Basic School Choice

Question 1 The New York school district currently uses for high school enrollment

(A) The Boston algorithm (also called the *immediate acceptance* algorithm)

(B) The Deferred Acceptance with schools proposing

(C) The Deferred Acceptance with students proposing

(D) None of the above

Question 2 The Boston school district currently uses for high school enrollment

(A) The Boston algorithm (also called the *immediate acceptance* algorithm)

(B) The Deferred Acceptance with schools proposing

(C) The Deferred Acceptance with students proposing

(D) None of the above

Question 3 We consider the problem of assigning students to public schools. There are three students (i_1, i_2 and i_3), and three schools (s_1, s_2 and s_3), each with one seat.

The true preferences of the students (or their parents') over schools are:

P_{i_1}	P_{i_2}	P_{i_3}
s_1	s_1	s_2
s_2	s_2	s_1
s_3	s_3	s_3

And the schools order students in the following way:

P_{s_1}	P_{s_2}	P_{s_3}
i_1	i_2	i_1
i_2	i_1	i_2
i_3	i_3	i_3

1. Compute the matching we would obtain using the **deferred acceptance** algorithm with students proposing, when parents submit their true preferences.

2. Compute the matching we would obtain using the **immediate acceptance** algorithm with students proposing, when parents submit their true preferences.

3. When running the immediate acceptance algorithm, is it in the best interest for all parents to submit their true preferences? Is there a profitable deviation by one of the parents (if yes, give an example of such a deviation).

Question 4 Both New York City and Boston use the deferred Acceptance algorithm to match studens to public high schools. Yet, the procedure is not **exactly** the same in both cities. What is the main difference? Are the two matching mechanisms (for Boston and for NYC) strategyproof?

Question 5 We consider the **Immediate Acceptance** algorithm with n students and n schools. The capacity of each school is 1 and we assume that each student finds all schools acceptable.

Let $(P_{s_1}, P_{s_2}, \ldots, P_{s_n})$ be schools' (strict) priority orderings over students and let Let $(P_{i_1}, P_{i_2}, \ldots, P_{i_n})$ be students' **true** (strict) preferences over schools.

We consider the Nash equilibria of the game where students' strategies consist of a preference list over schools submitted to the matching algorithm. To distinguish strategies from preferences we denote a typical strategy by student i with the letter Q, i.e., Q_i denote a strategy of student i.

Let $\varphi_i(Q)$ be the school that student i is assigned to when the submitted preference profile is $Q = (Q_{i_1}, Q_{i_2}, \ldots, Q_{i_n})$.

A **Nash equilibrium** is a profile of submitted preference profile Q^* such that, for each student i, for each Q_i,

$$\varphi_i(Q_i^*, Q_{-i}^*) \ R_i \ \varphi_i(Q_i, Q_{-i}^*).$$

where Q_{-i} denotes the submitted preference profile (i.e., the strategy) all all the other students except student i and R_i is the weak version of i's true preferences.

Show that if Q^* is a Nash equilibrium then the assignment $\varphi(Q)$ is stable.

For simplicity, assume that for each student i, Q_i^* ranks all schools (so no student is unmatched at $\varphi(Q^*)$).

(Hint: take Q giving an unstable matching, show that is cannot be a Nash equilibrium, i.e., there is Q' and i such that $\varphi_i(Q_i', Q_{-i}) \ P_i \varphi_i(Q_i, Q_{-i})$).

Question 6 Consider three students i_1, i_2, i_3 and three schools s_1, s_2, s_3. The capacity of each school is 1. Preferences and priorities are:

P_{i_1}	P_{i_2}	P_{i_3}
s_2	s_2	s_3
s_3	s_1	s_2
s_1	s_3	s_1

P_{s_1}	P_{s_2}	P_{s_3}
i_1	i_3	i_1
i_2	i_2	i_3
i_3	i_1	i_2

1. Use the DA algorithm to find the student-optimal stable matching.
2. Is the student-optimal stable matching the only stable matching? (If "no" show another stable matching).
3. Is it Pareto efficient for the students?
4. Compute the TTC matching.
5. Is the matching (obtained with TTC) stable? (if "no" show a blocking pair).
6. Are there other assignments that are Pareto efficient for the students? (If "yes" show such a matching).
7. Compute the matching we would obtain with TTC but reversing the roles of schools and students (i.e., if a school is in a cycle it is assigned the student it is pointing to).

Question 7 There are 5 students, i_1, i_2, i_3, i_4 and i_5. Their (true) preferences are as follows.

P_{i_1}	P_{i_2}	P_{i_3}	P_{i_4}	P_{i_5}
s_1	s_1	s_1	s_2	s_3
s_2	s_2	s_3	s_1	s_1
s_3	s_3	s_2	s_3	s_2
s_4	s_4	s_4	s_4	s_4

All schools have the same priority: $i_1 \; P_s \; i_2 \; P_s \; i_3 \; P_s \; i_4 \; P_s \; i_5$, for $s = s_1, s_2, s_3, s_4$. School s_2 has a capacity of 2 and all the other schools have a capacity of 1.

1. Compute the assignment when running the Immediate Acceptance algorithm.
2. Is there a student that can be better off by misrepresenting her preferences (if so, say which student and give a profitable misrepresentation).
3. Compute the assignment when running TTC.

Question 8 We consider five students, i_1, i_2, i_3, i_4 and i_5. For school s the priority over these students is $P_s : i_1, i_2, i_3, i_4, i_5$. The school can accomodate up to three students and the school's priority ordering over groups of students is responsive.

We have

(A) $\{i_1, i_3\}$ has a higher priority than $\{i_2, i_4\}$

(B) $\{i_2, i_4\}$ has a higher priority than $\{i_1, i_3\}$

(C) We cannot compare $\{i_1, i_3\}$ and $\{i_2, i_4\}$.

Question 9 Consider the following school choice problem. The preferences and the priorities are given below (students' are denoted by i and schools by s). Each school has a capacity equal to 2.

P_{i_1}	P_{i_2}	P_{i_3}	P_{i_4}	P_{i_5}	P_{i_6}	P_{i_7}	P_{i_8}
s_1	s_2	s_4	s_2	s_3	s_1	s_3	s_4
s_2	s_4	s_1	s_4	s_4	s_4	s_4	s_3
s_4	s_3	s_3	s_3	s_1	s_2	s_1	s_2
s_3	s_1	s_2	s_1	s_2	s_3	s_2	s_1

Schools s_1, s_3 have the priority $i_1, i_2, i_3, i_4, i_5, i_6, i_7, i_8$

Schools s_2, s_4 have the priority $i_2, i_4, i_3, i_1, i_5, i_6, i_8, i_7$.

The Deferred Acceptance and the Immediate Acceptance algorithms give the same assignment (both with students proposing).

(A) True

(B) False

Question 10 In school choice problems schools often have weak priorities. Ties between students are usually broken by assigning to each student a random number and rank them accordingly (if there are ties). For instance, if for a school i_1 has a higher priority than i_2 and i_3 but students i_2 and i_3 have the same priority and i_2's random number is 284 and i_3' random number is 19 then after breaking ties the school's priority is i_1, i_3, i_2.

Economists advice schools district to use a

(A) Single tie-breaking: for each school each student is assigned the same random number.

(B) Multiple tie-breaking: student's random number differ across schools.

Question 11 The New York school district currently uses for high school enrollment

(A) The Boston algorithm (also called the *immediate acceptance* algorithm)

(B) The Deferred Acceptance with schools proposing

(C) The Deferred Acceptance with students proposing

(D) None of the above

Question 12 The Boston school district currently uses for high school enrollment

(A) The Boston algorithm (also called the *immediate acceptance* algorithm)

(B) The Deferred Acceptance with schools proposing

(C) The Deferred Acceptance with students proposing

(D) None of the above