Basic Matching model

Question 1 A matching in a marriage model such that nobody is matched to an unacceptable partner will be stable if

(A) I cannot find another matching that makes all men better off (i.e., any matching that makes a man better off necessarily makes another man worse off).

(B) each woman is matched to her most preferred women

(C) For each man, each woman he prefers to his match is matched to a man she prefers.

(D) Each man and each woman is matched to an acceptable partner.

Question 2 When we use the deferred acceptance algorithm with women proposing

(A) It is a dominant strategy for women to submit their true preferences

(B) It is a dominant strategy for men to submit their true preferences

(C) It is a dominant strategy for men and women to submit their true preferences

(D) Both men and women can be better off submitting a preference ordering that is not their true preferences

Question 3 In the marriage matching market, the matching obtained when running the deferred acceptance algorithm with men proposing gives

(A) The most preferred stable matching for men

(B) The most preferred matching for men

(C) The most preferred stable matching for women

(D) The most preferred matching for women

Question 4 Consider a marriage problem with strict preferences. In a men-optimal stable matching, each woman is matched with the least preferred man she could match in any stable matching.

(A) True.

(B) False.

Question 5 In the marriage matching market, if the matching obtained when running the deferred acceptance algorithm with men proposing is the same as the matching obtained when running the algorithm with women proposing, then

(A) there might be other stable matchings.

(B) there is a unique stable matching.

(C) the matching is not stable.

(D) that cannot be happen, the man-optimal and the woman-optimal matchings are necessarily different.

Question 6 We have 3 men (m_1, m_2 and m_3) and 3 women with the following preferences

P_{m_1}	P_{m_2}	P_{m_3}
w_1	w_1	w_1
w_2	w_2	w_3
w_3	w_3	w_2
m_1	m_2	m_3

P_{w_1}	P_{w_2}	P_{w_3}
m_2	m_1	m_1
m_1	m_3	m_2
m_3	m_2	m_3
w_1	w_2	w_3

Consider the matching μ given by

$$\mu(m_1) = w_1 \ , \mu(m_2) = w_2 \ \text{ and } \mu(m_3) = w_3 \ .$$

The matching μ is

(A) stable

(B) Not stable

Question 7 Take any marriage problem in which agents on one side, say, women, have the same preferences over the men. Is there a unique stable matching? If yes, provide a sketch of the proof. If not, provide an example.

Question 8 We consider the following marriage problem with four men and five women. The preferences are as follows,

P_{m_1}	P_{m_2}	P_{m_3}	P_{m_4}
w_1	w_1	w_4	w_5
w_5	w_3	w_2	w_3
w_4	w_5	w_3	w_2
w_3	w_2	w_5	w_1
w_2	w_4	w_1	w_4

P_{w_1}	P_{w_2}	P_{w_3}	P_{w_4}	P_{w_5}
m_4	m_1	m_1	m_4	m_4
m_2	m_2	m_3	m_2	m_2
m_1	m_3	m_4	m_1	m_3
m_3	m_4	m_2	m_3	m_1

1. Find the man-optimal matching
2. Find the woman-optimal matching.
3. Consider the Deferred Acceptance algorithm with women proposing. Does any man or woman have an incentive to misrepresent his/her true preferences (when everybody else is truthful)?

Question 9 We consider a matching problem with two men and two women. The preferences are the following:

P_{m_1}	P_{m_2}
w_1	w_2
w_2	w_1

P_{w_1}	P_{w_2}
m_1	m_2
m_2	m_1

Assume we use the Deferred Acceptance algorithm with women proposing. Is it a dominant strategy for men to report truthfully their preferences?

Question 10 A woman is achievable for a man if

(A) The man is acceptable for the woman.

(B) The man is acceptable for the woman and the woman is acceptable for the man.

(C) There exists a stable matching where they are matched together.

(D) She prefers the man to her partner in the man-optimal matching.

(E) She prefers the man to her partner in the woman-optimal matching.

Question 11 Suppose that for some marriage problem the man-optimal matching μ_M is Pareto efficient *for the women*, i.e., there is no matching μ such that all women weakly prefer μ to μ_M with at least one women strictly preferring μ to μ_M. Show that in this case $\mu_W = \mu_M$.