Surprising Strong Assumptions

by Sven Nilsen, 2018

Alice and Bob are talking about logic.

> Alice: If Santa Claus is real, then snow exists.
>
> Bob: I agree.
>
> Alice: If snow exists, I still can not prove that Santa Claus is real.
>
> Bob: That sounds reasonable. I believe snow exists, but I do not not believe that Santa Claus is real.
>
> Alice: Instead of assuming that snow exists, I will weaken my assumption to either snow exists or Santa Claus is real, not both, but at least one of them. Can I prove that snow exists and Santa Claus is not real?
>
> Bob: No. A weaker assumption can not prove more conclusions.
>
> Alice: Got you! This is the same as saying that Santa Claus is not real and snow exists.
>
> Bob: Aww… are you sure?
>
> Alice: Check it for yourself using the `pocket_prover` library from crates.io. You do not need to know logical axioms to use it, because it checks the proof for every value.

Actually, Alice is making a stronger assumption here. The following logical statement is true for all values of `a` and `b`:

$$(a \to b) \wedge (a \neg= b) = \neg a \wedge b$$

Using the example from the conversation between Alice and Bob:

$$(\text{santa\_claus\_exists} \to \text{snow\_exists}) \wedge (\text{santa\_claus\_exists} \neg= \text{snow\_exists}) = \neg\text{santa\_claus\_exists} \wedge \text{snow\_exists}$$

Here is another hilarious example, proving that when you combine the assumption of plutomaniacs that rich people are happy, and the assumption of unhappy lottery millionaires that you are either rich or happy, it proves that everybody are poor and happy!

$$(\text{rich} \to \text{happy}) \wedge (\text{rich} \neg= \text{happy}) = \neg\text{rich} \wedge \text{happy}$$

What is going on here is an example of entangled functions. Assuming that you are rich, you are both happy and unhappy, which is impossible, so you can not be rich. Since you are not rich, you must be happy. The entanglement happens in a precise way such that the values can be determined.