MPCS58020 HW1 Yecheng Li

1. $P(3\text{-engine}) = P(2\text{ functional}) + P(3\text{ functional})$

$= p^2(1-p)\cdot C_3^1 + p^3$

$= 3p^2 - 2p^3$

$P(5\text{-engine}) = P(3\text{-functional}) + P(4\text{ functional}) + P(5\text{ functional})$

$= p^3(1-p)^2 C_5^2 + p^4(1-p)C_5^1 + p^5$

$= 6p^5 - 15p^4 + 10p^3$

$\Rightarrow 3p^2 - 2p^3 > 6p^5 - 15p^4 + 10p^3$

$6p^5 - 15p^4 + 12p^3 - 3p^2 < 0$

$p^2(p-1)^2(2p-1) < 0$

$\Rightarrow 0 < p < \frac{1}{2}$

2. (a). $P(\text{alive}) = P(\text{feed}) + P(\text{don't feed})$

$= 90\% \times 85\% + 10\% \times 20\%$

$= 0.785$

(b). $P(\text{die} \mid \text{don't feed}) = (10\% \times 80\%)/(1-0.785)$

$= \frac{16}{43} \approx 0.372$

3. Assume the company should charge the amount of money x

then: $(x-A)p + x(1-p) = 0.1A$

$\Rightarrow x = (0.1+p)A$

4. (a) if $x<0$, $f(x)=0$

$\Rightarrow F(x)=0$ if $x<0$

if $x\geq 0$, $f(x)=e^{-x}$ $\rightarrow$ but when $x=0$, $f(x)=1$?

$\Rightarrow F(x)=\int_0^x e^{-x}\,dx = 1-e^{-x}$

Therefore: $F(x)=\begin{cases} 0 & \text{if } x<0 \\ 1-e^{-x} & \text{if } x\geq 0 \end{cases}$

(b). $E[X]=\int_{-\infty}^{\infty} xf(x)\,dx$

$=\int_{-\infty}^{0} 0\,dx+\int_0^{\infty} xe^{-x}\,d(x)$

$=0-(x+1)e^{-x}\Big|_0^{\infty}$

$=1$

5. $F(150)=\int_{100}^{150}\frac{100}{x^2}\,dx$

$=-\frac{100}{x}\Big|_{100}^{150}$

$=\frac{1}{3}$

$\Rightarrow P(\text{no tube broken within 150 hrs})=(1-\frac{1}{3})^5=\frac{32}{243}$

$\Rightarrow \bar{P}=1-P=\frac{211}{243}$

6. $f(x<y)=\int_0^{\infty}\int_x^{\infty} 2e^{-(x+2y)}\,dy\,dx$

$=\int_0^{\infty}\left(-e^{-(x+2y)}\Big|_x^{\infty}\right)dx$

$=\int_0^{\infty} e^{-3x}\,dx$

$=-\frac{e^{-3x}}{3}\Big|_0^{\infty}$

$=\frac{1}{3}$

7. ① $f_X(x) = \int_0^1 (\frac{9}{10}xy^2 + \frac{1}{5})\,dy$

$= (\frac{3}{10}xy^3 + \frac{1}{5}y)\Big|_0^1$

$= \frac{3}{10}x + \frac{1}{5}$

$\Rightarrow E[X] = \int_0^2 x f_X(x)\,dx$

$= \int_0^2 (\frac{3}{10}x^2 + \frac{1}{5}x)\,dx$

$= (\frac{1}{10}x^3 + \frac{1}{10}x^2)\Big|_0^2$

$= \frac{6}{5}$

② $f_Y(y) = \int_0^2 (\frac{9}{10}xy^2 + \frac{1}{5})\,dx$

$= (\frac{9}{20}x^2y^2 + \frac{1}{5}x)\Big|_0^2$

$= \frac{9}{5}y^2 + \frac{2}{5}$

$\Rightarrow E[Y] = \int_0^1 y f_Y(y)\,dy$

$= \int_0^1 (\frac{9}{5}y^3 + \frac{2}{5}y)\,dy$

$= (\frac{9}{20}y^4 + \frac{1}{5}y^2)\Big|_0^1$

$= \frac{13}{20}$

③ $E[XY] = \int_0^1\int_0^2 (\frac{9}{10}x^2y^3 + \frac{1}{5}xy)\,dx\,dy$

$= \int_0^1 (\frac{3}{10}x^3y^3 + \frac{1}{10}x^2y)\Big|_0^2\,dy$

$= \int_0^1 (\frac{12}{5}y^3 + \frac{2}{5}y)\,dy$

$= (\frac{3}{5}y^4 + \frac{1}{5}y^2)\Big|_0^1$

$= \frac{4}{5}$

$\Rightarrow Cov(X, Y) = E[XY] - E[X]E[Y]$

$= \frac{4}{5} - \frac{6}{5} \times \frac{13}{20}$

$= \frac{1}{50}$