

- A full rank matrix, \mathbf{A} , is one in which has N linearly independent column vectors. Two key implications arise from this:
 - Transforming the vectors space spanning \mathbb{R}^N , the resultant set also spans \mathbb{R}^N . So we say that the matrix \mathbf{A} is invertible.
 - A logical extension is that \mathbf{A} is a unique mapping, i.e. $\mathbf{y} = \mathbf{A}\mathbf{x}$ has a unique solution, \mathbf{x} , for all \mathbf{y} .
- Matrices with $\text{rank} < N$ are referred to as *singular*.

Matrixrank

2

3

$$AB = \sum_{k=0}^{M-1} a_{i,k} b_{k,j}$$

$$\begin{bmatrix} A_{1,1} & A_{1,2} & A_{1,3} \\ A_{2,1} & A_{2,2} & A_{2,3} \\ A_{3,1} & A_{3,2} & A_{3,3} \end{bmatrix} = \begin{bmatrix} 1 & 2 & 7 \\ 0.5 & 5 & 15.5 \\ 1 & 3 & 10 \end{bmatrix}$$

R

N

R

N

ARNXN

A spans RN

A **X**

=

y

singular matrix?

$$\begin{bmatrix} A_{1,1} & A_{1,2} & A_{1,3} \\ A_{2,1} & A_{2,2} & A_{2,3} \\ A_{3,1} & A_{3,2} & A_{3,3} \end{bmatrix} = \begin{bmatrix} 1 & 6 & 7 \\ 0.5 & 5 & 15.5 \\ 1 & 3 & 10 \end{bmatrix}$$

singular matrix?

ERN

$$1A.:_1 + 3A.:_2 = A.:_3$$

Singular. 3rd col is linear combination of first two

Full rank, all columns are linearly independent

Matrix rank

- A full rank matrix, $\mathbf{A} \in \mathbb{R}^{N \times N}$, is one in which has N linearly independent column vectors. Two key implications arise from this:
 - Transforming the vectors space spanning \mathbb{R}^N , the resultant set also spans \mathbb{R}^N . So we say that the matrix \mathbf{A} spans \mathbb{R}^N .
 - A logical extension is that \mathbf{A} is a unique mapping, i.e. $\mathbf{Ax} = \mathbf{y}$ has a unique solution, \mathbf{x} , for all $\mathbf{y} \in \mathbb{R}^N$.
- Matrices with rank $< N$ are referred to as *singular*.

Singular matrix?

$$\begin{bmatrix} A_{1,1} & A_{1,2} & A_{1,3} \\ A_{2,1} & A_{2,2} & A_{2,3} \\ A_{3,1} & A_{3,2} & A_{3,3} \end{bmatrix} = \begin{bmatrix} 1 & 2 & 7 \\ 0.5 & 5 & 15.5 \\ 1 & 3 & 10 \end{bmatrix}$$

Singular. 3rd col is linear combination of first two

$$1A_{:,1} + 3A_{:,2} = A_{:,3}$$

Singular matrix?

$$\begin{bmatrix} A_{1,1} & A_{1,2} & A_{1,3} \\ A_{2,1} & A_{2,2} & A_{2,3} \\ A_{3,1} & A_{3,2} & A_{3,3} \end{bmatrix} = \begin{bmatrix} 1 & 6 & 7 \\ 0.5 & 5 & 15.5 \\ 1 & 3 & 10 \end{bmatrix}$$

Full rank, all columns are linearly independent

Eigendecomposition

- Definition: $\mathbf{A}\mathbf{v} = \lambda\mathbf{v}$
- Decomposition: $\mathbf{A} = \mathbf{Q}\mathbf{\Lambda}\mathbf{Q}^T$ where $\mathbf{Q} = \begin{bmatrix} v_1^{(1)} & \dots & v_1^{(N)} \\ \vdots & \ddots & \vdots \\ v_N^{(1)} & \dots & v_N^{(N)} \end{bmatrix}$ and $\mathbf{\Lambda} = \begin{bmatrix} \lambda_1 & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & \lambda_N \end{bmatrix}$
- Applies only to square matrices

