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Measuring Hysteresis in Unemployment Rates with Long Memory Models

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Abstract

We address the question of unemployment hysteresis within the context of ARFIMA models. Our results suggest that in the post-1973 era, hysteresis is considerably less of a stylized fact for the unemployment rates of key OECD economies.

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1. Introduction

As a possible explanation of persistently high European unemployment rates since the early 1970s, Blanchard and Summers (1986) raised the issue of whether unemployment in these countries is characterized by what they called “hysteresis.” By hysteresis they had in mind the lack of any tendency in these unemployment rate series to revert to some mean value after a shock. Accordingly, the natural rate of unemployment is not unique under unemployment hysteresis.

One time series implication of such behavior is that the unemployment rate may be a unit root process. It is well known that for unit root processes past shocks have permanent effects on future values of the series. In their empirical analysis Blanchard and Summers (1986) reported strong evidence in favor of hysteresis for several European unemployment rate series.

The purpose of this paper is to cast the unemployment hysteresis question within the context of estimated autoregressive fractionally integrated moving average (ARFIMA) models. Sowell (1992) showed how unit root testing can be carried out within this relatively general framework, one which nests both unit root and stationary models. Another benefit is that the ARFIMA approach allows for hysteresis effects for an important class of non-unit root alternatives. Further, point estimates of the ARFIMA fractional integration parameter serve as comparative measures of unemployment hysteresis across countries.

The paper proceeds as follows. In Section 2 we briefly summarize the ARFIMA modeling strategy and point out an important property of the cumulative impulse response functions of these models. To allow for the possibility that the ARFIMA dynamics shifted after the 1973 global oil price shock, in Section 3 we carry out tests of structural stability for the unemployment rate series. In Section 4 we present our ARFIMA results and we conclude in Section 5.

2. Autoregressive Fractionally Integrated Moving Average Models

The discrete time ARFIMA (p,d,q) model is given by

$$\Phi(L)(1-L)^d X_t = \Theta(L)\epsilon_t, \ \epsilon \sim WN(0,\sigma^2), \tag{1}$$

where $d \in \mathbb{R}$, L is the lag operator, the integer orders of $\Phi(L)$ and $\Theta(L)$ are p and q, respectively, all roots of $\Phi(L)$ and $\Theta(L)$ lie outside the unit circle, and $(1 - L)^d X_t$ is a stationary stochastic process. Stationarity of X_t requires $d < 0.5$. For any real number d, the fractional difference operator $(1 - L)^d$ is defined through the binomial expansion

$$(1 - L)^d = 1 - dL + \frac{d(d-1)}{2!}L^2 - \frac{d(d-1)(d-2)}{3!}L^3 + \cdots. \tag{2}$$

If X_t is a unit-root process, then d = 1 and the infinite sum in (2) collapses to the first two terms.

For $0 < d < 0.5$, ARFIMA processes are characterized by a hyperbolic decaying autocovariance function; see Brockwell and Davis (1991, pp. 525-526). This stands in sharp contrast to the exponential decay displayed by the autocovariance function of conventional stationary ARMA models. Thus, ARFIMA models with $0 < d < 0.5$ are said to exhibit long memory. For $0.5 < d < 1$, X_t is nonstationary. But the limiting value of the cumulative impulse functions of such models, is 0, implying that shocks do not have permanent effects; see Diebold et al. (1991, p. 1261). Tsay and Chung (1995) call ARFIMA models with $1 < d < 1.5$ strong long memory processes. In this case, past shocks to the series have permanent effects but the first differenced series is stationary with a long memory autocovariance function. Consequently, if an unemployment rate time series follows a strong long memory process, its associated natural rate of unemployment is not unique.

Estimation of stationary ARFIMA models by exact maximum likelihood requires writing the spectral density function $f_x(\omega)$ in terms of the parameters of the model and then calculating the autocovariance function $\gamma(k)$ at lag k by

$$\gamma(k) = (1/2\pi) \int_0^{2\pi} f_x(\omega)\, e^{\iota\omega k}\, d\omega. \tag{3}$$

The exact maximum likelihood estimator of d, for d > 0, has an asymptotic normal distribution; see Dahlhaus (1989). This allows for hypothesis testing based on the Wald statistic

$$\frac{\hat{d} - d}{SE(\hat{d})}, \tag{4}$$

where d is the value of the fractional integration parameter under the null hypothesis and $SE(\hat{d})$ is the maximum likelihood estimate of the standard error of $\hat{d}$. The rejection region is set up based upon a one-sided alternative hypothesis.

3. Structural Stability

We analyze seasonally adjusted monthly unemployment rate series from the OECD Main Economic Indicators database for five G-7 countries: Canada, Germany, Japan, the United Kingdom, and the United States; for the two other G-7 countries, either only quarterly data were available (Italy) or the monthly series was extremely short (France). The sample period runs from the beginning of 1960 to July 1994 for most of the five unemployment rate series studied. Over this sample each of these economies was exposed to the 1973 oil price shock, an event which may have significantly shifted the behavior of the ARFIMA models for each country. Since failure to account for the possibility of such instability in the ARFIMA parameters can lead to biased test statistics, we first check to see whether there is significant evidence indicating structural change for each unemployment rate series before and after the 1973 global oil price shock.

To formally test the stability of the model structure over the pre- and post-oil shock sub-samples, we perform a Wald-type test; see, e.g., Hamilton (1994, p. 425) for details. Let $\mathbf{b}_1$ and $\mathbf{b}_2$ be the estimated vectors of the $k = p + q + 1$ parameters of Φ, Θ, and d for sub-periods 1 and 2, respectively. Let V_1, V_2 be the corresponding estimated variance-covariance matrices of the $\mathbf{b}$ vectors and let n_1, n_2 be the sub-sample sizes. We construct the statistic

$$\lambda = (\mathbf{b}_1 - \mathbf{b}_2)'(n_1^{-1}V_1 + n_2^{-1}V_2)^{-1}(\mathbf{b}_1 - \mathbf{b}_2), \tag{5}$$

which is asymptotically $\chi^2(k)$ distributed under the null hypothesis of structural stability, i.e.,

$$H_0\text{: } \mathbf{b}_1 = \mathbf{b}_2. \tag{6}$$

The asymptotic distribution of λ follows from the asymptotic normality of the vectors $\mathbf{b}$ and the $\sqrt{n}$ convergence of the estimates as explained in Wu and Crato (1995).

The results of our structural stability results are quite strong: for each unemployment rate series we reject the null hypothesis (6) with a p-value less than 0.01, indicating a significant shift in the estimated parameters before and after the 1973 oil price shock.[1] Therefore, for each series we split our sample into pre- and post-1973 oil price sub-samples for estimation of the ARFIMA models and the hypothesis testing that follows.

[1] Full details on the structural stability tests and all estimation results for the paper are available from the authors upon request.

4. Hysteresis Test Results

In Table 1 we present estimates of the fractional integration parameter along with Wald statistics for testing the unit root null hypothesis. We select the order (p,d,q) of the ARFIMA models through use of the Akaike Information Criterion (AIC). The results of Crato and Ray (1996) show that when the ARFIMA model is mixed, i.e., either $p > 0$, or $q > 0$, or both, the AIC performs better than other standard model selection criteria. Note that for each unemployment rate series examined the AIC-identified ARFIMA model is mixed; the SIC-selected models, which are not reported here, are also mixed.

Consider first the pre-1973 oil price shock results. The one-sided p-value for testing the unit root null hypothesis is less than 0.05 for only one country, Canada. In this case the estimated fractional integration parameter is greater than 1, however, indicating strong long memory and permanent persistence of shocks. For the four other countries, the null hypothesis of a unit root can not be rejected at the 5% significance level. Thus, the pre-1973 oil price shock evidence suggests that the unemployment rates in each of these countries exhibit hysteresis.

The post-oil price shock evidence differs dramatically from the results for the earlier sample period for three countries, Canada, Germany, and the United States. Our results strongly suggest that none of these unemployment rate series displays hysteresis effects for this latter sample period. In the case of Canada there once again is strong rejection of the unit root null hypothesis, but with a shift from a strong long memory process to a long memory process with an estimated fractional integration parameter significantly less than 1 but greater than 0. The unit root null hypothesis is rejected at the 5% significance level for both Germany and the United States. Further, the p-value for the $d = 0$ null hypothesis is close to 0.50 for Germany, suggesting that the German unemployment rate has actually been stationary since 1974.

Hysteresis appears to be a feature of unemployment rate dynamics for both Japan and the United Kingdom in the post-oil price shock period. In neither case can the unit root null hypothesis be rejected at the 5% significance level.

In light of the fact that the unemployment hysteresis literature was opened by Blanchard and Summers (1986) in search of an explanation for high European unemployment rates since the 1970s, we consider our most interesting results to be those for Germany and Japan. First, in the post-oil price shock period the German unemployment rate both increased significantly and essentially lost all hysteresis-type dynamics. It would appear, then, that the shift to larger unemployment rates in Germany during this period is not well explained by hysteresis. Second, the Japanese unemployment rate has consistently displayed strong hysteresis effects since the early 1960s. Out of the five countries considered, Japan also is the one that has had the lowest average unemployment rate over this time frame. This suggests that hysteresis in unemployment rate dynamics does not necessarily lead to persistently high unemployment rates.

5. Conclusions

Our use of the relatively general ARFIMA approach has allowed us to distinguish between long memory mean reverting behavior and hysteresis-type dynamics in the OECD unemployment rates studied. Through tests based upon exact maximum likelihood estimates of the fractional integration parameter, we have documented a significant shift away from hysteresis in the unemployment rates for Canada, Germany, and the United States in the post-1973 oil price shock period. For Japan and the United Kingdom, however, there is strong evidence in favor of hysteresis across both sample periods.

Table 1

Exact Maximum Likelihood Estimates of Fractional Integration Parameters for OECD Monthly Unemployment Rates in Pre- and Post-1973 Oil Price Shock Periods

Monthly Unemployment Rate	Period	Order (p,d,q) of Selected ARFIMA Model	$\hat{d}$	SE($\hat{d}$)	One-Sided p-value for Testing H_0: d = 1
Canada	1960:01-1973:12	(1,d,0)	1.16	0.08	0.023
	1974:01-1994:07	(3,d,0)	0.56	0.13	0.000
Germany	1962:01-1973:12	(3,d,0)	0.97	0.15	0.421
	1974:01-1994:06	(2,d,2)	0.01	0.25	0.000
Japan	1960:01-1973:12	(0,d,1)	1.21	0.26	0.210
	1974:01-1994:07	(2,d,1)	1.19	0.13	0.072
United Kingdom	1960:01-1973:12	(2,d,1)	0.61	0.31	0.104
	1974:01-1994:07	(2,d,2)	0.51	0.36	0.087
United States	1960:01-1973:12	(2,d,3)	0.99	0.17	0.477
	1974:01-1993:12	(3,d,3)	0.61	0.18	0.015

ARFIMA estimation results of the fractional integration parameter d for the seasonally adjusted monthly unemployment rate series from the OECD Main Economic Indicators database. For $\hat{d} > 1$, the alternative hypothesis is $d > 1$; for $\hat{d} < 1$, the alternative hypothesis is $d < 1$.

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