Brazilian Journal of Probability and Statistics (2000), 14, pp. 185–206.

PARAMETRIC AND SEMIPARAMETRIC ESTIMATIONS OF STATIONARY UNIVARIATE ARFIMA MODELS

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Summary
In this paper we consider the estimation of the fractional parameter d and the autoregressive and moving average parameters of an ARFIMA(p, d, q) process with $d \in (0.0, 0.5)$. A recent study related to this is found in Smith et al (1997). We follow the efforts made by these authors considering different methods of estimating the parameter d which is estimated from the semiparametric and parametric approaches and also, different samples sizes. The methodology presented here is applied to two sets of data: the Nile River minima and the annual rainfall at Fortaleza - Brazil.

Key Words: Fractional differencing; long memory; periodogram; smoothed regression.

1 Introduction

The ARFIMA model has recently become a useful tool in the analyses of time series in different fields such as astronomy, hydrology, computer science and many others. It can characterize "long-range dependence or positive memory " when $d \in (0.0, 0.5)$, and "intermediate or negative memory" when $d \in (-0.5, 0.0)$. A good review of long memory process may be found in Beran (1994). There are many estimators of the parameter d proposed in the literature. They are grouped mainly into two categories: The parametric and semiparametric methods. In the first group one finds, for example, Fox and Taqqu (1986), Dahlhaus (1989), Sowell (1992) and Ludeña (2000) in the second category are Geweke and Porter-Hudak (1983), Reisen (1993, 1994), Chen, et al. (1994), Robinson (1995) and Lobato and Robinson (1996) and others. Some recent simulation studies comparing different techniques of estimation in long memory process may be found in Taqqu, Teverovsky and Bellcore (1995), Bisaglia et al. (1998), Taqqu and Teverovsky (1996), Reisen and Lopes (1999) and Hurvich and Deo (1999). Most of these works are related to the estimation of d only. When dealing with the ARFIMA (p,d,q) model, all the parameters, including the autoregressive and moving average ones - besides the differencing parameter, have to be estimated. Reisen, Abraham and Lopes (2001) considered the iterative estimation procedure by Hosking (1981) to estimate the parameters of the process. Smith, Taylor and Yadav (1997) looked at the bias in both the fractional integration parameter d and the short-run AR and MA parameters in ARFIMA models using the Gaussian Likelihood (ML) method (Sowell (1992)) and two semiparametric estimation methods (those of Geweke and Porter-Hudak (GPH)(1983) and Lobato and Robinson (1996)(APER)). Their results suggest that provided the correct ARFIMA model is fitted the ML procedure is probably superior to the GPH and APER procedures. However, as pointed out by many authors, the ML estimates are inconsistent if the short-run parameters are misspecified. Cheung and Dielbold (1994) analyze the behaviour of the finite sample size efficiency of the ML and the Whittle methods (Fox and Taqqu (1986)) showing that the performance of the two estimators is nearly identical for the sample size $n > 150$. Here, we deal with some well known estimation methods of the parameter d based on semiparametric and parametric approaches. The outline of this paper is as follows: in Section 2, we summarize some results related to the ARFIMA(p, d, q) model and the estimation of the parameters of this process. Section 3 we use simulated results to evaluate the bias of the estimators of coefficients. In Section 4, long memory and short memory models are used to analyse minimal water levels of the Nile River and the annual rainfall at Fortaleza, Brazil. Some concluding remarks are given in Section 5.

2 The ARFIMA(p,d,q) model

Let $\{\epsilon_t\}$ be a white noise process with $E(\epsilon_t) = 0, V(\epsilon_t) = \sigma_\epsilon^2$ and denote the back-shift operator, B, such that $BX_t = X_{t-1}$. Let $\Phi(B) = 1 - \phi_1 B - \ldots - \phi_p B^p$ and $\Theta(B) = 1 - \theta_1 B - \ldots - \theta_q B^q$ be polynomials of orders p and q, respectively, with roots outside of the unit circle. If $\{X_t\}$ is a linear process satisfying

$$\Phi(B)(1-B)^d X_t = \Theta(B)\epsilon_t, \quad d \in (-0.5, 0.5), \tag{2.1}$$

then $\{X_t\}$ is called an ARFIMA(p,d,q) process where d is the degree of differencing.

The process defined in (2.1) is stationary and invertible, and its spectral density, $f(w)$, is given by

$$f(w) = f_u(w)(2\sin(w/2))^{-2d}\ ,\ w \in [-\pi, \pi] \tag{2.2}$$

where the function $f_u(w)$ is the spectral density of an ARMA(p,q) process. Hosking (1981) and Reisen (1994) describe ARFIMA models in detail.

2.1 Estimators of d

A number of alternative estimators have been developed for estimating the ARFIMA model and here we consider five methods. The semiparametric methods are: The regression of the periodogram proposed by Geweke and Porter-Hudak (1983), hereafter denoted by $\hat{d}_p$, the regression estimator using the smoothed periodogram function suggested by Reisen (1994) $(\hat{d}_{sp})$, the average periodogram of Robinson (1994a) and Lobato and Robinson (1996)$(\hat{d}_{lr})$ and the univariate case of Robinson (1995, page 1049) $(\hat{d}_{rb})$. For $\hat{d}_p$ and $\hat{d}_{sp}$ methods, the number of observations in the regression equation is a function of the sample size n, that is, $m = n^\alpha$, with $0.0 < \alpha < 1.0$ and the truncation point in the lag Parzen window in the smoothed periodogram estimator is $\nu = n^\beta$, with $0.0 < \beta < 1.0$. For $d \in (-0.5, 0.0)$, Geweke and Porter-Hudak (1983) show that the residuals are approximately independent and identically distributed with a Gambel distribution. Reisen (1994) demonstrated that the error of the approximated regression equation using the smoothed periodogram estimator is approximately independent normally distributed. The $\hat{d}_{rb}$ estimator is a modified form of the log periodogram estimator in which we regress $\{logI(w_j)\}$ on $y_j = ln(2\sin(w_j/2))^2$ $(j = l+1, m)$ where l is the lower truncation point or trimming number which tends to infinity more slowly than m. Our choice of the bandwidth parameter m is now based on the expression give by Robinson (1994b, page 445) which takes the form

$$m = \begin{cases} A(d,\tau)\, n^{\frac{2\tau}{2\tau+1}}, & 0 \le d \le 0.25 \\ A(d,\tau)\, n^{\frac{\tau}{\tau+1-2d}}, & 0.25 < d \le 0.5 \end{cases}$$

where we use $\tau = 0.5$ and $A(d, \tau) = 1.0$. The above bandwidth asymptotically minimizes the mean squared error for the unlogged averaged periodogram estimator proposed by Robison (1994a) and Lobato and Robinson (1996). We decided to use this m because it satisfies the conditions $m/n \to 0$ and $mlogm/n \to 0$ as m and $n \to \infty$ (condition 1 in Hurvich, Deo and Brodsky (1998)). The upper limit of $\hat{d}_{rb}$ depends on the unknown parameters. In this situation, this problem could be turned around by replacing the unknown parameter d by either the estimate obtained from the Geweke and Porter-Hudak (1983) or Reisen (1994) methods, that is, obtained by $\hat{d}_p$ or $\hat{d}_{sp}$, respectively. The appropriate choice of the optimal m has been the subject of many papers such as Hurvich, Deo and Brodsky (1998) and Hurvich and Deo (1999).

The semiparametric estimator suggested by Robinson (1994a) and Lobato and Robinson (1996), $\hat{d}_{lr}$, is the weighted averages of the unlogged periodogram. This estimator is based on the number of frequencies m and a constant $q \in (0.0, 1.0)$. Lobato and Robinson (1996) presented a Monte Carlo simulation study to investigate the sensitivity of the the choice of m and q.

The parametric method considered, hereafter denoted by $\hat{d}_{wt}$, was proposed by Fox and Taqqu (1986), by adapting the approach suggested by Whittle (1953). For computational purposes the estimator $\hat{d}_{wt}$ is obtained by using the discrete form (see Dahlhaus (1989, page 1753))

$$L_n(\zeta) = \frac{1}{2n} \sum_{j=1}^{n-1} \left\{ logf(w_j, \zeta) + \frac{I(w_j)}{f(w_j, \zeta)} \right\} \tag{2.3}$$

where $f(w, \zeta)$ is the spectral density at frequency w and ζ denotes the vector of unknown parameters. For the ARFIMA(p,d,q) process ζ includes the parameter d and the unknown coefficients in the autoregressive and moving average parts. For more details see Fox and Taqqu (1986). The Whittle estimator is the value of ζ which minimizes the function 2.3.

3 Simulation study

We simulate data from ARFIMA(p, d, q) models, with $0 < d < .5$, and $p, q = 0, 1$, using the algorithm in Hosking (1984). The process $\{\epsilon_t\}$ is iid N (0, 1) and it was generated using the random number generator RNNOR in IMSL- FORTRAN. For each model, with given (d, ϕ, θ), we generate $n = 150, 300$.

Estimates of the parameters (d, ϕ, θ)

For smoothed periodogram and periodogram estimators, i.e., $\hat{d}_{sp}$ and $\hat{d}_p$ respectively, $m = n^{0.5}$ (as suggested in Geweke-Hudak, 1983, Reisen, 1994, and widely adopted in the literature) and $\beta = 0.9$ in the Parzen lag window (different values of β are considered in Reisen, 1994). In the Robinson estimator, $\hat{d}_{rb}$, the lower truncation point, i.e. the trimming number, is l =2 and the number of periodogram ordinates, i.e. the upper limit, is given in the table. In the Lobato and Robinson (1996) method, we fixed $m = 32$ and 64 (which corresponds approximately to n^{α} where $\alpha \in (0.6, 0.85)$) and $q = 0.5$ (these values of m and q were also considered by the authors).

In the Whittle method, the parameters of the process are estimated simultaneusly by the use of the subroutine BCONF in IMSL- FORTRAN. In the case of semiparametric methods, the AR and MA parameters were estimated by the subroutine NSLE-IMSL, after the series being differenciated by the estimate of d.

The results for all estimation procedures are based upon the same 1000 replications of the error process. We calculate empirical values of the mean, the standard deviation (sd), the bias and the mean square error (mse). The largest values of the bias and the mse are presented in boldface.

ARFIMA$(0, d, 0)$: Table 1 shows the results of estimating d in the ARFIMA$(0, d, 0)$ model, for n =150 and 300 and $d = 0.2$, 0.3 and 0.45. Other values of d were considered and the results are available upon request. The Whittle method ($\hat{d}_{wt}$) seems to be more accurate (smaller bias and mse) than the other methods which also give good results. In the semiparametric approaches, the $\hat{d}_{lr}$ estimator, for $m = 64$, usually presents smaller mse except when d gets close to the non-stationary condition where $\hat{d}_{rb}$ is superior. The mean values of $\hat{d}_{lr}$ underestimate the true parameter and as d gets closer to the non-stationary condition, there is a large negative bias. The choice of the number of frequencies is crucial for estimating d especially when using the Robinson estimator. For d= 0.2 and 0.3, $\hat{d}_{rb}$ has bigger mse compared to the other methods. In this case, the regression is built from $l = 2, ..., m$ frequencies, i.e. less observations are involved in obtaining $\hat{d}_{rb}$. For $d = 0.45, \hat{d}_{rb}$ improves and is very competitive with $\hat{d}_{wt}$. This may be due to the fact that more observations are involved in the regression equation for this estimator. $\hat{d}_{sp}$ performs better than $\hat{d}_p$ in terms of sd and mse. However, the bias of $\hat{d}_{sp}$ is larger than that of $\hat{d}_p$ even when n is larger. In general, the bias of all procedures are relativelly small for $n = 150$, except for $\hat{d}_{lr}$ when d gets close to 0.5. The estimates get better when the sample size increases.

ARFIMA(p, d, q) models

Now we consider models which involve non-zero AR and/or MA (short-run) parameters. The long memory parameter d is estimated taking into account the additional uncertainty due to the contemporary estimation of the autorregressive or moving average parameters. In addition to the results for $\hat{d}$ the tables also give the mean of the estimate of the short-run

Table 1
Results of estimating d in the $ARFIMA(0, d, 0)$ model

n **= 150**

d		$\hat{d}_{wt}$	$\hat{d}_{sp}$ $m = 12$	$\hat{d}_p$ $m = 12$	$\hat{d}_{rb}$	$\hat{d}_{lr}$ $m = 64$	$\hat{d}_{lr}$ $m = 32$
0.2	mean $(\hat{d})$	0.1983	0.1396	0.2110	0.2252	0.1607	0.1568
	sd $(\hat{d})$	0.0749	0.1915	0.2470	0.4289	0.0734	0.1051
	bias $(\hat{d})$	-0.0017	**-0.0604**	0.0110	0.0252	-0.0393	-0.0432
	mse $(\hat{d})$	0.0056	0.0402	0.0610	**0.1841**	0.0069	0.0129
					$m = 12$		
0.3	mean $(\hat{d})$	0.3073	0.2361	0.3248	0.3263	0.2368	0.2403
	sd $(\hat{d})$	0.0719	0.1957	0.2612	0.3210	0.0636	0.0818
	bias $(\hat{d})$	0.0073	**-0.0639**	0.0248	0.0263	-0.0632	-0.0597
	mse $(\hat{d})$	0.0052	0.0423	0.0687	**0.1035**	0.0080	0.0102
					$m = 16$		
0.45	mean $(\hat{d})$	0.4768	0.3724	0.4500	0.4615	0.3385	0.3267
	sd $(\hat{d})$	0.0379	0.1879	0.2275	0.1108	0.0486	0.0665
	bias $(\hat{d})$	0.0268	-0.0776	0.0000	0.0115	-0.1115	**-0.1233**
	mse $(\hat{d})$	0.0021	0.0412	**0.0516**	0.0124	0.0148	0.0196
					$m = 65$		

n **= 300**

d		$\hat{d}_{wt}$	$\hat{d}_{sp}$ $m = 17$	$\hat{d}_p$ $m = 17$	$\hat{d}_{rb}$	$\hat{d}_{lr}$ $m = 64$	$\hat{d}_{lr}$ $m = 32$
0.2	mean $(\hat{d})$	0.2033	0.1562	0.2018	0.2075	0.1637	0.1592
	sd $(\hat{d})$	0.0495	0.1501	0.1970	0.3088	0.0723	0.1050
	bias $(\hat{d})$	0.0033	**-0.0438**	0.0018	0.0075	-0.0363	-0.0408
	mse $(\hat{d})$	0.0024	0.0244	0.0387	**0.0952**	0.0065	0.0127
					$m = 17$		
0.3	mean $(\hat{d})$	0.3006	0.2491	0.3010	0.3036	0.2548	0.2373
	sd $(\hat{d})$	0.0479	0.1499	0.1871	0.2481	0.0576	0.0867
	bias $(\hat{d})$	0.0006	-0.0509	0.0010	0.0036	-0.0452	**-0.0627**
	mse $(\hat{d})$	0.0023	0.0250	0.0349	**0.0614**	0.0054	0.0114
					$m = 23$		
0.45	mean $(\hat{d})$	0.4722	0.4020	0.4594	0.4556	0.3566	0.3364
	sd $(\hat{d})$	0.0351	0.1631	0.2040	0.0835	0.0425	0.0640
	bias $(\hat{d})$	0.0222	-0.0480	0.0094	0.0056	-0.0934	**-0.1136**
	mse $(\hat{d})$	0.0017	0.0288	**0.0416**	0.0070	0.0105	0.0170
					$m = 115$		

parameter, and the corresponding sd and bias.

The results of the ARFIMA(1,d,0) model are given in Tables 2-3. Other cases of d and ϕ were considered but not presented here to save space. They are available upon request. When $d = 0.2$ and $\phi < 0$, the Whittle estimator gives, in general, better results than the others in terms of sd and mse. In addition, the bias of d is predominantly negative and positive for AR parameter. The semiparametric methods are very competitive in terms of the mse except the $\hat{d}_{rb}$ estimator which presents the largest values. The bias of $\hat{d}_{lr}$ when $m = 64$ and $n = 150$ is relatively large and it decreases as n increases. For positive value of ϕ, the Whittle method lost his superiority showing the largest values of the bias and mse. Similar behaviour has $\hat{d}_{rb}$. This may be explained by the fact that the AR part makes larger contribution to the spectrum of the process when ϕ is positive. $\hat{d}_{sp}$ performs a bit better than the others and as d and ϕ increase and $\hat{d}_{lr}$ improves substantially, specially for $m = 64$ (compare Table 2 to 3, $n = 300$). In general, the $\hat{d}_{sp}$ has less impact in terms of the bias and mse, with the short-memory term than the other estimators. For example, in Table 2 where $d = 0.2$ and $n = 300$ the width of bias and mse of $\hat{d}_{sp}$ are 0.0955 and 0.0065 respectively, while in the Whittle method these quantities are 0.3318 and 0.2255. The bias of $\hat{\phi}$ is larger when ϕ is positive, specially for the $\hat{d}_{wt}$ and $\hat{d}_{rb}$ methods.

We show the results for the ARFIMA(0,0.45,1) model (Table 4). Other cases such as the ARFIMA(0,0.2,1) and ARFIMA(0,0.3,1) models are available upon request. We observed that, in the $\hat{d}_{wt}$ estimator when $d = 0.2$, 0.3 (results not presented here) the sign of the MA components has the opposite effect for estimating the parameters compared to the ARFIMA$(1, d, 0)$ model. When θ is negative the largest bias is for $\hat{d}_{wt}$ and it improves when θ becomes positive, for example, in the ARFIMA(0,0.2,-0.6) model where $n = 300$, the bias and mse of $\hat{d}_{wt}$ are 0.1979 and 0.1485 respectively, while in the ARFIMA(0,0.2,0.6) model these quantities are -0.0196 and 0.0311. However, as d increases ($d = 0.45$) $\hat{d}_{wt}$ has similar behaviour as in the ARFIMA$(1, d, 0)$ model, i.e. this estimator has large bias and mse for positive θ (compare Tables 3 and 4). However, we can see that estimation results for the Whittle method are much higher in the case of the ARFIMA$(1, d, 0)$ model than in the case of ARFIMA$(0, d, 1)$. In the semiparametric approaches, the bias and mse are relativelly small for negative θ except for $\hat{d}_{rb}$. However, for large and positive MA parameter values the bias of the estimates increase substantially, specially for the Robinson (1995) and Lobato and Robinson's estimators, i.e. $\hat{d}_{rb}$ and $\hat{d}_{lr}$ respectively. As in the ARFIMA$(1, d, 0)$ model, $\hat{d}_{sp}$ and $\hat{d}_p$ estimators have less impact with MA component compared to the others and the first one outperforms the second method in terms of the mse.

Table 2
Results of estimating the parameters in the ARFIMA(1,0.2,0) model

$n = 150$

ϕ		$\hat{d}_{wt}$	$\hat{d}_{sp}$	$\hat{d}_{p}$	$\hat{d}_{rb}$	$\hat{d}_{lr}$ $m = 64$	$\hat{d}_{lr}$ $m = 32$
-0.6	mean ($\hat{d}$)	0.1885	0.1139	0.1808	0.1630	-0.0546	0.1241
	sd ($\hat{d}$)	0.0920	0.2071	0.2619	0.3636	0.1149	0.1532
	bias ($\hat{d}$)	-0.0115	-0.0861	-0.0192	-0.0370	**-0.2546**	-0.0759
	mse($\hat{d}$)	0.0086	0.0502	0.0688	**0.1333**	0.0780	0.0292
	mean($\hat{\phi}$)	-0.5927	-0.5280	-0.5496	-0.4836	-0.4182	-0.5467
	sd($\hat{\phi}$)	0.0966	0.1652	0.1958	0.3076	0.1227	0.1230
	bias($\hat{\phi}$)	0.0073	0.0720	0.0504	0.1164	**0.1818**	0.0533
-0.2	mean ($\hat{d}$)	0.1855	0.1350	0.2026	0.1945	0.0263	0.1234
	sd ($\hat{d}$)	0.1260	0.1916	0.2590	0.4243	0.0933	0.1030
	bias ($\hat{d}$)	-0.0145	-0.0650	0.0026	-0.0055	**-0.1737**	-0.0766
	mse($\hat{d}$)	0.0161	0.0408	0.0669	**0.1794**	0.0388	0.0165
	mean($\hat{\phi}$)	-0.1840	-0.1304	-0.1712	-0.1124	-0.0310	-0.1366
	sd($\hat{\phi}$)	0.1401	0.1960	0.2380	0.3704	0.0792	0.1209
	bias($\hat{\phi}$)	0.0160	0.0696	0.0288	0.0876	**0.1690**	0.0634
0.2	mean($\hat{d}$)	0.1424	0.1496	0.2165	0.2121	0.2720	0.2135
	sd ($\hat{d}$)	0.2289	0.1977	0.2648	0.4452	0.0532	0.0855
	bias ($\hat{d}$)	-0.0576	-0.0504	0.0165	0.0121	**0.0720**	0.0135
	mse($\hat{d}$)	0.0556	0.0415	0.0702	**0.1978**	0.0080	0.0075
	mean($\hat{\phi}$)	0.2364	0.2333	0.1777	0.2141	0.1149	0.1671
	sd($\hat{\phi}$)	0.2314	0.1975	0.2462	0.3712	0.0591	0.1018
	bias($\hat{\phi}$)	0.0364	0.0333	-0.0223	0.0141	**-0.0851**	-0.0329
0.6	mean ($\hat{d}$)	0.4691	0.2679	0.3302	0.3764	0.4273	0.3707
	sd ($\hat{d}$)	0.3390	0.1852	0.2459	0.3480	0.0229	0.0468
	bias ($\hat{d}$)	**0.2691**	0.0679	0.1302	0.1764	0.2273	0.1707
	mse($\hat{d}$)	**0.1870**	0.0388	0.0773	0.1519	0.0522	0.0313
	mean($\hat{\phi}$)	0.3381	0.4996	0.4423	0.4035	0.3674	0.4173
	sd($\hat{\phi}$)	0.2960	0.1720	0.2171	0.2864	0.0675	0.0761
	bias($\hat{\phi}$)	**-0.2619**	-0.1004	-0.1577	-0.1965	-0.2326	-0.1827

Table 2
Continuation

$n = 300$

ϕ		$\hat{d}_{wt}$	$\hat{d}_{sp}$	$\hat{d}_{p}$	$\hat{d}_{rb}$	$\hat{d}_{lr}$ $m = 64$	$\hat{d}_{lr}$ $m = 32$
-0.6	mean ($\hat{\mathrm{d}}$)	0.1926	0.1287	0.1780	0.1670	0.1058	0.1408
	sd ($\hat{\mathrm{d}}$)	0.0651	0.1624	0.2173	0.2751	0.0761	0.1037
	bias ($\hat{\mathrm{d}}$)	-0.0074	-0.0713	-0.0220	-0.0330	**-0.0942**	-0.0592
	mse($\hat{\mathrm{d}}$)	0.0043	0.0314	0.0476	**0.0766**	0.0146	0.0142
	mean($\hat{\phi}$)	-0.5987	-0.5393	-0.5504	-0.5117	-0.5424	-0.5605
	sd($\hat{\phi}$)	0.0841	0.1281	0.1792	0.2534	0.0713	0.0784
	bias($\hat{\phi}$)	0.0013	0.0607	0.0496	**0.0883**	0.0576	0.0395
-0.2	mean ($\hat{\mathrm{d}}$)	0.1902	0.1483	0.1900	0.1944	0.1363	0.1514
	sd ($\hat{\mathrm{d}}$)	0.0734	0.1621	0.1999	0.3159	0.0793	0.1052
	bias ($\hat{\mathrm{d}}$)	-0.0098	-0.0517	-0.0100	-0.0056	**-0.0637**	-0.0486
	mse($\hat{\mathrm{d}}$)	0.0055	0.0289	0.0399	**0.0995**	0.0103	0.0134
	mean($\hat{\phi}$)	-0.1913	-0.1384	-0.1632	-0.1209	-0.1472	-0.1570
	sd($\hat{\phi}$)	0.0911	0.1671	0.1939	0.3139	0.0864	0.1167
	bias($\hat{\phi}$)	0.0087	0.0616	0.0368	**0.0791**	0.0528	0.0430
0.2	mean($\hat{\mathrm{d}}$)	0.1841	0.1658	0.2051	0.2172	0.2277	0.1830
	sd ($\hat{\mathrm{d}}$)	0.1306	0.1618	0.2081	0.2578	0.0636	0.0896
	bias ($\hat{\mathrm{d}}$)	-0.0159	**-0.0342**	0.0051	0.0172	0.0277	-0.0170
	mse($\hat{\mathrm{d}}$)	0.0173	0.0273	0.0432	**0.0666**	0.0048	0.0083
	mean($\hat{\phi}$)	0.2094	0.2344	0.2012	0.2004	0.1637	0.2104
	sd($\hat{\phi}$)	0.1398	0.1668	0.2053	0.2567	0.0738	0.1041
	bias($\hat{\phi}$)	0.0094	0.0344	0.0012	0.0004	**-0.0363**	0.0104
0.6	mean ($\hat{\mathrm{d}}$)	0.5159	0.2242	0.2806	0.2978	0.3794	0.2806
	sd ($\hat{\mathrm{d}}$)	0.3609	0.1562	0.1968	0.3276	0.0308	0.0754
	bias ($\hat{\mathrm{d}}$)	**0.3159**	0.0242	0.0806	0.0978	0.1794	0.0806
	mse($\hat{\mathrm{d}}$)	**0.2298**	0.0249	0.0451	0.1166	0.0331	0.0122
	mean($\hat{\phi}$)	0.3080	0.5536	0.5002	0.4784	0.4166	0.5080
	sd($\hat{\phi}$)	0.3178	0.1464	0.1814	0.2713	0.0505	0.0799
	bias($\hat{\phi}$)	**-0.2920**	-0.0464	-0.0998	-0.1216	-0.1834	-0.0920

Table 3
Results of estimating the parameters in the ARFIMA(1,0.45,0) model

$n = 150$

ϕ		$\hat{d}_{wt}$	$\hat{d}_{sp}$	$\hat{d}_{p}$	$\hat{d}_{rb}$	$\hat{d}_{lr}$ $m=64$	$\hat{d}_{lr}$ $m=32$
-0.6	mean ($\hat{d}$)	0.5896	0.3890	0.4670	0.0801	0.0619	0.2889
	sd ($\hat{d}$)	0.1075	0.1995	0.2572	0.1310	0.1096	0.0773
	bias ($\hat{d}$)	0.1396	-0.0610	0.0170	-0.3699	**-0.3881**	-0.1611
	mse($\hat{d}$)	0.0310	0.0434	0.0662	0.1539	**0.1626**	0.0319
	mean($\hat{\phi}$)	-0.8075	-0.5409	-0.5612	-0.2432	-0.2490	-0.4999
	sd($\hat{\phi}$)	0.1882	0.1524	0.2086	0.2064	0.1356	0.0951
	bias($\hat{\phi}$)	-0.2075	0.0591	0.0388	0.3568	**0.3510**	0.1001
-0.2	mean ($\hat{d}$)	0.6050	0.3670	0.4568	0.3045	0.2630	0.3056
	sd ($\hat{d}$)	0.1098	0.2005	0.2668	0.1264	0.0637	0.0756
	bias ($\hat{d}$)	0.1550	-0.0830	0.0068	-0.1455	**-0.1866**	-0.1444
	mse($\hat{d}$)	0.0361	0.0470	**0.0710**	0.0371	0.0389	0.0265
	mean($\hat{\phi}$)	-0.3114	-0.1082	-0.1645	-0.0499	-0.0161	-0.0639
	sd($\hat{\phi}$)	0.1110	0.2075	0.2570	0.1216	0.0732	0.1042
	bias($\hat{\phi}$)	-0.1114	0.0918	0.0355	0.1501	**0.1839**	0.1361
0.2	mean ($\hat{d}$)	0.7005	0.3952	0.4840	0.6139	0.4002	0.3576
	sd ($\hat{d}$)	0.1335	0.1990	0.2444	0.1090	0.0309	0.0618
	bias ($\hat{d}$)	**0.2505**	-0.0548	0.0340	0.1639	-0.0498	-0.0924
	mse($\hat{d}$)	**0.0805**	0.0425	0.0608	0.0387	0.0034	0.0123
	mean($\hat{\phi}$)	-0.0239	0.2460	0.1684	0.0412	0.2413	0.2832
	sd($\hat{\phi}$)	0.1355	0.2011	0.2335	0.0890	0.0657	0.0853
	bias($\hat{\phi}$)	**-0.2239**	0.0460	-0.0316	-0.1588	0.0413	0.0832
0.6	mean ($\hat{d}$)	0.8686	0.5217	0.6014	1.0373	0.4738	0.4436
	sd ($\hat{d}$)	0.0706	0.2022	0.2734	0.1219	0.0108	0.0261
	bias ($\hat{d}$)	0.4186	0.0717	0.1514	**0.5873**	0.0238	-0.0064
	mse($\hat{d}$)	0.1802	0.0459	0.0975	**0.3597**	0.0007	0.0007
	mean($\hat{\phi}$)	0.1976	0.5015	0.4274	0.0570	0.5670	0.5937
	sd($\hat{\phi}$)	0.0967	0.1843	0.2399	0.0883	0.0635	0.0598
	bias($\hat{\phi}$)	-0.4024	-0.0985	-0.1726	**-0.5430**	-0.0330	-0.0063

Table 3
Continuation

$n = 300$

ϕ		$\hat{d}_{wt}$	$\hat{d}_{sp}$	$\hat{d}_{p}$	$\hat{d}_{rb}$	$\hat{d}_{lr}$ $m = 64$	$\hat{d}_{lr}$ $m = 32$
-0.6	mean ($\hat{d}$)	0.5323	0.4085	0.4699	0.2349	0.3197	0.3314
	sd ($\hat{d}$)	0.0785	0.1683	0.2002	0.0770	0.0536	0.0678
	bias ($\hat{d}$)	0.0823	-0.0415	0.0199	**-0.2151**	-0.1303	-0.1186
	mse($\hat{d}$)	0.0129	0.0300	0.0404	**0.0522**	0.0199	0.0187
	mean($\hat{\phi}$)	-0.7204	-0.5497	-0.5759	-0.4335	-0.5118	-0.5189
	sd($\hat{\phi}$)	0.1755	0.1229	0.1324	0.0881	0.0655	0.0703
	bias($\hat{\phi}$)	-0.1204	0.0503	0.0241	**0.1665**	0.0882	0.0811
-0.2	mean ($\hat{d}$)	0.5230	0.4062	0.4707	0.3436	0.3389	0.3341
	sd ($\hat{d}$)	0.0800	0.16392	0.2046	0.0865	0.0466	0.0622
	bias ($\hat{d}$)	0.0730	-0.0438	0.0207	-0.1064	-0.1111	**-0.1159**
	mse($\hat{d}$)	0.0117	0.0287	**0.0422**	0.0190	0.0145	0.0173
	mean($\hat{\phi}$)	-0.2568	-0.1496	-0.1886	-0.0924	-0.0940	-0.0872
	sd($\hat{\phi}$)	0.0815	0.1615	0.1870	0.0849	0.0719	0.0860
	bias($\hat{\phi}$)	-0.0568	0.0504	0.0114	0.1076	0.1060	**0.1128**
0.2	mean ($\hat{d}$)	0.4298	0.4793	0.5878	0.3810	0.3485	0.6229
	sd ($\hat{d}$)	0.1397	0.1650	0.2030	0.0753	0.0379	0.0596
	bias ($\hat{d}$)	**0.1729**	-0.0202	0.0293	0.1378	-0.0690	-0.1015
	mse($\hat{d}$)	**0.0494**	0.0276	0.0420	0.0247	0.0062	0.0138
	mean($\hat{\phi}$)	0.0459	0.2236	0.1798	0.0664	0.2711	0.3070
	sd($\hat{\phi}$)	0.1264	0.1740	0.2030	0.0658	0.0596	0.0804
	bias($\hat{\phi}$)	**-0.1541**	0.0236	-0.0202	-0.1335	0.0711	0.1070
0.6	mean ($\hat{d}$)	0.8140	0.4864	0.5709	1.0030	0.4534	0.4006
	sd ($\hat{d}$)	0.1020	0.1847	0.2346	0.1071	0.0179	0.0414
	bias ($\hat{d}$)	**0.3640**	0.0364	0.1209	**0.5530**	0.0034	-0.0494
	mse($\hat{d}$)	0.1427	0.0353	0.0695	**0.3174**	0.0003	0.0042
	mean($\hat{\phi}$)	0.2520	0.5396	0.4659	0.0965	0.5932	0.6405
	sd($\hat{\phi}$)	0.1195	0.1688	0.2113	0.0770	0.0432	0.0506
	bias($\hat{\phi}$)	-0.3480	-0.0604	-0.1341	**-0.5035**	-0.0068	0.0405

Table 4
Results of estimating the parameters in the ARFIMA(0,0.45,1) model

$n = 150$

θ		$\hat{d}_{wt}$	$\hat{d}_{sp}$	$\hat{d}_p$	$\hat{d}_{rb}$	$\hat{d}_{lr}$ $m = 64$	$\hat{d}_{lr}$ $m = 32$
-0.6	mean ($\hat{\mathrm{d}}$)	0.6027	0.4031	0.4834	0.7837	0.4340	0.3629
	sd ($\hat{\mathrm{d}}$)	0.1265	0.1969	0.2555	0.1099	0.0230	0.0599
	bias ($\hat{\mathrm{d}}$)	0.1528	-0.0469	0.0334	**0.3337**	-0.0160	-0.0871
	mse($\hat{\mathrm{d}}$)	0.0393	0.0409	0.0662	**0.1234**	0.0008	0.0112
	mean($\hat{\theta}$)	-0.4731	-0.6061	-0.5530	-0.3881	-0.6077	-0.6402
	sd($\hat{\theta}$)	0.1214	0.1330	0.1813	0.1315	0.0666	0.0684
	bias($\hat{\theta}$)	0.1269	-0.0061	0.0470	**0.2119**	-0.0077	-0.0402
-0.2	mean ($\hat{\mathrm{d}}$)	0.7068	0.4033	0.4810	0.5919	0.3906	0.3485
	sd ($\hat{\mathrm{d}}$)	0.1690	0.2017	0.2488	0.1096	0.0327	0.0611
	bias ($\hat{\mathrm{d}}$)	**0.2568**	-0.0467	0.0310	0.1419	-0.0594	-0.1015
	mse($\hat{\mathrm{d}}$)	**0.0944**	0.0427	0.0627	0.0321	0.0046	0.0140
	mean($\hat{\theta}$)	0.0412	-0.2181	-0.1468	-0.0731	-0.2482	-0.2794
	sd($\hat{\theta}$)	0.2120	0.1891	0.2367	0.1034	0.0738	0.0861
	bias($\hat{\theta}$)	**0.2412**	-0.0181	0.0532	0.1269	-0.0482	-0.0794
0.2	mean ($\hat{\mathrm{d}}$)	0.8761	0.3723	0.4483	0.2986	0.2404	0.2933
	sd ($\hat{\mathrm{d}}$)	0.0910	0.2068	0.2778	0.1150	0.0693	0.0763
	bias ($\hat{\mathrm{d}}$)	**0.4261**	-0.0777	-0.0017	-0.1514	-0.2096	-0.1567
	mse($\hat{\mathrm{d}}$)	**0.1898**	0.0487	0.0770	0.0361	0.0487	0.0304
	mean($\hat{\theta}$)	0.5991	0.1456	0.2211	0.0479	-0.0054	0.0518
	sd($\hat{\theta}$)	0.1287	0.2194	0.2846	0.0941	0.0656	0.0969
	bias($\hat{\theta}$)	**0.3991**	-0.0544	0.0211	-0.1521	-0.2054	-0.1482
0.6	mean ($\hat{\mathrm{d}}$)	0.7330	0.2531	0.3383	-0.0976	-0.1372	0.0380
	sd ($\hat{\mathrm{d}}$)	0.1131	0.2087	0.2704	0.1076	0.1194	0.1330
	bias ($\hat{\mathrm{d}}$)	0.2830	-0.1969	-0.1117	-0.5476	**-0.5872**	-0.4120
	mse($\hat{\mathrm{d}}$)	0.0928	0.0822	0.0854	0.3114	**0.3591**	0.1874
	mean($\hat{\theta}$)	0.7880	0.4130	0.4861	0.0082	-0.0426	0.1758
	sd($\hat{\theta}$)	0.0714	0.2049	0.2638	0.1308	0.1215	0.1429
	bias($\hat{\theta}$)	0.1880	-0.1874	-0.1139	0.4418	**-0.6426**	-0.4242

Table 4
Continuation

$n = 300$

θ		$\hat{d}_{wt}$	$\hat{d}_{sp}$	$\hat{d}_{p}$	$\hat{d}_{rb}$	$\hat{d}_{lr}$ $m = 64$	$\hat{d}_{lr}$ $m = 32$
-0.6	mean ($\hat{d}$)	0.5442	0.4091	0.4620	0.6770	0.3789	0.3403
	sd ($\hat{d}$)	0.1309	0.1550	0.1982	0.0808	0.0361	0.0639
	bias ($\hat{d}$)	0.0942	-0.0409	0.0120	**0.2270**	-0.0711	-0.1097
	mse($\hat{d}$)	0.0257	0.0393	**0.0581**	0.0064	0.0161	0.0260
	mean($\hat{\theta}$)	-0.5261	-0.6129	-0.5819	-0.4777	-0.6325	-0.6501
	sd($\hat{\theta}$)	0.0961	0.0895	0.1215	0.0742	0.0480	0.0510
	bias($\hat{\theta}$)	0.0739	-0.0129	0.0181	**0.1223**	-0.0325	-0.0501
-0.2	mean ($\hat{d}$)	0.5778	0.4125	0.4602	0.5591	0.3668	0.3398
	sd ($\hat{d}$)	0.1367	0.1611	0.2009	0.0758	0.0428	0.0662
	bias ($\hat{d}$)	**0.1278**	-0.0375	0.0102	0.1091	-0.0832	-0.1102
	mse($\hat{d}$)	**0.0350**	0.0273	0.0404	0.0176	0.0087	0.0165
	mean($\hat{\theta}$)	-0.0780	-0.2183	-0.1747	-0.1007	-0.2644	-0.2860
	sd($\hat{\theta}$)	0.1490	0.1482	0.1878	0.0774	0.0593	0.0716
	bias($\hat{\theta}$)	**0.1220**	-0.0183	0.0253	0.0993	-0.0644	-0.0860
0.2	mean ($\hat{d}$)	0.7223	0.3986	0.4533	0.3316	0.3217	0.3232
	sd ($\hat{d}$)	0.1825	0.1707	0.2119	0.0758	0.0528	0.0653
	bias ($\hat{d}$)	**0.2723**	-0.0514	0.0033	-0.1184	-0.1283	-0.1268
	mse($\hat{d}$)	**0.1074**	0.0317	0.0448	0.0197	0.0192	0.0203
	mean($\hat{\theta}$)	0.4568	0.1502	0.2064	0.0764	0.0629	0.0662
	sd($\hat{\theta}$)	0.2040	0.1857	0.2292	0.0696	0.0663	0.0815
	bias($\hat{\theta}$)	**0.2568**	-0.0498	0.0064	-0.1236	-0.1371	-0.1338
0.6	mean ($\hat{d}$)	0.6732	0.3272	0.3811	-0.0254	0.0827	0.2199
	sd ($\hat{d}$)	0.1106	0.1683	0.2102	0.0775	0.0971	0.0975
	bias ($\hat{d}$)	0.2232	-0.1228	-0.0689	**-0.4754**	-0.3673	-0.2301
	mse($\hat{d}$)	0.0620	0.0434	0.0488	**0.2320**	0.1443	0.0624
	mean($\hat{\theta}$)	0.7674	0.4808	0.5265	0.0680	0.2137	0.3706
	sd($\hat{\theta}$)	0.0870	0.1669	0.2080	0.0958	0.0975	0.1017
	bias($\hat{\theta}$)	0.1674	-0.1192	-0.0735	**-0.5320**	-0.3863	-0.2294

Results for the ARFIMA$(1, 0.45, 1)$ model are presented in Table 5 for the three (ϕ,θ) pairs (0.6,-0.6), (0.6,-0.2) and (0.2,-0.2). The bias in the fractional parameter is generally positive, i.e. the estimators overestimate the true value of d and consequently the AR and/or MA coefficients are underestimated. Again, the Whittle estimator is outperformed by the $\hat{d}_{sp}$, $\hat{d}_p$ and $\hat{d}_{lr}$ methods. Except the Robinson estimator, the other semiparametric methods have relatively small and competitive bias . As expected, the $\hat{d}_{sp}$ estimator always has smaller mse compared to the popular Geweke and Porter-Hudak method.

4 Applications

The objective of this section, based on the methodology described in the previous sections, is to model and compare the ARIMA and ARFIMA approaches in the following sets of data.

The Nile River Minima

The Nile river minima data has been widely discussed (see for example Beran, 1994) and was one of several that led to the discovery of the so-called Hurst-effect. The data are the minimal levels of the Nile river for the years 622 - 1281 measured at the Roda Gauge near Cairo.

The sample mean of the data is 1148 and the standard deviation is 89.05. As reported in other papers, the series may exhibit long memory behaviour; Beran (1994) considered an ARFIMA model and the estimates were based on the self-similarity parameter H where $H = d + 0.5$.

We subtract the sample mean from the data and consider an ARFIMA model with d estimated by the methods seen earlier. The number of frequencies in the Robinson estimator was obtained as given in Section 2 by replacing the unknown parameter d by $\hat{d}_{wt}$ and $\hat{d}_{sp}$ (both values satisfy the stationarity conditions). Hence, the number of observations in the regression equations were 101 and 186 respectively. The Whittle approach gives the estimates of all parameters simultaneously. Different ARFIMA(p,d,q) models were considered and the estimates of the AR and MA parameters were approximately zero. In the semiparametric approaches, the order of the ARMA process was identified after the series being differenciated by the estimate of d. The choices of the model were made by testing the AR and MA coefficients, the AIC (Akaike Information Criteria) and SBC (Schwartz Criteria) and also by performing a residual analysis. All five estimators of d led to the ARFIMA(0,d,0) model. The Box-Jenkins models considered were ARMA(1,1), ARIMA(1,1,1) and AR(2) and the estimates of the parameters were obtained by using MINITAB package.

Table 6 gives the results related to the identification and estimation of the models.

From the table we see that, the fitted ARFIMA$(0, d, 0)$ with $\hat{d}_{wt}$ seems

Table 5

Results of estimating the parameters in the ARFIMA(1,0.45,1) model

$n = 150$

$n = 300$ ϕ	θ		$\hat{d}_{wt}$	$\hat{d}_{sp}$	$\hat{d}_{p}$	$\hat{d}_{rb}$	$\hat{d}_{lr}$ $m = 64$	$\hat{d}_{lr}$ $m = 32$
0.6	-0.6	mean ($\hat{d}$)	0.8476	0.5390	0.6146	1.3329	0.4879	0.4529
		sd ($\hat{d}$)	0.0987	0.1872	0.2441	0.1610	0.0056	0.0225
		bias ($\hat{d}$)	0.3976	0.0890	0.1646	**0.8829**	0.0379	0.0029
		mse($\hat{d}$)	0.1678	0.0429	0.0865	**0.8054**	0.0015	0.0005
		mean($\hat{\phi}$)	0.2304	0.4860	0.4124	-0.2036	0.5540	0.5891
		sd($\hat{\phi}$)	0.1516	0.1986	0.2444	0.1596	0.0904	0.0899
		bias($\hat{\phi}$)	-0.3696	-0.1140	-0.1876	**-0.8036**	-0.0460	-0.0109
		mean($\hat{\theta}$)	-0.4468	-0.6022	-0.6070	-0.6635	-0.5866	-0.5869
		sd($\hat{\theta}$)	0.1458	0.0919	0.0936	0.1543	0.0883	0.0884
		bias($\hat{\theta}$)	**0.1532**	-0.0022	-0.0070	-0.0635	0.0134	0.0131
0.6	-0.2	mean ($\hat{d}$)	0.8665	0.5278	0.6150	1.1842	0.4822	0.4495
		sd ($\hat{d}$)	0.0774	0.2034	0.2584	0.1246	0.0079	0.0242
		bias ($\hat{d}$)	0.4165	0.0778	0.1650	**0.7342**	0.0322	-0.0005
		mse($\hat{d}$)	0.1795	0.0473	0.0938	**0.5546**	0.0011	0.0006
		mean($\hat{\phi}$)	0.2147	0.4715	0.3834	-0.1212	0.5534	0.5821
		sd($\hat{\phi}$)	0.2077	0.2271	0.2825	0.2059	0.1133	0.0998
		bias($\hat{\phi}$)	-0.3853	-0.1285	-0.2166	**-0.7212**	-0.0466	-0.0179
		mean($\hat{\theta}$)	-0.1610	-0.2405	-0.2516	-0.2772	-0.2072	-0.2097
		sd($\hat{\theta}$)	0.2160	0.1461	0.1533	0.2428	0.1343	0.1315
		bias($\hat{\theta}$)	0.0390	-0.0405	-0.0516	**-0.0772**	-0.0072	-0.0097
0.2	-0.2	mean ($\hat{d}$)	0.7378	0.4048	0.4904	0.7804	0.4327	0.3739
		sd ($\hat{d}$)	0.1418	0.2081	0.2536	0.1216	0.0237	0.0511
		bias ($\hat{d}$)	0.2878	-0.0452	0.0404	**0.3304**	-0.0173	-0.0761
		mse($\hat{d}$)	0.1029	0.0452	0.0658	**0.1239**	0.0009	0.0084
		mean($\hat{\phi}$)	0.0414	0.2185	0.1688	-0.1022	0.1967	0.2771
		sd($\hat{\phi}$)	0.3409	0.2802	0.2848	0.2193	0.1771	0.1631
		bias($\hat{\phi}$)	0.1586	0.0185	-0.0312	**-0.3022**	0.0771	-0.0369
		mean($\hat{\theta}$)	-0.0915	-0.2154	-0.1894	-0.2089	-0.2141	-0.1896
		sd($\hat{\theta}$)	0.4559	0.2155	0.2412	0.2712	0.1914	0.1806
		bias($\hat{\theta}$)	**0.1085**	-0.0154	0.0106	-0.0089	-0.0141	0.0104

Table 5
Continuation

$n = 300$

ϕ	θ		$\hat{d}_{wt}$	$\hat{d}_{sp}$	$\hat{d}_{p}$	$\hat{d}_{rb}$	$\hat{d}_{lr}$ $m=64$	$\hat{d}_{lr}$ $m=32$
0.6	-0.6	mean $(\hat{\mathrm{d}})$	0.7785	0.4916	0.5436	1.1772	0.4614	0.4021
		sd $(\hat{\mathrm{d}})$	0.1367	0.1639	0.1961	0.0915	0.0146	0.0395
		bias $(\hat{\mathrm{d}})$	0.3285	0.0416	0.0936	**0.7272**	0.0114	-0.0479
		mse$(\hat{\mathrm{d}})$	0.1266	0.0285	0.0471	**0.5372**	0.0003	0.0038
		mean$(\hat{\phi})$	0.2880	0.5404	0.4928	-0.1148	0.5835	0.6373
		sd$(\hat{\phi})$	0.1522	0.1606	0.1949	0.0927	0.0503	0.0541
		bias$(\hat{\phi})$	-0.3120	-0.0596	-0.1072	**-0.7148**	-0.0165	0.0373
		mean$(\hat{\theta})$	-0.5006	-0.6058	-0.6059	-0.6721	-0.5971	-0.5998
		sd$(\hat{\theta})$	0.1073	0.0549	0.0577	0.0650	0.0549	0.0545
		bias$(\hat{\theta})$	**0.0994**	-0.0058	-0.0059	-0.0721	0.0029	0.0002
0.6	-0.2	mean $(\hat{\mathrm{d}})$	0.8021	0.4765	0.5306	1.0633	0.4581	0.4007
		sd $(\hat{\mathrm{d}})$	0.1185	0.1720	0.2031	0.0871	0.0145	0.0438
		bias $(\hat{\mathrm{d}})$	0.3521	0.0265	0.0806	**0.6133**	0.0081	-0.0493
		mse$(\hat{\mathrm{d}})$	0.1380	0.0302	0.0477	**0.3837**	0.0003	0.0043
		mean$(\hat{\phi})$	0.2478	0.5494	0.4956	-0.0876	0.5927	0.6404
		sd$(\hat{\phi})$	0.1840	0.1739	0.2115	0.1551	0.0603	0.0565
		bias$(\hat{\phi})$	-0.3522	-0.0506	-0.1044	**-0.6876**	-0.0073	0.0404
		mean$(\hat{\theta})$	-0.1958	-0.2143	-0.2171	-0.3387	-0.1920	-0.1999
		sd$(\hat{\theta})$	0.1290	0.0896	0.0960	0.1520	0.0827	0.0817
		bias$(\hat{\theta})$	0.0042	-0.0143	-0.0171	**-0.1387**	0.0080	0.0001
0.2	-0.2	mean $(\hat{\mathrm{d}})$	0.7229	0.4002	0.4684	0.7084	0.3904	0.3464
		sd $(\hat{\mathrm{d}})$	0.1868	0.1569	0.1856	0.0786	0.0345	0.0602
		bias $(\hat{\mathrm{d}})$	**0.2729**	-0.0498	0.0184	0.2580	-0.0596	-0.1036
		mse$(\hat{\mathrm{d}})$	**0.1093**	0.0270	0.0347	0.0730	0.0047	0.0143
		mean$(\hat{\phi})$	0.1521	0.2481	0.1622	-0.1600	0.2770	0.3400
		sd$(\hat{\phi})$	0.4132	0.2417	0.2821	0.1697	0.1284	0.1418
		bias$(\hat{\phi})$	-0.0479	0.0481	-0.0378	**-0.3600**	0.0770	0.1400
		mean$(\hat{\theta})$	0.0167	-0.1954	-0.2196	-0.3359	-0.1762	-0.1566
		sd$(\hat{\theta})$	0.5486	0.1528	0.1871	0.1849	0.1390	0.1350
		bias$(\hat{\theta})$	**0.2167**	0.0046	-0.0196	-0.1359	0.0238	0.0434

Table 6
Identification and estimation of the Nile River data

	ARFIMA(0,d,0)							
	$\hat{d}_{wt}$	$\hat{d}_{sp}$	$\hat{d}_p$	$\hat{d}_{rb}$ $m = 186$	$\hat{d}_{lr}$ $m = 32$	ARMA(1,1)	ARIMA(1,1,1)	AR(2)
mean ($\hat{d}$)	0.399	0.439	0.500	0.352	0.323	-	-	-
sd($\hat{d}$)	(0.0498)	(0.0649)	(0.1570)	(0.0543)	(0.03)	-	-	-
95(%)C.I.($\hat{d}$)	(0.301; 0.497)	(0.312; 0.566)	(0.192; 0.808)	(0.246; 0.458)	(0.264; 0.382)	-	-	-
$\hat{\phi}_1$	-	-	-	-	-	0.8701	0.3571	0.4829
sd($\hat{\phi}_1$)	-	-	-	-	-	(0.0293)	(0.0427)	(0.0384)
$\hat{\phi}_2$	-	-	-	-	-	-	-	0.1595
sd($\hat{\phi}_2$)	-	-	-	-	-	-	-	(0.0384)
$\hat{\theta}$	-	-	-	-	-	0.4978	0.9144	-
sd($\hat{\theta}$)	-	-	-	-	-	(0.0515)	(0.0176)	-
$\hat{\sigma}_\epsilon^2$	4840	4856	4899	4842	4846	5066	4992	5186
AIC	5550.9	5553.1	5558.9	5551.3	5551	5582.8	5575	5598
SBC	5555.5	5557.6	5563.5	5555.7	5556.3	5591.7	5588.7	5607.0
MBP	0.731	0.542	0.218	0.820	0.80	0.114	0.597	0.001

MBP- the modified Box-Pierce Chi-square Statistic (11 df) to be more appropriate for this data. $\hat{d}_{rb}$ and $\hat{d}_{lr}$ give similar values. For $m = 64$, we found $\hat{d}_{lr} = 0.3566$ with $sd = 0.014$. The estimates of d and the associated confidence intervals (CI) suggest that the data are long memory; $\hat{d}_p$ indicates that the series can be non-stationary as well. These results are in agreement with the results in Beran (1994).

The annual rainfall at Fortaleza

Now we follow the same methodology applied in the previous example to the annual rainfall at Fortaleza data. This series has 152 observations measured, in mm, for the years 1849 to 1996.The sample mean is 1437.9 and sd is 1423.9. This series was also analysed using Stochastic Cycles and Bayesian approaches by Harvey and Souza (1987) and Brasil and Souza (1993) respectively.

The results related to the identification and estimation models are in Table 7. The series was identified as an AR(1) model using the MINITAB-package (other Box-Jenkins models were considered and the adequate model was chosen looking at the sample correlogram, the residual analysis, AIC, SBC and the Modified Box-Pierce Chi-Square Statistic).

The semiparametric estimates of d indicate that the process is not long-memory, i.e. intermediate memory ($d < 0$) or $d = 0.0$ (short memory). We considered the absolute value of $\hat{d}_{sp}$ to obtain the bandwidth m in Robinson's estimator. For the Whittle estimator, different ARFIMA(p, d, q) models were considered and the ARFIMA$(1, d, 0)$ and ARFIMA$(0, d, 0)$ models shown to be more appropriate.

The hypothesis test $d = 0$ was also performed based on the asymptotic Normal distribution of each estimator of d with the level of significance $\alpha = 5\%$ (a comprehensive study related to the hypothesis tests in ARFIMA models is presented in Reisen and Lopes (1999)). Also, the residual analysis of the fitted models was considered and it indicated the same pattern for the errors, i.e they are approximately independent Normal distributed.

Both semiparametric methods $\hat{d}_p$ and $\hat{d}_{lr}$ (m =32) indicated the series is short memory, i.e, $d = 0$. The estimates are -0.111 ($sd = 0.257$) and 0.146($sd = 0.104$) respectively. $\hat{d}_{lr}$, for $m = 64$, gave 0.203 with $sd = 0.092$ and $\hat{d}_{rb}$ = -0.3186 ($sd = 0.3629$).

Since there is no evidence of long memory property in this series, we state the following comments based on Table 7.

i. The intermediate memory behaviour is well noticed when using the $\hat{d}_{sp}$, $\hat{d}_{rb}$ and $\hat{d}_{wt}$ in the case when AR coefficient is included.

ii. Using the ARFIMA$(0, d, 0)$ model, the $\hat{d}_{wt}$ suggests long memory, i.e $\hat{d}_{wt} = 0.2044$. This is expected in that the estimated coefficient of the AR(1) model, i.e., $\hat{\phi} = 0.2714$, is diluted in coefficients of the

Table 7
Identification and estimation of the annual rainfall at Fortaleza - Brazil

Parameters (Estimates)	ARFIMA(0,$\hat{d}_{wt}$,0)	ARFIMA(1,$\hat{d}_{wt}$,0)	ARFIMA(1,$\hat{d}_{sp}$,0)	AR(1)
mean ($\hat{\text{d}}$)	0.2044	-0.5920	-0.2236	-
sd($\hat{\text{d}}$)	0.0104	0.0104	0.1020	-
$\hat{\phi}_1$	-	0.8428	0.5179	0.2714
sd($\hat{\phi}_1$)	-	0.0463	0.0727	0.0808
$\hat{\sigma}^2_\epsilon$	239363	230596	231385	231343
AIC	1773.1	1769.8	1768.3	1770.3
SBC	1776	1775.8	1776.2	1776.2
MBP	0.917	0.968	0.977	0.935

MBP- the modified Box-Pierce Chi-square Statistic (22 df)

binomial expansion $(1-B)^{0.2044}$ of the observations X_t, X_{t-1}, X_{t-2}, ...

iii. When considering a model of the form ARFIMA$(1, d, 0)$, the value of $\hat{d}$ produces impact in the estimate of the AR coefficient, i.e., $\hat{\phi}$. It is well noticed in the results (see, for example, the fitted ARFIMA(1,$\hat{d}_{wt}$,0) and ARFIMA(1,$\hat{d}_{sp}$,0) models). Now the estimated coefficient of the AR(1) model is diluted in the coefficients of the expansion $(1-\hat{\phi}B)(1-B)^{\hat{d}}$. Negative value of d leads to positive value of ϕ.

iv. It is evidence that, based on the Modified Box-Pierce Chi-Square Statistic, the hypothesis of adequate model is not rejected for all cases.

As is well known in time series analysis, or more generally in data analysis, there may be several adequate models that can be used to represent a given data set. Sometimes, the choice can be very diffilcult. In the case of the series above our main motivation was to apply the methodology of the ARFIMA process and hence we will not suggest here the appropriate model for this series. It could be a subject for future researches including cyclical modelling and forecasting issues.

5 Conclusions

We present some simulation results and discuss the estimation of all parameters of the ARFIMA model. When dealing with AR and MA parts the bias and the *mse* of the estimates increase substantially and the Whittle method is, now, outperformed by the semiparametric approaches. The estimates improve as the sample size increases.

Two sets of data, the Nile river minima and the annual rainfall at Fortaleza were analysed from the ARFIMA and ARIMA models.

Acknowledgments

V.A. Reisen and E. M. Toscano were partially supported by CNPq, FAPEMIG and FUNDEP, Brazil and B. Abraham was partially supported by grants from NSERC. We would like to thank Dr. B. Ray (NJIT) for providing her Fortran program of the Whittle estimator and providing some additional support. We also thank Prof. Gutemberg Brasil (DEST-UFES) for the annual rainfall data. We thank the anonymous referee.

Part of this work was done while V. A. Reisen was visiting the University of Waterloo, Canada and the Departamento de Estatistica, UFMG, Brazil.

(Received March, 2000. Revised February, 2001.)

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