

# Matrix Completion

# Objectives

- define the matrix completion problem
- approach missing data using low-rank models
- introduce iterative singular value thresholding

Use "patterns" to fill in missing entries 2

Ratings

matrix

$$\underline{X} \in \mathbb{R}^{N \times m}$$

5	4	9	1	X	N movies
9	6	X	X	7	
X	10	X	2	4	
3	7	3	X	X	
8	X	X	6	2	

m users →

We need to infer them from  
the existing data (predict)

Can we predict the missing entries?

★ Model: assume  $\underline{X}$  is well approximated with a small number of patterns

$$\underline{X} \approx \sum_{i=1}^r \underline{t}_i \underline{s}_i^T = \underline{T} \underline{S}$$

→ small number of factors affecting the ratings  
genres, actors, director...  
hobbies, age, address...

Matrix completion: use known data to find patterns and predict missing entries 3

$\Omega = \{(i,j) : x_{ij} \text{ given}\}$  indices of known values

☆ (low-rank 指明用户行为可以被少数几个基本模式“解释”)( $M$  需要和  $X$  中的已知值一样, rest of them should lead to rank 3)

1) Rank minimization  $\underline{X} = \underset{\underline{M}}{\operatorname{argmin}} \operatorname{rank}(\underline{M})$  s.t.  $\underline{M}_{ij} = \underline{x}_{ij} \quad \forall i, j \in \Omega$

minimum number of patterns matching given values  
Intractable!

2) Nuclear norm minimization

$\underline{X} = \underset{\underline{M}}{\operatorname{argmin}} \|\underline{M}\|_* \quad \text{s.t. } \underline{M}_{ij} = \underline{x}_{ij} \quad \forall i, j \in \Omega$

subject to

Computationally tractable

Recommended

Nuclear / trace norm

$$\|\underline{M}\|_* = \sum_k \sigma_k$$

(sum of singular values is the nuclear norm)

↓ And we want to minimize the sum

# Iterative Singular Value Thresholding

is one possible algorithm

Initialize

$$\underline{M}^{(0)} = \underline{0}$$

Initialize  $M$  to all-zero  
Set threshold or  $r$       num of ranks

Iterate

$$\text{for } k = 1, 2, 3, \dots$$

$$\underline{M}^{(k)} = \underline{M}^{(k-1)}$$

$$\underline{M}_{\text{sr}}^{(k)} = \underline{X}_{\text{sr}} \quad (\text{fill in known values})$$

$$[\underline{U}, \Sigma, \underline{V}] = \text{svd}(\underline{M}^{(k)})$$

$$\hat{\Sigma}_{ii} = \Sigma_{ii} \cdot \begin{cases} 1 & \Sigma_{ii} > \text{threshold} \\ 0 & \Sigma_{ii} \leq \text{threshold} \end{cases}$$

弱模式变成0  
强模式变成1

- or -

$$\hat{\Sigma}_{ii} = \begin{cases} \Sigma_{ii}, & i \leq r \\ 0, & i \geq r+1 \end{cases}$$

第  $k$  轮的最佳猜测

$$\underline{M}^{(k)} = \underline{U} \left( \sum_{i=1}^r \hat{\Sigma}_{ii} \right) \underline{V}^T$$

消退过的新  $\Sigma$

$$\text{if } \|\underline{M}^{(k)} - \underline{M}^{(k-1)}\|_F < \varepsilon$$

stop

else

next  $k$

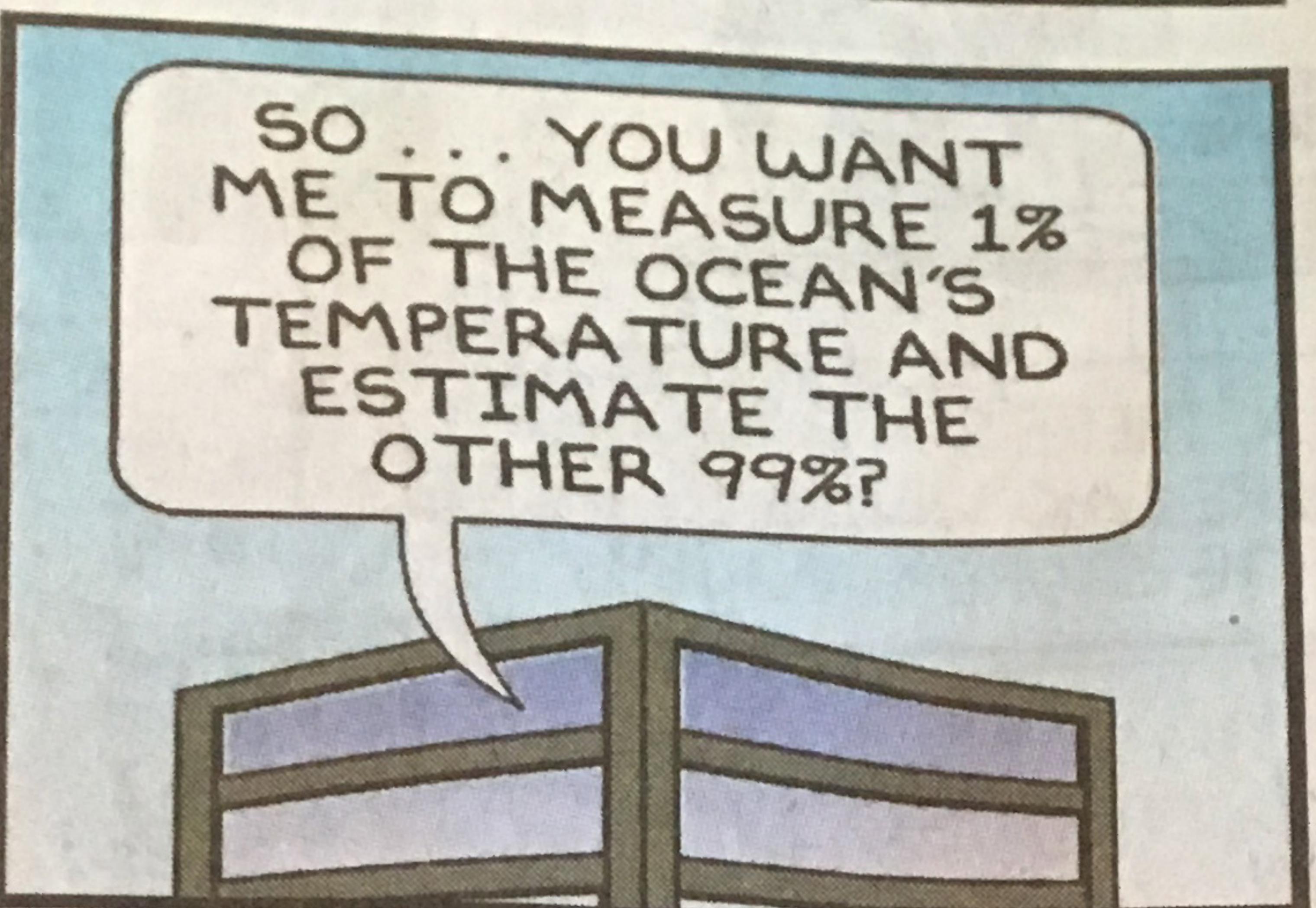
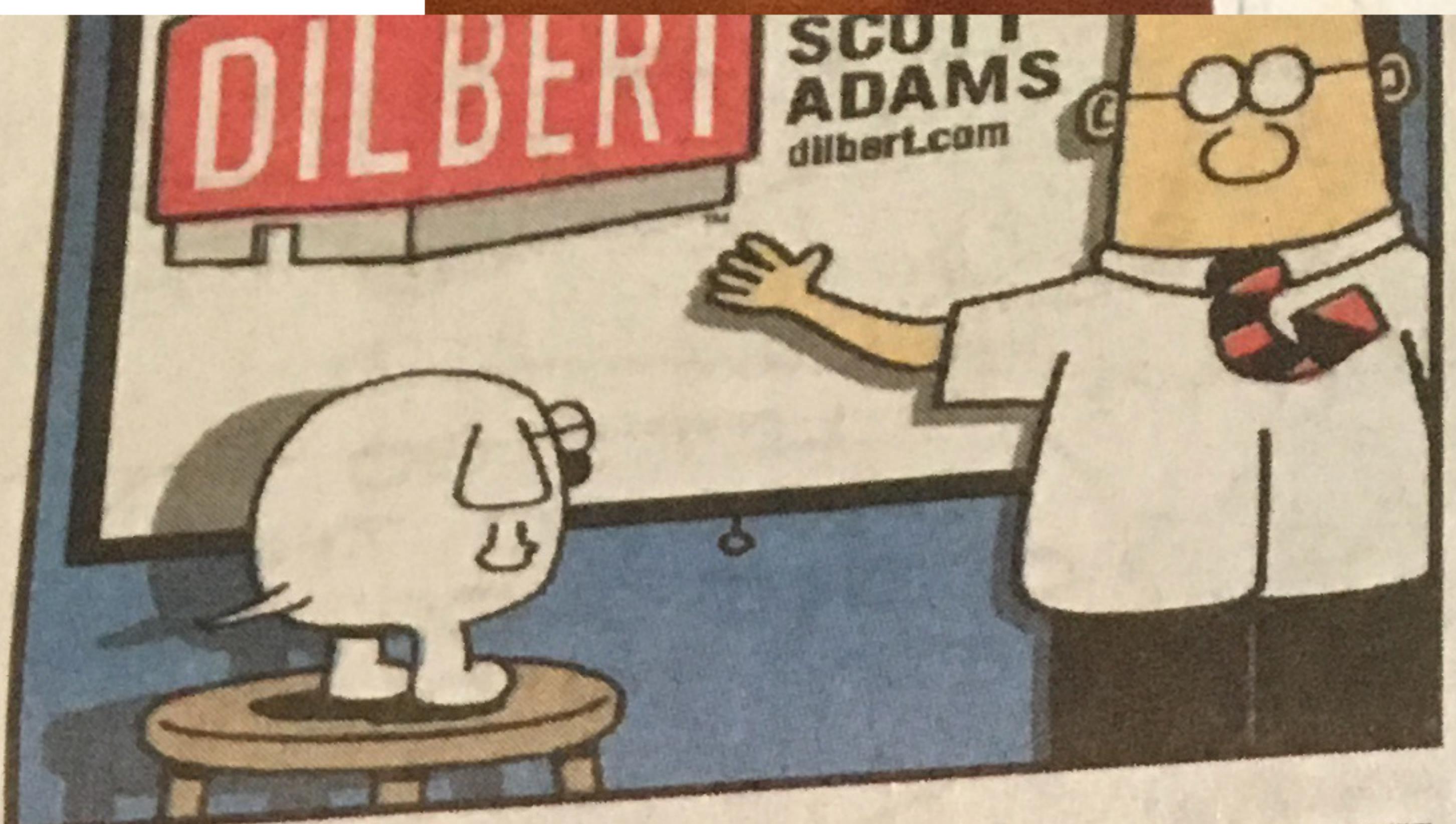
↓  
把上轮的  $M^{(k)}$  中上半  
的  $M^{(k)}$  相比，如果  
变化很小说明 converge  
停止，否则继续

Matrix completion is an open problem

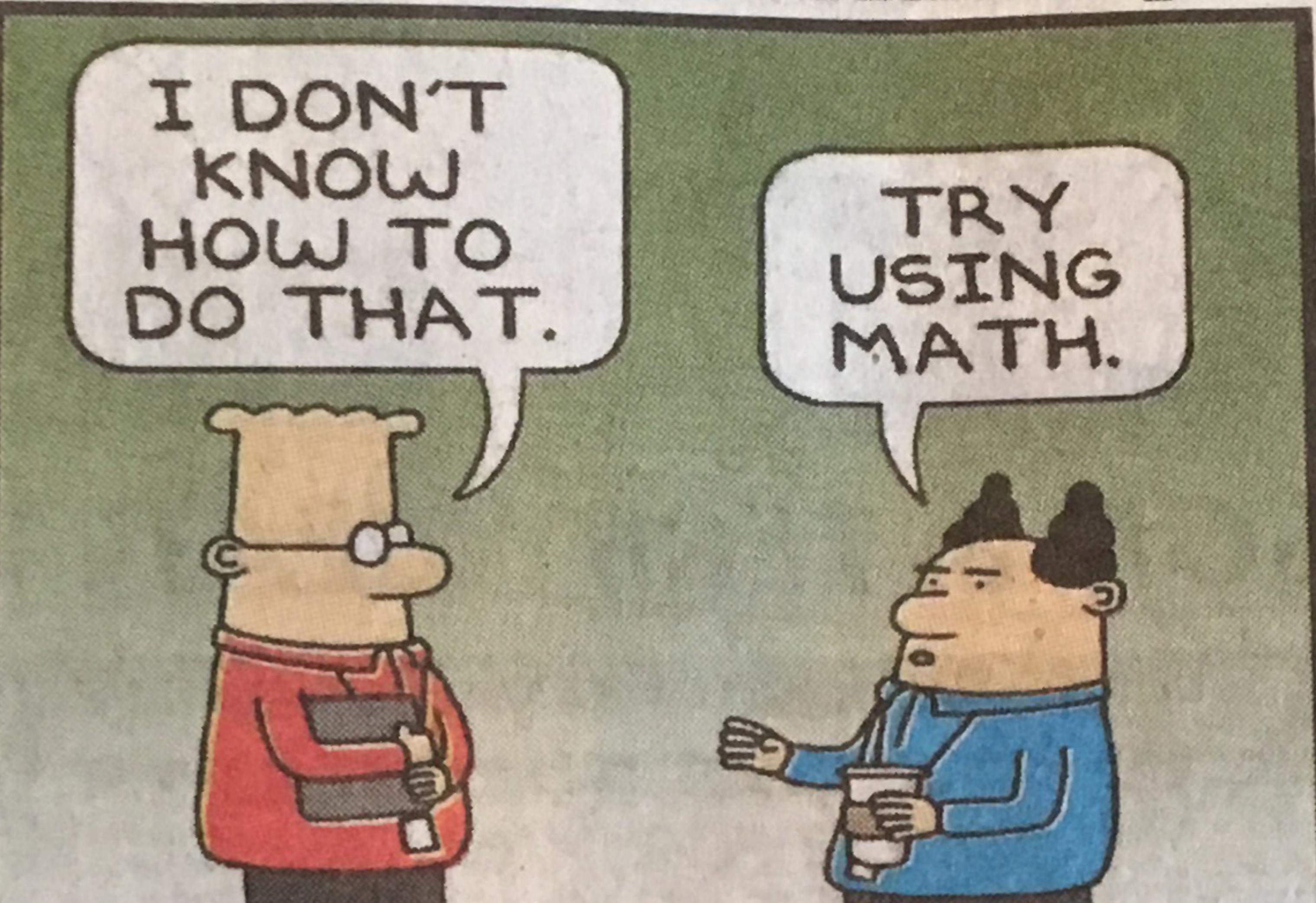
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- choosing  $r$  or threshold in ISVT
- multiple algorithms:
  - convergence
  - complexity
  - noise
- results depend on distribution of missing entries
- applications include missing pixels in images, position from partial distance info, ...

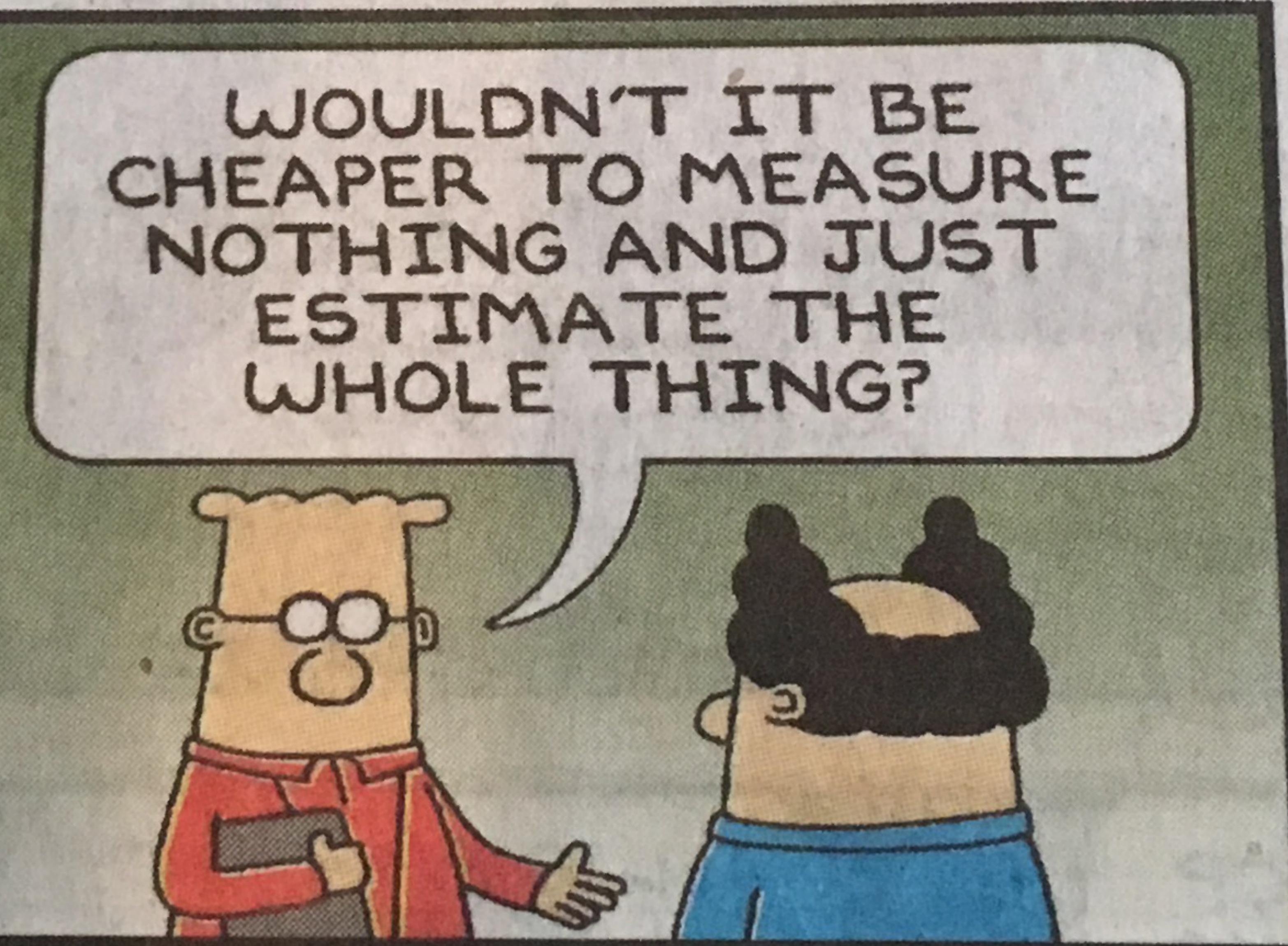
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3-10-19

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