

Product rule, independence, conditional independence

- Product rule:
$$P(x^{(1)}, \dots, x^{(n)}) = P(x^{(1)}) \prod_{i=2}^n P(x^{(i)} | x^{(1)}, \dots, x^{(i-1)})$$
- Independence condition:
$$P(x, y) = P(x)P(y)$$
- Conditional independence condition:
$$P(x, y | z) = P(x | z) P(y | z)$$

Expected value and covariance functions

- Expectation: $\mathbb{E}_{x \sim P}[f(x)] = \sum_x P(x)f(x)$
- Variance: $Var(f(x)) = \mathbb{E}_x[(f(x) - \mathbb{E}[f(x)])^2]$
- Covariance: $Cov(f_1(x), f_2(x)) = \mathbb{E}[(f_1(x) - \mathbb{E}[f_1(x)]) \cdot (f_2(x) - \mathbb{E}[f_2(x)])]$
- Covariance of random vector, \mathbf{x} :
$$Cov(\mathbf{x}, \mathbf{x}) = \mathbb{E}[(\mathbf{x} - \mathbb{E}[\mathbf{x}])(\mathbf{x} - \mathbb{E}[\mathbf{x}])^T]$$
$$= \mathbb{E}[\mathbf{x}\mathbf{x}^T - \mathbb{E}[\mathbf{x}]\mathbb{E}[\mathbf{x}]^T]$$