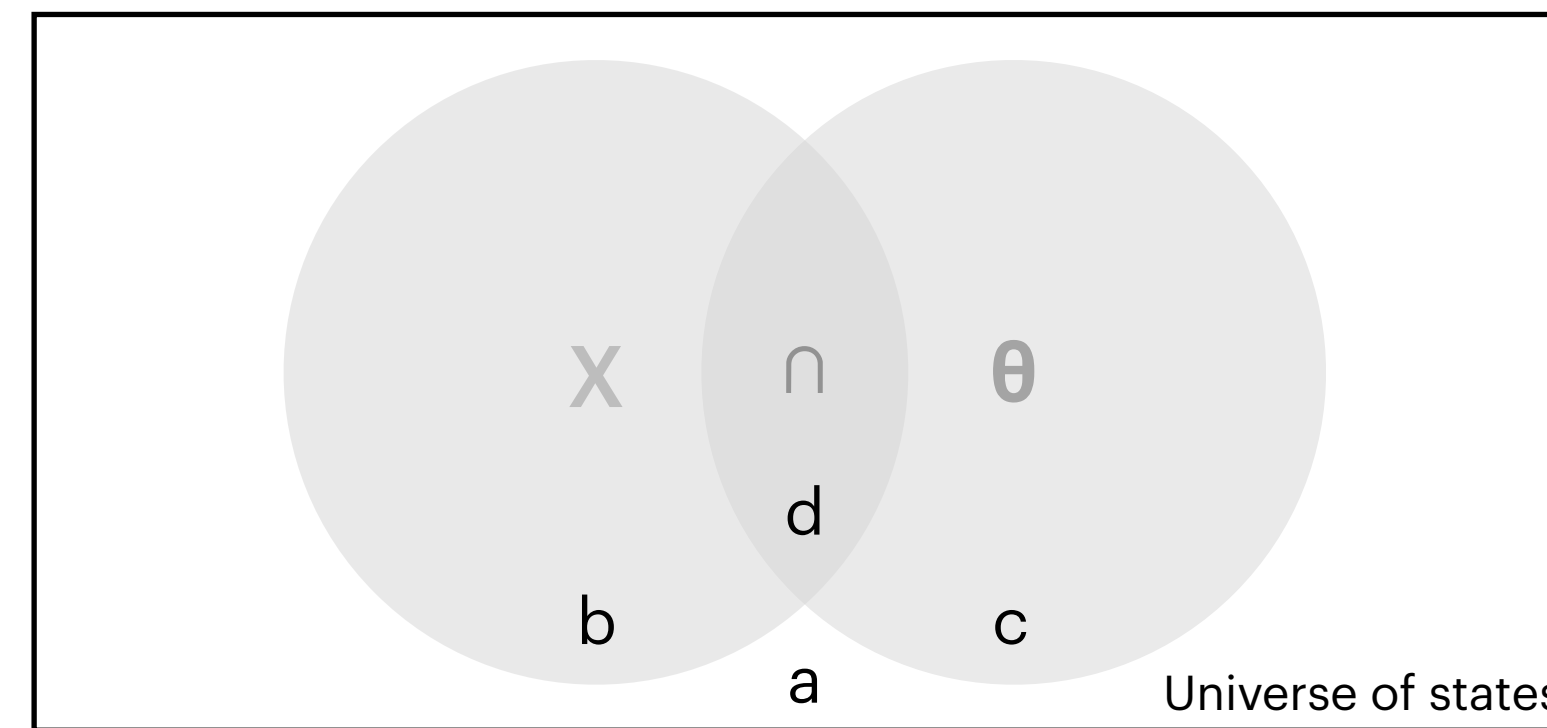


Bayes' Rule

- Intuitive statement: $P(x|\theta)P(\theta) = P(\theta|x)P(x)$

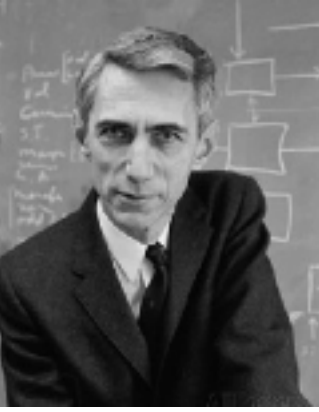
- Proof

$$\frac{\cancel{d}}{\cancel{c}} \cdot \frac{\cancel{c}}{a} = \frac{d}{\cancel{b}} \cdot \frac{\cancel{b}}{a} = \frac{d}{a}$$



- Bayes' Rule follows by deduction: $P(\theta|x) = \frac{P(x|\theta)P(\theta)}{P(x)}$ $posterior = \frac{likelihood \times prior}{evidence}$
- Prescription for how to update a model given new evidence (i.e., new data)

Similarity measures between distributions



Claude
Shannon

- Shannon postulated that any measure of the informativeness of an event, x , should satisfy three conditions:
 1. An event with probability 1 yields no information
 2. The probability of an event and the information it yields vary inversely with each other
 3. The total information coming from independent events is purely additive
- Which he used to define self-information: $I(x) = -\log P(x)$
- Shannon entropy: $H(P) = \mathbb{E}_{x \sim P}[I(x)] = -\mathbb{E}_{x \sim P}[\log P(x)]$
- Kullback-Leibler (KL) divergence: $D_{KL}(P || Q) = \mathbb{E}_{x \sim P} \left[\log \frac{P(x)}{Q(x)} \right]$
- Cross entropy: $H(P, Q) = H(P) + D_{KL}(P || Q) = -\mathbb{E}_{x \sim P}[\log Q(x)]$