Applied Economics Letters, 1994, **1**, 139–141

A reappraisal of parity reversion for UK real exchange rates

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Received: 17 June 1994

We apply a new approach to test the long-run purchasing power parity theory of real exchange rate movements for the UK. The question of whether real exchange rates have a unit root or are mean reverting is set in the more general framework of fractionally differenced time-series models. Our results suggest that in the current period of floating rates, UK real exchange rates return to parity in the long run.

1. INTRODUCTION

Viewed as a long-run equilibrium concept, one implication of the purchasing power parity (PPP) theory of exchange-rate fluctuations is that, over sufficiently long time horizons, the real exchange rate will revert to its parity value. A useful way of testing such a long-run version of PPP is by consideration of the univariate time-series properties of real exchange rates. In particular, if current shocks to the exchange rate have permanent effects, then there is no long-run equilibrium value to which the series returns. Such permanent deviations from parity would occur if the real exchange rate has an autoregressive unit root, as would be the case if it follows a random walk. If the effects of current shocks to the real exchange rate dissipate fully in the long run, then parity reversion holds.

A number of authors have questioned the long-run PPP hypothesis. Using standard tests for unit roots (e.g. the Dickey-Fuller (1981) test), they typically fail to reject the null hypothesis that real exchange rate series contain an autoregressive unit root. On the basis of analysis of estimated integrated autoregressive moving average (ARIMA) models fitted to real exchange rate data, it has also been shown that exchange rates exhibit strong persistence and appear to follow a martingale process.

There are problems with such approaches to testing long-run PPP, however. First, the conventional tests for unit roots tend to have low power against stationary alternatives; see, e.g. DeJong *et al.* (1992). Second, ARIMA models are relatively restrictive with respect to admissible low-frequency dynamics. Accordingly, use of such models may mistake slow parity reversion for non-reverting martingale behaviour.

We address both of these issues by setting the parity reversion question within the context of fractionally integrated autoregressive moving average (ARFIMA) models. A generalization of the ARMA model, the ARFIMA model allows for long memory and less restrictive forms of parity reversion. The differencing parameter of an ARFIMA model can assume real values, and thus is not restricted to the integer domain. Such fractional integration is a more general way to capture long-run dependence than the unit root specification. Further, Diebold and Rudebusch (1991) showed that Dickey-Fuller tests have low power against fractionally integrated alternatives. As such, the heretofore ubiquitous rejection of long-run PPP on the basis of unit root tests may simply be a reflection of ARFIMA type dynamics in real exchange rate movements.

In our use of fractionally differenced models we follow Diebold *et al.* (1991). They used an approximate spectral likelihood approach on a long-run dataset from the gold standard period. In contrast, we use Sowell's (1992) exact maximum likelihood procedure on data from the recent flexible exchange rate period to test for the degree of integration for nine real bilateral UK-based exchange rate series.

In Section II we briefly summarize fractionally differenced models and explain how they can be used to test long-run parity reversion. In Section III we present our results. Section IV concludes.

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II. FRACTIONAL UNIT ROOTS

Let the time series under consideration be represented by $[X_t]$ and its mean by μ. The discrete time autoregressive fractionally integrated moving average (ARFIMA) model of order (p,d,q) is given by

$$\Phi(L)\,(1-L)^d(X_t-\mu)=\Theta(L)\,\varepsilon_t,\ \varepsilon\sim WN(0,\sigma^2) \tag{1}$$

where $d \in \mathbb{R}$, L is the lag operator, $\Phi(L) = 1 - \phi_1 L - \ldots - \phi_p L^p$, $\Theta(L) = 1 + \phi_1 L + \ldots + \phi_q L^q$, all roots of $\Phi(L)$ and $\Theta(L)$ lie outside the unit circle, and $(1-L)^d X_t$ is a stationary stochastic process. The real number d is called the fractional integration or fractional difference parameter. Stationarity of $[X_t]$ requires $d < 0.5$. For any real number d, the fractional difference operator $(1-L)^d$ is defined through the binomial expansion:

$$(1-L)^d = 1 - dL + \frac{d(d-1)}{2!}L^2 - \frac{d(d-1)(d-2)}{3!}L^3 + \ldots \tag{2}$$

The ARFIMA model was introduced independently by Granger and Joyeux (1980) and by Hosking (1981). If $d = 0$, Equation 1 reduces to a stationary autoregressive moving average (ARMA) process. If d is a natural number, then Equation 1 is a non-stationary integrated ARMA model.

ARFIMA models have an interesting impulse response function; see, e.g. Diebold and Nerlove (1990). In fact, for $d < 1.5$, the first differences of the ARFIMA process $[X_t]$ have the following Wold representation:

$$X_t - X_{t-1} = \sum_{k=0}^{\infty} \psi_k \varepsilon_{t-k},\ \psi_0 = 1 \tag{3}$$

From Equation 3 the cumulative impulse response at horizon h is calculated as

$$S_h = \sum_{k=0}^{h} \psi_k \tag{4}$$

showing how a unit shock in period t affects the level of X_t h periods ahead. It can be shown that if $d < 1$ then $S_h \to 0$ as $h \to \infty$, i.e. mean reversion exists in the long run. In contrast, S_h converges to a finite non-zero value if $d = 1$ and diverges if $d > 1$. The remarkable property of the long-run behaviour of S_h is an ARFIMA model for exchange rates is that non-stationarity ($d \geq 1/2$) is compatible with parity reversion ($d < 1$).

This suggests testing long-run PPP by identifying and estimating an ARFIMA model for a given real exchange rate series. If the estimated fractional difference parameter $\hat{d}$ is less than 1, then PPP holds.

Sowell (1992) developed an exact maximum likelihood procedure for estimation of stationary ARFIMA models. The exact maximum likelihood estimator of d, for $d > 0$, has an asymptotic normal distribution; see Dahlhaus (1989). The asymptotic properties for the $d < 0$ case have not been established. But Monte Carlo simulations by Cheung and Diebold (1990) and Sowell (1992) showed that the maximum likelihood estimator has similar small-sample properties for both positive and negative d, suggesting that the conventional asymptotic properties apply when $d < 0$. This allows for hypothesis testing based on the Wald statistic:

$$\frac{\hat{d}-d}{\mathrm{SE}(\hat{d})} \tag{5}$$

where d is the value of the fractional integration parameter under the null hypothesis and $\mathrm{SE}(\hat{d})$ is the maximum likelihood estimate of the standard error of $\hat{d}$.

III. TESTING THE STATIONARITY OF BILATERAL UK REAL EXCHANGE RATES

The data we consider are those in a new coherent set of real exchange rates constructed by Sarantis and Stewart (1993). Sterling pound exchange rates against the following currencies were used: the US dollar, Japanese yen, deutschmark, French franc, Italian lira, Canadian dollar, Dutch guilder, Swiss franc and

Table 1. *Exact maximum likelihood estimates of fractional integration parameter for log-level of real bilateral sterling exchange rates*

Country	Selection criterion	Order (p,d,q) of selected ARFIMA model	Estimated d	Standard error of estimated d
Canada	AIC	$(1,d,0)$	0.23	0.14
	SIC	$(0,d,0)$	1.16	0.12
Netherlands	AIC	$(0,d,0)$	0.88	0.10
	SIC	$(0,d,0)$	0.88	0.10
Germany	AIC	$(1,d,0)$	0.24	0.19
	SIC	$(1,d,0)$	0.24	0.19
France	AIC	$(1,d,1)$	–0.20	0.27
	SIC	$(1,d,0)$	0.20	0.18
Italy	AIC	$(0,d,1)$	0.87	0.12
	SIC	$(0,d,0)$	0.85	0.13
Japan	AIC	$(2,d,0)$	–0.46	0.26
	SIC	$(2,d,0)$	–0.46	0.26
Switzerland	AIC	$(1,d,0)$	–0.29	0.46
	SIC	$(0,d,0)$	0.32	0.27
Sweden	AIC	$(1,d,0)$	0.10	0.18
	SIC	$(1,d,1)$	0.98	0.12
USA	AIC	$(1,d,0)$	0.22	0.14
	SIC	$(1,d,0)$	0.22	0.14

Swedish kroner. The observations are quarterly, beginning in 1973:1 and ending in 1990:3, and thus correspond to the period of flexible exchange rates. Via augmented Dickey-Fuller tests, Sarantis and Stewart (1993) were unable to reject the unit root null hypothesis for these exchange-rate series.

We estimated 16 different ARFIMA(p,d,q) models, with $p,q = 0,1,2,3$ for the level and first differences of each of the real exchange-rate series (Table 1). Models were identified by two well-known selection criteria, the Aikaike information criterion (AIC) and Schwarz information criterion (SIC); for details see Brockwell and Davis (1991). This is consistent with the recommendation of Schmidt and Tschernig (1993), who in an extensive simulation study found that no single selection criterion systematically dominates the other in identification of ARFIMA models. The estimated values for the differencing parameter d for the log-level of the real exchange rates are displayed in Table 1. In some cases, alternative models were selected by the two criteria, leading to different estimates of d for the same series.

In five out of nine cases (Germany, France, Japan, Switzerland and USA), the results are very clear: we obtain stationary models ($d < 1/2$), and never reject parity reversion ($d < 1$) while rejecting the unit root hypothesis ($d = 1$) at conventional significance levels. In two cases (Canada and Sweden), we have conflicting results: the AIC chooses a stationary model, indicates parity reversion and rejects the unit root null hypothesis, but the SIC selects a model for which neither the unit root nor the parity reversion null hypotheses are rejected. In the two final cases (Italy and Netherlands), we obtain results that point to the direction of parity reversion but that are also compatible with the existence of a unit root.

These results are very clear. For all the real exchange-rate series examined, we have evidence in favour of long-run parity reversion.

IV. CONCLUSIONS

Using a test derived from the analysis of fractionally differenced models we have addressed the PPP hypothesis in the floating exchange rate period in the UK case. Use of fractionally differenced models allows for a relatively more general framework than that used in most earlier studies. In our approach, rejection of stationarity does not necessarily imply the presence of an autoregressive unit root.

The fractionally differencing parameter d of the series was estimated by exact maximum likelihood. In most cases, the estimated parameters indicated that the real exchange rates were stationary. In all cases, the estimates did not reject parity reversion, since the estimates of the fractional integration parameter were compatible with mean-reverting fractional models.

ACKNOWLEDGEMENTS

We thank Nicholas Sarantis and Chris Stewart for their comments and for providing the data used in this study. We are also grateful to Fallaw Sowell for generously supplying his GQSTFRAC FORTRAN ARFIMA estimation program.

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