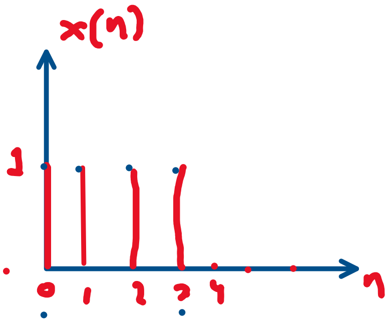
Sistemas LIT

sábado, 8 de marzo de 2025 10:01 a. m.

Problema: Calcular la salida y(n) del siguiente sistema LIT:

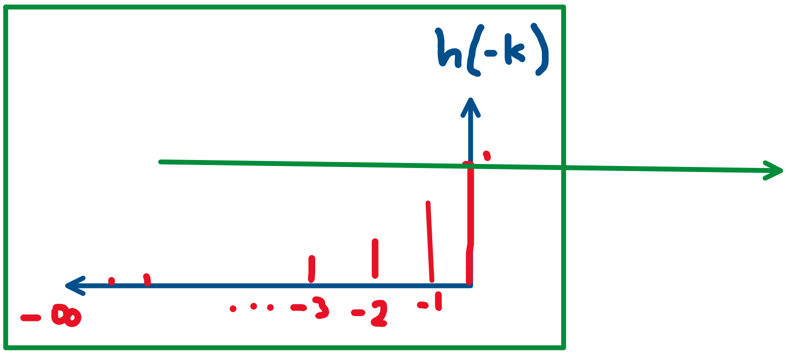
- Entrada: x(n) = u(n) - u(n-4) (pulso rectangular de anchura 4)
- Respuesta al impulso: h(n) = (0.7)^n·u(n)

$$h(n) = \begin{cases} 0 & n<0 \\ 0.7^n & n \geq 0 \end{cases}$$



$$y(n) = \sum_{k=-\infty}^{\infty} x(k)\, h(n-k)$$

$$= \sum_{k=0}^{3} (1)\, h(n-k) = \sum_{k=0}^{3} (0.7)^{n-k}\, u(n-k)$$



Caso 1. $n<0$ $\boxed{y(n)=0}$

Caso 2. $0 \leq n \leq 3$

$$y(n) = \sum_{k=0}^{n} (0.7)^{n-k} = 0.7^n \sum_{k=0}^{n} (0.7)^{-k} = 0.7^n \boxed{\sum \frac{1}{0.7}^{k}}$$

$$\rightarrow S = \sum_{k=0}^{m} r^k = \frac{1-r^{m+1}}{1-r}, \quad \text{para } |r|<1 \qquad r = a = \frac{1}{0.7}$$

$$\sum 0.7^{-k} = \frac{1-(0.7)^{-(n+1)}}{1-\frac{1}{0.7}} = \frac{1-0.7^{-(n+1)}}{\frac{0.7-1}{0.7}} = \frac{0.7\left[1-0.7^{-(n+1)}\right]}{-0.3}$$

$0.7 - 0.7^{-n}$ $\quad 0.7^{n+1}$ $\quad 1$

$$= \frac{0.7 - 0.7^{-n}}{-0.3} = \frac{0.7^{n+1} - 1}{0.3}$$

$$\boxed{y(n) = 0.7^{n} \frac{0.7^{n+1} - 1}{0.3} = \frac{0.7^{2n+1} - 0.7^{n}}{0.3} \quad 0 \leq n \leq 3}$$

Caso 3. $k = 0, 1, 2, 3$

$$y(n) = \sum_{k=0}^{3} (0.7)^{n-k} = 0.7^{n} \sum_{k=0}^{3} 0.7^{-k}$$

$$\sum_{k=0}^{3} 0.7^{-k} = \frac{1 - (0.7)^{-4}}{1 - 0.7^{-1}} = \frac{1 - 0.7^{-4}}{-\frac{0.3}{0.7}} = \frac{0.7^{4} - 1}{0.3}$$

$$= \frac{0.2401 - 1}{0.3} \approx -2.533$$

$$\boxed{y(n) = 0.7^{n}(-2.533) \quad n \geq 3}$$