Assigning Probabilities to Logical Quantifiers

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Using the technique for assigning probabilities to boolean functions, I proved the probabilities for `and` and `or`:

$$P(and(x_0, x_1)) = P(x_0) \cdot P(x_1)$$

$$P(or(x_0, x_1)) = 1 - (1-P(x_0)) \cdot (1-P(x_1))$$

This is used to generalize the notion of `and` and `or` to the logical quantifiers "for all" (using the symbol `∀`) and "there exists" (using the symbol `∃`).

$$P(\forall\ i\ \{\ x_i\ \}) = \prod\ i\ \{\ P(x_i)\ \}$$

$$P(\exists\ i\ \{\ x_i\ \}) = 1 - \prod\ i\ \{\ 1-P(x_i)\ \}$$