# A Sample Mathematics Paper

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**Abstract**

Mathematical model of my authoritarian violence paper

## 1  Multilevel model with instrumental variable

Unit indexed by $i$, group indexed by $j$. Model with treatment $T$ and IV $z$ both at the group level. There are also covariates at the group level, and none at the individual level.

$$y_i \sim N(\alpha_{j[i]}, \sigma_y^2) \tag{1}$$

$$\begin{bmatrix} \alpha_j \\ T_j \end{bmatrix} \sim N\left( \begin{bmatrix} \gamma_0 + \gamma_1 T_j + \gamma_2 x_j \\ \mu_0 + \mu_1 z_j \end{bmatrix}, \begin{bmatrix} \sigma_\alpha^2 & \rho\sigma_\alpha\sigma_T \\ \rho\sigma_\alpha\sigma_T & \sigma_T^2 \end{bmatrix} \right) \tag{2}$$

In our paper,

$$T_j = \text{legislature in country-year}_j \tag{3}$$

$$z_j = \text{inherited parties in country-year}_j \tag{4}$$

$$x_j = \text{other covariates in country-year}_j \tag{5}$$

Notice that in the model, $T_j$ is not assumed to be uncorrelated with the error term of $\alpha_j$ (the non-diagonal covariance is non-zero), but $z_j$ is (implicitly since it's un-modeled). Writing it out in non-multilevel form:

$$\alpha_j = \gamma_0 + \gamma_1 T_j + \gamma_2 x_j + \epsilon_\alpha = N(\gamma_0 + \gamma_1 T_j + \gamma_2 x_j, \sigma_\alpha) \tag{6}$$

$$T_j = \mu_0 + \mu_1 + \epsilon_T = N(\mu_0 + \mu_1, \sigma_T) \tag{7}$$

In the multilevel model, we allow $T_j$ to be correlated with $\epsilon_\alpha$ via $\rho\sigma_\alpha\sigma_T$

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