

# Sub-Types as Contextual Notation

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In equational form, one can not direct the reader's attention toward some variables:

$$\text{add}(a, b) = c$$

In path semantics, one can define a sub-type in a way to focus on any “natural” part of the equation:

$$\begin{aligned} a &: [\text{add } b] \, c \\ b &: [\text{add}(a)] \, c \\ c &: (= \text{add}(a, b)) \\ (a, b) &: [\text{add}] \, c \\ \text{add}(a, b) &: (= c) \end{aligned}$$

This syntax has an advantage when using multiple equations:

$$\begin{aligned} \text{add}(a, b) &= c \\ \text{even}(a) &= \text{true} \\ \text{even}(b) &= \text{true} \\ \text{even}(c) &= \text{true} \end{aligned}$$

The equations above can be reduced to a single line in path semantics:

$$\text{add}(a : \text{even}, b : \text{even}) = c : \text{even}$$

Although much rarer, one can also use nested form to express the same equations:

$$b : \text{even} \wedge [\text{add}(a : \text{even})] \, c : \text{even}$$