Random variables

- Examples: UV index, temperature, hair color, skin color, coin flip result etc..

- Probability distribution

  - Definition: How likely a random variable (or set of RVs) assumes each of its possible values. We call this a probability density function (pdf) for continuous RVs, and a probability mass function (pmf) for discrete valued RVs. In this class we are primarily concerned with PMFs.

  - A PMF, $P(x)$, must be:

    - Bounded: $\forall x \ in \ \mathrm{X} : 0 \leq P(x) \leq 1$

    - Normalized: $\sum_{x \in \mathrm{X}} P(x) = 1$

Gaussian distribution

- Univariate:
$$N(x; \mu, \sigma^2) = (2\pi\sigma^2)^{-1/2} exp(-\frac{(x-\mu)^2}{2\sigma^2})$$

- Multivariate:
$$N(\mathbf{x}; \boldsymbol{\mu}, \boldsymbol{\Sigma}) = \sqrt{\frac{1}{(2\pi)^N \det \boldsymbol{\Sigma}}} \exp(-\frac{1}{2}(\mathbf{x}-\boldsymbol{\mu})^T \boldsymbol{\Sigma}^{-1}(\mathbf{x}-\boldsymbol{\mu})) \quad where \quad \mathbf{x} \in \mathbb{R}^N$$