Existential Path of `if` expression

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The existential path of `if` uses the following rule:

$$\exists(\backslash(x) = \text{if } f_1(x) \ \{ \ f_0(x) \ \}) \iff \exists f_0\{f_1\}$$

This is a partial function, and a shorthand for:

$$\exists(\backslash(x) = \text{if } f_1(x) \ \{ \ f_0(x) \ \}) \iff \exists f_0\{[f_1] \text{ true}\}$$

If you have an if-else expression:

$$\exists(\backslash(x) = \text{if } f_1(x) \ \{ \ f_0(x) \ \} \text{ else } \{ \ f_2(x) \ \}) \iff \exists f_0\{f_1\} \lor \exists f_2\{\neg f_1\}$$

This is a total function, and a shorthand for:

$$\exists(\backslash(x) = \text{if } f_1(x) \ \{ \ f_0(x) \ \} \text{ else } \{ \ f_2(x) \ \}) \iff \backslash(x') = (\exists f_0\{f_1\})(x') \lor (\exists f_2\{\text{not} \cdot f_1\})(x')$$

For example:

$$\begin{aligned}
&\exists(\backslash(x) = \text{if even}(x) \ \{ \ \text{add}(1, x) \ \} \text{ else } \{ \ \text{add}(2, x) \ \}) \\
&\backslash(x') = (\exists \text{add}(1)\{\text{even}\})(x') \lor (\exists \text{add}(2)\{\text{not} \cdot \text{even}\})(x') \\
&\backslash(x') = (\exists \text{add}(1)\{\text{linear}(0, 2)\})(x') \lor (\exists \text{add}(2)\{\text{not} \cdot \text{even}))(x') \\
&\backslash(x') = (\exists \text{add}(1)\{\text{linear}(0, 2)\})(x') \lor (\exists \text{add}(2)\{\text{odd})(x') \\
&\backslash(x') = (\exists \text{add}(1)\{\text{linear}(0, 2)\})(x') \lor (\exists \text{add}(2)\{\text{linear}(1, 2)\})(x') \\
&\backslash(x') = \text{linear}(1, 2)(x') \lor \text{linear}(3, 2)(x') \\
&\backslash(x') = \text{odd}(x') \lor \text{linear}(3, 2)(x')
\end{aligned}$$

$$\begin{aligned}
&\text{even} \iff \text{linear}(0, 2) \\
&\text{odd} \iff \text{linear}(1, 2) \\
&\text{linear}(a, b: (> 0)) = \backslash(x) = \text{if } x < a \ \{ \text{ false } \} \text{ else } \{ \ ((x - a) \ \% \ b) == 0 \ \}
\end{aligned}$$

When you have nested if expressions, the output is constrained by combining the conditions with the `∧` operator (logical AND):

$$\exists(\backslash(x) = \text{if } f_1(x) \ \{ \text{ if } f_2(x) \ \{ \ f_0(x) \ \} \ \}) \iff \exists f_0\{f_1 \land f_2\}$$

In an else-if expression constraints are set by chaining the conditions with the `∧` operator:

$$\begin{aligned}
\exists(\backslash(x) = \ &\text{if } f_1(x) \ \{ \ f_0(x) \ \} \\
&\text{else if } f_3(x) \ \{ \ f_2(x) \ \} \\
&\text{else if } f_5(x) \ \{ \ f_4(x) \ \} \\
&\text{else } \{ \ f_6(x) \ \} \\
) \iff \ &\exists f_0\{f_1\} \lor \exists f_2\{\neg f_1 \land f_3\} \lor \exists f_4\{\neg f_1 \land \neg f_3 \land f_5\} \lor \exists f_6\{\neg f_1 \land \neg f_3 \land \neg f_5\}
\end{aligned}$$