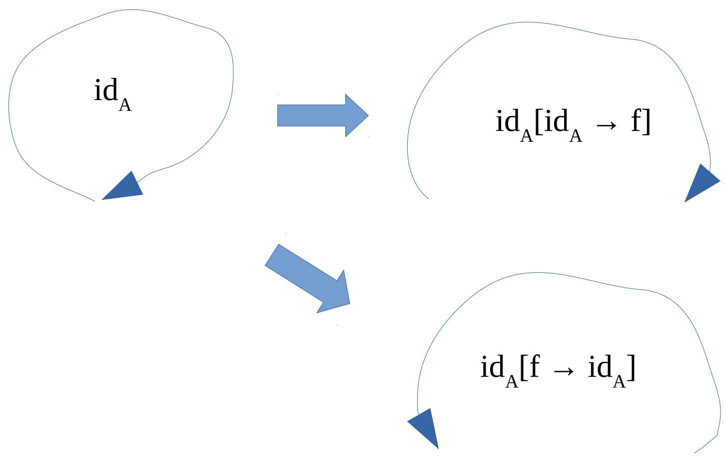
Asymmetric Inverse

by Sven Nilsen, 2017

Asymmetric path semantics can express operations on functions that connect, disconnect and reverse start and end points of paths.



These operations are valid for all functions that have inverses.

$f : A \rightarrow B$
$f^{-1} : B \rightarrow A$

$id_A : A \rightarrow A$
$id_B : B \rightarrow B$

The standard way of thinking of these operations is in terms of composition:

$id_A <=> f^{-1} . f$
$id_B <=> f . f^{-1}$

Intuitively, if we start with an identity function and move the end point using a function, then we get a path that is logically equivalent to the function itself:

$id_A[id_A \rightarrow f] <=> f$
$id_B[id_B \rightarrow f^{-1}] <=> f^{-1}$

Something interesting happens when we do it on the input instead of the output:

$id_A[f \rightarrow id_A] <=> f^{-1}$
$id_B[f^{-1} \rightarrow id_B] <=> f$

What happens is that the path must necessarily map from the output of \`f\` to it's input, therefore the path becomes the inverse function. The cool thing is that we can refer to the inverse of a function without using any other definition of the inverse!