Universal Existential Paths

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An existential path is a total function that determines the truth value of whether there exists some input to a function that returns some output:

$$\exists f := \backslash(x') = \exists x \{ f(x) = x' \}$$

$$x' = f(x)$$
$$f : X \to X$$

This can be generalized using domain constraint notation:

$$\exists f\{T_X\} := \backslash(x') = \exists x : T_X \{ f(x) = x' \}$$

In domain constraint notation you can create the universal `f{}`, so one can also create a universal existential path:

$$\exists f\{\} := \backslash(T_x) = \backslash(x') = \exists x : T_X \{ f(x) = x' \}$$

All universal existential paths are reducible to normal existential paths using `$true_1$`:

$$\exists f\{[true_1]\ true\} \iff \exists f\{true_1\} \iff \exists f$$

$$true_1 := \backslash(\_) = true$$

A sub-existential path is when you pass any other function than `$true_1$` to a universal existential path:

$$\exists f\{[g(c)]\ true\} \iff \exists f\{g(c)\}$$

$$\exists f\{g(c)\} : B \to bool$$
$$f : A \to B$$
$$g : C \to A \to bool$$

When one passes a function of more than one argument to a universal existential path, one creates a higher order existential path:

$$\exists f\{g\} : C \to B \to bool$$

Name	**Notation**	**Type**
Sub-existential path	$\exists f\{g(c)\}$	$B \to bool$
Higher order existential path	$\exists f\{g\}$	$C \to B \to bool$
Universal existential path	$\exists f\{\}$	$T_C \to B \to bool$