Lecture 6: Kidney exchange

Thilo Klein

ZEW Mannheim

Introduction

As of January, 2016 there are in the US a bit more than 100,000 people waiting for a kidney transplant.

Each year,

- nearly 4,000 patients become too sick to receive a transplant;
- almost 5,000 patients die waiting for a kidney.

Dialysis costs \$80,000 or more per year.

$\rightarrow$ Maximizing the number of transplants can save lives (and money).

Introduction

As of January, 2016 there are in the US a bit more than 100,000 people waiting for a kidney transplant.

Each year,

- nearly 4,000 patients become too sick to receive a transplant;
- almost 5,000 patients die waiting for a kidney.

Dialysis costs \$80,000 or more per year.

$\rightarrow$ Maximizing the number of transplants can save lives (and money).

Introduction

As of January, 2016 there are in the US a bit more than 100,000 people waiting for a kidney transplant.

Each year,

- nearly 4,000 patients become too sick to receive a transplant;
- almost 5,000 patients die waiting for a kidney.

Dialysis costs $80,000 or more per year.

→ Maximizing the number of transplants can save lives (and money).

Introduction

As of January, 2016 there are in the US a bit more than 100,000 people waiting for a kidney transplant.

Each year,

- nearly 4,000 patients become too sick to receive a transplant;
- almost 5,000 patients die waiting for a kidney.

Dialysis costs $80,000 or more per year.

$\rightarrow$ Maximizing the number of transplants can save lives (and money).

There are two possibilities to get a kidney:

- From a deceased donor.

  Managed using a waiting list that takes into account:
  - Waiting time
  - State of the patient
  - other factors (e.g., location, availability of the patient, etc).

- From a living donor.

  The human body has two kidneys but most of us can live with only one.

There are two possibilities to get a kidney:

- From a deceased donor.

  Managed using a waiting list that takes into account:
  - Waiting time
  - State of the patient
  - other factors (e.g., location, availability of the patient, etc).

- From a living donor.

  The human body has two kidneys but most of us can live with only one.

There are two possibilities to get a kidney:

- From a deceased donor.

  Managed using a waiting list that takes into account:

  - Waiting time
  - State of the patient
  - other factors (e.g., location, availability of the patient, etc).

- From a living donor.

  The human body has two kidneys but most of us can live with only one.

In 2014, 17,107 kidney transplants took place in the US:

- 11,570 transplants from deceased donors;
- 5,537 transplants from living donors.

Donations from the deceased are "easy": once we have a kidney we just have to identify the compatible patient with the highest priority.

Living donors are more difficult: often a relative who offers a kidney. A problem arises when the kidney is not compatible.

In 2014, 17,107 kidney transplants took place in the US:

- 11,570 transplants from deceased donors;
- 5,537 transplants from living donors.

Donations from the deceased are "easy": once we have a kidney we just have to identify the compatible patient with the highest priority.

Living donors are more difficult: often a relative who offers a kidney. A problem arises when the kidney is not compatible.

In 2014, 17,107 kidney transplants took place in the US:

- 11,570 transplants from deceased donors;
- 5,537 transplants from living donors.

Donations from the deceased are “easy”: once we have a kidney we just have to identify the compatible patient with the highest priority.

Living donors are more difficult: often a relative who offers a kidney. A problem arises when the kidney is not compatible.

Kidney compatibility

Two factors determine whether a kidney is compatible for a patient:

- Tissue type compatibility

  Relates to the immunological system (set of markers on cells used to detect what is foreign).

  Can be controlled for using immunosuppressants.

- Blood type compatibility

  The blood comes in different types, depending on some proteins we have (or don't have) on our red cells.

  Cannot be bypassed.

Kidney compatibility

Two factors determine whether a kidney is compatible for a patient:

- Tissue type compatibility

  Relates to the immunological system (set of markers on cells used to detect what is foreign).

  Can be controlled for using immunosuppressants.

- Blood type compatibility

  The blood comes in different types, depending on some proteins we have (or don't have) on our red cells.

  Cannot be bypassed.

Kidney compatibility

Two factors determine whether a kidney is compatible for a patient:

- Tissue type compatibility

  Relates to the immunological system (set of markers on cells used to detect what is foreign).

  Can be controlled for using immunosuppressants.

- Blood type compatibility

  The blood comes in different types, depending on some proteins we have (or don't have) on our red cells.

  Cannot be bypassed.

Kidney compatibility

Two factors determine whether a kidney is compatible for a patient:

- Tissue type compatibility

  Relates to the immunological system (set of markers on cells used to detect what is foreign).

  Can be controlled for using immunosuppressants.

- Blood type compatibility

  The blood comes in different types, depending on some proteins we have (or don't have) on our red cells.

  Cannot be bypassed.

Blood types

There are 4 major blood types: A, B, AB and O. They correspond to some proteins ("markers") on our red cells:

- A type blood $\rightarrow$ red cells have the marker A
- B type blood $\rightarrow$ red cells have the marker B
- AB type blood $\rightarrow$ red cells have the markers A and B
- O type blood $\rightarrow$ red cells do not have marker A or B.

Blood types

There are 4 major blood types: A, B, AB and O. They correspond to some proteins ("markers") on our red cells:

- A type blood $\rightarrow$ red cells have the marker A
- B type blood $\rightarrow$ red cells have the marker B
- AB type blood $\rightarrow$ red cells have the markers A and B
- O type blood $\rightarrow$ red cells do not have marker A or B.

Blood types

There are 4 major blood types: A, B, AB and O. They correspond to some proteins ("markers") on our red cells:

- A type blood $\rightarrow$ red cells have the marker A
- B type blood $\rightarrow$ red cells have the marker B
- AB type blood $\rightarrow$ red cells have the markers A and B
- O type blood $\rightarrow$ red cells do not have marker A or B.

Blood types

There are 4 major blood types: A, B, AB and O. They correspond to some proteins ("markers") on our red cells:

- A type blood $\rightarrow$ red cells have the marker A
- B type blood $\rightarrow$ red cells have the marker B
- AB type blood $\rightarrow$ red cells have the markers A and B
- O type blood $\rightarrow$ red cells do not have marker A or B.

Blood types

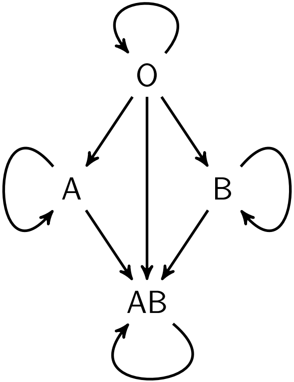
There are 4 major blood types: A, B, AB and O. They correspond to some proteins ("markers") on our red cells:

- A type blood $\rightarrow$ red cells have the marker A
- B type blood $\rightarrow$ red cells have the marker B
- AB type blood $\rightarrow$ red cells have the markers A and B
- O type blood $\rightarrow$ red cells do not have marker A or B.

Blood types

There are 4 major blood types: A, B, AB and O. They correspond to some proteins ("markers") on our red cells:

- A type blood $\rightarrow$ red cells have the marker A
- B type blood $\rightarrow$ red cells have the marker B
- AB type blood $\rightarrow$ red cells have the markers A and B
- O type blood $\rightarrow$ red cells do not have marker A or B.



$X \longrightarrow Y$ means
type X can give to type Y

- O can only receive from O.
- A can receive from O and A.
- B can receive from O and B.
- AB can receive from O, A, B, and AB.

Questions

When a donor's kidney is not compatible with the patient:

- Can we do something with the kidney?
- Can we "compensate" the patient for giving her donor's kidney to someone else?

Questions

When a donor's kidney is not compatible with the patient:

- Can we do something with the kidney?
- Can we "compensate" the patient for giving her donor's kidney to someone else?

Questions

When a donor's kidney is not compatible with the patient:

- Can we do something with the kidney?
- Can we "compensate" the patient for giving her donor's kidney to someone else?

Questions

When a donor's kidney is not compatible with the patient:

- Can we do something with the kidney?
- Can we "compensate" the patient for giving her donor's kidney to someone else?

List exchange

A first solution is to implement list exchanges (a.k.a. indirect exchanges).

If a the kidney of a patient's donor is not compatible with the patient, then:

- Treat the kidney as a deceased kidney: give it to the highest priority compatible patient in the waiting list;
- Give the patient a high priority position on the waiting list.

List exchange

A first solution is to implement list exchanges (a.k.a. indirect exchanges).

If a the kidney of a patient's donor is not compatible with the patient, then:

- Treat the kidney as a deceased kidney: give it to the highest priority compatible patient in the waiting list;
- Give the patient a high priority position on the waiting list.

List exchange

A first solution is to implement list exchanges (a.k.a. indirect exchanges).

If a the kidney of a patient's donor is not compatible with the patient, then:

- Treat the kidney as a deceased kidney: give it to the highest priority compatible patient in the waiting list;
- Give the patient a high priority position on the waiting list.

List exchange

A first solution is to implement list exchanges (a.k.a. indirect exchanges).

If a the kidney of a patient's donor is not compatible with the patient, then:

- Treat the kidney as a deceased kidney: give it to the highest priority compatible patient in the waiting list;
- Give the patient a high priority position on the waiting list.

List exchange

A first solution is to implement list exchanges (a.k.a. indirect exchanges).

If a the kidney of a patient's donor is not compatible with the patient, then:

- Treat the kidney as a deceased kidney: give it to the highest priority compatible patient in the waiting list;
- Give the patient a high priority position on the waiting list.

Drawback

List exchanges is harmful for type O patients:

- Such patients can only receive a type O kidney;
- Very few type O kidneys will be offered by a living donor:

  If the donor in a patient-donor pair is of type O it will usually be possible to give her kidney to the patient in the pair. That is, patient-donor pairs with a O donor do not usually show up in kidney exchange programs.

Drawback

List exchanges is harmful for type O patients:

- Such patients can only receive a type O kidney;
- Very few type O kidneys will be offered by a living donor:

  If the donor in a patient-donor pair is of type O it will usually be possible to give her kidney to the patient in the pair. That is, patient-donor pairs with a O donor do not usually show up in kidney exchange programs.

Drawback

List exchanges is harmful for type O patients:

- Such patients can only receive a type O kidney;
- Very few type O kidneys will be offered by a living donor:

  If the donor in a patient-donor pair is of type O it will usually be possible to give her kidney to the patient in the pair. That is, patient-donor pairs with a O donor do not usually show up in kidney exchange programs.

Trading kidneys

The basic idea of trading kidneys is very simple.

- Alice (a patient) and Arthur (her donor), but his kidney is not compatible for her;
- Bob (a patient) and Barbara (his donor), but her kidney is not compatible for him;
- However, Barbara kidney's is compatible for Alice, Arthur's kidney is compatible for Bob.

Trading kidneys

The basic idea of trading kidneys is very simple.

- Alice (a patient) and Arthur (her donor), but his kidney is not compatible for her;
- Bob (a patient) and Barbara (his donor), but her kidney is not compatible for him;
- However, Barbara kidney's is compatible for Alice, Arthur's kidney is compatible for Bob.

Trading kidneys

The basic idea of trading kidneys is very simple.

- Alice (a patient) and Arthur (her donor), but his kidney is not compatible for her;
- Bob (a patient) and Barbara (his donor), but her kidney is not compatible for him;
- However, Barbara kidney's is compatible for Alice, Arthur's kidney is compatible for Bob.

Trading kidneys

The basic idea of trading kidneys is very simple.

- Alice (a patient) and Arthur (her donor), but his kidney is not compatible for her;
- Bob (a patient) and Barbara (his donor), but her kidney is not compatible for him;
- However, Barbara kidney's is compatible for Alice, Arthur's kidney is compatible for Bob.

Trading kidneys

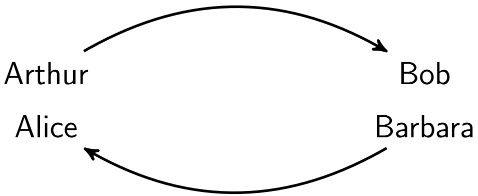
The basic idea of trading kidneys is very simple.

- Alice (a patient) and Arthur (her donor), but his kidney is not compatible for her;
- Bob (a patient) and Barbara (his donor), but her kidney is not compatible for him;
- However, Barbara kidney's is compatible for Alice, Arthur's kidney is compatible for Bob.

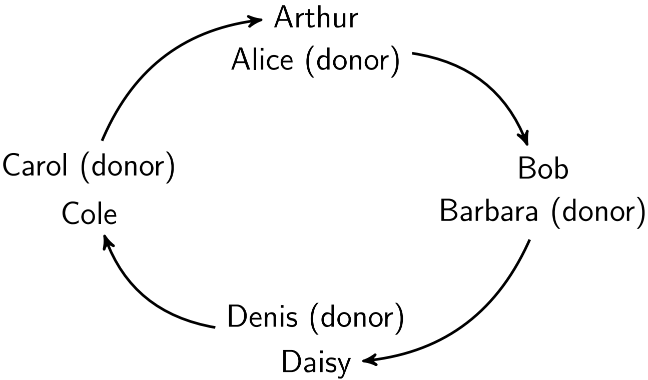
Trading kidneys

The basic idea of trading kidneys is very simple.

- Alice (a patient) and Arthur (her donor), but his kidney is not compatible for her;
- Bob (a patient) and Barbara (his donor), but her kidney is not compatible for him;
- However, Barbara kidney's is compatible for Alice, Arthur's kidney is compatible for Bob.

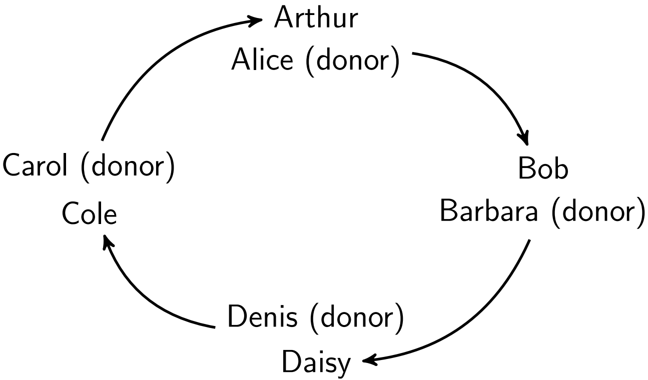


There's no need to restrict to two couples. . .



With such trades no monetary payment is needed.

There's no need to restrict to two couples...



With such trades no monetary payment is needed.

The difficulty lies in combining in the same mechanism

- trades;
- the waiting list.

It looks like a problem of assigning objects (i.e., kidneys) with mixed endowment where we would have:

- **Private endowments**: donors' kidneys are "privately held" by the patients;
- **Public endowments**: kidneys from deceased donors.

But it is different from a (classic) problem with mixed endowment: kidneys from deceased donors cannot be considered as available (but not yet assigned).

→ kidneys from deceased donors are not part of the public endowment.

The difficulty lies in combining in the same mechanism

- trades;
- the waiting list.

It looks like a problem of assigning objects (i.e., kidneys) with mixed endowment where we would have:

- **Private endowments**: donors' kidneys are "privately held" by the patients;
- **Public endowments**: kidneys from deceased donors.

But it is different from a (classic) problem with mixed endowment: kidneys from deceased donors cannot be considered as available (but not yet assigned).

$\rightarrow$ kidneys from deceased donors are not part of the public endowment.

The difficulty lies in combining in the same mechanism

- trades;
- the waiting list.

It looks like a problem of assigning objects (i.e., kidneys) with mixed endowment where we would have:

- **Private endowments**: donors' kidneys are "privately held" by the patients;
- **Public endowments**: kidneys from deceased donors.

But it is different from a (classic) problem with mixed endowment: kidneys from deceased donors cannot be considered as available (but not yet assigned).

→ kidneys from deceased donors are not part of the public endowment.

The difficulty lies in combining in the same mechanism

- trades;
- the waiting list.

It looks like a problem of assigning objects (i.e., kidneys) with mixed endowment where we would have:

- **Private endowments**: donors' kidneys are "privately held" by the patients;
- **Public endowments**: kidneys from deceased donors.

But it is different from a (classic) problem with mixed endowment: kidneys from deceased donors cannot be considered as available (but not yet assigned).

→ kidneys from deceased donors are not part of the public endowment.

The difficulty lies in combining in the same mechanism

- trades;
- the waiting list.

It looks like a problem of assigning objects (i.e., kidneys) with mixed endowment where we would have:

- **Private endowments**: donors' kidneys are "privately held" by the patients;
- **Public endowments**: kidneys from deceased donors.

But it is different from a (classic) problem with mixed endowment: kidneys from deceased donors cannot be considered as available (but not yet assigned).

$\rightarrow$ kidneys from deceased donors are not part of the public endowment.

The difficulty lies in combining in the same mechanism

- trades;
- the waiting list.

It looks like a problem of assigning objects (i.e., kidneys) with mixed endowment where we would have:

- **Private endowments**: donors' kidneys are "privately held" by the patients;
- **Public endowments**: kidneys from deceased donors.

But it is different from a (classic) problem with mixed endowment: kidneys from deceased donors cannot be considered as available (but not yet assigned).

$\rightarrow$ kidneys from deceased donors are not part of the public endowment.

So sum up, we need to design an exchange procedure that combines at the same time:

- Trades between donors and patients;
- The management of the waiting list for cadaveric donors.

So sum up, we need to design an exchange procedure that combines at the same time:

- Trades between donors and patients;
- The management of the waiting list for cadaveric donors.

So sum up, we need to design an exchange procedure that combines at the same time:

- Trades between donors and patients;
- The management of the waiting list for cadaveric donors.

The kidney exchange algorithm

We first need to describe the model:

- We have a set of patients. Some come with a donor, others come alone.
- There is a set of kidneys (proposed by the living donors).
- Each patient has a (strict) preference relation over kidneys and the option of entering the waiting list.

  Those preferences are not about *taste*, but rather about the fitness relation between the patient and the kidney.

  For a patient, a kidney is acceptable if it is compatible for the her. Otherwise the kidney is unacceptable.

The kidney exchange algorithm

We first need to describe the model:

- We have a set of patients. Some come with a donor, others come alone.
- There is a set of kidneys (proposed by the living donors).
- Each patient has a (strict) preference relation over kidneys and the option of entering the waiting list.

  Those preferences are not about *taste*, but rather about the fitness relation between the patient and the kidney.

  For a patient, a kidney is acceptable if it is compatible for the her. Otherwise the kidney is unacceptable.

The kidney exchange algorithm

We first need to describe the model:

- We have a set of patients. Some come with a donor, others come alone.
- There is a set of kidneys (proposed by the living donors).
- Each patient has a (strict) preference relation over kidneys and the option of entering the waiting list.

  Those preferences are not about *taste*, but rather about the fitness relation between the patient and the kidney.

  For a patient, a kidney is acceptable if it is compatible for the her. Otherwise the kidney is unacceptable.

The kidney exchange algorithm

We first need to describe the model:

- We have a set of patients. Some come with a donor, others come alone.
- There is a set of kidneys (proposed by the living donors).
- Each patient has a (strict) preference relation over kidneys and the option of entering the waiting list.

  Those preferences are not about *taste*, but rather about the fitness relation between the patient and the kidney.

  For a patient, a kidney is acceptable if it is compatible for the her. Otherwise the kidney is unacceptable.

The kidney exchange algorithm

We first need to describe the model:

- We have a set of patients. Some come with a donor, others come alone.
- There is a set of kidneys (proposed by the living donors).
- Each patient has a (strict) preference relation over kidneys and the option of entering the waiting list.

  Those preferences are not about *taste*, but rather about the fitness relation between the patient and the kidney.

  For a patient, a kidney is acceptable if it is compatible for the her. Otherwise the kidney is unacceptable.

The kidney exchange algorithm

We first need to describe the model:

- We have a set of patients. Some come with a donor, others come alone.
- There is a set of kidneys (proposed by the living donors).
- Each patient has a (strict) preference relation over kidneys and the option of entering the waiting list.

  Those preferences are not about *taste*, but rather about the fitness relation between the patient and the kidney.

  For a patient, a kidney is acceptable if it is compatible for the her. Otherwise the kidney is unacceptable.

The kidney exchange algorithm

We first need to describe the model:

- We have a set of patients. Some come with a donor, others come alone.
- There is a set of kidneys (proposed by the living donors).
- Each patient has a (strict) preference relation over kidneys and the option of entering the waiting list.

  Those preferences are not about *taste*, but rather about the fitness relation between the patient and the kidney.

  For a patient, a kidney is acceptable if it is compatible for the her. Otherwise the kidney is unacceptable.

The core idea of the algorithm is like for the Top Trading Cycle algorithm: each patient points to the most preferred kidney (or the waiting list if all kidneys are unacceptable).

We can have two types of situations:

- A cycle between donor-patient pairs (k_i, p_i), $i = 1, \dots m$:

$$(k_1 — p_1)(k_2 — p_2)(k_3 — p_3) \cdots\cdots (k_m — p_m)\ (k_1 — p_1)$$

- A chain, with donor-patient pairs (k_i, p_i), $i = 1, \dots m$, ending with the waiting list.

$$(k_1 — p_1)(k_2 — p_2)(k_3 — p_3) \cdots\cdots (k_m — p_m) \quad W$$

The core idea of the algorithm is like for the Top Trading Cycle algorithm: each patient points to the most preferred kidney (or the waiting list if all kidneys are unacceptable).

We can have two types of situations:

- A cycle between donor-patient pairs (k_i, p_i), $i = 1, \ldots m$:

$$(k_1 — p_1)(k_2 — p_2)(k_3 — p_3) \cdots\cdots (k_m — p_m)\ (k_1 — p_1)$$

- A chain, with donor-patient pairs (k_i, p_i), $i = 1, \ldots m$, ending with the waiting list.

$$(k_1 — p_1)(k_2 — p_2)(k_3 — p_3) \cdots\cdots (k_m — p_m) \quad W$$

The core idea of the algorithm is like for the Top Trading Cycle algorithm: each patient points to the most preferred kidney (or the waiting list if all kidneys are unacceptable).

We can have two types of situations:

- A cycle between donor-patient pairs (k_i, p_i), $i = 1, \dots m$:

$$(k_1 — p_1)(k_2 — p_2)(k_3 — p_3) \cdots\cdots (k_m — p_m)\ (k_1 — p_1)$$

- A chain, with donor-patient pairs (k_i, p_i), $i = 1, \dots m$, ending with the waiting list.

$$(k_1 — p_1)(k_2 — p_2)(k_3 — p_3) \cdots\cdots (k_m — p_m) \quad W$$

The core idea of the algorithm is like for the Top Trading Cycle algorithm: each patient points to the most preferred kidney (or the waiting list if all kidneys are unacceptable).

We can have two types of situations:

- A cycle between donor-patient pairs (k_i, p_i), $i = 1, \dots m$:

$$(k_1 - p_1)(k_2 - p_2)(k_3 - p_3) \cdots\cdots (k_m - p_m)\,(k_1 - p_1)$$

- A chain, with donor-patient pairs (k_i, p_i), $i = 1, \dots m$, ending with the waiting list.

$$(k_1 - p_1)(k_2 - p_2)(k_3 - p_3) \cdots\cdots (k_m - p_m) \quad W$$

The core idea of the algorithm is like for the Top Trading Cycle algorithm: each patient points to the most preferred kidney (or the waiting list if all kidneys are unacceptable).

We can have two types of situations:

- A cycle between donor-patient pairs (k_i, p_i), $i = 1, \ldots m$:

$$(k_1 — p_1)(k_2 — p_2)(k_3 — p_3) \cdots\cdots (k_m — p_m)\,(k_1 — p_1)$$

- A chain, with donor-patient pairs (k_i, p_i), $i = 1, \ldots m$, ending with the waiting list.

$$(k_1 — p_1)(k_2 — p_2)(k_3 — p_3) \cdots\cdots (k_m — p_m) \quad W$$

Top Trading Cycles and Chains

The algorithm is a multistep procedure where all steps are identical. For each $h = 1, 2, \dots,$ proceed as follows

- **Step h.1**

  Each patient points to her most preferred acceptable kidney.

  If there are no acceptable kidney the patient points to the waiting list.

- **Step h.2**

  If there are one or more cycles, proceed as follows, for each cycle:

  - each patient is assigned the kidney she is pointing to;
  - all patients and all kidneys (once re-assigned) are removed from the problem.

  Then to to step **(h+1)**

  If there are no cycle, go to step **h.3**

Top Trading Cycles and Chains

The algorithm is a multistep procedure where all steps are identical. For each $h = 1, 2, \ldots,$ proceed as follows

- **Step h.1**

  Each patient points to her most preferred acceptable kidney.

  If there are no acceptable kidney the patient points to the waiting list.

- **Step h.2**

  If there are one or more cycles, proceed as follows, for each cycle:

  - each patient is assigned the kidney she is pointing to;
  - all patients and all kidneys (once re-assigned) are removed from the problem.

  Then to to step **(h+1)**

  If there are no cycle, go to step **h.3**

Top Trading Cycles and Chains

The algorithm is a multistep procedure where all steps are identical. For each $h = 1, 2, \ldots,$ proceed as follows

- **Step h.1**
  Each patient points to her most preferred acceptable kidney.

  If there are no acceptable kidney the patient points to the waiting list.

- **Step h.2**
  If there are one or more cycles, proceed as follows, for each cycle:

  - each patient is assigned the kidney she is pointing to;
  - all patients and all kidneys (once re-assigned) are removed from the problem.

  Then to to step **(h+1)**

  If there are no cycle, go to step **h.3**

Top Trading Cycles and Chains

The algorithm is a multistep procedure where all steps are identical. For each $h = 1, 2, \ldots,$ proceed as follows

- **Step h.1**
  Each patient points to her most preferred acceptable kidney.

  If there are no acceptable kidney the patient points to the waiting list.

- **Step h.2**
  If there are one or more cycles, proceed as follows, for each cycle:

  - each patient is assigned the kidney she is pointing to;
  - all patients and all kidneys (once re-assigned) are removed from the problem.

  Then to to step **(h+1)**

  If there are no cycle, go to step **h.3**

Top Trading Cycles and Chains

The algorithm is a multistep procedure where all steps are identical. For each $h = 1, 2, \ldots,$ proceed as follows

- **Step h.1**

  Each patient points to her most preferred acceptable kidney.

  If there are no acceptable kidney the patient points to the waiting list.

- **Step h.2**

  If there are one or more cycles, proceed as follows, for each cycle:

  - each patient is assigned the kidney she is pointing to;
  - all patients and all kidneys (once re-assigned) are removed from the problem.

  Then to to step **(h+1)**

  If there are no cycle, go to step **h.3**

Top Trading Cycles and Chains

The algorithm is a multistep procedure where all steps are identical. For each $h = 1, 2, \ldots,$ proceed as follows

- **Step h.1**

  Each patient points to her most preferred acceptable kidney.

  If there are no acceptable kidney the patient points to the waiting list.

- **Step h.2**

  If there are one or more cycles, proceed as follows, for each cycle:

  - each patient is assigned the kidney she is pointing to;
  - all patients and all kidneys (once re-assigned) are removed from the problem.

  Then to to step **(h+1)**

  If there are no cycle, go to step **h.3**

Top Trading Cycles and Chains

The algorithm is a multistep procedure where all steps are identical. For each $h = 1, 2, \ldots,$ proceed as follows

- **Step h.1**
  Each patient points to her most preferred acceptable kidney.

  If there are no acceptable kidney the patient points to the waiting list.

- **Step h.2**
  If there are one or more cycles, proceed as follows, for each cycle:

  - each patient is assigned the kidney she is pointing to;
  - all patients and all kidneys (once re-assigned) are removed from the problem.

  Then to to step **(h+1)**

  If there are no cycle, go to step **h.3**

Top Trading Cycles and Chains

The algorithm is a multistep procedure where all steps are identical. For each $h = 1, 2, \ldots,$ proceed as follows

- **Step h.1**
  Each patient points to her most preferred acceptable kidney.

  If there are no acceptable kidney the patient points to the waiting list.

- **Step h.2**
  If there are one or more cycles, proceed as follows, for each cycle:

  - each patient is assigned the kidney she is pointing to;
  - all patients and all kidneys (once re-assigned) are removed from the problem.

  Then to to step **(h+1)**

  If there are no cycle, go to step **h.3**

Top Trading Cycles and Chains

The algorithm is a multistep procedure where all steps are identical. For each $h = 1, 2, \ldots,$ proceed as follows

- **Step h.1**
  Each patient points to her most preferred acceptable kidney.

  If there are no acceptable kidney the patient points to the waiting list.

- **Step h.2**
  If there are one or more cycles, proceed as follows, for each cycle:

  - each patient is assigned the kidney she is pointing to;
  - all patients and all kidneys (once re-assigned) are removed from the problem.

  Then to to step **(h+1)**

  If there are no cycle, go to step **h.3**

Top Trading Cycles and Chains

The algorithm is a multistep procedure where all steps are identical. For each $h = 1, 2, \ldots,$ proceed as follows

- **Step h.1**

  Each patient points to her most preferred acceptable kidney.

  If there are no acceptable kidney the patient points to the waiting list.

- **Step h.2**

  If there are one or more cycles, proceed as follows, for each cycle:

  - each patient is assigned the kidney she is pointing to;
  - all patients and all kidneys (once re-assigned) are removed from the problem.

  Then to to step **(h+1)**

  If there are no cycle, go to step **h.3**

Top Trading Cycles and Chains

The algorithm is a multistep procedure where all steps are identical. For each $h = 1, 2, \dots,$ proceed as follows

- **Step h.1**
  Each patient points to her most preferred acceptable kidney.

  If there are no acceptable kidney the patient points to the waiting list.

- **Step h.2**
  If there are one or more cycles, proceed as follows, for each cycle:

  - each patient is assigned the kidney she is pointing to;
  - all patients and all kidneys (once re-assigned) are removed from the problem.

  Then to to step **(h+1)**

  If there are no cycle, go to step **h.3**

- **Step h.3**

  Select one chain and allocate kidneys in the following way:

  - The last patient in the chain is added to the waiting list;
  - The other patients in the chain (if any) are assigned the kidney they are pointing to.

  For all patient involved in the selected chain, the assignment is final.

  A chain selection rule determines whether the selected chain is removed from the problem.

  Go to step **(h+1)**.

- The algorithm stops when all patients have been either assigned to a kidney or added to the waiting list.

- **Step h.3**

  Select one chain and allocate kidneys in the following way:

  - The last patient in the chain is added to the waiting list;
  - The other patients in the chain (if any) are assigned the kidney they are pointing to.

  For all patient involved in the selected chain, the assignment is final.

  A chain selection rule determines whether the selected chain is removed from the problem.

  Go to step **(h+1)**.

- The algorithm stops when all patients have been either assigned to a kidney or added to the waiting list.

- **Step h.3**

  Select one chain and allocate kidneys in the following way:

  - The last patient in the chain is added to the waiting list;
  - The other patients in the chain (if any) are assigned the kidney they are pointing to.

  For all patient involved in the selected chain, the assignment is final.

  A chain selection rule determines whether the selected chain is removed from the problem.

  Go to step **(h+1)**.

- The algorithm stops when all patients have been either assigned to a kidney or added to the waiting list.

- **Step h.3**

  Select one chain and allocate kidneys in the following way:

  - The last patient in the chain is added to the waiting list;
  - The other patients in the chain (if any) are assigned the kidney they are pointing to.

  For all patient involved in the selected chain, the assignment is final.

  A chain selection rule determines whether the selected chain is removed from the problem.

  Go to step **(h+1)**.

- The algorithm stops when all patients have been either assigned to a kidney or added to the waiting list.

- **Step h.3**

  Select one chain and allocate kidneys in the following way:

  - The last patient in the chain is added to the waiting list;
  - The other patients in the chain (if any) are assigned the kidney they are pointing to.

  For all patient involved in the selected chain, the assignment is final.

  A chain selection rule determines whether the selected chain is removed from the problem.

  Go to step **(h+1)**.

- The algorithm stops when all patients have been either assigned to a kidney or added to the waiting list.

- **Step h.3**

  Select one chain and allocate kidneys in the following way:

  - The last patient in the chain is added to the waiting list;
  - The other patients in the chain (if any) are assigned the kidney they are pointing to.

  For all patient involved in the selected chain, the assignment is final.

  A chain selection rule determines whether the selected chain is removed from the problem.

  Go to step **(h+1)**.

- The algorithm stops when all patients have been either assigned to a kidney or added to the waiting list.

- **Step h.3**

  Select one chain and allocate kidneys in the following way:

  - The last patient in the chain is added to the waiting list;
  - The other patients in the chain (if any) are assigned the kidney they are pointing to.

  For all patient involved in the selected chain, the assignment is final.

  A chain selection rule determines whether the selected chain is removed from the problem.

  Go to step **(h+1)**.

- The algorithm stops when all patients have been either assigned to a kidney or added to the waiting list.

- **Step h.3**

  Select one chain and allocate kidneys in the following way:

  - The last patient in the chain is added to the waiting list;
  - The other patients in the chain (if any) are assigned the kidney they are pointing to.

  For all patient involved in the selected chain, the assignment is final.

  A chain selection rule determines whether the selected chain is removed from the problem.

  Go to step **(h+1)**.

- The algorithm stops when all patients have been either assigned to a kidney or added to the waiting list.

Remarks

Proposition (Roth, Sönmez and Ünver, 2005)

For finite number of patients and kidneys there must always exist either a cycle or a chain.

$\rightarrow$ At each step (cycle or chain) the number of remaining patients decreases. So the algorithm eventually stops.

The algorithm describes a family of algorithms: it depends on the chain selection rule we chose to apply.

In September, 2004, the Renal Transplant Oversight Committee of New England approved the creation of a clearinghouse for kidney exchange.

Remarks

Proposition (Roth, Sönmez and Ünver, 2005)

For finite number of patients and kidneys there must always exist either a cycle or a chain.

→ At each step (cycle or chain) the number of remaining patients decreases. So the algorithm eventually stops.

The algorithm describes a family of algorithms: it depends on the chain selection rule we chose to apply.

In September, 2004, the Renal Transplant Oversight Committee of New England approved the creation of a clearinghouse for kidney exchange.

Remarks

Proposition (Roth, Sönmez and Ünver, 2005)

For finite number of patients and kidneys there must always exist either a cycle or a chain.

$\rightarrow$ At each step (cycle or chain) the number of remaining patients decreases. So the algorithm eventually stops.

The algorithm describes a family of algorithms: it depends on the chain selection rule we chose to apply.

In September, 2004, the Renal Transplant Oversight Committee of New England approved the creation of a clearinghouse for kidney exchange.

Remarks

Proposition (Roth, Sönmez and Ünver, 2005)

For finite number of patients and kidneys there must always exist either a cycle or a chain.

$\rightarrow$ At each step (cycle or chain) the number of remaining patients decreases. So the algorithm eventually stops.

The algorithm describes a family of algorithms: it depends on the chain selection rule we chose to apply.

In September, 2004, the Renal Transplant Oversight Committee of New England approved the creation of a clearinghouse for kidney exchange.

Remarks

Proposition (Roth, Sönmez and Ünver, 2005)

For finite number of patients and kidneys there must always exist either a cycle or a chain.

$\rightarrow$ At each step (cycle or chain) the number of remaining patients decreases. So the algorithm eventually stops.

The algorithm describes a family of algorithms: it depends on the chain selection rule we chose to apply.

In September, 2004, the Renal Transplant Oversight Committee of New England approved the creation of a clearinghouse for kidney exchange.

Chain selection rules

First issue: Each donor-patient pair can be in only one cycle. But a pair can be part of several chains.

In such a case, the chain selection rule must select only one of the two possible chains.

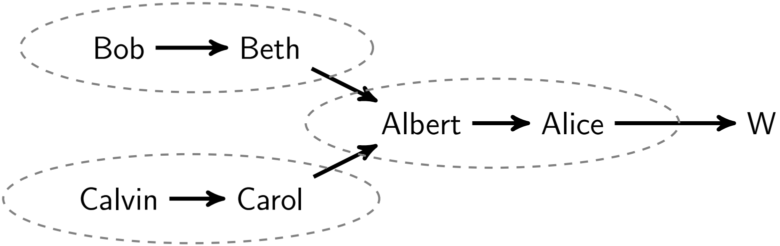
Chain selection rules

First issue: Each donor-patient pair can be in only one cycle. But a pair can be part of several chains.

In such a case, the chain selection rule must select only one of the two possible chains.

Chain selection rules

First issue: Each donor-patient pair can be in only one cycle. But a pair can be part of several chains.



In such a case, the chain selection rule must select only one of the two possible chains.

Second issue:

Should we remove from the problem all the patients and kidneys involved in a chain?

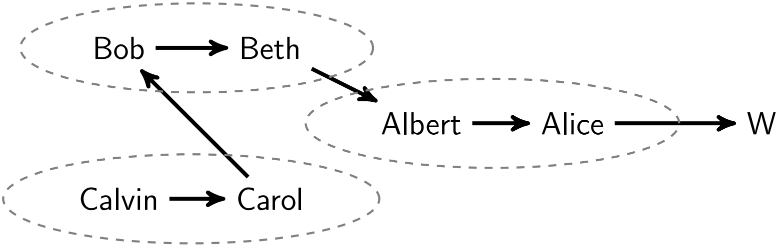
Suppose we select the chain starting with Bob. If Bob's kidney is also acceptable for Carol, then in a next step Carol can point to Bob.

Second issue:
Should we remove from the problem all the patients and kidneys involved in a chain?

Suppose we select the chain starting with Bob. If Bob's kidney is also acceptable for Carol, then in a next step Carol can point to Bob.

Second issue:
Should we remove from the problem all the patients and kidneys involved in a chain?

Suppose we select the chain starting with Bob. If Bob's kidney is also acceptable for Carol, then in a next step Carol can point to Bob.



Roth, Sönmez and Ünver propose and analyze different selection rules:

Rule a Choose the smallest chains and remove from the problem the patients and kidneys in those chains once the assignment is determined.

Rule b Choose the longest chain and remove remove from the problem the patients and kidneys in that chain (pick one chain at random if it is not unique).

Rule c Choose the longest chain and keep in the problem the patients and kidneys in that chain (pick one at random if it is not unique).

Roth, Sönmez and Ünver propose and analyze different selection rules:

Rule a Choose the smallest chains and remove from the problem the patients and kidneys in those chains once the assignment is determined.

Rule b Choose the longest chain and remove remove from the problem the patients and kidneys in that chain (pick one chain at random if it is not unique).

Rule c Choose the longest chain and keep in the problem the patients and kidneys in that chain (pick one at random if it is not unique).

Roth, Sönmez and Ünver propose and analyze different selection rules:

Rule a Choose the smallest chains and remove from the problem the patients and kidneys in those chains once the assignment is determined.

Rule b Choose the longest chain and remove remove from the problem the patients and kidneys in that chain (pick one chain at random if it is not unique).

Rule c Choose the longest chain and keep in the problem the patients and kidneys in that chain (pick one at random if it is not unique).

Roth, Sönmez and Ünver propose and analyze different selection rules:

Rule a Choose the smallest chains and remove from the problem the patients and kidneys in those chains once the assignment is determined.

Rule b Choose the longest chain and remove remove from the problem the patients and kidneys in that chain (pick one chain at random if it is not unique).

Rule c Choose the longest chain and keep in the problem the patients and kidneys in that chain (pick one at random if it is not unique).

Rule d Choose the chain that starts with the highest priority patient and remove from the problem the patients and kidneys in that chain.

Rule e Choose the chain that starts with the highest priority patient and keep in the problem the patients and kidneys in that chain.

Rule f Prioritize patient-donor pairs so that pairs with a type O donor have a higher priority (than the pairs whose donor is not of type O). Then choose the chain that starts with the highest priority pair.
If the starting pair in the chain has a type O donor then remove from the problem the patients and kidneys in that chain. Otherwise keep in the problem all the patients and donors that are in the chain.

Rule d Choose the chain that starts with the highest priority patient and remove from the problem the patients and kidneys in that chain.

Rule e Choose the chain that starts with the highest priority patient and keep in the problem the patients and kidneys in that chain.

Rule f Prioritize patient-donor pairs so that pairs with a type O donor have a higher priority (than the pairs whose donor is not of type O). Then choose the chain that starts with the highest priority pair.
If the starting pair in the chain has a type O donor then remove from the problem the patients and kidneys in that chain. Otherwise keep in the problem all the patients and donors that are in the chain.

Rule d Choose the chain that starts with the highest priority patient and remove from the problem the patients and kidneys in that chain.

Rule e Choose the chain that starts with the highest priority patient and keep in the problem the patients and kidneys in that chain.

Rule f Prioritize patient-donor pairs so that pairs with a type O donor have a higher priority (than the pairs whose donor is not of type O). Then choose the chain that starts with the highest priority pair.
If the starting pair in the chain has a type O donor then remove from the problem the patients and kidneys in that chain. Otherwise keep in the problem all the patients and donors that are in the chain.

Definition

A kidney exchange mechanism is efficient if, for any problem (set of patients, donors and their preferences it always select an assignment such that:
There does not exist another assignment that is weakly preferred by all patients (and strictly preferred by at least one patient).

Theorem

Consider a chain selection rule such that any chain selected at a nonterminal step remains in the procedure (and thus the kidney of the first donor in the chain remains available for the following steps).
Then the Top Trading Cycle and Chains mechanism, implemented with any such chain selection rule, is efficient.

Definition

A kidney exchange mechanism is efficient if, for any problem (set of patients, donors and their preferences it always select an assignment such that:
There does not exist another assignment that is weakly preferred by all patients (and strictly preferred by at least one patient).

Theorem

Consider a chain selection rule such that any chain selected at a nonterminal step remains in the procedure (and thus the kidney of the first donor in the chain remains available for the following steps).
Then the Top Trading Cycle and Chains mechanism, implemented with any such chain selection rule, is efficient.

Incentives

Do patients (e.g., their physicians) have an incentive to report truthfully their preferences?

Theorem

An assignment mechanism that uses the Top Trading Cycles and Chains algorithm with the chain selection rules a, d, e, or f is strategyproof.

Incentives

Do patients (e.g., their physicians) have an incentive to report truthfully their preferences?

Theorem

An assignment mechanism that uses the Top Trading Cycles and Chains algorithm with the chain selection rules a, d, e, or f is strategyproof.

Incentives

Do patients (e.g., their physicians) have an incentive to report truthfully their preferences?

Theorem

An assignment mechanism that uses the Top Trading Cycles and Chains algorithm with the chain selection rules a, d, e, or f is strategyproof.

On the number of exchanges

The Top Trading Cycles and Chains algorithm seems to be a good way to increase the number of transplants.

But in real-life settings this algorithm cannot be used as it is: the cycles may involve too large a number of patient-donor pairs.

Exchanges require that surgeries for

- Extractions of donors' kidneys
- Transplantations of the kidneys

all occur at the same time. Otherwise, a donor may refrain from donating once her patient received a kidney.

Remark: Extraction and Transplants in chains can be done sequentially.

On the number of exchanges

The Top Trading Cycles and Chains algorithm seems to be a good way to increase the number of transplants.

But in real-life settings this algorithm cannot be used as it is: the cycles may involve too large a number of patient-donor pairs.

Exchanges require that surgeries for

- Extractions of donors' kidneys
- Transplantations of the kidneys

all occur at the same time. Otherwise, a donor may refrain from donating once her patient received a kidney.

Remark: Extraction and Transplants in chains can be done sequentially.

On the number of exchanges

The Top Trading Cycles and Chains algorithm seems to be a good way to increase the number of transplants.

But in real-life settings this algorithm cannot be used as it is: the cycles may involve too large a number of patient-donor pairs.

Exchanges require that surgeries for

- Extractions of donors' kidneys
- Transplantations of the kidneys

all occur at the same time. Otherwise, a donor may refrain from donating once her patient received a kidney.

Remark: Extraction and Transplants in chains can be done sequentially.

On the number of exchanges

The Top Trading Cycles and Chains algorithm seems to be a good way to increase the number of transplants.

But in real-life settings this algorithm cannot be used as it is: the cycles may involve too large a number of patient-donor pairs.

Exchanges require that surgeries for

- Extractions of donors' kidneys
- Transplantations of the kidneys

all occur at the same time. Otherwise, a donor may refrain from donating once her patient received a kidney.

Remark: Extraction and Transplants in chains can be done sequentially.

On the number of exchanges

The Top Trading Cycles and Chains algorithm seems to be a good way to increase the number of transplants.

But in real-life settings this algorithm cannot be used as it is: the cycles may involve too large a number of patient-donor pairs.

Exchanges require that surgeries for

- Extractions of donors' kidneys
- Transplantations of the kidneys

all occur at the same time. Otherwise, a donor may refrain from donating once her patient received a kidney.

Remark: Extraction and Transplants in chains can be done sequentially.

We may need to consider two-way or three-way kidney exchanges (i.e., involving only 2 or 3 pairs).

How much do we loose?

In standard assignments problems this question can be difficult (if not impossible to answer).

For kidney exchange, *most* of the preferences is given by blood type: there is a strong structure on the preferences.

Roth, Ünver and Sönmez show that if there is no restriction on the length of cycles then the maximum length of a cycle is equal to number of blood types.

So restricting the length of the cycles *may* not be too harmful.

We may need to consider two-way or three-way kidney exchanges (i.e., involving only 2 or 3 pairs).

How much do we loose?

In standard assignments problems this question can be difficult (if not impossible to answer).

For kidney exchange, *most* of the preferences is given by blood type: there is a strong structure on the preferences.

Roth, Ünver and Sönmez show that if there is no restriction on the length of cycles then the maximum length of a cycle is equal to number of blood types.

So restricting the length of the cycles *may* not be too harmful.

We may need to consider two-way or three-way kidney exchanges (i.e., involving only 2 or 3 pairs).

How much do we loose?

In standard assignments problems this question can be difficult (if not impossible to answer).

For kidney exchange, *most* of the preferences is given by blood type: there is a strong structure on the preferences.

Roth, Ünver and Sönmez show that if there is no restriction on the length of cycles then the maximum length of a cycle is equal to number of blood types.

So restricting the length of the cycles *may* not be too harmful.

We may need to consider two-way or three-way kidney exchanges (i.e., involving only 2 or 3 pairs).

How much do we loose?

In standard assignments problems this question can be difficult (if not impossible to answer).

For kidney exchange, *most* of the preferences is given by blood type: there is a strong structure on the preferences.

Roth, Ünver and Sönmez show that if there is no restriction on the length of cycles then the maximum length of a cycle is equal to number of blood types.

So restricting the length of the cycles *may* not be too harmful.

We may need to consider two-way or three-way kidney exchanges (i.e., involving only 2 or 3 pairs).

How much do we loose?

In standard assignments problems this question can be difficult (if not impossible to answer).

For kidney exchange, *most* of the preferences is given by blood type: there is a strong structure on the preferences.

Roth, Ünver and Sönmez show that if there is no restriction on the length of cycles then the maximum length of a cycle is equal to number of blood types.

So restricting the length of the cycles *may* not be too harmful.

We may need to consider two-way or three-way kidney exchanges (i.e., involving only 2 or 3 pairs).

How much do we loose?

In standard assignments problems this question can be difficult (if not impossible to answer).

For kidney exchange, *most* of the preferences is given by blood type: there is a strong structure on the preferences.

Roth, Ünver and Sönmez show that if there is no restriction on the length of cycles then the maximum length of a cycle is equal to number of blood types.

So restricting the length of the cycles *may* not be too harmful.

The distribution of blood types may also play a role: O–A, O–B or O–AB are more frequent that A–O, B–O or AB–O pairs. (X–Y means patient of type X, donor of type Y).

Simulations suggest that up to 3-way there the gain becomes marginal.

♯ pairs	2-way	up to 3-way	up to 4-way	unrestricted
25	8.86	11.272	11.824	11.992
50	21.792	27.266	27.986	28.09
100	49.708	59.714	60.354	60.39

The distribution of blood types may also play a role: O–A, O–B or O–AB are more frequent that A–O, B–O or AB–O pairs. (X–Y means patient of type X, donor of type Y).

Simulations suggest that up to 3-way there the gain becomes marginal.

♯ pairs	2-way	up to 3-way	up to 4-way	unrestricted
25	8.86	11.272	11.824	11.992
50	21.792	27.266	27.986	28.09
100	49.708	59.714	60.354	60.39

The distribution of blood types may also play a role: O–A, O–B or O–AB are more frequent that A–O, B–O or AB–O pairs. (X–Y means patient of type X, donor of type Y).

Simulations suggest that up to 3-way there the gain becomes marginal.

♯ pairs	2-way	up to 3-way	up to 4-way	unrestricted
25	8.86	11.272	11.824	11.992
50	21.792	27.266	27.986	28.09
100	49.708	59.714	60.354	60.39

The distribution of blood types may also play a role: O–A, O–B or O–AB are more frequent that A–O, B–O or AB–O pairs. (X–Y means patient of type X, donor of type Y).

Simulations suggest that up to 3-way there the gain becomes marginal.

♯ pairs	2-way	up to 3-way	up to 4-way	unrestricted
25	8.86	11.272	11.824	11.992
50	21.792	27.266	27.986	28.09
100	49.708	59.714	60.354	60.39

We can compare the percentage gains, from k to $(k+1)$-way exchange:

$\sharp$ pairs	$2 \to 3$-way	$3 \to 4$-way	4-way $\to$ unrestricted
25	27.2%	4.9%	1.4%
50	25.1%	2.6%	0.4%
100	20.1%	1.1%	0.1%

In other words:

- the logistics of kidney exchange drastically reduces the number of pairs that can be involves in an exchange.
- The specificities of kidney transplants almost eliminate the need to worry about logistic constraints.

We can compare the percentage gains, from k to $(k+1)$-way exchange:

$\sharp$ pairs	$2 \to 3$-way	$3 \to 4$-way	4-way $\to$ unrestricted
25	27.2%	4.9%	1.4%
50	25.1%	2.6%	0.4%
100	20.1%	1.1%	0.1%

In other words:

- the logistics of kidney exchange drastically reduces the number of pairs that can be involves in an exchange.
- The specificities of kidney transplants almost eliminate the need to worry about logistic constraints.

We can compare the percentage gains, from k to $(k+1)$-way exchange:

$\sharp$ pairs	$2 \to$ 3-way	$3 \to$ 4-way	4-way $\to$ unrestricted
25	27.2%	4.9%	1.4%
50	25.1%	2.6%	0.4%
100	20.1%	1.1%	0.1%

In other words:

- the logistics of kidney exchange drastically reduces the number of pairs that can be involves in an exchange.
- The specificities of kidney transplants almost eliminate the need to worry about logistic constraints.

We can compare the percentage gains, from k to $(k+1)$-way exchange:

$\sharp$ pairs	$2 \to 3$-way	$3 \to 4$-way	4-way $\to$ unrestricted
25	27.2%	4.9%	1.4%
50	25.1%	2.6%	0.4%
100	20.1%	1.1%	0.1%

In other words:

- the logistics of kidney exchange drastically reduces the number of pairs that can be involves in an exchange.
- The specificities of kidney transplants almost eliminate the need to worry about logistic constraints.

Take-away

- Kidneys for transplants can come from deceased donors and living donors.
- Most living donors show up with a patient (e.g., relative, friend). But often the donor's kidney is not compatible with the patient.
- In almost all countries it is prohibited to sell or buy a kidney.
- An assignment mechanism can be used to redistribute kidneys (from living donors) to patients.

  We have a kidney exchange, no need to put a price on kidneys.

- The kidney exchange problem differs from the traditional assignment problem with mixed endowments:
  - patients that have a living donor have a private endowment (the kidney).
  - Kidneys from deceased donors are not public endowment: they are not available yet. We replace it with a waiting list.
- The Top Trading Cycles and Chains (TTCC) algorithm manages at the same time:
  - The exchange of kidneys from living donors
  - The waiting list for kidneys from deceased donors.
- The TTCC algorithm defines a family of algorithms. Variations depend on the chain selection rule:
  - Which chain (leading to the waiting list) is selected;
  - Whether the chain is removed once selected.

- With some chain selection rules TTCC is efficient. Some chain selection rules also guarantee strategyproofness.
- Simultaneity of the exchange can create hurdles. We need $2k$ simultaneous surgeries for a k-way exchange.
- Blood types are the main parameters deciding kidney compatibility. Statistical analysis show that allowing for 4-way (or more) exchanges does not increase significantly the number of exchanges.