Are Australia's Savings and Investment Fractionally Cointegrated?

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Abstract This paper uses an Autoregressive Fractionally Integrated Moving Average (ARFIMA) process to determine if Australia's savings and investment are fractionally cointegrated. The study finds the two series to be fractionally cointegrated implying that deviations from equilibrium are persistent.

Keywords: Australia, savings, investment, long memory, fractional cointegration

JEL Classification: C22, C32, E21, E22

1. Introduction

This paper applies an Autoregressive Fractionally Integrated Moving Average (ARFIMA) process to determine if gross investment (I) and gross savings(S) in Australia are fractionally cointegrated. In the recent past, Australia's gross savings has fallen short of gross investment giving rise to a policy debate on the question of the adequacy of saving in Australia. The general view is that Australia's national savings is low by international standards. The outcome of this ARFIMA study given the presence of a long run relationship between these macroeconomic aggregates will help address these policy related issues.

The chosen methodology for this analysis is the ARFIMA process. This is chosen because many economic time series exhibit a high degree of persistence. Sowell (1990), Diebold and Rudebusch (1991) among others show that traditional unit root tests have low power against a fractional alternative because they are restricted to integer order I(1) or I(0). Fractional integration (long memory models) permits the integration order of a series to take on any fraction. Long-memory, or long-term dependence implies that a series is dependent on its values in the distant past. Such series are characterized by distinct cyclical patterns not dissimilar from those evident in the preceding spectral analysis.

2. Data

All data are quarterly, seasonally adjusted and run from 1959:3 to 2005:4. The Australian Bureau of Statistics is the sole data source. Both the gross investment and gross savings series are constructed from ABS 5406.0: Australian National Accounts.

Gross investment is constructed by adding to gross capital expenditure in the Australian National Accounts the value of investment in inventories. Further, gross savings is calculated by adding the aggregate measures net national savings and depreciation (capital used up in production).

3. The GPH Test for Fractional Cointegraton

We apply the Geweke and Porter-Hudak (1983) test based on spectral regression estimates. The central feature of this test is the fractional differencing parameter d which in turn is based on the slope of the spectral density function around the angular frequency $= 0$.

A time series, y, is said to follow an autoregressive fractionally integrated moving average (ARFIMA process of order (p, d, q)) with mean μ if:

$$\phi(L)(1-L)^d (y_t - \mu) = \theta(L)\varepsilon_t \qquad \varepsilon_t \ \square \ i.i.d(0, \sigma_u^2) \tag{1}$$

where $\phi(L)$ is an autoregressive coefficient of order p and $\theta(L)$ is a moving average coefficient of order q and ε_t is a white noise process. $(1-L)^d$ is the fractional differencing operator defined as follows:

$$(1-L)^d = \frac{\sum_{k=0}^{\alpha} \Gamma(k-d)L^k}{\Gamma(-d)\Gamma(k+1)} \tag{2}$$

with $\Gamma(.)$ denoting the generalised factorial function. The parameter d is permitted to assume any real value. If d is not an integer, the series can be said to be fractionally integrated. In the time domain, the series can be expected to exhibit a hyperbolically decaying ACF. In the frequency domain, the process y is both stationary and invertible if all roots of $\phi(L)$ and $\theta(L)$ are outside the unit circle and $0 > d > 1/2$. The series is non stationary and possesses an infinite variance for $d \geq 1/2$. See Granger and Joyeux (1980) and Hosking (1981).

Geweke and Porter-Hudak (GPH) have developed a non parametric test for estimating the fractional differencing parameter, d. The GPH test is carried out on the first differences of the series to ensure that stationarity and invertibility are achieved. Geweke and Porter-Hudak show that the differencing parameter, d, which is also called a long memory parameter, can be estimated consistently from the least squares regression:

$$\ln(I(\omega_j)) = \theta + \lambda \ln(4\sin^2(\omega_j/2)) + \upsilon_j ; \quad j = 1,....J \tag{3}$$

where θ is a constant, $\omega_j = 2\pi j / T (j = 1,....T-1)$ denotes the Fourier frequencies of the sample, $J = f(T^\mu)$. J is an increasing function of T where T are the number of observations and $0 < \mu < 1$. $I(\omega_j)$ is the periodogram of the time series at frequency ω_j. For a sample series

with T observations, the periodogram at harmonic frequency ω_j, $I(\omega_j)$ is computed in the following manner:

$$\hat{I}(\omega_j) = \frac{1}{2\pi} \sum_{k=-T+1}^{T-1} \hat{\gamma}_j e^{-i\omega_j k}, i^2 = -1, \text{or equivalently,}$$

$$\hat{I}(\omega_j) = \frac{1}{2\pi}(\hat{\gamma}_0 + 2\sum_{k=1}^{T-1} \gamma_k \cos(\omega_j k)), \tag{4}$$

where γ_j is order j auto-covariance of sample series,

$$\hat{\gamma}_j = \begin{cases} T^{-1} \sum_{t=j+1}^{T} (y_t - \bar{y})(y_{t-j} - \bar{y}), j = 0,1,2,...,T-1 \\ \hat{\gamma}_{-j}, j = -1,-2,...,-T+1 \end{cases}. \tag{5}$$

The existence of a fractional order of integration can be tested by examining the statistical significance of the differencing parameter, d. The estimated d values can be interpreted as follows: the process is a long memory process if $0 < d < 1$. If $0.5 > d > 1$ the process is non-stationary and exhibits long memory; if $0 < d < 0.5$ the process is stationary and exhibits long memory. The process is stationary and has short memory if $-\frac{1}{2} < d < 0$.[1]

4. Empirical Results

The GPH test results for fractional integration and cointegration are reported in Table 1 (see Table1). Results are reported for the $\tilde{d}$ estimates $J = T^{0.45}, T^{0.5}, T^{0.55}$. Different values of μ are used in order to check the sensitivity of the results to changes in μ. Fractional cointegration requires testing for fractional integration in the error correction term of the cointegrating regression. As the I and S series were found to be non stationary, the GPH test is carried out on the first differences of the series.

The results reported in Table 1 suggest that the first differences of the S and I series are long memory mean reverting processes. The hypothesis of fractional cointegration requires a test of the error correction term (ε) obtained from the cointegrating regression of the individual series. For the cointegrating relationship between savings and investment, the fractional cointegration tests reported in Table 1 show that $0 < d < .5$ for the error correction term (ε) implying that the series are stationary and exhibit long memory. The fact that the two series are fractionally cointegrated implies that a long run relation exists between the two series.

5. Conclusions

A FARIMA test of Australian gross investment and savings indicate that a long run relation exists between the two series. Despite the fact that a long run relation exists between the two series, the error correction term possesses long memory which implies that deviations from equilibrium are highly persistent. The relation between the two series suggests that Australia could successfully adopt policies that focus on increasing investment through increasing domestic savings. Our results also suggest that a low level of domestic savings can constrain domestic investment. These results are in contrast to Schmidt (2003) who in a study of Australia's savings and investment states that investment is strongly exogenous and therefore policies that aim on increasing investment through savings are unlikely to be successful. In conclusion it can be pointed that it is important for Australia to take measures to increase its level of domestic savings.

Endnotes

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1. See Granger and Joyeux (1980) and Hosking (1981) for a detailed discussion.

References

Diebold, F. and G. Rudebusch. 1991. "On the Power of Dickey-Fuller Tests against Fractional Alternatives," *Economics Letters,* 35, 155-60

Geweke, J. and S. Porter-Hudak. 1983. "The Estimation and Application of Long Memory Time Series Models," *Journal of Time Series*, 4, 221-238.

Granger, C. and R. Joyeux. 1980. "An Introduction to Long Memory Time Series Models and Fractional Differencing," *Journal of Time Series Analysis*, 1, 15-39.

Schmidt, M. 2003. "Savings and Investment in Australia," *Applied Economics*, 35, 99-106

Sowell, F. 1990. "The Fractional Unit Root Distribution," *Econometrica*, 58, 495-505

Table 1: GPH Test Results for Fractional Integration and Fractional Cointegration

	d=.45	d=.5	d=.55
Results for Fractional Integration			
ΔI	0.044	0.014	0.087
	(0.307)	(0.247)	(0.182)
ΔS	0.223	0.223	0.187
	(0.414)	(0.341)	(0.248)
Results for Fractional Cointegration			
ε	0.228	0.167	0.111
	(0.146)	(0.124)	(0.152)

Note: $\tilde{d}$ =0.45, $\tilde{d}$ =0.5 and $\tilde{d}$ = 0.55 give the $\tilde{d}$ estimates corresponding to the GPH spectral regression of sample size J = $T^{0.45}$, J = $T^{0.5}$, J = $T^{0.55}$. The standard errors are reported in parenthesis. The term in parenthesis is the OLS standard error.