Confidence Factors of Probabilistic Constraints

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A probabilistic constraint is a confidence in some evidence `C(A)` ("C" for confidence) supporting a variable having a certain sub-type. If that belief it is false, it means it is unknown whether the variable has the sub-type, falling back to probability `P(A)` in the absence of evidence. When updating beliefs on the evidence, it leads to an observed probability `P'(A)`:

$$P'(A) = C(A) + (1 - C(A)) \cdot P(A)$$

Solving for `C(A)`:

$$C(A) = (P'(A) - P(A)) / (1 - P(A))$$

One curious property of the range of `C(A)` when `P'(A)` is observed in the standard probability range from 0 to 1, is that it can take negative values:

$$C(A) = (0 - P(A)) / (1 - P(A))$$
$$P'(A) = 0$$
$$C(A) = -P(A) / (1 - P(A))$$
$$C(A) = P(A) / (P(A) - 1)$$

This is the lowest value of the valid range of `C(A)` and corresponds to when the constraint is never observed to be satisfied, interpreted as a "negative constraint".

The highest value of the valid range of `C(A)` is when the constraint is always observed to be satisfied:

$$C(A) = (1 - P(A)) / (1 - P(A))$$
$$P'(A) = 1$$
$$C(A) = 1$$

For example: Alice rolls a six-sided dice and reports the number of eyes to Bob, but her reports are not accurate. 60% of the time, Alice rolls another dice and reports the eyes of the second dice to Bob, instead of the first one that Bob asked her to do.

For an observer just wanting random numbers from 1-6, reporting a different dice is the exact same as rolling a single dice. However, Bob cares about whether the report tells the eyes from the first dice. He can trust Alice to report 40% of the eyes from the first dice, and otherwise there is a 1/6 chance that the second dice shows the same eyes as the first one.

$$P'(A) = C(A) + (1 - C(A)) \cdot P(A)$$
$$P'(A) = 0.4 + (1 - 0.4) \cdot 1/6$$
$$C(A) = 0.4$$
$$P(A) = 1/6$$
$$P'(A) = 0.5$$

Bob expects that 50% of the report contains true values for the first dice.

If Alice never reports the true values of the first dice, she would roll the first dice and then roll the second dice repeatedly until it showed a different number of eyes:

$$C(A) = P(A) / (P(A) - 1)$$
$$C(A) = 1/6 / (1/6 - 1)$$
$$P(A) = 1/6$$
$$C(A) = -0.2$$

The confidence Bob has in Alice reporting the first dice is -20%.

If Alice ignores the first dice completely, Bob would expect that some eyes on the second dice matches the first dice one out of six times:

$$P'(A) = C(A) + (1 - C(A)) \cdot P(A)$$
$$P'(A) = 0 + (1 - 0) \cdot P(A)$$
$$C(A) = 0$$
$$P(A) = 1/6$$
$$P'(A) = 1/6$$

Confidence factors are useful because one can convert to and from probabilities. It is in particular useful for constraints, since the absence of a constraint means it is unknown whether the constraint is satisfied. In a probabilistic path, the power-set of constraints are summed over to produce a probability of a function returning some output from the probability of the constraints.