Commutativity

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A commutative operator `g` for function `f` has the following property:

$$f[swap \rightarrow id] <=> f[g]$$

For example, multiplication of natural numbers has `id` as commutative operator:

$$mul[swap \rightarrow id] <=> mul[id]$$

Multiplication of non-zero real numbers has another commutative operator `inv`. This is an example of a “fake” commutative operator, since it also has `id` as commutative operator:

$$b \cdot a = 1/(1/a \cdot 1/b)$$

$$mul[swap \rightarrow id] <=> mul[inv]$$

Multiplication of square matrices has a commutative operator `transpose`:

$$BA = (A^T B^T)^T$$

$$mul[swap \rightarrow id] <=> mul[transpose]$$

Multiplication of invertible square matrices has also a commutative operator `inverse`. Notice that `id` is not a commutative operator here:

$$BA = (A^{-1} B^{-1})^{-1}$$

$$mul[swap \rightarrow id] <=> mul[inverse]$$

Anti-commutative multiplication has `neg`:

$$b \cdot a = - a \cdot b$$

$$mul[swap \rightarrow id] <=> mul[neg]$$