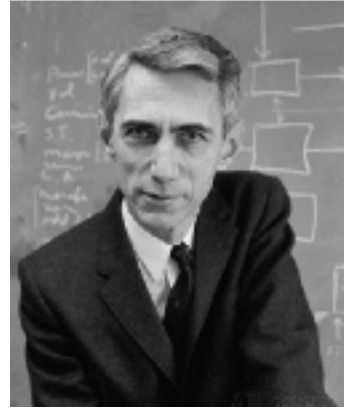
Similarity measures between distributions



Claude Shannon

- Shannon postulated that any measure of the informativeness of an event, *x*, should satisfy three conditions:
  1. An event with probability 1 yields no information
  2. The probability of an event and the information it yields vary inversely with each other
  3. The total information coming from independent events is purely additive

  - Which he used to define *self-information*: $I(x) = -\log P(x)$

- Shannon entropy: $H(P) = \mathbb{E}_{x\sim P}[I(x)] = -\mathbb{E}_{x\sim P}[\log P(x)] = -\sum_{x\sim P} P(x)\log P(x)$

- Kullback-Leibler (KL) divergence: $D_{KL}(P||Q) = \mathbb{E}_{x\sim P}\left[\log \frac{P(x)}{Q(x)}\right]$

- Cross entropy: $H(P,Q) = H(P) + D_{KL}(P||Q) = -\mathbb{E}_{x\sim P}[\log Q(x)] = -\sum_{x\sim P} P(x)\log Q(x)$

Maximum likelihood estimation

- Estimates the parameters, $\boldsymbol{\theta}$, of a distribution using a likelihood function, $\mathscr{L}(\mathbf{D};\boldsymbol{\theta})$, given some data, ***D.***

- We maximize the likelihood function by minimizing its -logarithm:

$$\mathscr{L}(\boldsymbol{\theta}\,|\,\mathbf{D}) = P(\mathbf{D};\boldsymbol{\theta})$$

$$= \left(\prod_{i=1}^{M} P(\mathbf{y}_i\,|\,\mathbf{x}_i;\boldsymbol{\theta})\right)^{\frac{1}{M}}$$

$$= \frac{1}{M}\sum_{i=1}^{M} \log P(\mathbf{y}_i\,|\,\mathbf{x}_i;\boldsymbol{\theta})$$

Note: technically natural log, but true to within a constant

$$\hat{\boldsymbol{\theta}} = \underset{\boldsymbol{\theta}}{\operatorname{argmin}} \sum_{i=1}^{M} -\log P(\mathbf{y}_i\,|\,\mathbf{x}_i;\boldsymbol{\theta})$$

- This expresses an optimization problem; the form of $p(\mathbf{D};\boldsymbol{\theta})$ dictates how we solve it
  - In deep learning, this function is a neural network; we compute its gradient w.r.t. $\boldsymbol{\theta}$, and then estimate $\hat{\boldsymbol{\theta}}$ using stochastic gradient descent (SGD).