

Maximum likelihood estimation

- Estimates the parameters, θ , of a distribution using a likelihood function, $\mathcal{L}(\mathbf{D}; \theta)$, given some data, \mathbf{D} .

- We maximize the likelihood function by minimizing its -logarithm:

$$\mathcal{L}(\theta | \mathbf{D}) = P(\mathbf{D}; \theta)$$

$$= \left(\prod_{i=1}^M P(\mathbf{y}_i | \mathbf{x}_i; \theta) \right)^{\frac{1}{M}}$$

$$= \frac{1}{M} \sum_{i=1}^M \log P(\mathbf{y}_i | \mathbf{x}_i; \theta)$$

Note: technically natural log, but true to within a constant

$$\hat{\theta} = \underset{\theta}{\operatorname{argmin}} \sum_{i=1}^M -\log P(\mathbf{y}_i | \mathbf{x}_i; \theta)$$

- This expresses an optimization problem; the form of $p(\mathbf{D}; \theta)$ dictates how we solve it
 - In deep learning, this function is a neural network; we compute its gradient w.r.t. θ , and then estimate $\hat{\theta}$ using stochastic gradient descent (SGD).



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Refresher: Neural Networks