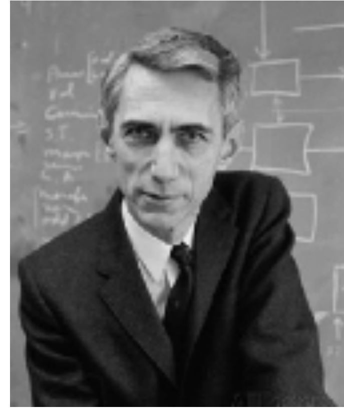
Similarity measures between distributions



Claude Shannon

- Shannon postulated that any measure of the informativeness of an event, x, should satisfy three conditions:
    1. An event with probability 1 yields no information
    2. The probability of an event and the information it yields vary inversely with each other
    3. The total information coming from independent events is purely additive

- Which he used to define self-information: $I(x) = -\log P(x)$

- Shannon entropy: $H(P) = \mathbb{E}_{x\sim P}[I(x)] = -\mathbb{E}_{x\sim P}[\log P(x)]$

- Kullback-Leibler (KL) divergence: $D_{KL}(P||Q) = \mathbb{E}_{x\sim P}\left[\log \frac{P(x)}{Q(x)}\right]$

- Cross entropy: $H(P, Q) = H(P) + D_{KL}(P||Q) = -\mathbb{E}_{x\sim P}[\log Q(x)]$

Maximum likelihood estimation

- Estimates the parameters, $\boldsymbol{\theta}$, of a distribution using a likelihood function, $\mathscr{L}(\mathbf{D}; \boldsymbol{\theta})$, given some data, ***D.***

- We maximize the likelihood function by minimizing its -logarithm:

$$\begin{aligned}\mathscr{L}(\boldsymbol{\theta} \mid \mathbf{D}) &= P(\mathbf{D}; \boldsymbol{\theta}) \\ &= \left(\prod_{i=1}^{M} P(\mathbf{y}_i \mid \mathbf{x}_i; \boldsymbol{\theta}) \right)^{\frac{1}{M}} \\ &= \frac{1}{M} \sum_{i=1}^{M} \log P(\mathbf{y}_i \mid \mathbf{x}_i; \boldsymbol{\theta})\end{aligned}$$

Note: technically natural log, but true to within a constant

$$\hat{\boldsymbol{\theta}} = \underset{\boldsymbol{\theta}}{\operatorname{argmin}} \sum_{i=1}^{M} -\log P(\mathbf{y}_i \mid \mathbf{x}_i; \boldsymbol{\theta})$$

- This expresses an optimization problem; the form of $p(\mathbf{D}; \boldsymbol{\theta})$ dictates how we solve it
  - In deep learning, this function is a neural network; we compute its gradient w.r.t. $\boldsymbol{\theta}$, and then estimate $\hat{\boldsymbol{\theta}}$ using stochastic gradient descent (SGD).