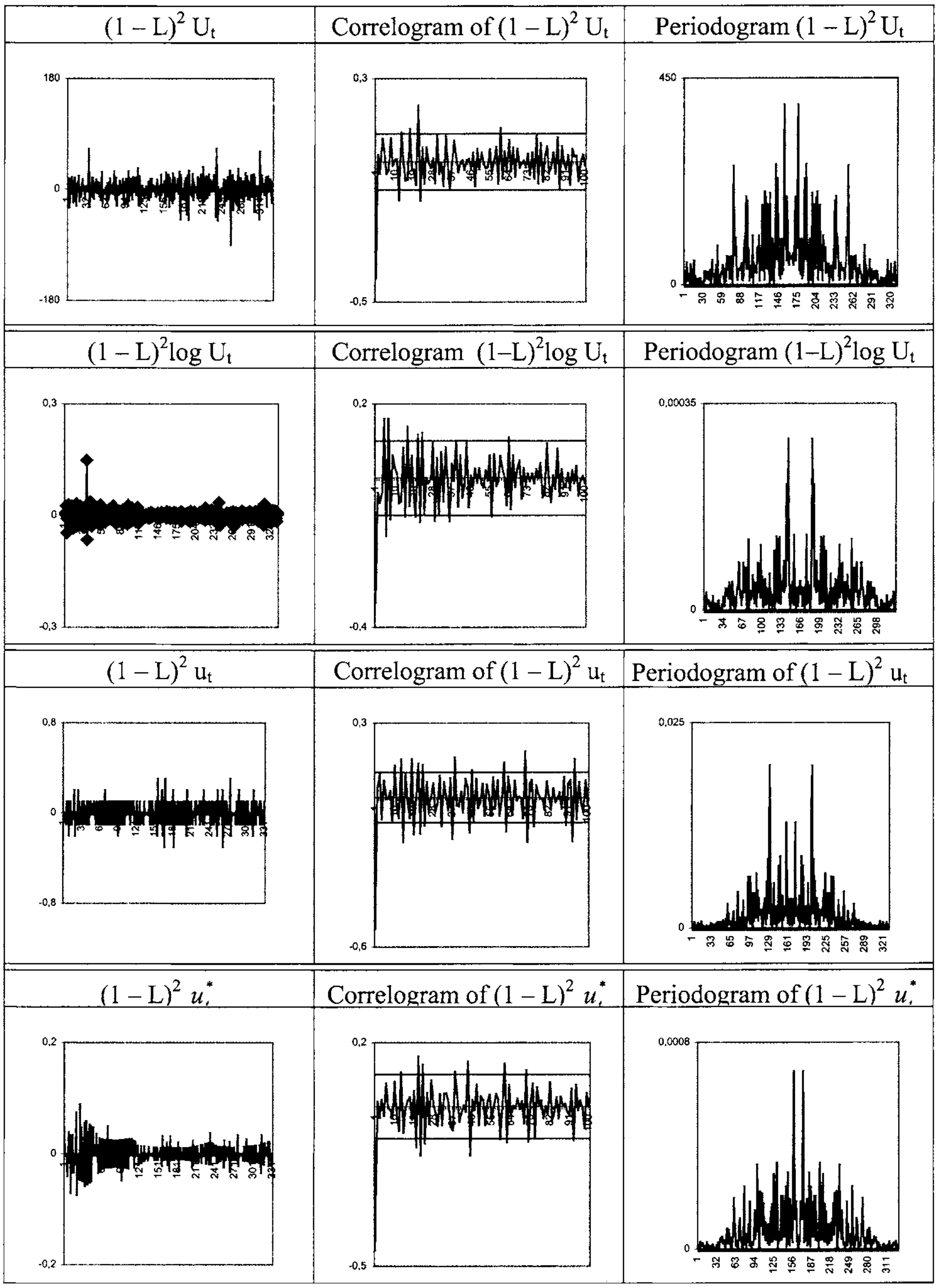

Figure 3. The large sample standard error under the null hypothesis of no autocorrelation is $T^{-1/2}$ or roughly 0.10 for series of length considered here.

Table I. Estimates of d based on the R/S statistic (i.e., $\overline{d}_1$ in (4)).

	U_t	Log U_t	u_t	u_t^*
Time series	0.356	0.354	0.354	0.353
First differences	0.236	0.233	0.220	0.217
Second differences	–0.068	–0.084	–0.156	–0.140

Table II. Log autocovariance estimates of d (i.e., $\overline{d}_2$ in (5)).

Series	Intervals for j	$\overline{d}_2$	Order of integration of the original series
$(1-L)U_t$	(102, 134)	0.48	1.48
$(1-L)\log U_t$	(54, 73)	0.47	1.47
	(105, 134)	0.58	1.58
$(1-L)u_t$	(104, 135)	0.46	1.46
$(1-L)u_t^*$	(106, 135)	0.48	1.48
	(183, 189)	0.40	1.40

Next we look at log-autocovariance estimate given by $\overline{d}_2$ in (5). There is an obvious problem with the application of this estimate for the original series, that for large j, many of the $\overline{\gamma}_j$ are negative, while the model involves $\log \overline{\gamma}_j$ and thus calls for all positive $\overline{\gamma}_j$ for large enough j up to the sample size. The correlograms of the original series indicate that there are no significant positive autocorrelations for lags greater than 1/3 of the sample. Thus, the estimate is not operational here. The same happens for the second differenced series where we are not able to find more than three persistent significant positive autocorrelation values and thus the estimate cannot be performed. Table II sets out results of $\overline{d}_2$ for the first differenced series for some intervals of 'large' j values with all $\overline{\gamma}_j$ positive. For example, taking $(1-L)U_t$, we observe positive and significant autocorrelation values when j is between 102 and 134, so one could 'trim out' the $\overline{\gamma}_j$ for $j > 134$ and take r in (5) for $r = 135 - 33 = 102$ to $r = 135 - 1 = 134$, with $T - 1$ the upper limit of summation in $\overline{d}_2$ replaced by 134. The estimate in such a case is 0.48, implying an order of integration for the unemployment of 1.48. Similar estimates were performed for the remaining series, and we observe across Table II that the values oscillate between 0.40 for $(1-L)u_t^*$ when $j = (183, 189)$ and 0.58 for $(1-L)\log U_t$ with $j = (105, 134)$. In general, all these estimates are far greater than zero, suggesting that when taking first differences, the series may still have a component of long memory behaviour.

In what follows we concentrate on u_t^*, (i.e., the logistic transformation of the CC series as a percentage of the workforce) as the measure of the U.K. unemployment. Similar results were obtained when using the other series. The minimum distance autocovariance estimate of Robinson (1994a), (i.e., $\overline{d}_3$ in (7)), was computed next. This estimate is also based on the time domain and the results for the original time series, (u_t^*), and its first and second differences were completely meaningless for the three series, with $\overline{d}_3$ jumping abruptly from –0.50 to 0.50 in all cases.

In view of the poor results obtained when estimating semi-parametrically d with the time domain procedures, we next consider the methods based on the frequency domain. We firstly look at the log-periodogram regression estimate of Robinson (1994a), i.e., $\overline{d}_4$ in (9). Results displayed in Figure 4 correspond to d_4 for values $q = 0$, 1 and 5 of the trimming number and J initially, (in the left-hand side of the figure), from 50 to 150. The estimates are very sensitive to q when J is constrained between 50 and 100. However, if J is greater than 100, the estimates seem fairly stable across the different values of q. Thus, in the right-hand side of the figure, we display the same estimates but for a range of values of J from 110 to 150. When looking at the original time series, the estimates are monotonically decreasing with J, and all the values are above 0.5, i.e., outside the stationary region. This monotonic decrease and the fact that $\overline{d}_4 > 0.5$ reinforce the nonstationary character of this series. If we look at the first differences for the same range of values of J, $\overline{d}_4$ remains relatively stable across q, with the estimates fluctuating between 0.42 and 0.50. The most stable behaviour is observed when J ranges between 130 and 140, with $\overline{d}_4$ approximately 0.47 and thus, suggesting an estimate of 1.47. Finally, looking at the second differenced series, $\overline{d}_4$ ranges between –0.58 and –0.45, with the most stable behaviour observed around –0.53 and thus, implying an estimate of 1.47, (that is, the same value as the one obtained for the first differences).

The averaged periodogram estimate of Robinson (1994b), i.e., $\overline{d}_5$ in (10) was also computed for values of $q = 0.25$, 0.33 and 0.50. The left-hand side of Figure 5 displays the results of $\overline{d}_5$ for the three series and J from 50 to 170. When looking at the original series, all the estimates are above 0.5, showing once more the nonstationary nature of the series. Taking first differences, the estimates seem very sensitive to the choice of q, and the same happens with the second differences when J ranges between 50 and 110. Thus, as in the previous estimate, we concentrate (in the right-hand side of the figure) only on a range of values of J where $\overline{d}_5$ remains relatively stable.

The first plot shows $\overline{d}_5$ when $q = 0.50$ for the first differenced series. The estimates are fairly stable, fluctuating between 0.39 and 0.41, and thus implying an order of integration for unemployment of approximately 1.40. The second plot shows $\overline{d}_5$ for $(1 - L)^2 u_t^*$ with $J \in [110, 170]$. The estimates also appear rather stable, ranging from –0.75 to –0.27. In the last plot of the figure, we concentrate on a smaller interval for J, constrained between 120 and 150. We see that $\overline{d}_5$ lies then between –0.47 and –0.34, with most of the estimates slightly above –0.40 when

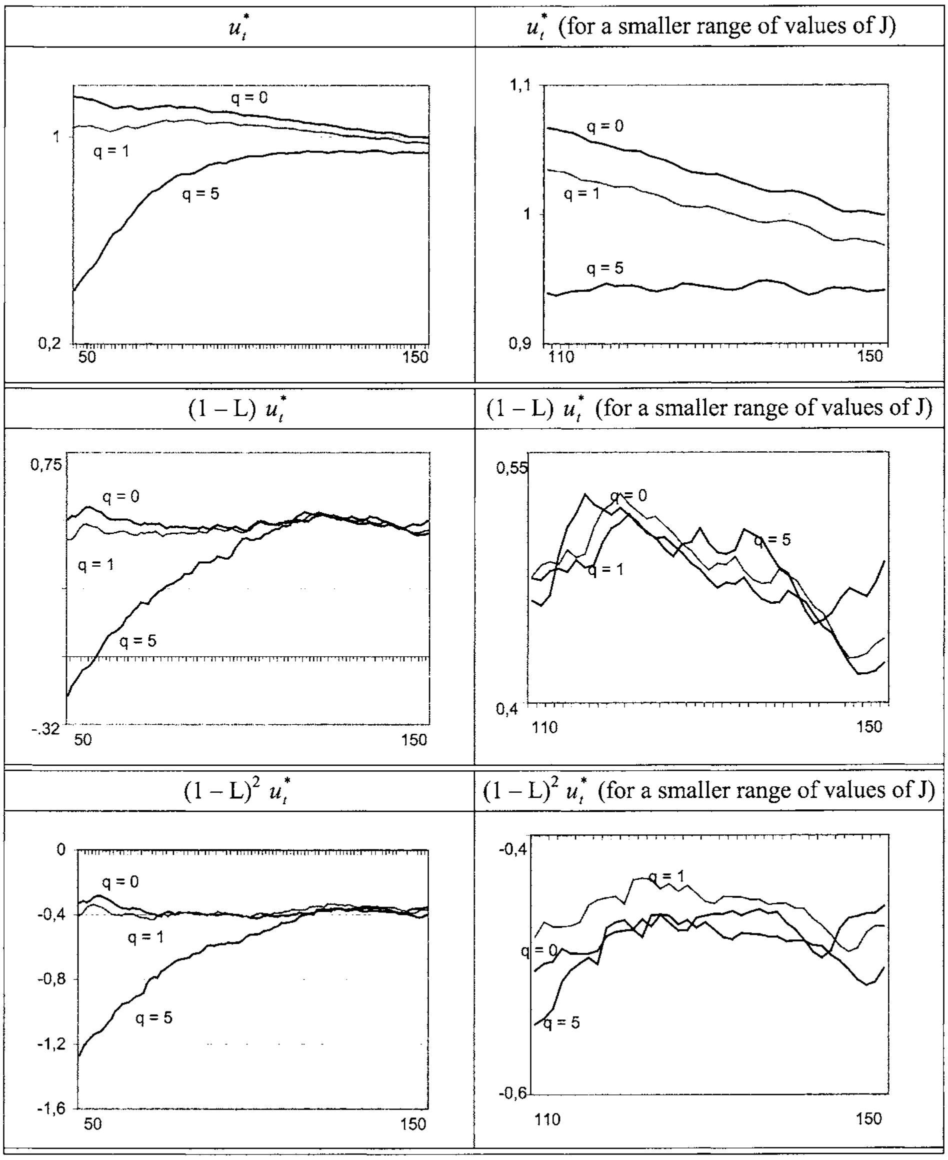

Figure 4. Log-periodogram regression estimates of d (i.e., $\overline{d}_4$ in (9)).

$q = 0.25$ but slightly below –0.40 when $q = 0.33$ and thus, suggesting estimates of about 1.60.

Finally, the quasi maximum likelihood estimate of Robinson (1995b) was also computed. Results for the three series and for a range of values of m from 50 to 200 are displayed in the left-hand side of Figure 6. As we expected, the estimates for the original series are all 0.50. Taking first differences, the estimates lie on 0.50

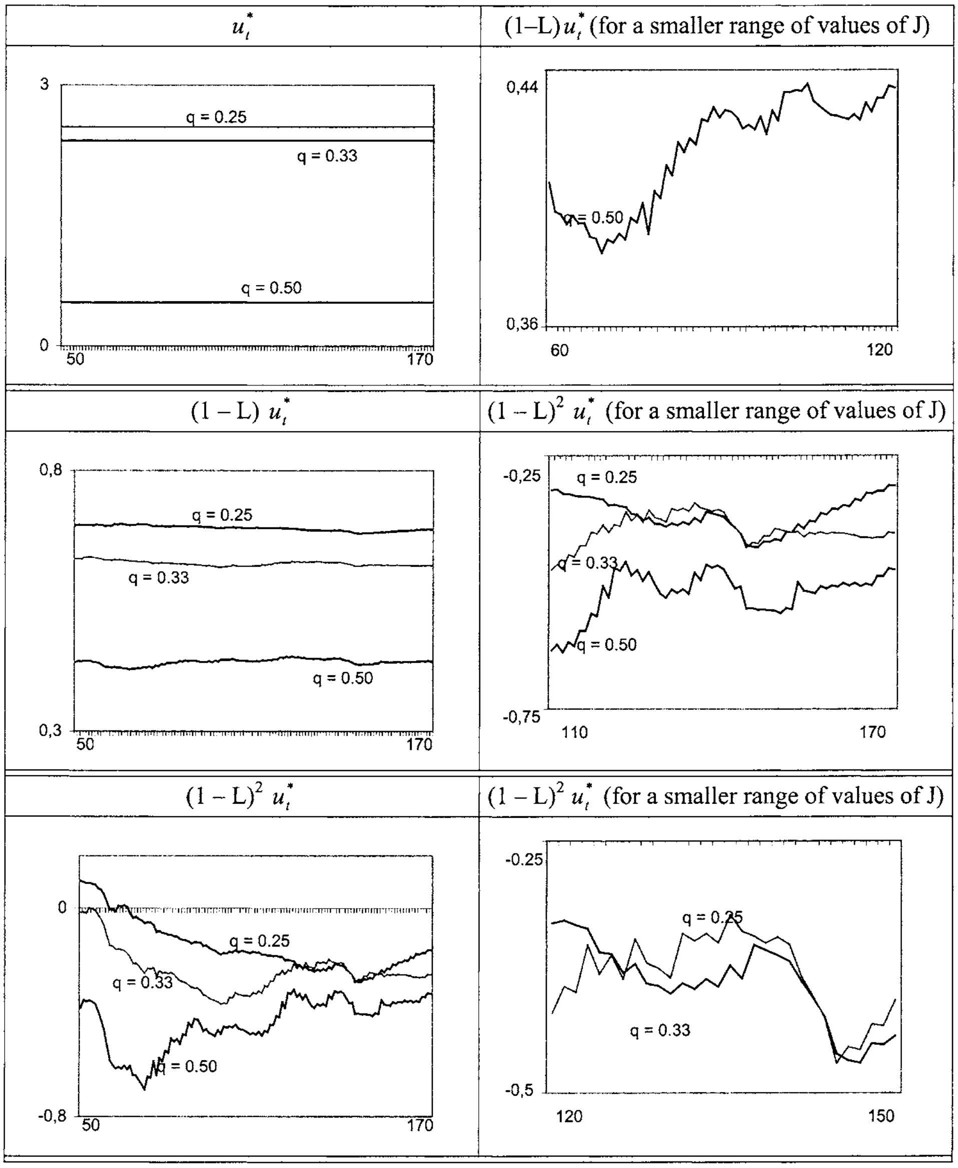

Figure 5. Averaged periodogram estimates of d (i.e., $\overline{d}_5$ in (10)).

for $m < 130$, and then start decreasing slowly with m. The estimates based on the second differenced series are all below zero and a fair degree of stability is observed when m is greater than 100. The right-hand side of the figure concentrates firstly on the first differences with $m = [130, 150]$. We see that $\overline{d}_6$ remains relatively stable around 0.48, implying estimates of d of about 1.48. The second plot shows the results for the second differenced series with $m = [60, 100]$. We see that $\overline{d}_6$ lies

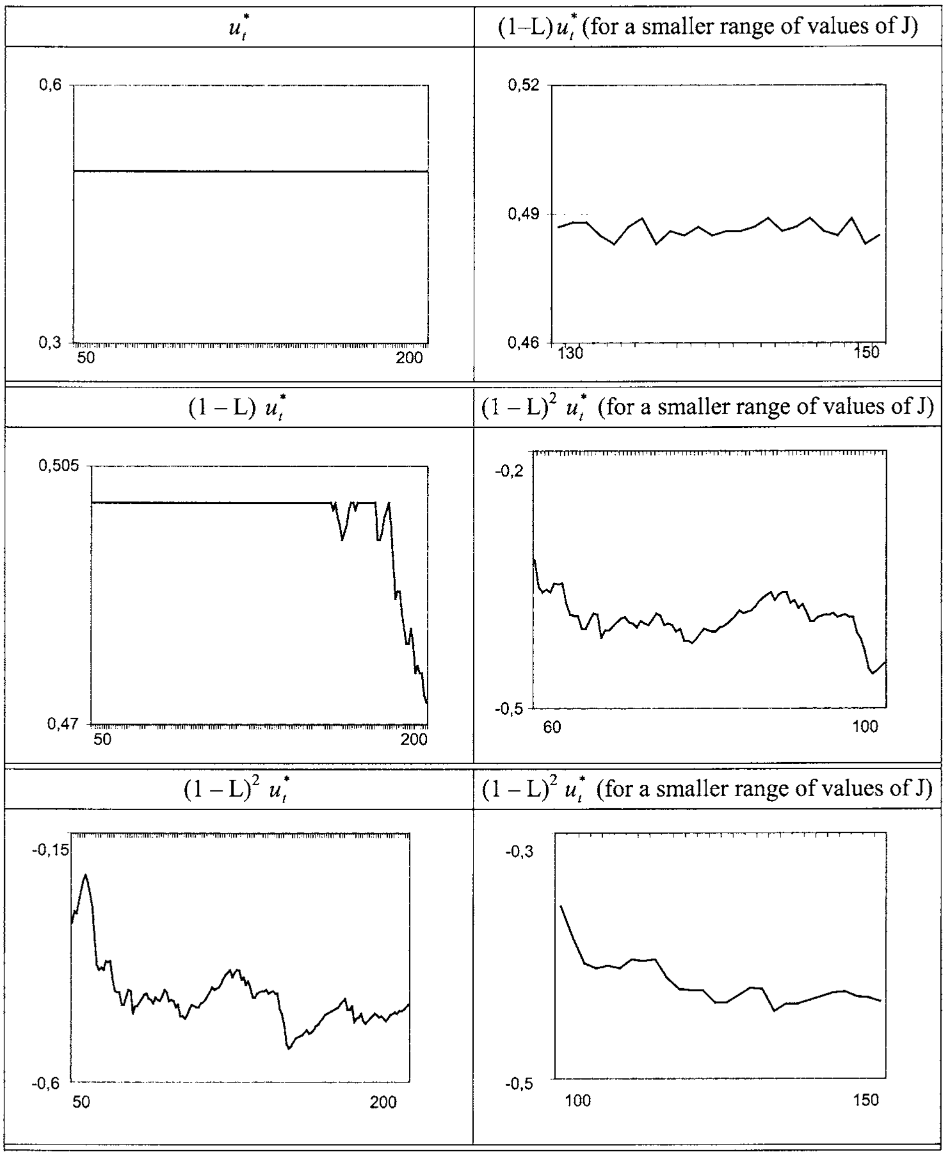

Figure 6. Quasi maximum likelihood estimates of d (i.e., $\overline{d}_6$ in (11)).

then around –0.40, suggesting estimates of approximately 1.60. However, taking $m = [100, 150]$, we see in the last plot of the figure that $\overline{d}_6$ lies around –0.45, implying orders of integration of 1.55.

3. Concluding Remarks

We have presented in this article a variety of semiparametric methods of estimating the fractional differencing parameter for four different measures of the U.K. unemployment. In particular, the number of people claiming unemployment benefits, known as the Claimant Count (CC), U_t; its logarithmic transformation, $\log U_t$; the CC series as a percentage of the workforce, u_t; and its logistic transformation, u_t^*. The methods for estimating d were based on both the time and the frequency domain. Using the time domain procedures, the log-autocovariance estimate of Robinson (1994a) was not operational for the original and the second differenced series. Performed on the first differenced series, we obtained estimates ranging between 0.40 and 0.50, implying orders of integration slightly below 1.50. The minimum distance autocovariance estimate was also performed but the results were completely meaningless for all the series. In view of the poor results obtained when using the time domain approaches, the frequency domain procedures were implemented. The log-periodogram regression estimate of Robinson (1995a) suggests values ranging between 1.40 and 1.50. Similarly, the averaged periodogram estimate (Robinson, 1994b) and the quasi maximum likelihood estimate of Robinson (1995b) both suggest orders of integration ranging between 1.40 and 1.60.

We can conclude by saying that the order of integration of the U.K. unemployment clearly seems to be higher than 1, and thus, the standard approach of taking first differences does not guarantee that the series are $I(0)$ stationary. In fact, these results show that the order of integration is about 1.50. That is, even taking first differences, the series still have a high degree of long memory, with d located around the boundary case between stationarity and nonstationarity. To corroborate this final result, we also performed formal tests for stationarity on the first differenced series. In particular, we employed KPSS-tests with the recent findings of Hobijn et al. (1998), and thus including use of the quadratic spectral kernel and automatic bandwidth selection. The results, however, were rather mixed, depending on the series and the inclusion or not of deterministic components. In many cases, the null hypothesis of stationarity was rejected against unit roots and, in those cases where the null was not rejected, it might be possibly due to the fractional structure underlying the series. How this new testing procedure performs against fractionally integrated alternatives is a matter that still remains to be investigated.

Note

[1] See Wallis (1987) for a justification based on the logistic transformation being defined between $\pm\infty$ so that standard distributions apply.

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