



- A full rank matrix,  $\mathbf{A}$ , is one in which has  $N$  linearly independent column vectors. Two key implications arise from this:
  - Transforming the vectors space spanning  $\mathbb{R}^N$ , the resultant set also spans  $\mathbb{R}^N$ . So we say that the matrix  $\mathbf{A}$  is invertible.
  - A logical extension is that  $\mathbf{A}$  is a unique mapping, i.e.  $\mathbf{y} = \mathbf{A}\mathbf{x}$  has a unique solution,  $\mathbf{x}$ , for all  $\mathbf{y}$ .
- Matrices with  $\text{rank} < N$  are referred to as *singular*.

**Matrixrank**



$$AB = \sum_{k=0}^{M-1} a_{i,k} b_{k,j}$$

$$\begin{bmatrix} A_{1,1} & A_{1,2} & A_{1,3} \\ A_{2,1} & A_{2,2} & A_{2,3} \\ A_{3,1} & A_{3,2} & A_{3,3} \end{bmatrix} = \begin{bmatrix} 1 & 2 & 7 \\ 0.5 & 5 & 15.5 \\ 1 & 3 & 10 \end{bmatrix}$$

R

N

R

N



ARNXN

Aspans RN

**A** **X**

**=**

**y**

singular matrix?

$$\begin{bmatrix} A_{1,1} & A_{1,2} & A_{1,3} \\ A_{2,1} & A_{2,2} & A_{2,3} \\ A_{3,1} & A_{3,2} & A_{3,3} \end{bmatrix} = \begin{bmatrix} 1 & 6 & 7 \\ 0.5 & 5 & 15.5 \\ 1 & 3 & 10 \end{bmatrix}$$

singular matrix?

ERN

$$1A.:_1 + 3A.:_2 = A.:_3$$



Singular. 3rd col is linear combination of first two

Full rank, all columns are linearly independent

# Matrix rank

- A full rank matrix,  $\mathbf{A} \in \mathbb{R}^{N \times N}$ , is one in which has  $N$  linearly independent column vectors. Two key implications arise from this:
  - Transforming the vectors space spanning  $\mathbb{R}^N$ , the resultant set also spans  $\mathbb{R}^N$ . So we say that the matrix  $\mathbf{A}$  spans  $\mathbb{R}^N$ .
  - A logical extension is that  $\mathbf{A}$  is a unique mapping, i.e.  $\mathbf{Ax} = \mathbf{y}$  has a unique solution,  $\mathbf{x}$ , for all  $\mathbf{y} \in \mathbb{R}^N$ .
- Matrices with rank  $< N$  are referred to as *singular*.

Singular matrix?

$$\begin{bmatrix} A_{1,1} & A_{1,2} & A_{1,3} \\ A_{2,1} & A_{2,2} & A_{2,3} \\ A_{3,1} & A_{3,2} & A_{3,3} \end{bmatrix} = \begin{bmatrix} 1 & 2 & 7 \\ 0.5 & 5 & 15.5 \\ 1 & 3 & 10 \end{bmatrix}$$

Singular. 3rd col is linear combination of first two

$$1A_{:,1} + 3A_{:,2} = A_{:,3}$$

Singular matrix?

$$\begin{bmatrix} A_{1,1} & A_{1,2} & A_{1,3} \\ A_{2,1} & A_{2,2} & A_{2,3} \\ A_{3,1} & A_{3,2} & A_{3,3} \end{bmatrix} = \begin{bmatrix} 1 & 6 & 7 \\ 0.5 & 5 & 15.5 \\ 1 & 3 & 10 \end{bmatrix}$$

Full rank, all columns are linearly independent

# Eigendecomposition

- Definition:  $\mathbf{A}\mathbf{v} = \lambda\mathbf{v}$
- Decomposition:  $\mathbf{A} = \mathbf{Q}\mathbf{\Lambda}\mathbf{Q}^T$  where  $\mathbf{Q} = \begin{bmatrix} v_1^{(1)} & \dots & v_1^{(N)} \\ \vdots & \ddots & \vdots \\ v_N^{(1)} & \dots & v_N^{(N)} \end{bmatrix}$  and  $\mathbf{\Lambda} = \begin{bmatrix} \lambda_1 & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & \lambda_N \end{bmatrix}$
- Applies only to square matrices

