Distance metrics

- The two distance metrics used most often in machine learning are the Manhattan (L1) and Euclidean (L2) distances, which can be defined using the Lp norm:

  - Manhattan: $$\|\mathbf{x_1} - \mathbf{x_2}\|_1 = \sum_i |x_i^{(1)} - x_i^{(2)}|$$

  - Euclidean: $$\|\mathbf{x_1} - \mathbf{x_2}\|_2 = \sqrt{\sum_i (x_i^{(1)} - x_i^{(2)})^2}$$

Matrix rank

- A full rank matrix, $\mathbf{A} \in \mathbb{R}^{N \times N}$, is one in which has *N* linearly independent column vectors. Two key implications arise from this:
  - Transforming the vectors space spanning $\mathbb{R}^N$, the resultant set also spans $\mathbb{R}^N$. So we say that the matrix $\mathbf{A}$ spans $\mathbb{R}^N$.
  - A logical extension is that ***A*** is a unique mapping, i.e. $\mathbf{A}\mathbf{x} = \mathbf{y}$ has a unique solution, ***x***, for all ***y*** $\in \mathbb{R}^N$.
- Matrices with rank < N are referred to as ***singular***.

Singular matrix?

$$\begin{bmatrix} A_{1,1} & A_{1,2} & A_{1,3} \\ A_{2,1} & A_{2,2} & A_{2,3} \\ A_{3,1} & A_{3,2} & A_{3,3} \end{bmatrix} = \begin{bmatrix} 1 & 2 & 7 \\ 0.5 & 5 & 15.5 \\ 1 & 3 & 10 \end{bmatrix}$$

Singular matrix?

$$\begin{bmatrix} A_{1,1} & A_{1,2} & A_{1,3} \\ A_{2,1} & A_{2,2} & A_{2,3} \\ A_{3,1} & A_{3,2} & A_{3,3} \end{bmatrix} = \begin{bmatrix} 1 & 6 & 7 \\ 0.5 & 5 & 15.5 \\ 1 & 3 & 10 \end{bmatrix}$$