Product rule, independence, conditional independence

- Product rule: $P(x^{(1)}, \ldots, x^{(n)}) = P(x^{(1)}) \prod_{i=2}^{n} P(x^{(i)} \mid x^{(1)}, \ldots, x^{(i-1)})$

- Independence condition: $P(x, y) = P(x)P(y)$

- Conditional independence condition: $P(x, y \mid z) = P(x \mid z)\, P(y \mid z)$

Expected value and covariance functions

- Expectation: $$\mathbb{E}_{x \sim P}[f(x)] = \sum_x P(x) f(x)$$

- Variance: $$Var(f(x)) = \mathbb{E}_x\left[\left(f(x) - \mathbb{E}[f(x)]\right)^2\right]$$

- Covariance: $$Cov(f_1(x), f_x(x)) = \mathbb{E}\left[\left(f_1(x) - \mathbb{E}[f_1(x)]\right) \cdot \left(f_2(x) - \mathbb{E}[f_2(x)]\right)\right]$$

- Covariance of random vector, ***x***: $$\begin{aligned} Cov(\mathbf{x}, \mathbf{x}) &= \mathbb{E}\left[(\mathbf{x} - \mathbb{E}[\mathbf{x}])(\mathbf{x} - \mathbb{E}[\mathbf{x}])^T\right] \\ &= \mathbb{E}\left[\mathbf{x}\mathbf{x}^T - \mathbb{E}[\mathbf{x}]\mathbb{E}[\mathbf{x}]^T\right] \end{aligned}$$