Existential Path of Function Composition

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Finding the existential path of function composition follows this rule:

$$∃(f_0 \cdot f_1) <=> ∃f_0\{∃f_1\}$$

This is a shorthand version for:

$$∃(f_0 \cdot f_1) <=> ∃f_0\{[∃f_1] \text{ true}\}$$

This means the function `$f_0$` is constrained by the output of function `$f_1$`.

For example, for compositions of linear number sequences you get another number sequence. Since the existential path of such sequences are known, one can derive the existential path of the function composition:

$$\text{sequence}(a_0, b_0) \cdot \text{sequence}(a_1, b_1) <=> \text{sequence}(a_0 + b_0 \cdot a_1, b_0 \cdot b_1)$$
$$∃\text{sequence}(a_0, b_0)\{∃\text{sequence}(a_1, b_1)\} <=> ∃\text{sequence}(a_0 + b_0 \cdot a_1, b_0 \cdot b_1)$$
$$∃\text{sequence}(a_0, b_0)\{∃\text{sequence}(a_1, b_1)\} <=> \text{linear}(a_0 + b_0 \cdot a_1, b_0 \cdot b_1)$$

$$∃\text{sequence}(a, b) <=> \text{linear}(a, b)$$
$$\text{sequence}(a, b: (> 0)) = \backslash(x) = a + b \cdot x$$
$$\text{linear}(a, b: (> 0)) = \backslash(x) = \text{if } x < a \text{ \{ false \} else \{ } ((x - a) \ \% \ b) == 0 \text{ \}}$$