The Law of Impulse Displacement

by Sven Nilsen, 2018

In this paper I represent a law of impulse displacement that can be used by agents to quickly and cheaply consider alternative actions with varied force on an actuator such as a rotationally stable wing flap. The method assumes predicting Newtonian interactions with normalized virtual particles.

When two Newtonian particles starting in same physical state are influenced by same characteristic impulse function, modeled as a normalized virtual particle transformed by a scale force and time interval, they end up in a relative physical state (a dual number equation):

$$\Delta s_0 / (\Delta t_0^2\ F_0) + \Delta v_0 / (\Delta t_0\ F_0)\ \varepsilon = \Delta s_1 / (\Delta t_1^2\ F_1) + \Delta v_1 / (\Delta t_1\ F_1)\ \varepsilon$$

This means that the observed physical state resulting from using actuators in a certain way, can be used to predict the physical state when using the same actuators in a slightly different way. For example, the agent can use more or less force or execute the same instructions slower or faster.

By compensating the time interval for the change in force, one obtains two simple equations:

$$\Delta t_0 = \Delta t / F_0$$
$$\Delta t_1 = \Delta t / F_1$$

$$\Delta s_0 / (\Delta t_0^2\ F_0) + \Delta v_0 / (\Delta t_0\ F_0)\ \varepsilon = \Delta s_1 / (\Delta t_1^2\ F_1) + \Delta v_1 / (\Delta t_1\ F_1)\ \varepsilon$$
$$\Delta s_0\ F_0^2 / (\Delta t^2\ F_0) + \Delta v_0\ F_0 / (\Delta t\ F_0)\ \varepsilon = \Delta s_1\ F_1^2 / (\Delta t^2\ F_1) + \Delta v_1\ F_1 / (\Delta t\ F_1)\ \varepsilon$$
$$\Delta s_0\ F_0 / \Delta t^2 + \Delta v_0 / \Delta t\ \varepsilon = \Delta s_1\ F_1 / \Delta t^2 + \Delta v_1 / \Delta t\ \varepsilon$$
$$\Delta s_0\ F_0 / \Delta t + \Delta v_0\ \varepsilon = \Delta s_1\ F_1 / \Delta t + \Delta v_1\ \varepsilon$$

$$\mathbf{\Delta s_0\ F_0 = \Delta s_1\ F_1}$$
$$\mathbf{\Delta v_0 = \Delta v_1}$$

This means the particles have the same velocity, but they are located at different positions after the physical interaction (in addition to their displacements in time). The space displacement is determined by the scale force used to control the impulse.

It might seem, at first sight, that using higher force results in less displacement, but remember that these positions do not have the same time. Since one particle is ahead of the other in time, its velocity will move it forward while the other has not yet reached the final velocity. At the moment both particles have final velocity, the displacement between them will be:

$$\Delta s_0' = \Delta s_0 - \Delta s_1 + \Delta v\,(\Delta t_1 - \Delta t_0)$$

A normalized virtual particle has time interval `Δt = 1`, so a faster way to compute is the following:

$$\Delta s_0' = \Delta s_0\ (1 - F_0/F_1) + \Delta v_0\ (1/F_1 - 1/F_0)$$

Displacement relative to another particle can be found using only the scale forces. Since the displacement is the only difference, this equation can be used to compare effects of using the actuator.