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The Performance of the GPH Estimator of the Fractional Difference Parameter: Simulation Results

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Abstract. This paper investigates the behavior of the GPH estimator of the fractional difference parameter suggested by Geweke and Porter-Hudak (1983), through Monte Carlo simulations. The simulation results indicate that when considering a stationary AR(1) generating process the GPH estimator of the fractional difference parameter has serious bias which increases with the value of the autoregressive parameter, even for relatively large samples. The results suggest that tests and point estimates based on this procedure can be seriously misleading with hypothesis tests yielding incorrect inferences and thus must be used with great care.

Key words: Fractional difference, long memory, periodogram

1. Introduction

The persistence of macroeconomic shocks and the specification of statistical models of economic fluctuations have been the subject of recent empirical investigation. Currently there is a debate over whether macroeconomic time series can best be described by transitory deviations from a deterministic trend or whether variables, such as aggregate output or real exchange rates, contain a substantial permanent component. The latter implying that a shock to this variable will persist and will not necessarily be reversed in the future through reversion to the trend.

Studies by Stock and Watson (1986), Perron and Phillips (1987), Campbell and Deaton (1989), among others, have concluded that real output and other economic time series can be described by a low-order ARMA process with a single unit root. Nelson and Plosser (1982) and Schwert (1987) apply a variety of unit root tests to a large number of economic variables and find strong evidence form the existence of unit roots. Studies that have attempted to quantify the extent of shock persistence include Watson (1986), Campbell and Mankiw (1987a, 1987b) and Clark (1987).[1]

Recent attention has been given to the Fractionally Integrated Autoregressive Moving Average model as being a more general and useful tool for examining the persistent nature of time series.[2] The Fractionally-Integrated model is designed to capture a variety of long-run low-frequency components and nests the unit-root hypothesis as a special and restrictive case. The findings of large low frequency components in a number of economic time series, and the low power of Dickey-Fuller (1979) Unit-root tests against fractional alternatives, found by Diebold and Rudebusch (1991a), suggest that the use of the fractionally integrated (differenced) ARMA methodology may add additional empirical insights about the properties of economic time series.

Geweke and Porter-Hudak (1983) proposed an attractive simple regression procedure for the estimation of the fractional difference parameter. This paper investigates the behavior of the GPH estimator of the fractional difference parameter through Monte Carlo simulations. In particular, we investigate the following question: If the true time series data is generated from a nonfractionally integrated stationary AR(1) process with a large autoregressive parameter close to one, how useful is the GPH regression procedure in detecting the stationarity of the process? Through simulations we examine the robustness of the difference parameter estimate with respect to alternative values of the autoregressive parameter and the power of this procedure in determining the existence of unit roots. The simulation results indicate that the GPH estimator of the fractional difference parameter can have serious bias, even for relatively large samples. Due to the bias of the estimates, the statistical power of this procedure in the presence of a large AR(1) parameter value is very low. These results suggest that tests and point estimates based on this procedure can be seriously misleading with hypothesis tests yielding incorrect inferences and thus must be used with great care.

The remainder of this paper is organized as follows. Section 2 discusses the Fractionally Integrated ARMA representation and the GPH regression model. Section 3 presents simulation results with regard to the statistical power of the tests used and the biasedness of the estimates of the GPH regression model. Section 4 ends the paper with a conclusion.

2. Fractionally Integrated ARMA Representation and the GPH Regression Model

As discussed in Granger (1980), many economic series are generated from stationary processes but their autocorrelation functions decay much more slowly than those from conventional stationary processes. In other words, economic series are quite often more closely characterized by a stationary process with a long memory component as opposed to the conventional short memory ARMA representations. Hosking (1981) and Granger and Joyeux (1980) extend the class of ARMA models by allowing non-integer d values, providing for parsimonious yet flexible modeling of low-frequency dynamics of long memory processes. They are named long memory processes because of their ability to display significant dependence between observations widely separated in time. This has become known as the *Autoregressive Fractionally Integrated Moving Average* model (hereafter, ARFIMA).[3]

The fractionally integrated ARMA process is a generalized notion of the conventional representation of the ARIMA (p, d, q) process where the value of d has been restricted to be a non-negative integer.[4] Standard ARMA processes are often labeled short-memory processes because the autocorrelation (or dependence) between the current observation and τ lags decay rapidly as τ increases. It can be shown, that for a non-zero $d < .5$, a stationary ARFIMA process has an autocorrelation function, $\rho(\tau)$, approximated by τ^{2d-1} for large τ, which decays at a hyperbolic rate. The autocorrelation function, $\mathcal{P}(\tau)$, for the conventional stationary ARMA process, where $d = 0$, is approximated by ρ^{τ} for $\tau = 1, 2, \ldots$, where $-1 < r < 1$, which decays, in absolute value, at a faster exponential rate.

The intuition of long memory models, and the limitation of assuming an integer-d, is illustrated in frequency domain. The series displays long memory if its spectral density increases without limit as angular frequency, λ, tends to zero. In the frequency domain, the spectral density of the conventional stationary ARMA (p, q) series is bounded at zero angular frequency, while the spectral density of an ARFIMA series, that has a non-zero d component, behaves like λ^{-2d} as $\lambda \to 0$. As such, d parameterizes the low-frequency behavior. A wide range of spectral behavior are possible near the origin when the integer-d restriction is relaxed. Granger (1986) has shown that the "typical spectral shape" of economic variables is monotonically declining with frequency (with the exception of peaks at seasonals) with high power at low-frequencies. This shape is well captured by the fractionally integrated process. While levels of economic variables tend to have high power at low frequencies, differences of economic time series tend to have very lower power at low frequencies indicating over-differencing. If this is true, then data may be characterized by the fractionally integrated process with $d < 1/2$.

Geweke and Porter-Hudak (1983) proposed a simple procedure for the estimation of the fractional difference parameter d. Consider an ARFIMA(p, d, q) representation of a series Y_t:

$$\Phi(L)(1 - L)^d Y_t = \Theta(L)\epsilon_t,\ \epsilon_t \sim (0,\ \sigma_\epsilon^2) \tag{1}$$

where $\Phi(L) = 1 - \phi_1 L - \cdots - \phi_p L^p$, $\Theta(L) = 1 - \Theta_1 L - \cdots \Theta_q L^q$, L is the lag operator, all roots of $\Phi(L)$ and $\Theta(L)$ lie outside the unit circle, and ϵ_t is white noise. In (1), d is not necessarily an integer, and thus, the possibility of fractional differencing is allowed. Stationarity and invertibility requires $|d| < 1/2$. Multiplying both sides of (1) by $\Phi^{-1}(L)$ yields,

$$(1 - L)^d Y_t = \Phi^{-1}(L)\ \Theta(L)\epsilon_t \tag{2}$$

If Y_t has a value of d greater than 1/2, the invertible and stationary ARFIMA representation can be achieved by first differencing the series. Suppose that the series has a value of d greater than 1/2. Letting $X_t = (1 - L)Y_t$, equation 2 can be rewritten as,

$$(1 - L)^{\tilde{d}} X_t = u_t$$

where $\tilde{d} = d - 1$, and $u_t = \Phi^{-1}(L)\Theta(L)\epsilon_t$. If $\tilde{d} = 0$ then $d = 1$, which indicates the process Y_t has a unit root. The right-hand side of the equation, u_t is a well-defined stationary process. Let $f_u(\lambda)$ be the spectral density of the stationary process, u_t. Then the spectral density of X_t is given by

$$f_x(\lambda) = [2 \sin (\lambda/2)]^{-2\tilde{d}} f_u(\lambda) \tag{3}$$

Given a sample of size T, let $\lambda_j = 2\pi j/T$, $j = (0, \cdots, T{-}1)$, denote the harmonic ordinates of the sample. Taking logarithms of (3), adding and subtracting $\ln\{f_u(0)\}$, and evaluating at the harmonic ordinates, we obtain

$$\ln\{f_x(\lambda_j)\} = \ln\{f_u(0)\} - \tilde{d} \ln[4 \sin^2 (\lambda_j/2)] + \ln\{f_u(\lambda_j)/f_u(0)\} \tag{4}$$

The important step in the development of the GPH estimator of d follows from the assertion that the last term in (4) is negligible, or at least approximately constant, for low-frequency ordinates near zero. Then, for some integer $K < T$, $\ln\{I_x(\lambda_j)\}$ denotes the periodogram at ordinate λ_j, where $j \leq K$, we can write approximately

$$\ln\{I_x(\lambda_j)\} = \ln\{f_u(0)\} - \tilde{d} \ln[4 \sin^2 (\lambda_j/2)] + \ln\{I_x(\lambda_j)/f_x(0)\}$$

This formation can be expressed as a simple liner regression equation,

$$\ln\{I_x(\lambda_j)\} = \beta_0 + \beta_1 \ln[4 \sin^2 (\lambda_j/2)] + v_j, j = 1, \cdots K, \tag{5}$$

where β_0 is the constant, $ln\{f_u(0)\}$, $\beta_1 = -\tilde{d}$ *and* $v_j = ln\{I_x(\lambda_j)/f_x(\lambda_j)\}$ are independently and identically distributed across the harmonic frequences.

If we let the number of low-frequency ordinates, K, used above be a function of sample size T, i.e., $K = g(T)$, then under regularity conditions on $g(\cdot)$, Geweke and Porter-Hudak (1983) show that the negative of the OLS estimate of β_1 provides a consistent and asymptotically normal estimate of $\tilde{d}$. Note that the estimation of d does not require the knowledge of the order of p or q. We call equation 5 the GPH regression. Based on theoretical considerations and Monte Carlo simulations, Geweke and Porter-Hudak (1983) recommend using a sample size of $g(T) = T^\alpha$ with $\alpha = 0.5$. While the variance of the estimate of β_1 is given by the usual OLS estimator, Geweke and Porter-Hudak (1983) show the theoretical asymptotic variance of the regression error, v_j, is equal to $\pi^2/6$, which can be imposed to increase efficiency. It should be noted that if a researcher believes that a series Y_t is stationary so that differencing is not necessary, the GPH regression would be performed on the log periodogram of the series Y_t directly. This being the case, the estimate of β_1 will become the estimate of $-d$.

3. Simulation

The unit-root test in a time domain approach is based on the autoregressive model, $(1 - bL)Y_{t+1} = z_0 + e_{t+1}$ with a null of $b = 1$ against the alternative $|b| < 1$. In the context of an ARFIMA specification, no matter how close b is to 1, the long memory parameter d will equal 0. If one believes that the series is stationary, but its autocovariance decays very slowly, then one may conjecture that such a process may be well approximated by a long memory parameter, d, slightly less than 1/2 but greater than zero. Diebold and Rudebusch (1989) found the fractional difference parameter d to lie between the interval 1/2 and 1 for Real GNP, a series which previous research—based on Dickey-Fuller (1979) tests—had suggested contained a unit root. This motivates an investigation into the distributional properties of GPH estimates of d relative to the true distribution of d.

To explore the behavior of the GPH estimator of the fractional difference parameter, series of sample size 200 and 500 are generated from a first-order autoregressive process, $(1 - bL)Y_{t+1} = e_{t+1}$, (so that the true value is $d = 0$) where $e_{t+1} \sim N(0, 1)$ and $Y_1 = 0$.

We estimate equation 5 for sample sizes of 200 and 500.[5] As suggested by Geweke and Porter-Hudak, we use the exact Fourier transform to obtain the dependent variable of equation 5 rather than the fast Fourier transform technique. The estimations are performed over 1000 replications for each true parameter, $b = 0.0, 0.3, 0.7, 0.9, 0.95$, and 0.99. Two null hypothesis tests at the 5% significance level are performed: (1) H_0: $d = 1$ vs. H_A: $d < 1$, (2) H_0: $d = 0$ vs. H_A: $d \neq 0$. These hypotheses tests help one gain an insight into the distribution of the sample estimate as well as the power of the tests. In an effort to describe the distributions of the estimates of d, values of simple statistics such as mean, minimums, maximums, and standard deviations of estimates, over 1000 replications, are reported.

Since the spectral density (1) is defined on $\tilde{d} \in (-.5, .5)$, under the null that the series has a unit-root component, it would be appropriate to run the GPH regression (5) on the differenced series. However, if the data is in fact generated from a stationary process with $d \in (-.5, .5)$, the regression model (5) is not valid for a first differenced series. This is because differencing results in a value of $\tilde{d}$ being less than -0.5. A priori, the range of the true d is not known, although for testing the null of $d = 1$, it is appropriate to use the differenced series. We estimate the parameter d for differenced as well as nondifferenced series. The sample sizes are equal to T^{α}, $\alpha = .5$ and $\alpha = .55$. The estimates of the order of fractional integration are robust over these two sample sizes.

Table 1 reports results of the null hypothesis that H_0: $d = 1$. Since the null implies a unit-root, the GPH regression is applied on the differenced series.[6] In terms of rejecting the null H_0: $d = 1$, in table 1, the simulation results show very low power (i.e., the probability of rejecting a false null is small) for values of the autoregressive parameter greater than $b = .9$, with some improvement obtained with a larger sample (for example,

Table 1. Simulation results (differenced).

	b					
	0.00	0.30	0.70	0.90	0.95	0.99
Reject H_0:d=1 (H_A:d<1)	90.4% (95.8%)	94.3% (99.5%)	93.9% (98.8%)	52.4% (96.0%)	21.6% (70.8%)	7.3% (10.5%)
Simple statistics of estimate d						
Mean	0.253	0.199	0.260	0.598	0.790	0.971
	(0.292)	(0.219)	(0.183)	(0.415)	(0.631)	(0.934)
Std. Deviation	0.291	0.269	0.236	0.227	0.230	0.230
	(0.254)	(0.224)	(0.190)	(0.174)	(0.172)	(0.174)
Minimum	−0.857	−0.681	−0.600	−0.199	−0.040	0.170
	(−0.766)	(−0.766)	(−0.514)	(−0.269)	(−0.227)	(0.090)
Maximum	1.034	1.013	1.066	1.204	1.408	1.584
	(0.981)	(0.981)	(0.934)	(0.983)	(1.115)	(1.438)
Skewness	−0.227	−0.157	−0.191	−0.218	−0.298	−0.279
	(−0.150)	(−0.073)	(−0.127)	(−0.132)	(−0.145)	(−0.131)
Kurtosis	0.088	0.141	0.192	0.195	0.415	0.243
	(−0.262)	(−0.198)	(0.470)	(0.301)	(0.549)	(0.528)

Note: The figures are the percentage of cases rejecting the null based on 1000 replications of sample sizes 200 and 500 after differencing. Numbers in parentheses are for the case of 500 observtions. Each sample is generated from $(1 - bL)Y_t = \epsilon_t$, where $\epsilon_t \sim N(0, 1)$ and $Y_1 = 0$.

500 observations in our simulations). The standard deviations of estimates of d are within the range of .227 (.172) and .291 (.254) for the case of 200 (500) observations for values of the true autoregressive parameter ranging between 0 and .99. These are very close to the asymptotic standard errors of .24 and .2 for 200 and 500 observations respectively.[7] The empirical means of the sampling distribution of the fractional difference parameter d indicate that the GPH regression tends to generate a larger estimate of d as the true value of the AR(1) coefficient increases. Thus, a striking feature of the results is that there is a very substantial bias in the estimate of d and this bias increases monotonically with the value of the autoregressive parameter. Furthermore, even though the sample size is more than doubled, (increasing the sample from 200 to 500 observations), this bias decreases by only a very small amount. The bias of course yields tests of the null hypotheses that $d = 1$ very unreliable.

Since the true generating process in our simulation is in fact stationary, we further investigate the biasedness of the GPH estimator by applying the GPH regression to nondifferenced data. These results are reported in table 2. Compared with the case of differenced data, the bias is slightly smaller. However, the bias is still large, so that the rejection percentage of the true null that H_0: $d = 0$ reaches, for example, 71.3% with 200 observations when the autoregressive parameter is equal to .9. In contrast, the rejection percentage for a false null of H_0: $d = 1$ is 58.2%.

Table 2. Simulation results (nondifferenced).

	b					
	0.00	0.30	0.70	0.90	0.95	0.99
Reject H_0:d=0 (H_a:d≠0)	5.6% (5.9%)	6.3% (5.8%)	10.2% (8.4%)	71.3% (58.8%)	91.1% (93.8%)	97.4% (99.9%)
Reject H_0:d=1 (H_A:d<1)	100.0% (100.0%)	100.0% (100.0%)	97.3% (100.0%)	58.2% (97.4%)	24.3% (75.2%)	7.4% (11.3%)
Simple statistics of estimate d						
Mean	−0.009	0.011	0.166	0.559	0.770	0.968
	(−0.002)	(0.005)	(0.075)	(0.371)	(0.607)	(0.926
Std. Deviation	0.233	0.232	0.230	0.236	0.238	0.237
	(0.172)	(0.172)	(0.173)	(0.177)	(0.178)	(0.178)
Minimum	−1.025	−0.990	−0.725	−0.487	−0.196	−0.145
	(−0.567)	(−0.590)	(−0.500)	(−0.332)	(−0.005)	(0.225)
Maximum	0.605	0.620	0.762	1.227	1.437	1.646
	(0.490)	(0.495)	(0.564)	(0.873)	(1.177)	(1.502)
Skewness	−0.479	−0.446	−0.376	−0.395	−0.400	−0.364
	(0.018)	(0.013)	(0.001)	(−0.067)	(−0.043)	(−0.133)
Kurtosis	1.059	0.888	0.582	0.856	0.852	0.596
	(0.010)	(0.025)	(0.029)	(0.260)	(0.203)	(0.258)

Note: The figures are the percentage of cases rejecting the null based on 1000 replications of sample sizes 200 and 500. Numbers in parentheses are for the case of 500 observations. Each sample is generated from $(1 - bL)Y_t = \epsilon_t$, where $\epsilon_t \sim N(0, 1)$ and $Y_1 = 0$.

The few studies that have used the GPH regression procedure have warned about making statistical inferences of point estimates of d, due to the relatively large standard error of d. However, our simulations not only echo this caution, but also indicate that a more serious problem lies in the biasedness of the estimate for alternative stationary processes.[8] Our simulations show that if the GPH procedure is used to test the null hypothesis that $d = 0$, this hypothesis will frequently be rejected for a first order autoregressive process with a large positive parameter.

The statistical power of the GPH estimator for the fractional difference parameter, d, is very poor even for modest values of the autoregressive coefficient and does not diminish even with large samples. We do not doubt the existence of generating processes with fractional d, but doubt the ability of the GPH procedure to detect extreme values of d.

The usefulness of this estimator may be more suitable for time series which are very smooth and follow a pattern which repeats itself over time. An example of such a series is consumption, which, in fact, Diebold and Rudebusch (1991a) have examined using the GPH procedure. On the other hand, based on the findings of this paper, the GPH estimator should perform poorly with time series which are much more volatile and characterized by sudden and unexpected jumps, such as exchange rates.

4. Conclusion

Recent attention has been given to the Fractionally Integrated Autoregressive Moving Average model as being a more general and useful tool for examining the persistent nature of time series. Geweke and Porter-Hudak (1983) have suggested a simple procedure for the estimation of the fractional difference parameter d. This paper has investigated the behavior of the GPH estimator of the fractional difference parameter through Monte Carlo simulations.

Employing simulations, we test the robustness of the difference parameter d with respect to alternative values of the autoregressive parameter and the power of this procedure. The simulation results indicate that, when considering a generating process with a large autoregressive parameter, the GPH estimator of the fractional difference parameter can have serious bias, even for relatively large samples. These results suggest that tests and point estimates based on this procedure can be seriously misleading with hypothesis tests yielding incorrect inferences and thus, must be used with great care.[9]

The results indicate that simply extracting the fractional integrating factor from a time series is not sufficient enough information to completely determine the time series properties of the series in question. Future research, which attempts to specify within the ARFIMA model specification the value of the autoregressive and moving average parameters, along with the order of fractional integration, is required to analyze time series processes that have autoregressive parameters close to unity. Of course such procedures are far more burdensome than the GPH procedure.

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Notes

1. Campbell and Mankiw (1987a), using an unrestricted ARIMA representation, find that a unit innovation results in a long-run response that is substantially larger than the initial innovation. This result is in contrast to Watson (1986) and Clark (1987) who find a smaller degree of persistence employing unobserved components models.
2. Diebold and Rudebusch (1989) examine the persistence in U.S. aggregate output by estimating fractionally integrated ARMA models. They find evidence of long memory, which induces persistence, though this long memory is not necessarily associated with a unit root. Diebold and Rudebusch (1991b) use a generalized long memory stochastic representation to examine whether the apparent excess smoothness of consumption is the result of the ARIMA representation's implicit restrictions on low-frequency dynamics.
3. Further discussion of fractional integration can be found in Diebold and Nerlove (1990). Porter-Hudak (1990) has recently applied the GPH procedure to seasonal times series.
4. Fractional integration allows a local generalization of the unit root hypothesis. Instead of forcing the difference parameter $d = 1$, this procedure allows $1/2 < d < 3/2$. See for example, Diebold and Rudebusch (1989, 1991b).
5. For 200 (500) observations, and $\alpha = 0.5$, this suggests that the first $13 - K = 13$ ($23 - K = 23$) periodogram ordinates are to be used.
6. Since the GPH regression is applied here to the differenced series, we add one to the estimate of the differenced parameter $\tilde{d}$ to obtain the parameter value d.
7. These are obtained by calculating the square root of $\sigma^2(X'X)^{-1}$, where the regression error variance, $\sigma^2 = \pi^2/6$.
8. The bias of the estimator, for large values of the autoregressive parameter, is most likely related to the assertion that the last term in (4) is negligible, or at least approximately constant, for low-frequency ordinates near zero. The noted bias would arise if the log spectrum is not approximately constant over the relevant range, but is instead, strongly downward sloping.
9. It should be noted, however, that the GPH procedure employed in this paper does not require the knowledge of the order of p and q in an ARMA model. A procedure which simultaneously determines the value of p, d, and q, may yield greater power than that reported in this study.

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