Intervals on Projective Reals

by Sven Nilsen, 2017

In path semantics one uses functions to describe sub-types. The major reason for this is to study intrinsic behavior and semantics of mathematics, as a complementary technique to dependently typed systems that adds information at type level that is not necessarily intrinsic. Path semantics is less confusing than some other mathematical notation because its structure is naturally occurring. The way mathematics is applied in e.g. engineering and normal programming on the other hand, uses often vague concepts that works because they are applied in a specific context. When the boundary of this context is explored, one quickly discovers that semantics matters. This is the case for projective reals, which are an extension of real numbers with a point at infinity.

The real numbers are a sub-type of projective reals:

real <=> proj_real ∧ (¬= ∞)

By the law of sub-type extension:

real ∨ (= ∞) <=> proj_real

Extending the sub-type means that constraints on real numbers are not the same as constraints on projective reals. For example, when one says “all real numbers”, it is not the same as “all projective real numbers” since the latter includes infinity.

This can get confusing for people when learning about projective reals for the first time, because they might get introduced to them e.g. through “posits” and “valids” as a different way of encoding floats. The common usage of floats is to approximate real numbers, but since infinity is treated differently in floats and posits, one has to understand the values that lies beyond the range of normal numbers.

Consider the following interval:

(-∞, +∞)

This interval describes all real numbers that are between negative infinity and positive infinity.

In the traditional way of thinking about numbers, the one commonly taught in school, negative infinity and positive infinity are two different values. However, in the theory of projective reals, negative infinity and positive infinity are located at one and the same point!

(∞, ∞)

Intervals on the projective reals behaves like a train moving on a circular track: It starts from one point and moves forward until it hits the end point. The interval above is equal to all the real numbers.

real <=> (∞, ∞)
proj_real <=> [∞, ∞] <=> [∞, ∞) <=> (∞, ∞]