

# Lecture 5: Course allocation

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# Introduction

“Traditional” assignment problems are either:

- ▶ one-to-one (e.g., assigning bedrooms)
- ▶ or many-to-one (e.g., assigning students to schools)

For those problems, we can identify well-behaved mechanisms (strategyproof, efficient or stable).

**Many-to-many** assignment problems are more complex. A typical example is **course allocation**:

- ▶ Each student has to enroll in several courses;
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# Preliminaries

A **course allocation problem** is given by:

- ▶ A (finite) set of **students**;
- ▶ A (finite) set of **courses**, and each course has a maximal capacity.
- ▶ Students have to be assigned a **schedule**: a collection of courses.

A course allocation problem is a special case of a **combinatorial assignment problem**.

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In a course allocation problem students have preferences over schedules.

For many-to-one assignment problems we usually assume that objects' priorities are **responsive**. This is a strong assumption for course allocation problems:

- ▶ Some courses are complements
- ▶ Some courses are substitute.

Responsive preferences do not allow us to capture such realistic constraints.

We will study what can be done with responsive preferences, but eventually we will have to drop that assumption.

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Another complication is about the choice of the assignment mechanism.

Unlike for one-to-one or many-to-one problems, we do not have much choice. Before that we need the following concept:

### Definition

A mechanism  $\varphi$  is **non-bossy** if for any profile of submitted preferences  $P$ , and for any individual  $i$ , and preference ordering  $P'_i$ ,

$$\underbrace{\varphi(P'_i, P_{-i})(i)}_{i\text{'s assignment}} = \varphi(P_i, P_{-i})(i) \quad \Rightarrow \quad \underbrace{\varphi(P'_i, P_{-i})}_{\text{the whole assignment}} = \varphi(P_i, P_{-i})$$

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## Theorem (Pápai)

*A many-to-many assignment mechanism is **strategyproof**, **non-bossy** and **Pareto efficient** if, and only if, it is a **Serial Dictatorship** mechanism.*

Note that here courses do not have priorities over students. So Serial Dictatorship and Top Trading Cycles (with public endowments) are the same.

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# Bidding for courses

A common assignment mechanisms in many business schools consists of running an **auction** (Columbia Business School, Haas Business School, etc.).

The basic structure of those auctions is:

1. Each student is endowed with a budget (of coins, tokens, points, etc.).
2. Students submits bids for each of the course they want to enroll in.
3. Bids are used to produce an ordering of students. A Serial Dictatorship mechanism is run, using this ordering.

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## Example

Alice, Bob, Carol and Erik need to take two courses among courses X, Y and Z. The capacities are 3, 2 and 4, respectively. The bids are:

<b>capacity</b>	3	2	4	
	X	Y	Z	
Alice	60	38	2	Alice
Bob	48	22	30	Bob
Carol	47	28	25	Carol
Erik	45	35	20	Erik



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Alice, Bob, Carol and Erik need to take two courses among courses  $X$ ,  $Y$  and  $Z$ . The capacities are 3, 2 and 4, respectively. The bids are:

<b>capacity</b>	2	2	4	
	$X$	$Y$	$Z$	
Alice	60	38	2	$X$
Bob	48	22	30	
Carol	47	28	25	
Erik	45	35	20	

Alice's bid for  $X$  is the highest, so she is assigned that course.

## Example

Alice, Bob, Carol and Erik need to take two courses among courses X, Y and Z. The capacities are 3, 2 and 4, respectively. The bids are:

<b>capacity</b>	1	2	4				
	X	Y	Z	Alice	Bob	Carol	Erik
Alice	60	38	2	X	X		
Bob	48	22	30				
Carol	47	28	25				
Erik	45	35	20				

Next highest bid is Bob's bid for X, so he is assigned that course.

## Example

Alice, Bob, Carol and Erik need to take two courses among courses X, Y and Z. The capacities are 3, 2 and 4, respectively. The bids are:

capacity	0	2	4				
	X	Y	Z	Alice	Bob	Carol	Erik
Alice	60	38	2	X	X	X	
Bob	48	22	30				
Carol	47	28	25				
Erik	45	35	20				

Next highest bid is Carol's bid for X, so she is assigned that course. Course X is now full.

Subsequent bids for course X will be ignored.

## Example

Alice, Bob, Carol and Erik need to take two courses among courses X, Y and Z. The capacities are 3, 2 and 4, respectively. The bids are:

capacity	0	1	4				
	X	Y	Z	Alice	Bob	Carol	Erik
Alice	60	38	2	X,Y	X	X	
Bob	48	22	30				
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Next highest bid is Alice's for course Y. So she gets it. She is no longer participating, she has 2 courses.

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Alice, Bob, Carol and Erik need to take two courses among courses X, Y and Z. The capacities are 3, 2 and 4, respectively. The bids are:

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Alice	60	38	2	X,Y	X	X	Y
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Next highest bid is Erik's for course Y. So he gets it. Course Y is now full.

Subsequent bids for course Y will be ignored.

## Example

Alice, Bob, Carol and Erik need to take two courses among courses X, Y and Z. The capacities are 3, 2 and 4, respectively. The bids are:

<b>capacity</b>	0	0	3				
	X	Y	Z	Alice	Bob	Carol	Erik
				X,Y	X,Z	X	Y
Alice	60	38	2				
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Next highest bid is Bob's for course Z. So he gets it. He is no longer participating, he has 2 courses.

## Example

Alice, Bob, Carol and Erik need to take two courses among courses X, Y and Z. The capacities are 3, 2 and 4, respectively. The bids are:

<b>capacity</b>	0	0	2				
	X	Y	Z	Alice	Bob	Carol	Erik
Alice	60	38	2	X,Y	X,Z	X,Z	Y
Bob	48	22	30				
Carol	47	28	25				
Erik	45	35	20				

Next highest bid is Carol's for course Z. So she gets it. She is no longer participating, she has 2 courses.

## Example

Alice, Bob, Carol and Erik need to take two courses among courses X, Y and Z. The capacities are 3, 2 and 4, respectively. The bids are:

<b>capacity</b>	0	0	1				
	X	Y	Z	Alice	Bob	Carol	Erik
				X,Y	X,Z	X,Z	Y,Z
Alice	60	38	2				
Bob	48	22	30				
Carol	47	28	25				
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Erik is the last one without a second course. Only Z is available, so he gets it.



## Example

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The auction is over.

# Issues

In principle, with a course bidding mechanism we could interpret bids as preferences with intensities.

But it is easy to see that it cannot be strategyproof:

My favorite course is the least demanded.

⇒ I submit the lowest possible bid for that course.

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Also, courses prices in the bidding mechanism cannot be considered as **market-clearing prices**.

If an auction is efficient it should produce competitive prices. It is not the case here.

### Example:

- ▶ Liza bids 1 for course  $X$  (her favorite), 50 for course  $Y$  and 50 for course  $Z$ .
- ▶ She knows  $X$  is underdemanded: she'll be enrolled for sure.
- ▶ Suppose she ends up being enrolled in courses  $Y$  and  $Z$ , and nobody is enrolled in course  $X$ .
- ▶ The market clearing price for course  $X$  should be 0.
- ▶ But Liza bids **above** the competitive price for  $X$ .

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# Deferred Acceptance with bids

Sönmez and Ünver argue that we need to separate:

- ▶ Inferring student's **preference orderings** over courses;
- ▶ Determining which student has **a bigger claim** (priority) over each course.

The propose a modified mechanism that separates these two problems.

For the presentation of the mechanism it is assumed that each student has to enroll in  $k$  courses.



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# Gale-Shapley Pareto-Dominant Market Mechanism

## Step 1

Students are randomly ordered. This order will be used to break ties between students, if needed.

## Step 2

Each student submits her preferences over courses (not over schedules).

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### Step 3

Each student submits a bid for each course.

**Step 4** Run the Deferred Acceptance algorithm where:

- ▶ students first propose to their  $k$  most preferred courses.
- ▶ When a student is rejected by  $p$  courses, she propose to her next  $p$  most preferred courses.
- ▶ Courses accept and reject students using the bids to prioritize students (e.g., the student with the highest bid for a course has the highest priority).
- ▶ If two students have identical bids for the same course their relative priority is given by the random ordering of Step 1.

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In this mechanism, revealing one's true preferences over courses (Step 2) is a **dominant strategy**.

However, choosing how much to bid is still a strategic decision. We need to consider **equilibria** of the bidding game (Step 3).

## Proposition

*With the Gale-Shapley Pareto-Dominant Market Mechanism, if students choose bids that maximize their expected payoffs then the course assignment (and the prices) correspond to a market equilibrium.*

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*The course assignment obtained using the Gale-Shapley Pareto-Dominant Market Mechanism **Pareto dominates** the assignment of any competitive equilibrium whenever the bids used constitute an equilibrium.*

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# The Harvard Business School method

The Harvard Business School (HBS) takes a different approach:

Budish and Cantillon (2012) studied it, they found that:

- ▶ The mechanism is not strategyproof or efficient;
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# The HBS draft mechanism

The mechanism used at HBS is a modification of the Random Serial Dictatorship

## Step 1:

Each student submits a preference list over courses.

**Step 2:** Students are assigned a random number (no two students have the same number).

## Step $k$ , $k \geq 3$ , $k$ odd

Each student who still needs a course is assigned her most preferred course among the courses that are still available, starting with the student with the **highest random number**.

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## Example

Suppose that the random order is

Alice, Bob, Carol, Denis

The HBS Draft mechanism works as follows (once the students submitted their preferences):

1. Run the Serial Dictatorship with this order:  
Alice, Bob, Carol, Denis
2. Run the Serial Dictatorship with this order:  
Denis, Carol, Bob, Alice
3. Run the Serial Dictatorship with this order:  
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4. And so on.

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# Strategic behavior

The HBS Draft mechanisms is **not** strategyproof: there is a conflict between

- ▶ students' preferences, and
- ▶ courses popularity.

Being truthful may not be longer an option if the most preferred course is not very popular.

In this case a student can be better off declaring:

- ▶ The very popular course as top choice;
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## Example

Each student has to take 2 courses, each course has a capacity of 2 students.

$P_{\text{Alice}}$	$P_{\text{Bob}}$	$P_{\text{Carol}}$
$c_1$	$c_2$	$c_1$
$c_2$	$c_1$	$c_3$
$c_3$	$c_3$	$c_4$
$c_4$	$c_4$	$c_2$

Suppose all students are truthful. Then:

- ▶ For any ordering of the students Bob is sure to get  $c_2$ .
- ▶ At the end of the 1st round  $c_1$  is no longer available: taken by Alice and Carol.
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$c_2$	$c_2$	$c_3$
$c_3$	$c_3$	$c_4$
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- ▶ Bob gets  $c_1$  if he's 1st or 2nd. So he gets  $c_1$  with probability  $\frac{2}{3}$ .
- ▶ If Alice is last in the queue: one seat left for  $c_2$ . The top choices in the 2nd step are:  $c_3$  (Alice),  $c_2$  (Bob),  $c_3$  (Carol). So Bob gets  $c_2$ .
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- ▶ If Bob is last in the queue, he picks  $c_2$  in the first step and then  $c_3$  in the 2nd step.

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$c_1$	$c_1$	$c_1$
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- ▶ If Bob is last in the queue, he picks  $c_2$  in the first step and then  $c_3$  in the 2nd step.

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- ▶ If Bob is last in the queue, he picks  $c_2$  in the first step and then  $c_3$  in the 2nd step.

Suppose Bob deviates, and we have

$P_{\text{Alice}}$	$P'_{\text{Bob}}$	$P_{\text{Carol}}$
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- ▶ Bob gets  $c_1$  if he's 1st or 2nd. So he gets  $c_1$  with probability  $\frac{2}{3}$ .
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To sum up.

- ▶ If Bob submits  $P_{\text{Bob}} = c_2, c_1, c_3, c_4$   
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## Proposition (Budish and Cantillon)

- (a) *Students should not reverse the relative ranking of two courses in their submitted preference lists (with respect to their true preferences) if by doing so they do not obtain the preferred course for sure.*
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Budish and Cantillon analyzed a series of data consisting of:

- ▶ a poll conducted in May, 2005, asking students their true preferences over courses;
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They observe that:

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- ▶ On average, 64% of the students would benefit by trading at least one of the courses obtained with the HBS Draft mechanism.
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# Course assignment at Wharton

The bidding mechanism, the Gale-Shapley Pareto-Dominant Market mechanism or the HBS draft mechanisms implicitly assume that students' preferences over courses are **responsive**.

But students may well see some courses as **substitutable** or **complement**.

Ideally, we would like to have a **combinatorial** assignment mechanism. There are two problems, though:

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# Approximate Competitive Equilibrium from Equal Incomes

Budish (2011) proposed a solution, using a “traditional” economics approach, the **competitive equilibrium**, with a twist: instead of an **exact** equilibrium he seeks only an **approximate** equilibrium.

The approach is like for the bidding mechanism: endow students with a budget of “tokens”.

In a competitive equilibrium we need to find prices for each course such that:

# student asking for a course  $\leq$  course capacity,

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# Example

Budget is 1,000 tokens and I need to enroll in 2 courses among the following 3 courses:

- ▶ Marketing;
- ▶ Corporate Finance;
- ▶ Accounting.

Let  $p_M$ ,  $p_C$  and  $p_A$  be the prices for those courses.  
My demand could be

$p_M$	$p_C$	$p_A$	Demand
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- ▶ An **approximate** equilibrium: prices do not necessarily clear the markets.

For instance, the prices could be that there some courses are oversubscribed.

For HBS, he calculates that with 900 students choosing 5 courses (among 50 possible courses), there are only 11 instances of oversubscription.

- ▶ Students are not all endowed with the same budget. But differences can be (very) small.

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# How to report complex demands?

To see whether the ACEEI can be implemented Budish and Kessler set up an experiment.

A typical experimental setup would be:

- ▶ Endow subjects with some preferences;
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Budish and Kessler propose the following mechanism:

1. Students give a score between 1 and 100 for any course they are interested in.  
(1 for the least desired courses, 100 for the most desired courses).
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Budish and Kessler propose the following mechanism:

1. Students give a score between 1 and 100 for any course they are interested in.  
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## Example

A student reports the following scores:

Course	<i>A</i>	<i>B</i>	<i>C</i>	<i>D</i>
Score	90	50	40	20

This means the student prefers *A* and *D* ( $90 + 20 = 110$ ) to *B* and *C* ( $50 + 40 = 90$ ).

If the student puts an adjustment of +25 to *B* and *D*, then

$$\underbrace{B \text{ and } D}_{50+20+25=95} \quad \text{preferred to} \quad \underbrace{B \text{ and } C}_{50+40=90}$$

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# Results

The experiment was a success! Wharton adopted the mechanism in fall 2013.

In the experiments, subjects were also asked to play with the former bidding mechanism, and then compare the schedules they would obtain for both mechanisms.

Note that:

- ▶ Reporting preferences over combinations of courses is **difficult**
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8 sessions were run.

- ▶ For 6 sessions, the majority of the students preferred the schedule obtained with ACEEI. For 2 sessions there was a tie.
- ▶ Envy is almost impossible to eliminate in problems like course allocation.

ACEEI reduced envyness by about 30%.

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# Take-away

- ▶ Course allocation is a **many-to-many** problem:
  - ▶ Each student can enroll in many courses;
  - ▶ Many students enroll in the same courses.
- ▶ Many-to-many problems are more complex than many-to-one problems. The only strategyproof, non-bossy and Pareto efficient mechanism is the Serial Dictatorship.
- ▶ Auctions are often used to allocate courses.
  - ▶ Students are endowed with a budget (tokens, points, etc.)
  - ▶ Students bid for courses.
  - ▶ Bids are ordered and a Serial Dictatorship is run.

This mechanism is not strategyproof, and bids cannot be considered as market-clearing prices.

- ▶ A solution is to use a mix of bids (to determine students' priority rankings for each course) and the Deferred Acceptance algorithm (with students submitting preferences over courses).

This mechanism is efficient mechanism and bids correspond to market prices.

- ▶ Harvard Business School Draft: A series of Serial Dictatorship where students choose one course, with the order of dictators reversed for each run.
- ▶ HBS mechanism is not strategyproof, not efficient.
- ▶ A simple strategy is to ask first for popular courses, not necessarily the most preferred.

Empirical evidence shows that students use such strategies.

But strategic plays increases welfare compared to a strategyproof mechanism!

- ▶ Wharton put in place a new mechanism that allows students to express **substitutabilities** and **complementarities**.
- ▶ Students' preferences are elicited, then heavy computation determines students' demand (with a budget constraint) and calculate a **competitive market equilibrium**.
- ▶ A key feature: the equilibrium is **approximate** and not students have **unequal budgets** (but differences are small).
- ▶ Outcomes under the Wharton mechanism is preferred by students to the traditional bidding mechanism.
- ▶ Budish and Kessler developed a new methodology to elicit students' preferences.