

Expected value and covariance functions

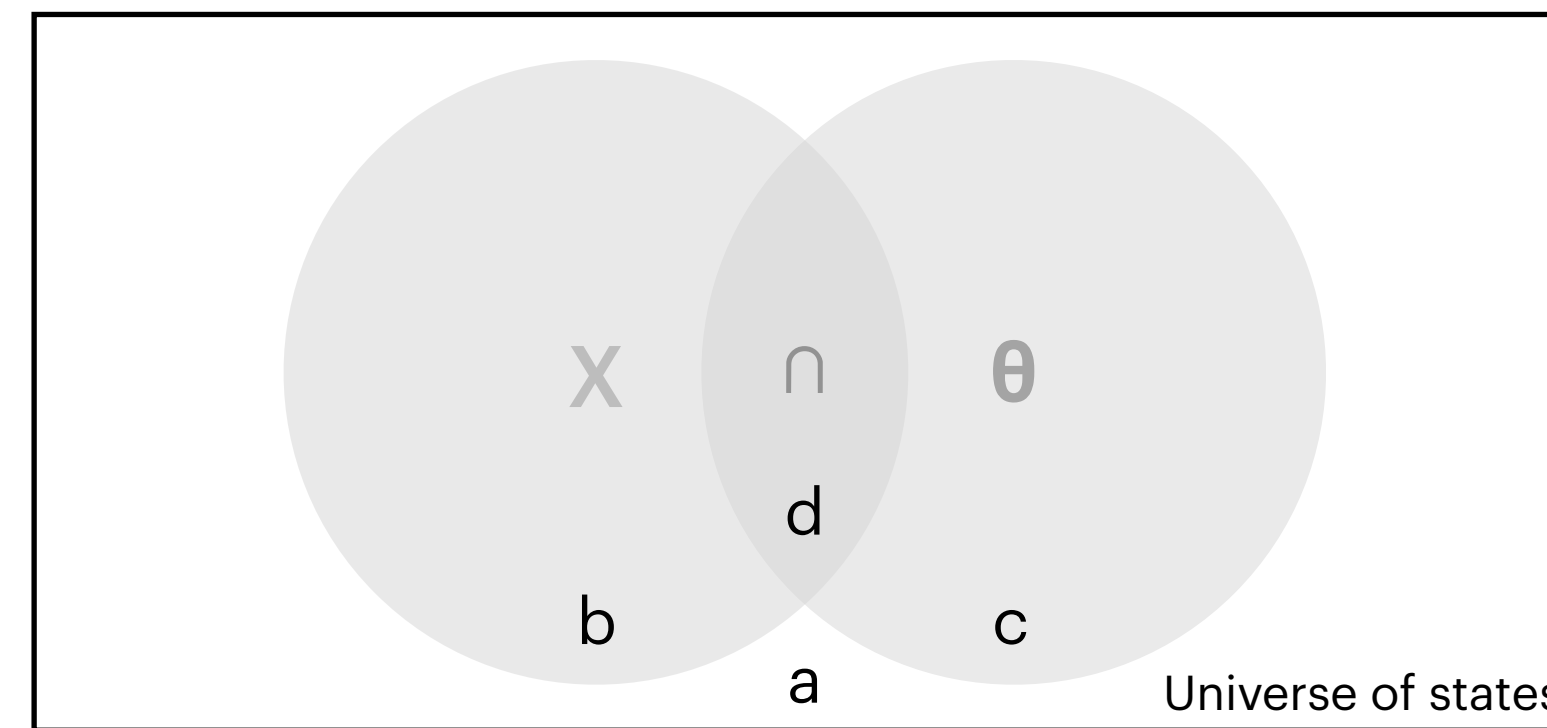
- Expectation: $\mathbb{E}_{x \sim P}[f(x)] = \sum_x P(x)f(x)$
- Variance: $Var(f(x)) = \mathbb{E}_x[(f(x) - \mathbb{E}[f(x)])^2]$
- Covariance: $Cov(f_1(x), f_2(x)) = \mathbb{E}[(f_1(x) - \mathbb{E}[f_1(x)]) \cdot (f_2(x) - \mathbb{E}[f_2(x)])]$
- Covariance of random vector, \mathbf{x} :
$$Cov(\mathbf{x}, \mathbf{x}) = \mathbb{E}[(\mathbf{x} - \mathbb{E}[\mathbf{x}]) (\mathbf{x} - \mathbb{E}[\mathbf{x}])^T]$$
$$= \mathbb{E}[\mathbf{x} \mathbf{x}^T - \mathbb{E}[\mathbf{x}] \mathbb{E}[\mathbf{x}]^T]$$

Bayes' Rule

- Intuitive statement: $P(x|\theta)P(\theta) = P(\theta|x)P(x)$

- Proof

$$\frac{\cancel{d}}{\cancel{c}} \cdot \frac{\cancel{c}}{a} = \frac{\cancel{d}}{\cancel{b}} \cdot \frac{\cancel{b}}{a} = \frac{d}{a}$$



- Bayes' Rule follows by deduction: $P(\theta|x) = \frac{P(x|\theta)P(\theta)}{P(x)}$ $posterior = \frac{likelihood \times prior}{evidence}$
- Prescription for how to update a model given new evidence (i.e., new data)