MPCS 58020 2015 Homework 4

Assignment date: Aug 12, 2015

Due date: Aug 26, 2015

Solve the following problems. Send solutions (programs, results, and analytical results) to the TA (rahaman@cs.uchicago.edu) with the subject line "[MPCS 58020 2015] HW3".

1. Create a 100 point discretely sampled Gaussian filter $g(t) = e^{-t^2}$ evaluated over $-3 \leq t \leq 3$. Also, create an ARMA(2,1) process x_t with $\phi_1 = .5$, $\phi_2 = .1$, and $\theta = .5$. Using this signal x_t and filter g_t, do the following:

   - Calculate the discrete convolution $x * g$ using periodic boundary conditions with a hand-coded routine (no convolution libraries) that carries out the calculation in physical space. Do this for signals of length 1000, 10000, 100000, and 1000000. Time your results.
   - Repeat in Fourier space. Verify that your results are the same and compare performance. You may use an FFT package in the language of your choice.

2. Derive an expression for the Fourier Transform of the following functions. Show all work.

   (a) $f(t) = Ae^{-at^2}$ where a and A are constants

   (b) $f(t) = sin(2\pi\omega t) + cos(2\pi\omega t)$

   (c) $\gamma(t) = \int_{-\infty}^{\infty} f(t')f(t+t')dt'$

   (d) $f(t) = \begin{cases} \frac{1}{a} & \text{for } -a/2 \leq t \leq a/2 \\ 0 & \text{otherwise} \end{cases}$

3. Create an instance x_t of an invertible ARMA(2,2) process with coefficients of your choice using $n = 10000$. Filter the process using a physical space filter defined as $-2te^{-t^2/2}$ (using either convolution or FFT). Submit the

following plots: a) x_t, b) filtered x_t, c) the filter in physical space, d) the filter in Fourier space. Explain what you observe in (b).

4. Derive an analytic expression for the autocovariance function for the ARMA(2,2) process defined in the previous problem. Compare a plot of this expression vs. the sample autocovariance from your sample signal x_t.

5. Show the calculation to determine if the following ARMA(2,2) process invertible:
$$\phi(B) = 1 - .2B + .5B^2$$
$$\theta(B) = 1 + .5B + .9B^2$$

6. Shumway problem 4.2

7. Shumway problem 4.4

8. Shumway problem 4.8

9. Shumway Problem 4.9

10. ~~Shumway Problem 4.12~~