Differential Existential Paths

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A differential existential path is defined as following:

$$\forall f : A \to A \{ \exists' f \Leftrightarrow \exists(\backslash(x) = f(x) - x) \}$$

When there is a domain constraint:

$$\forall f : A \to A \{ \exists' f\{T_A\} \Leftrightarrow \exists(\backslash(x : T_A) = f(x) - x) \}$$

Differential existential paths are used to analyze changes in dynamical systems that has an iterative function as law for motion. Like existential paths, one can learn something simply by looking at the expression.

Here is a very simple function that increases one unit per step:

f := \(x : real) = x + 1
∃'f := \(x) = x == 1

The differential existential path returns `true` for `1` only because the function increases the value with `1` everywhere. From this we can derive the following rule:

a : [∃'f] true => a : (= 1)

Here is another example:

f := \(x : real) = x^2
∃'f := \(x) = x >= -1/4

The lowest value of `-1/4` comes from taking the derivative and finding the critical point and then evaluate the function. Since the function is unbounded upwards we know it is a minimum value:

$g(x) = f(x) - x = x^2 - x$
$g'(x) = 2x - 1$

$2x - 1 = 0$
$2x = 1$
$x = 1/2$

$(1/2)^2 - 1/2 = 1/4 - 1/2 = 1/4 - 2/4 = -1/4$