Assignment#3

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Q1

1. The class expression is in NNF,so we can directly write:

   $$S_0 = \{x : \exists R.\exists E.\neg C \sqcap \forall R.C\}$$

   The application of $\rightarrow_\sqcap$ gives:

   $$S_1 = S_0 \cup \{x : \exists R.\exists E.\neg C, x : \forall R.C\}$$

   The application of $\rightarrow_\exists$ gives:

   $$S_2 = S_1 \cup \{(x, y) : R, y : \exists E.\neg C\}$$

   The application of $\rightarrow_\exists$ gives:

   $$S_3 = S_2 \cup \{(y, z) : E, z : \neg C\}$$

   The application of $\rightarrow_\forall$ gives:

   $$S_4 = S_3 \cup \{y : C\}$$

   No rule is applicable to S_4 and S_4 contains no clash. Thus, the concept is satisfiable. A model $\mathcal{I}$ of this concept is given by:

   $$\Delta^\mathcal{I} = \{x, y, z\}, \quad C^\mathcal{I} = \{y\}, \quad R^\mathcal{I} = \{(x, y)\}, \quad E^\mathcal{I} = \{(y, z)\}$$

2. From TBox $A \sqsubseteq B$ we can know there $\exists A'$ s.t. $A = A' \sqcap B$.

   And then we can convert (R some A) and (R only (not B)) into

   $$\exists R.(A' \sqcap B) \sqcap \forall R.\neg B$$

   It is NNF, the initial constraint is:

   $$S_0 = \{x : \exists R.(A' \sqcap B) \sqcap \forall R.\neg B\}$$

   The application of $\rightarrow_\sqcap$ gives:

   $$S_1 = S_0 \cup \{x : \exists R.(A' \sqcap B), x : \forall R.\neg B\}$$

   The application of $\rightarrow_\exists$ gives:

   $$S_2 = S_1 \cup \{(x, y) : R, y : A' \sqcap B\}$$

   The application of $\rightarrow_\forall$ gives:

   $$S_3 = S_2 \cup \{y : \neg B\}$$

   The application of $\rightarrow_\sqcap$ gives:

   $$S_4 = S_3 \cup \{y : B, y : A'\}$$

   S_4 contains clush: $\{y : B, y : \neg B\}$.

   No rule is applicable to S_4 , and it contains clushes. So the class expression *(R some A) and (R only (not B))* is unsatisfiable w.r.t *A SubClassOf B*.

3. To proof $(\exists R.A \sqcap \exists R.B) \sqsubseteq (\exists R.(A \sqcap B))$, is to proof $\neg((\exists R.A \sqcap \exists R.B) \sqsubseteq (\exists R.(A \sqcap B)))$ is unsatisfiable. Convert it into NNF:

   $$\exists R.A \sqcap \exists R.B \sqcap \forall R.(\neg A \sqcup \neg B)$$

and then we get:

$$S_0 = \{x : \exists R.\, A \sqcap \exists R.\, B \sqcap \forall R.\, (\neg A \sqcup \neg B)\}$$

The application of $\rightarrow_\sqcap$ gives:

$$S_1 = S_0 \cup \{x : \exists R.\, A, x : \exists R.\, B, x : \forall R.\, (\neg A \sqcup \neg B)\}$$

The application of $\rightarrow_\exists$ gives:

$$S_2 = S_1 \cup \{(x, y) : R, y : A\}$$

The application of $\rightarrow_\exists$ gives:

$$S_3 = S_2 \cup \{(x, z) : R, z : B\}$$

The application of $\rightarrow_\forall$ gives:

$$S_4 = S_3 \cup \{y : \neg A \sqcup \neg B, z : \neg A \sqcup \neg B\}$$

The application of $\rightarrow_\sqcup$ gives:

$$S_5 = S_4 \cup \{y : \neg A\}$$

S_5 contains clush: $\{y : A, y : \neg A\}$.

So we get another branch:

$$S_5^* = S_4 \cup \{y : \neg B\}$$

The application of $\rightarrow_\sqcup$ gives:

$$S_6 = S_5^* \cup \{z : \neg B\}$$

S_6 contains clush: $\{z : B, z : \neg B\}$.

So we get another branch:

$$S_6^* = S_5^* \cup \{z : \neg A\}$$

No rule is applicable to S_6^* and S_6^* contains no clash. Thus, the concept is satisfiable. Therefore, (R some A) and (R some B) is not subsumed by R some (A and B) .

Q2

Please see the source code `A#3-181220076.java`.