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ORIGINAL PAPER

Critical values of the augmented fractional Dickey–Fuller test

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Abstract This paper presents response surface estimates of finite sample critical values of the Augmented Fractional Dickey–Fuller test of Dolado et al. (Testing I(1) against I(d) alternatives in the presence of deterministic components. Universidad Carlos III, Madrid, mimeo, 2006). The null hypothesis that a series contains a unit root is tested against the alternative that it is fractionally integrated. It is shown that finite sample critical values depend critically on the lag order used to construct the test statistic as well as the hypothesized value of the fractional differencing parameter under the alternative. Non-linear non-parametric response surfaces provide a novel approach to constructing estimates of finite sample critical values.

Keywords Finite-sample critical value · Monte Carlo · Response surface · MARS

JEL Classification C12 · C15 · C22

1 Introduction

Dolado et al. (2002, 2006) provide a simple test for fractional integration that is similar in construction to the Augmented Dickey–Fuller test. Their test examines the null that a series contains a unit root against the alternative that it is fractionally integrated. When the differencing parameter (d) is pre-set and between zero and one-half, the test statistics are functions of fractional Brownian motion, while they are asymptotically standard normal when d lies between one-half and one. If estimates of the differencing parameter (denoted $\hat{d}$) are obtained from any $T^{1/2}$-consistent estimator (with $0 \leq \hat{d} < 1$), the asymptotic distribution of the test statistics are pivotal, and

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asymptotically standard normal. Dolado et al. (2002, 2006) demonstrate that test power is relatively high, particularly when the model contains errors that are not normally distributed.

The purpose of this paper is to provide finite sample estimates of critical values of these tests for use in applied work when the fractional alternative is relevant. Note that when testing for a stochastic unit root is more appropriate, Granger and Swanson (1997) provide guidance on how to proceed. Using response surface analysis, MacKinnon (1991, 1994, 1996), MacKinnon et al. (1999), MacKinnon (2000) provided estimates of finite sample critical values for a variety of unit root and cointegration tests. Cheung and Lai (1995a,b) and Cook (2001) extended the analysis to demonstrate that finite sample critical values and test power are sensitive to the lag length used to adjust for serial correlation when constructing the test statistic. Sephton (1995a) and Presno and Lopez (2003), among others, applied similar methods to estimate finite sample critical values for tests of stationarity.

The traditional approach to using response surface regressions to provide estimates of finite sample critical values relies on a Monte Carlo analysis in which a large number of simulations across various sample sizes and lag lengths are used in a linear regression that models critical values as a function of sample size and lag length. This approach is somewhat difficult to adopt in the case of the Dolado et al. (2002, 2006) tests for fractional integration, since they rely on the pre-set or estimated value of the fractional differencing parameter under the alternative hypothesis. Simulations across a very large number of values of the differencing parameter would be required to generate response surfaces that are sufficiently rich for use in applied research.

The novel approach considered here is the use of non-linear non-parametric methods to provide estimates of response surface regression coefficients. These response surfaces model the critical values as functions of not only the sample size and the number of lags included in the testing equation, but also the value of the fractional differencing parameter under the alternative hypothesis. Multivariate Adaptive Regression Spline (MARS) models due to Friedman (1991a,b) are used to capture the relationship between critical values, sample size, lag length, and the fractional differencing parameter. This approach provides finite sample critical values that are similar to those based on a traditional response surface regression analysis, but they have the added benefit of taking the fractional differencing parameter as in input.

The next section briefly discusses the Dolado et al. (2006) tests for fractional integration. The traditional approach to modeling finite sample critical values without adjustment for lag order is followed by a summary of the MARS modeling strategy and the lag-adjusted finite sample critical values. Comparison to the traditional approach suggests the MARS models provide reliable estimates for use in applied work. MATLAB procedures to construct the test statistics and calculate finite sample critical values are available for download from the author's website.

2 The augmented fractional Dickey–Fuller test

Dolado et al. (2002, 2006) derived a test for fractional integration (henceforth the FDF test) that is based on a regression of the first difference of a series against its fractional difference, deterministic components, and perhaps lagged changes in the dependent variable to capture serial correlation. For a series y_t, their test examines equations such as (1)

$$\Delta y_t = \mu + \phi \Delta^d y_{t-1} + A(L)\Delta y_t + \varepsilon_t \tag{1}$$

where Δ denotes the differencing operator, μ may be zero, contain a constant, and/or a trend, and $A(L)\ \Delta y_t$ captures lagged first-differences to account for serial correlation in the errors, ε_t, which assumed to be i.i.d. The t-statistic on ϕ can be used to test whether the coefficient is significantly different from zero, corresponding to the null hypothesis that the series contains a unit root. As with the Dickey–Fuller test, there are three variants of the FDF test (henceforth denoted by ϕ_{nc}, ϕ_c, and ϕ_{ct}) depending on the choice of deterministics to include in the testing equation. Dolado et al. (2006) demonstrate that the test statistics are invariant to the parameters characterizing the deterministic components as long as the test regressions contain appropriate adjustments for non-linear trends in the maintained hypothesis.

When the fractional differencing parameter is pre-set, the distribution of the t-statistic on ϕ is asymptotically standard normal when $0.5 < d < 1$ and a function of fractional Brownian motion when $0 \leq d < 0.5$. If estimated by a $\mathrm{T}^{1/2}$-consistent estimator (with $0 \leq \hat{d} < 1$), the asymptotic distribution of the test statistics are standard normal. The interest here is in constructing finite sample critical values of the test statistics that can be employed in applied research.

3 Finite sample critical values without adjustment for lag order

The traditional approach to estimating finite sample critical values involves simulating the distributions of the test statistics for various sample sizes and lag lengths, and then linking sample size and lag length to the simulated critical values via linear least squares, with or without adjustment for heteroskedasticity. The resulting equations provide estimates of the asymptotic critical values and the nature by which finite sample critical values differ from their asymptotic values as a function of the number of included lags and observations. Equations such as (2) are used to model these relationships:

$$\varsigma^{\alpha}(T_i) \ = \ \theta_{\infty}^{\alpha} + \theta_1^{\alpha}\left(T_i^{-1}\right) + \theta_2^{\alpha}\left(T_i^{-2}\right) + \upsilon_i \tag{2}$$

where $\varsigma^{\alpha}\ (T_i)$ is the critical value of the test statistic at the α percent level based on sample size T_i, θ_{∞}^{α} is the asymptotic value of the test, and θ_1^{α} and θ_2^{α} capture how the finite sample critical values differ from the asymptotic value. Additional powers of the inverse of T_i can be included in the response surface regressions to improve the fit of the equation.

Cheung and Lai (1995a) and Cook (2001) demonstrated that in the context of the Augmented Dickey–Fuller test, including adjustments for the number of lags in the testing regression leads to estimated critical values that are much different from those ignoring the adjustment. Recognizing that the effects of lag order on critical values should fall as the sample size rises, terms involving $\left({}^{k}/_{T_i}\right)$ and their powers can be included in the response surface equations, where k denotes lag order.

MacKinnon (2000) notes that the errors in Eq. (2) will generally be heteroskedastic, with the sample size (T_i) driving the error variance. Weighted least squares provides an attractive solution and takes advantage of the design of the simulation experiment. Denoting the variance of the error term in Eq. (2) for sample size T_i by $\sigma^2(T_i)$, it can be estimated by the sample variance of the estimated critical values across the various experiments for a given sample size and lag order. Weighted least squares, with its GMM interpretation, can then be used to obtain estimates of the parameters in Eq. (2).

The design of the simulation experiments here is as follows. First, the fractional differencing parameter was set between 0.01 to 0.99 in 0.01 increments.[1] Then for each of 18 different sample sizes (50, 60, 70, 80, 90, 100, 125, 150, 175, 200, 225, 250, 275, 300, 350, 400, 500, 1,000), the estimated quantiles (from 0.001 to 0.999) of the ϕ_{nc}, ϕ_c, and ϕ_{ct} tests are obtained from a simulation involving 10,000 iterations, each with lag orders spanning from zero to ten, using MATLAB and its internal random number generator based on the Marsaglia ziggurat algorithm, and by using exactly fractionally integrated rather than asymptotically fractionally integrated series. This is repeated 50 times for each value of the fractional differencing parameter, and provides an estimate of the average quantiles over the 50 experiments as well as their associated variances. These estimated variances provide the weights employed in the corresponding response surface regressions (using the reciprocal of the estimated standard deviation as the weighting variable). Note that since the regressors are the same for all values of T_i (given the associated lag order), the sample averages over the 50 experiments (denoted by $\bar{\varsigma}^{\alpha}$) can be used in the response surface regressions, as noted by MacKinnon (2000).

Without adjustment for lag order, experimentation with powers of T_i led to Eq. (3), where $\hat{\sigma}(T)$ is the estimated variance of the critical values over the 50 experiments for a given value of the fractional differencing parameter:

$$\frac{\bar{\varsigma}^{\alpha}(T)}{\hat{\sigma}(T)} = \theta_{\infty}^{\alpha}\left(\frac{(1)}{\hat{\sigma}(T)}\right) + \theta_1^{\alpha}\left(\frac{T^{-1}}{\hat{\sigma}(T)}\right) + \theta_2^{\alpha}\left(\frac{T^{-2}}{\hat{\sigma}(T)}\right) + \text{error} \tag{3}$$

Table 1 presents estimates of the response surface coefficients at the 5% level for the three test statistics, given several values of the fractional differencing parameter. All of the fits are very good, with R^2 0.99 or higher.

The GMM interpretation of the weighted least squares approach suggests that the sum of squared residuals from the estimated response surfaces provides a test of the function form against the alternative that the conditional mean of the critical value depends on the included regressors in some fashion. Given there 18 different sample sizes, this suggests the sum of squared residuals should be distributed as a χ^2 variate with degrees of freedom equal to 15 (18 less the number of estimated parameters in

[1] As Dolado et al. (2006) note, there is a discontinuity when the fractional differencing parameter is 0.50. For present purposes the fractional differencing parameter will span between 0.01–0.49, and 0.51–0.99, in 0.01 increments.

Table 1 WLS 5% critical value response surface estimates without adjustment for lags

	$d = 0.30$			$d = 0.60$			$d = 0.90$		
	NC	C	CT	NC	C	CT	NC	C	CT
Constant	−1.8031	−2.2780	−2.8956	−1.6849	−1.7541	−2.1423	−1.6451	−1.6499	−1.7543
	(0.0018)	(0.0023)	(0.0028)	(0.0021)	(0.0037)	(0.0072)	(0.0025)	(0.0023)	(0.0043)
TINV	−2.2081	−7.4199	−12.8059	−4.0480	−11.6627	−28.6052	−1.8446	−2.5289	−21.9525
	(0.4719)	(0.6097)	(0.7689)	(0.5442)	(0.9383)	(1.8994)	(0.6581)	(0.6214)	(1.1209)
TINV2	12.8881	128.2545	154.9310	106.4740	259.0988	635.3851	78.0331	78.0766	421.7736
	(23.5705)	(31.0003)	(37.9936)	(27.0255)	(47.0608)	(93.4341)	(31.8722)	(30.5206)	(55.8019)
RSS	0.2759	0.4303	0.8605	0.3299	1.0425	4.8775	0.1683	0.3767	1.2805
SEESQ	0.0184	0.0287	0.0574	0.0220	0.0695	0.3251	0.0283	0.0251	0.0854
RSQ	0.9998	0.9998	0.9998	0.9996	0.9996	0.9968	0.9994	0.9996	0.9990

Regressors include a constant and the first two powers of the inverse of the sample size. Standard errors appear in parentheses
d the value of the fractional differencing parameter, *RSS* residual sum of squares, *SEESQ* squared standard error of estimate, *RSQ* coefficient of determination

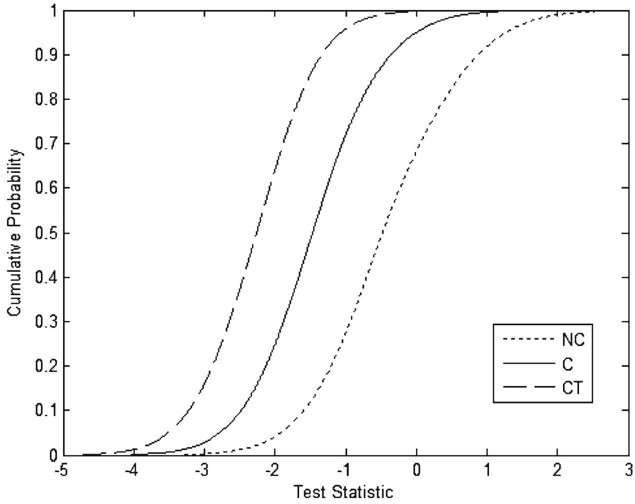

Fig. 1 Asymptotic distributions of test statistics, $D = 0.05$

each response surface), under the null that the specification is correct. The mean of the χ^2 distribution with 15 degrees of freedom is 15 and its variance is 30.

Without adjustment for lags, for every value of the fractional differencing parameter, the specification test did not indicate there were any deficiencies, with the average residual sum of squares over the 293,706 regressions (98 values of the fractional differencing parameter and three tests with 999 quantiles) 2.62 and the variance of 18.65. Attempts to exclude the squared reciprocal of the sample size led to an average test value of 11.58 with a variance of 411.31. While not statistically significant for every calculated quantile, the squared reciprocal will be included in the response surface regressions.

Figures 1, 2, 3 and 4 plot the asymptotic distributions of the test statistics for several values of the fractional differencing parameter. These are the fitted intercepts from the response surfaces, $\widehat{\theta}_{\infty}^{\alpha}$, for each of the 999 quantiles for each test statistic. As noted by Dolado et al. (2006), as the differencing parameter moves above 0.50, the distributions appear to be standard normal. Low values of the differencing parameter correspond to distributions that are substantially different from the normal. Figures 5, 6 and 7 plot the associated asymptotic densities for $d = 0.05$, $d = 0.35$, $d = 0.65$, and $d = 0.95$, whence it is clear that the distributions do appear to be standard normal for $d > 0.50$ but somewhat different when the series are fractionally integrated and stationary under the alternative hypothesis.

4 Estimates of lag adjusted critical values

The manner in which lag order and sample size enter equations like (3) is ad hoc and without much guidance. Asymptotic theory does suggest that as the sample size tends to infinity, the (lag-adjusted) finite sample critical values should approach their

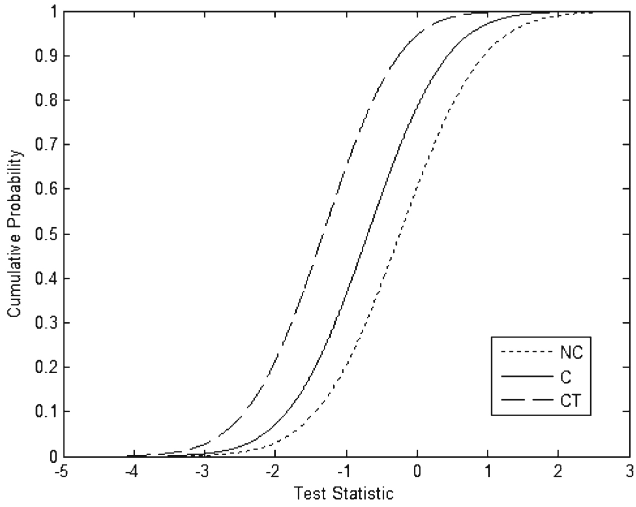

Fig. 2 Asymptotic distributions of test statistics, $D = 0.35$

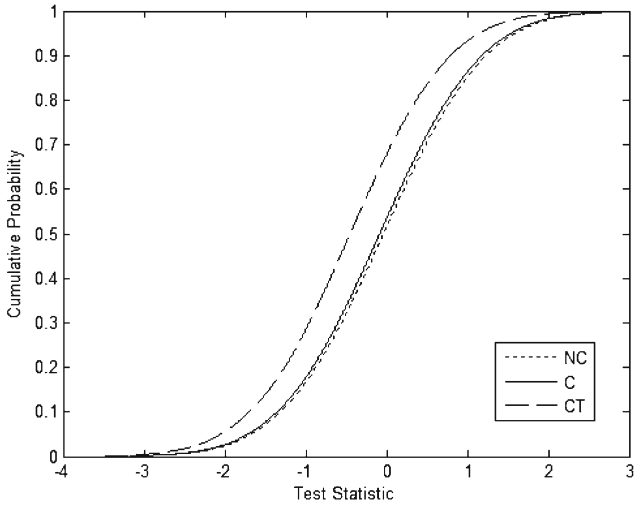

Fig. 3 Asymptotic distributions of test statistics, $D = 0.65$

asymptotic values. In the context of the Augmented Dickey–Fuller test, Cheung and Lai (1995a,b) reported that terms involving the ratio of the lag order to the sample size (and its square) led to satisfactory estimates of the response surfaces.

Ng and Perron (2001, 2005) demonstrated that the choice of lag length via model selection criteria is best applied to data spanning the same sample. In practice, one sets a maximum lag length and constructs the test statistics, choosing to test the null hypothesis using the statistic for the "optimal" lag length chosen by some decision

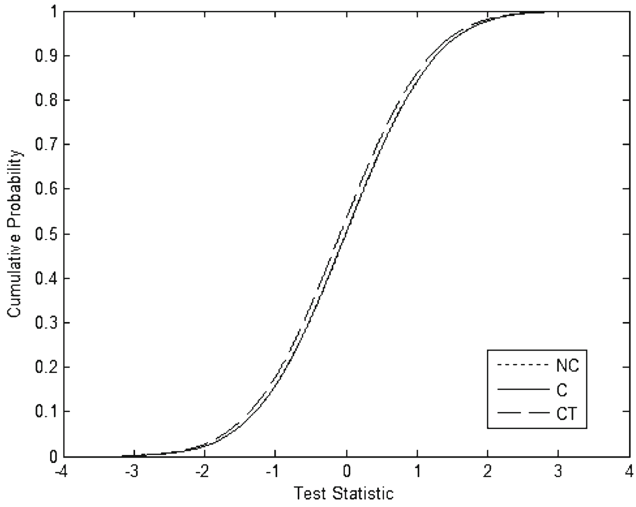

Fig. 4 Asymptotic distributions of test statistics, $D = 0.95$

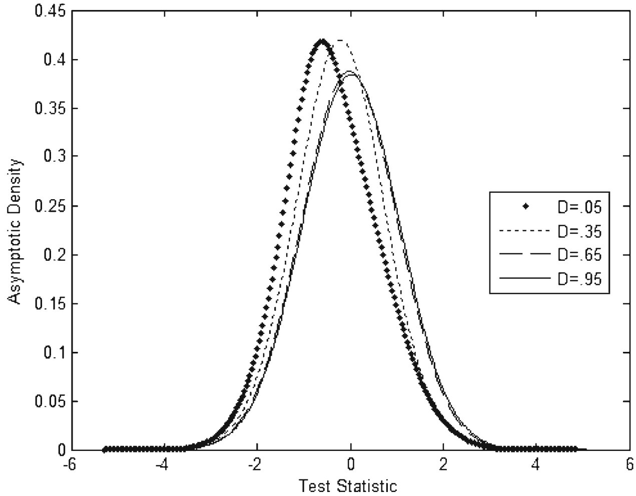

Fig. 5 Asymptotic density of NC test statistics, $D = 0.05, 0.35, 0.65, 0.95$

criterion.[2] In the present context this suggests the lag-adjusted critical values of the FDF test statistics should be constructed by first setting a maximum lag length which then determines the sample over which the specification search will proceed, rather than using a different sample size for every lag order considered. Since the largest sample size considered here spans 1,000 observations, the maximum lag order was set to ten. Dolado et al. (2002) have shown it is sufficient to include up to k lags of the

[2] Lopez et al. (2005) and Sephton (2006) demonstrate that this process does not always lead to robust inference.

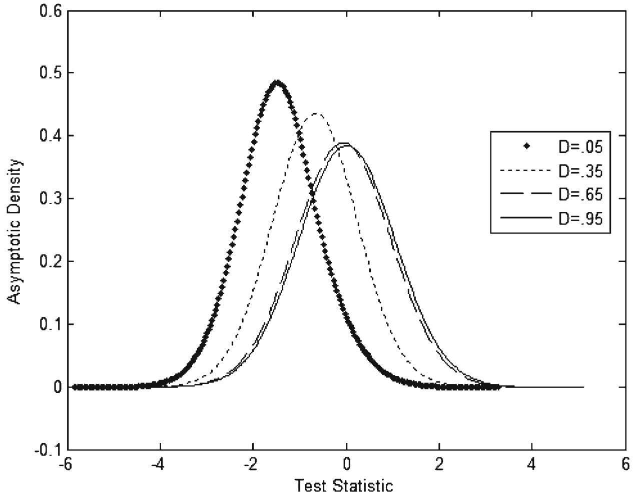

Fig. 6 Asymptotic density of C test statistics, $D = 0.05, 0.35, 0.65, 0.95$

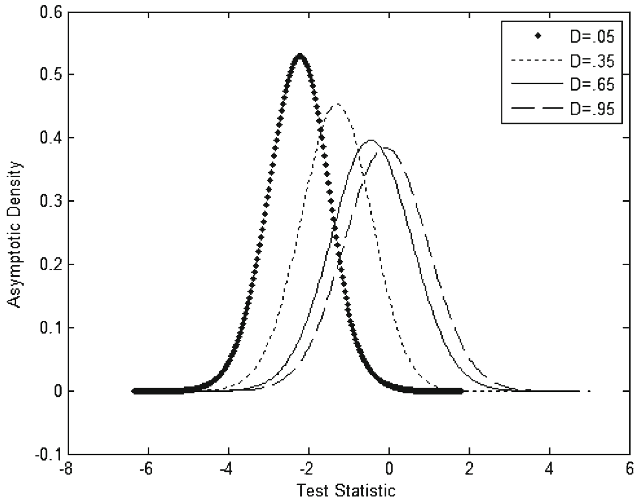

Fig. 7 Asymptotic density of CT test statistics, $D = 0.05, 0.35, 0.65, 0.95$

dependent variable such that $k \to \infty$ as $T \to \infty$ and $k^3/T \to 0$, as demonstrated by Said and Dickey (1984) in the context of unit root tests. Note that no serial correlation was added to the simulated data during the lag length selection process. Extending the results to examine how contaminated data affects the tests and their critical values may be of interest in future work.

Experimentation with various specifications involving the ratio of the lag length to the sample size led to estimates of the response surfaces which failed the GMM specification tests, a result similar to that reported by Harvey and Van Dijk (2006) in modeling finite sample critical values of seasonal unit root tests. Powers of the ratio of lag order to sample size as well as several non-linear transformations of the ratio failed to improve the statistical adequacy of the estimated response surfaces.

In the context of tests for multiple structural changes, Bai and Perron (2003) adopted non-linear regression methods to estimate finite sample critical values. Here, Multivariate Adaptive Regression Spline (MARS) models will be employed to provide estimates of the response surfaces. Friedman (1991a,b) introduced MARS, which is based on smoothing splines that fit linear and non-linear relationships among a set of predictors and a dependent variable. The MARS algorithm searches over all possible variables and all interactions among all variables to provide the best possible fit. It does so through the use of combinations of variables called "basis functions", which are similar to variable combinations created by using principal components analysis. Once MARS determines the optimal number of basis functions, a final linear least squares regression provides estimates of the fitted model.

When modeling the relationship between a single predictor X_t and the dependent variable Y_t, a general MARS model might take the form

$$Y_t = \sum_{k=1}^{M} \alpha_k B_k(X_t) + \varepsilon_t \tag{4}$$

where $B_k(X_t)$ is the kth basis function of X_t. Basis functions can be highly non-linear transformations of X_t, but note that Y_t is a linear (in the parameters) function of the basis functions. Estimates of the parameters α_k are chosen by minimizing the sum of squared residuals from Eq. (4). The advantage of MARS is in its ability to estimate the basis functions so that both the additive and the interactive effects of the predictors are allowed to determine the response variable.

MARS identifies basis functions using a series of knots, with an extensive search over their locations. For example, with a single predictor the sum of squared residuals would be

$$\sum_{i=1}^{T} \left\{ Y_t - \sum_{j=1}^{Q} \alpha_j \; X_t^j \; - \sum_{k=1}^{K} \beta_k (X_i - t_k)_+^Q \right\}^2 \tag{5}$$

where α_j and β_k are multiple regression coefficients on cubic ($Q = 3$) splines of X_t, and X_t relative to knot location t_k. The notation $(X_i - t_k)_+^Q$ indicates that the cubic spline of X_t relative to knot location t_k is included if the difference is positive, otherwise it is zero. From (5) it is clear that the addition of a knot can be viewed as adding the corresponding variable, $(X_i - t_k)_+^Q$. A forward and backward stepwise search is incorporated in the MARS algorithm, with the forward step purposely overfitting the data. Insignificant terms are deleted on the backward step of the routine.

The choice of which basis functions to retain on the backward step of the algorithm is based on an intensive search over all basis functions, with the first deleted basis function the one that is the least costly to remove in terms of a generalized mean squared error criterion. The process continues until all basis functions have been removed and the model is reduced to including only a constant. The specification with the lowest generalized mean squared error is then chosen as the final model. The generalized mean squared error criterion incorporates an adjustment for degrees-of-freedom (akin to the difference between R^2 *and* $\bar{R}^2$) since a larger, more complex model will generally have a better fit. This adjustment attempts to control for the fact that the model is chosen on the basis of a data-intensive search. The penalty can be preset, estimated via cross-validation, or based on some other method (such as a hold-out process in which a subsample of the data is used for training and then prediction errors determine the appropriate setting for the adjustment). For present purposes, model selection is based on tenfold cross-validation. The resulting MARS estimates do not differ substantially from those based on some other metric, such as the generalized cross validation (GCV) criterion of Craven and Wahba (1979).

MARS estimates can most readily be interpreted from an ANOVA (analysis of variance) representation of the model, where the fitted function is expressed as a linear combination of additive basis functions in single variables and interactions between variables. For additive and two-variable interactive models, MARS provides graphical plots which illustrate the optimal transformation of the variables chosen by the algorithm, much like the ACE (alternating conditional expectations) algorithm of Breiman and Friedman (1985).[3] As part of the MARS output, the relative contribution of each variable is determined, as are estimates of the model's generalized R^2. Sephton (2001) provides a readable summary of the modeling process.

MARS has been applied in a number of areas in economics, from predicting exchange rates and inflation to calculating the probability of recession and testing for non-linear causality and cointegration. It has been gainfully applied in medical research, the natural sciences and in management, and has been shown to be superior to many neural network models.[4]

One of the benefits to using this approach over others is that MARS can be used to provide estimates of finite sample critical values across a range of values of the fractional differencing parameter, taking the sample size and the number of lags used to construct the test statistic as predictors. For example, the simulated five percent critical value of the ϕ_{ct} test can be stacked across all 98 values of the fractional differencing parameter to provide a single response variable that can be modeled as a function of sample size, lag length, and the value of the differencing parameter.

MARS models with adjustment for lags at the one, five, and ten percent levels of significance were estimated using the inverse of the sample size, the ratio of the number of lags to the sample size, the fractional differencing parameter, and a constant as potential predictors. Up to three variable interaction was allowed, so that in principle, all predictors would be allowed to interact in a non-linear fashion. As many as 50 basis functions were allowed in the largest model so that the routine would be able to provide

[3] ACE finds the non-linear transformation of the predictors which maximizes the correlation between the dependent variable and the transformed predictors. A plot of the transformed series against the dependent variable is sometimes helpful in identifying a functional form to be used in parametric modeling. Hallman (1990) and Granger and Hallman (1991) employed ACE to examine non-linear cointegration.

[4] See Lewis and Stevens (1991), Sephton (1994, 1995a,b), De Gooijer et al. (1999), Le Blanc and Crowley (1999), Sephton (2001), York and Eaves (2001), Attoh-Okine et al. (2003), Zhang et al. (2003), Briand et al. (2004), Put et al. (2004), Deconinck et al. (2005), Mukkamala et al. (2005), Sephton (2005), York et al. (2005), and Zha and Chan (2005) for examples.

a very good fit to the simulation data. MARS models did very well at explaining the variability of the data, with most of the generalized R^2 values above 0.99.[5]

There is simply too much output to present from these models, but their real value is in calculating small sample critical values of the FDF tests. A MATLAB procedure to do so for any value of the fractional differencing parameter between 0.01 and 0.99 is available at the author's website http://web.business.queensu.ca/faculty/PSephton/newfdf/.

5 Concluding remarks

This article has provided estimates of the asymptotic distributions of the fractional Dickey–Fuller tests of Dolado et al. (2006) as well as response surfaces that can be used to estimate their finite sample critical values. Without adjustment for serial correlation, the response surfaces provide very accurate approximations across a wide range of values of the fractional differencing parameter. Multivariate adaptive regression spline models linking commonly used quantiles to sample size, lag order, and the fractional differencing parameter provide estimates of lag-adjusted critical values which effectively capture the simulated distributions of the test statistics. MATLAB procedures to calculate finite sample critical values based on MARS response surfaces are available for download from http://web.business.queensu.ca/faculty/PSephton/newfdf.

Subsequent work might extend the present analysis in several directions. Lobato and Velasco (2007) introduced an efficient Wald test for fractional unit roots and have demonstrated that the FDF test is relatively inefficient relative to their test when the degree of fractional integration is between 0.5 and 1.0. Providing response surface estimates of the critical values of their test for use in applied work seems a fruitful area of future pursuit. Their extension to other variants, for example, those which allow for breaking means and/or trends (Dolado et al. 2005), and to tests for fractional cointegration are also the subject of on-going work.

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[5] The one percent no constant test value was 0.952; the 5% no constant test value was 0.976; and the 10% no constant test value was 0.985. All other values were 0.999 or higher.

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