Composite & Prime Numbers in Path Semantics

by Sven Nilsen, 2017

Path semantical space connects various concepts naturally by which functions predict aspects of other functions. All normal paths seem to naturally reduce to simpler functions the higher you go in the space, while existential paths form a much flatter space with only two levels from any functions to a loop between `id` and `true$_1$`. In general, it is believed that normal paths are cheaper to compute because they reduce the amount of information to process. In this paper I show that this is not the case for existential paths, by using composite numbers as counter-example.

A composite number is defined as following:

x : composite
composite <=> ∃mul{(> 1), (> 1)}

mul : nat × nat → nat

Notice that this is a very simple existential path, that only requires knowledge of two functions `mul` and `>` to ask the question "what is a composite number?". In order to calculate the existential path, one needs the concept of prime numbers:

∃mul{(> 1), (> 1)} <=> (¬= 0) ∧ (¬= 1) ∧ ¬prime

One can obtain the definition of prime numbers from the definition of composite numbers:

prime <=> (¬= 0) ∧ (¬= 1) ∧ ¬∃mul{(> 1), (> 1)}

From a number theoretic perspective, it is not surprising that this shows up since it is a well studied area. From a path semantical perspective, this observation is more interesting. First, it tells that more complex concepts can hide in the existential paths of simpler functions. Second, since existential paths are used in the proof about `$P^{\exists f\{\}} = NP^{\forall f}$`, it means that if there is no other choice of existential paths that avoids bottlenecks, there could be a significant lower bound in performance for optimal algorithms.

Some open problems of hard difficulty:

- Find another example of higher complexity in the existential path, preferably with even higher complexity than prime numbers
- Derive the sparsity of existential paths that are more complex than their function
- Prove non-existence (or counter-prove) the lower bound on the `$P^{\exists f\{\}} = NP^{\forall f}$` algorithm