Type Checking Variable With Multiple Constraints

by Sven Nilsen, 2017

When you have a variable `a` with multiple constraints:

$$a : [f]\ b \wedge [g]\ c$$

$$f : A \to B$$
$$g : A \to C$$

This is equivalent to:

$$a : [f\{[g]\ c\}]\ b \wedge [g\{[f]\ b\}]\ c$$

When type checking, one must find the constrained existential paths:

$$b : [\exists f\{[g]\ c\}]\ \text{true}$$
$$c : [\exists g\{[f]\ b\}]\ \text{true}$$

Here is another example with 3 constraints:

$$a : [f]\ b \wedge [g]\ c \wedge [h]\ d$$

$$a : [f\{[g]\ c \wedge [h]\ d\}]\ b \wedge [g\{[f]\ b \wedge [h]\ d\}]\ c \wedge [h\{[f]\ b \wedge [g]\ c\}]\ d$$

$$b : [\exists f\{[g]\ c \wedge [h]\ d\}]\ \text{true}$$
$$c : [\exists g\{[f]\ b \wedge [h]\ d\}]\ \text{true}$$
$$d : [\exists h\{[f]\ b \wedge [g]\ c\}]\ \text{true}$$

$$b : [\exists f\{[g\{[h]\ d\}]\ c \wedge [h\{[g]\ c\}]\ d\}]\ \text{true}$$
$$c : [\exists g\{[f\{[h]\ d\}]\ b \wedge [h\{[f]\ b\}]\ d\}]\ \text{true}$$
$$d : [\exists h\{[f\{[g]\ c\}]\ b \wedge [g\{[f]\ b]\ c\}]\ \text{true}$$