Tutorial 5: Regression Model Interpretation

Jan Vogler (jan.vogler@duke.edu)

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Today's Agenda

1. Marginal effects and intercepts
2. Hypothesis testing
3. Multiple linear regression
4. Graphical representation
5. Tips for your final paper

1. Marginal effects and intercepts

An essential aspect of all linear models are the marginal effects that predictor variables (independent variables) are estimated to have on the response variable (dependent variable).

Note that the word "effect" may be problematic because it implies causality. However, without any additional assumptions or additional model features, linear models allow us to make statements with respect to correlation only. This means we can't say anything about causality when just having a linear model. So let us be very cautious when we use the word "marginal effect".

Every linear model has one response variable (dependent variable) and at least one predictor variable (independent variable) plus an intercept.

Let's assume that Y is our response variable and X is our only predictor variable. The model may look like this:

Y = 5 + 2X + error

How would we interpet the marginal effect of X?

The interpretation would be: For a 1-point increase in X we expect a 2-point increase in Y.

How would we interpret the intercept?

The intercept is the expected value of Y when X is at a value of 0.

Illustration of the marginal effect interpretation

Let's load another R dataset that can illustrate the interpretation of marginal effects. The "airquality" dataset. According to the documentation, this is "Daily air quality measurements in New York, May to September 1973."

More details can be found here:

```
data(airquality)
summary(airquality)
```

```
##      Ozone           Solar.R          Wind             Temp
##  Min.   :  1.00   Min.   :  7.0   Min.   : 1.700   Min.   :56.00
##  1st Qu.: 18.00   1st Qu.:115.8   1st Qu.: 7.400   1st Qu.:72.00
##  Median : 31.50   Median :205.0   Median : 9.700   Median :79.00
```

```
##  Mean   : 42.13    Mean   :185.9    Mean   : 9.958    Mean   :77.88
##  3rd Qu.: 63.25    3rd Qu.:258.8    3rd Qu.:11.500    3rd Qu.:85.00
##  Max.   :168.00    Max.   :334.0    Max.   :20.700    Max.   :97.00
##  NA's   :37        NA's   :7
##      Month            Day
##  Min.   :5.000    Min.   : 1.0
##  1st Qu.:6.000    1st Qu.: 8.0
##  Median :7.000    Median :16.0
##  Mean   :6.993    Mean   :15.8
##  3rd Qu.:8.000    3rd Qu.:23.0
##  Max.   :9.000    Max.   :31.0
##
```

Our question is: is there a linear relationship between the Ozone measures and the Solar.R measures?

Let us use linear regression to answer this question:

```
lm1=lm(Ozone ~ Solar.R, data=airquality)
```

The summary of this linear regression will return a t-value and a p-value for the intercept and all coefficients.

```
summary(lm1)
```

```
##
## Call:
## lm(formula = Ozone ~ Solar.R, data = airquality)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -48.292 -21.361  -8.864  16.373 119.136
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept) 18.59873    6.74790   2.756 0.006856 **
## Solar.R      0.12717    0.03278   3.880 0.000179 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 31.33 on 109 degrees of freedom
##   (42 observations deleted due to missingness)
## Multiple R-squared:  0.1213, Adjusted R-squared:  0.1133
## F-statistic: 15.05 on 1 and 109 DF,  p-value: 0.0001793
```

How would we interpret the finding with respect to the linear relationship between the two variables? The interpreation would look like this:

There is a positive linear relationship between Ozone and Solar.R. For a 1-point increase in Solar.R, we would expect a 0.13 increase in Ozone (in a multivariate model we would have to add: "holding all other variables constant").

Furthermore (already going into the next topic): The associated t-value is 3.880. This t-value imples a p-value of 0.0002. This $p < 0.001$ corresponds to a type-1 error rate of alpha < 0.001, meaning that the relationship is significant at all common levels of statistical significance.

How do we interpret the R-squared statistic? Our model explains a proportion of the total variation in the dependent variable. The R-squared statistic returns this proportion. How well does our model do?

2. Hypothesis testing

Let us use another dataset to conduct some hypothesis tests.

We will look at data from an article that was published in the journal "International Organization", the leading journal in the field of international relations. The article was written by Helen Milner and Keiko Kubota.

The article deals with the effect that democratization has on trade barriers. The authors believe that democratization has a negative effect on trade barriers in developing countries (that are scarce in capital). Their theory is based on the Stolper Samuelson theorem and the selectorate model by Bueno de Mesquita et al.

Let us try to emulate their test. In order to load their dataset you need to use the following command: install.packages("foreign")

Note that the working directory you set depends on where you have the file on your computer.

```r
setwd('C:/Users/Jan/OneDrive/Documents/GitHub/ps630_lab/W5')
# Sets the working directory
library(foreign)
# Allows you to read more data formats
LDC=read.dta("LDC_IO_replication.dta")
summary(LDC)
```

```
##     country         ctylabel              date        gatt_wto_new
##   Min.   :186.0   Length:5370        Min.   :1970    Min.   :0.0000
##   1st Qu.:423.0   Class :character   1st Qu.:1977    1st Qu.:0.0000
##   Median :628.0   Mode  :character   Median :1984    Median :0.0000
##   Mean   :605.9                      Mean   :1984    Mean   :0.4747
##   3rd Qu.:816.0                      3rd Qu.:1992    3rd Qu.:1.0000
##   Max.   :968.0                      Max.   :1999    Max.   :1.0000
##                                                      NA's   :698
##      aclpn            bpc1          dopen_wacz2        ecris2
##   Min.   :0.0000   Min.   :0.000   Min.   :0.0000   Min.   :0.0000
##   1st Qu.:0.0000   1st Qu.:0.000   1st Qu.:0.0000   1st Qu.:0.0000
##   Median :0.0000   Median :1.000   Median :0.0000   Median :0.0000
##   Mean   :0.3002   Mean   :0.591   Mean   :0.3097   Mean   :0.0641
##   3rd Qu.:1.0000   3rd Qu.:1.000   3rd Qu.:1.0000   3rd Qu.:0.0000
##   Max.   :1.0000   Max.   :1.000   Max.   :1.0000   Max.   :1.0000
##   NA's   :1183     NA's   :2734    NA's   :2580     NA's   :1967
##      fdignp         gdp_pc_95d         l1aclpn          l1bpc1
##   Min.   :-27.2356   Min.   :    0.0   Min.   :0.0000   Min.   :0.0000
##   1st Qu.:  0.0361   1st Qu.:  442.9   1st Qu.:0.0000   1st Qu.:0.0000
##   Median :  0.6644   Median : 1266.5   Median :0.0000   Median :1.0000
##   Mean   :  1.8962   Mean   : 2885.5   Mean   :0.2924   Mean   :0.5909
##   3rd Qu.:  2.0829   3rd Qu.: 3002.4   3rd Qu.:1.0000   3rd Qu.:1.0000
##   Max.   :184.5647   Max.   :44164.5   Max.   :1.0000   Max.   :1.0000
##   NA's   :2294       NA's   :1679      NA's   :1341     NA's   :2735
##     l1ecris2          newtar        polityiv_update2      signed
##   Min.   :0.0000   Min.   :  0.00   Min.   :-10.000    Min.   :0.0000
##   1st Qu.:0.0000   1st Qu.: 10.95   1st Qu.: -7.000    1st Qu.:0.0000
##   Median :0.0000   Median : 17.00   Median : -6.000    Median :0.0000
##   Mean   :0.0641   Mean   : 20.54   Mean   : -2.074    Mean   :0.1465
##   3rd Qu.:0.0000   3rd Qu.: 27.00   3rd Qu.:  6.000    3rd Qu.:0.0000
```

```
##   Max.   :1.0000   Max.   :102.20   Max.   : 10.000   Max.   :1.0000
##   NA's   :1967     NA's   :4463     NA's   :2003      NA's   :1362
##     yrsoffic         usheg            l1usheg          l1fiveop
##   Min.   : 0.000   Min.   :0.2434   Min.   :0.2434   Min.   :10.20
##   1st Qu.: 2.000   1st Qu.:0.2574   1st Qu.:0.2574   1st Qu.:10.90
##   Median : 5.000   Median :0.2663   Median :0.2655   Median :12.35
##   Mean   : 8.431   Mean   :0.2696   Mean   :0.2683   Mean   :12.03
##   3rd Qu.:12.000   3rd Qu.:0.2785   3rd Qu.:0.2784   3rd Qu.:12.72
##   Max.   :44.000   Max.   :0.3083   Max.   :0.2988   Max.   :13.20
##   NA's   :2361                      NA's   :179      NA's   :358
##     l1gdp_pc          avsw            avnewtar          l1avsw
##   Min.   :    0    Min.   :0.1398   Min.   : 0.00    Min.   :0.1398
##   1st Qu.:  442    1st Qu.:0.1505   1st Qu.: 0.00    1st Qu.:0.1505
##   Median : 1266    Median :0.1720   Median :17.43    Median :0.1613
##   Mean   : 2888    Mean   :0.3097   Mean   :14.91    Mean   :0.2974
##   3rd Qu.: 2999    3rd Qu.:0.5269   3rd Qu.:24.37    3rd Qu.:0.5054
##   Max.   :44165    Max.   :0.6667   Max.   :30.52    Max.   :0.6559
##   NA's   :1823                                       NA's   :179
##    l1avnewtar         lnpop            l1lnpop          l1office
##   Min.   : 0.00    Min.   :10.57    Min.   :10.62    Min.   : 0.000
##   1st Qu.: 0.00    1st Qu.:13.86    1st Qu.:13.86    1st Qu.: 2.000
##   Median :18.73    Median :15.32    Median :15.31    Median : 5.000
##   Mean   :15.01    Mean   :15.11    Mean   :15.10    Mean   : 8.431
##   3rd Qu.:24.37    3rd Qu.:16.40    3rd Qu.:16.39    3rd Qu.:12.000
##   Max.   :30.52    Max.   :20.95    Max.   :20.94    Max.   :44.000
##   NA's   :179      NA's   :490      NA's   :661      NA's   :2361
##  l1partyage2000      l1fdi            l1polity         l2polity
##   Min.   :  0.00   Min.   :-27.2356  Min.   :-10.000  Min.   :-10.00
##   1st Qu.: 10.00   1st Qu.:  0.0269  1st Qu.: -7.000  1st Qu.: -7.00
##   Median : 19.50   Median :  0.6382  Median : -6.000  Median : -7.00
##   Mean   : 24.18   Mean   :  1.7931  Mean   : -2.215  Mean   : -2.36
##   3rd Qu.: 32.00   3rd Qu.:  1.9904  3rd Qu.:  6.000  3rd Qu.:  5.00
##   Max.   :183.00   Max.   :184.5647  Max.   : 10.000  Max.   : 10.00
##   NA's   :3284     NA's   :2423      NA's   :2124     NA's   :2246
##     l3polity         l1signed          milit2            sp2
##   Min.   :-10.000  Min.   :0.0000   Min.   :0.0000   Min.   :0.0000
##   1st Qu.: -7.000  1st Qu.:0.0000   1st Qu.:0.0000   1st Qu.:0.0000
##   Median : -7.000  Median :0.0000   Median :0.0000   Median :0.0000
##   Mean   : -2.512  Mean   :0.1511   Mean   :0.1119   Mean   :0.1959
##   3rd Qu.:  5.000  3rd Qu.:0.0000   3rd Qu.:0.0000   3rd Qu.:0.0000
##   Max.   : 10.000  Max.   :1.0000   Max.   :1.0000   Max.   :1.0000
##   NA's   :2371     NA's   :1517
##     pers2           l1milit2           l1sp2           dictator1
##   Min.   :0.0000   Min.   :0.0000   Min.   :0.0000   Min.   :1.000
##   1st Qu.:0.0000   1st Qu.:0.0000   1st Qu.:0.0000   1st Qu.:2.000
##   Median :0.0000   Median :0.0000   Median :0.0000   Median :5.000
##   Mean   :0.1665   Mean   :0.1135   Mean   :0.1986   Mean   :4.737
##   3rd Qu.:0.0000   3rd Qu.:0.0000   3rd Qu.:0.0000   3rd Qu.:8.000
##   Max.   :1.0000   Max.   :1.0000   Max.   :1.0000   Max.   :8.000
##                    NA's   :179      NA's   :179      NA's   :1157
##   l1dictator1         yr70             yr80             l1ssch
##   Min.   :1.000    Min.   :0.0000   Min.   :0.0000   Min.   :0.0140
##   1st Qu.:2.000    1st Qu.:0.0000   1st Qu.:0.0000   1st Qu.:0.4562
##   Median :5.000    Median :0.0000   Median :0.0000   Median :0.8519
```

```
## Mean   :4.708   Mean   :0.3333   Mean   :0.3333   Mean   :1.0411
## 3rd Qu.:8.000   3rd Qu.:1.0000   3rd Qu.:1.0000   3rd Qu.:1.4652
## Max.  :8.000   Max.  :1.0000   Max.  :1.0000   Max.  :4.4422
## NA's  :1315                                      NA's  :3140
##    closedyr       _spline1        _spline2        _spline3
## Min.   : 0.000   Min.   :-24389   Min.   :-7854.0   Min.   :-9030.0
## 1st Qu.: 0.000   1st Qu.: -3375   1st Qu.:-2048.3   1st Qu.:-1629.3
## Median : 7.000   Median :  -343   Median : -260.2   Median : -165.6
## Mean   : 8.691   Mean   : -3075   Mean   :-1388.8   Mean   :-1340.9
## 3rd Qu.:15.000   3rd Qu.:     0   3rd Qu.:    0.0   3rd Qu.:    0.0
## Max.  :29.000   Max.  :     0   Max.  :    0.0   Max.  :    0.0
## NA's  :2580   NA's  :2580   NA's  :2580   NA's  :2580
## l1gatt_wto_new
## Min.   :0.000
## 1st Qu.:0.000
## Median :0.000
## Mean   :0.468
## 3rd Qu.:1.000
## Max.  :1.000
## NA's  :868
```

For information on the meaning of the variables see "LDCcodebook.pdf".

Let's have a look at our data.

We need a package for generating a scatterplot matrix that allows us to see relationships in our matrix.

install.packages("car")

```r
library(car)

LDC2=as.data.frame(LDC[,c("l1polity","l1signed","l1office","l1gdp_pc","l1lnpop","l1ecris2","l1bpc1","l1a
cor(LDC2)
```

```
##             l1polity l1signed l1office l1gdp_pc l1lnpop l1ecris2 l1bpc1
## l1polity           1       NA       NA       NA      NA       NA     NA
## l1signed          NA        1       NA       NA      NA       NA     NA
## l1office          NA       NA        1       NA      NA       NA     NA
## l1gdp_pc          NA       NA       NA        1      NA       NA     NA
## l1lnpop           NA       NA       NA       NA       1       NA     NA
## l1ecris2          NA       NA       NA       NA      NA        1     NA
## l1bpc1            NA       NA       NA       NA      NA       NA      1
## l1avnewtar        NA       NA       NA       NA      NA       NA     NA
##            l1avnewtar
## l1polity           NA
## l1signed           NA
## l1office           NA
## l1gdp_pc           NA
## l1lnpop            NA
## l1ecris2           NA
## l1bpc1             NA
## l1avnewtar          1
```

```r
LDC3=na.omit(LDC2)
cor(LDC3)
```

```
##                  l1polity      l1signed     l1office      l1gdp_pc      l1lnpop
## l1polity      1.000000000   0.01499208  -0.42901753   0.09129002   0.04404210
## l1signed      0.014992081   1.00000000  -0.04303356  -0.11240587   0.02321607
## l1office     -0.429017535  -0.04303356   1.00000000  -0.01936245  -0.17457461
## l1gdp_pc      0.091290020  -0.11240587  -0.01936245   1.00000000  -0.14082411
## l1lnpop       0.044042096   0.02321607  -0.17457461  -0.14082411   1.00000000
## l1ecris2      0.108433533   0.11269712  -0.10331828  -0.04446845   0.03580805
## l1bpc1       -0.171762855   0.10895350   0.06396247  -0.22176992  -0.08570695
## l1avnewtar   -0.008790383   0.03967319   0.03040646  -0.02357807   0.01262803
##                  l1ecris2        l1bpc1    l1avnewtar
## l1polity       0.10843353   -0.17176286  -0.008790383
## l1signed       0.11269712    0.10895350   0.039673186
## l1office      -0.10331828    0.06396247   0.030406457
## l1gdp_pc      -0.04446845   -0.22176992  -0.023578065
## l1lnpop        0.03580805   -0.08570695   0.012628033
## l1ecris2       1.00000000    0.03517381  -0.036064786
## l1bpc1         0.03517381    1.00000000   0.042851289
## l1avnewtar    -0.03606479    0.04285129   1.000000000
```

```r
scatterplotMatrix(~ l1polity + l1signed + l1office + l1gdp_pc + l1lnpop + l1ecris2 + l1bpc1 + l1avnewta
```
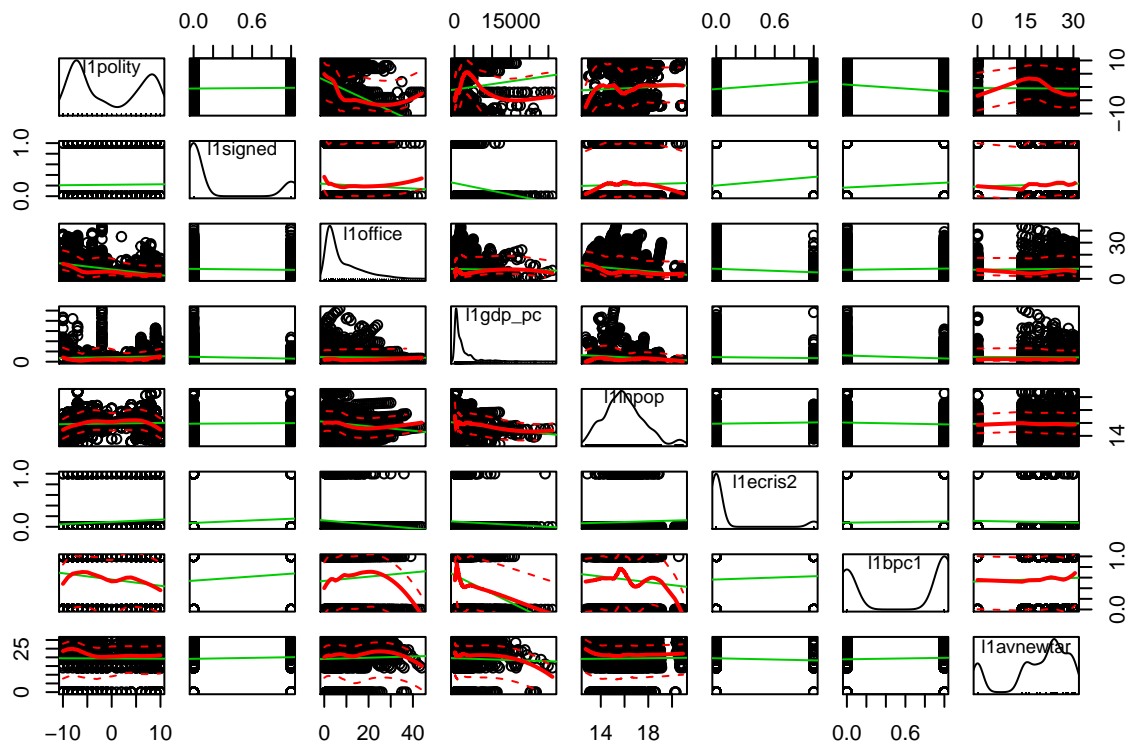

The results above indicate that there generally is a low level of multicollinearity among our variables.

6

Let us start with a simple model that is easy to interpet:

```
simple=lm(newtar~l1polity, data=LDC)
summary(simple)
```

```
##
## Call:
## lm(formula = newtar ~ l1polity, data = LDC)
##
## Residuals:
##     Min     1Q  Median     3Q     Max
## -23.425  -9.200  -3.425   5.275  80.475
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept) 21.92495    0.52865  41.474  < 2e-16 ***
## l1polity    -0.30001    0.07293  -4.113  4.3e-05 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 14.9 on 804 degrees of freedom
##   (4564 observations deleted due to missingness)
## Multiple R-squared:  0.02061,    Adjusted R-squared:  0.01939
## F-statistic: 16.92 on 1 and 804 DF,  p-value: 4.298e-05
```

What can we conclude from these statistics? What can we say about the hypothesis that there is a linear relationship between "l1polity" and "newtar"? What is the total variation that is explained by our model?

If there's too much information in this type of summary, try another one. We need another package: install.packages("arm")

```
library(arm)
```

```
## Warning: package 'arm' was built under R version 3.2.2

## Loading required package: MASS
## Loading required package: Matrix
## Loading required package: lme4

## Warning: package 'lme4' was built under R version 3.2.2

##
## arm (Version 1.8-6, built: 2015-7-7)
##
## Working directory is C:/Users/Jan/OneDrive/Documents/GitHub/ps630_lab/W5
##
##
## Attaching package: 'arm'
##
## The following object is masked from 'package:car':
##
##     logit
```

```
display(simple)
```

```
## lm(formula = newtar ~ l1polity, data = LDC)
##             coef.est coef.se
## (Intercept) 21.92     0.53
## l1polity    -0.30     0.07
## ---
## n = 806, k = 2
## residual sd = 14.90, R-Squared = 0.02
```

As you can see, this is narrowed down to just a few pieces of information. Sometimes reducing the amount of information that is displayed can be very useful.

3. Multiple linear regression

In the vast majority of cases there are good reasons to include multiple predictor variables.

The most important reasons to do so are:

1. Potential omitted variable bias
2. Theoretical reasons
3. Reviewers that demand you to include them

```
main=lm(newtar ~ l1polity + l1signed + l1office + l1gdp_pc + l1lnpop + l1ecris2 + l1bpc1 + l1avnewtar, o
summary(main)
```

```
##
## Call:
## lm(formula = newtar ~ l1polity + l1signed + l1office + l1gdp_pc +
##     l1lnpop + l1ecris2 + l1bpc1 + l1avnewtar, data = LDC)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -24.286  -7.694  -2.175   4.490  65.008
##
## Coefficients:
##                Estimate Std. Error t value Pr(>|t|)
## (Intercept) -4.901e+01  5.912e+00  -8.289 6.03e-16 ***
## l1polity    -2.053e-01  8.347e-02  -2.460 0.014151 *
## l1signed     4.758e-01  1.099e+00   0.433 0.665332
## l1office    -1.759e-01  6.989e-02  -2.516 0.012083 *
## l1gdp_pc    -1.281e-03  1.495e-04  -8.564  < 2e-16 ***
## l1lnpop      3.693e+00  3.217e-01  11.478  < 2e-16 ***
## l1ecris2    -5.736e+00  1.517e+00  -3.780 0.000171 ***
## l1bpc1       4.564e-01  9.681e-01   0.471 0.637462
## l1avnewtar   7.103e-01  8.413e-02   8.442  < 2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 12.16 on 685 degrees of freedom
```

```
##   (4676 observations deleted due to missingness)
## Multiple R-squared:  0.3781, Adjusted R-squared:  0.3708
## F-statistic: 52.05 on 8 and 685 DF,  p-value: < 2.2e-16
```

In the multiple linear regression, how would our expectation for the average tariff level change if our Polity Score increased from -10 to 10 and we had an economic crisis?

How well does our model do compared to the simple linear regression? Do we observe an improvement in the total variation that is explained by our model?

Again, it would be possible to reduce the amount of information with another command:

```
display(main)
```

```
## lm(formula = newtar ~ l1polity + l1signed + l1office + l1gdp_pc +
##     l1lnpop + l1ecris2 + l1bpc1 + l1avnewtar, data = LDC)
##             coef.est coef.se
## (Intercept) -49.01     5.91
## l1polity     -0.21     0.08
## l1signed      0.48     1.10
## l1office     -0.18     0.07
## l1gdp_pc      0.00     0.00
## l1lnpop       3.69     0.32
## l1ecris2     -5.74     1.52
## l1bpc1        0.46     0.97
## l1avnewtar    0.71     0.08
## ---
## n = 694, k = 9
## residual sd = 12.16, R-Squared = 0.38
```

We can access different elements of our model. Let's have a look at what those are:

```
names(main)
```
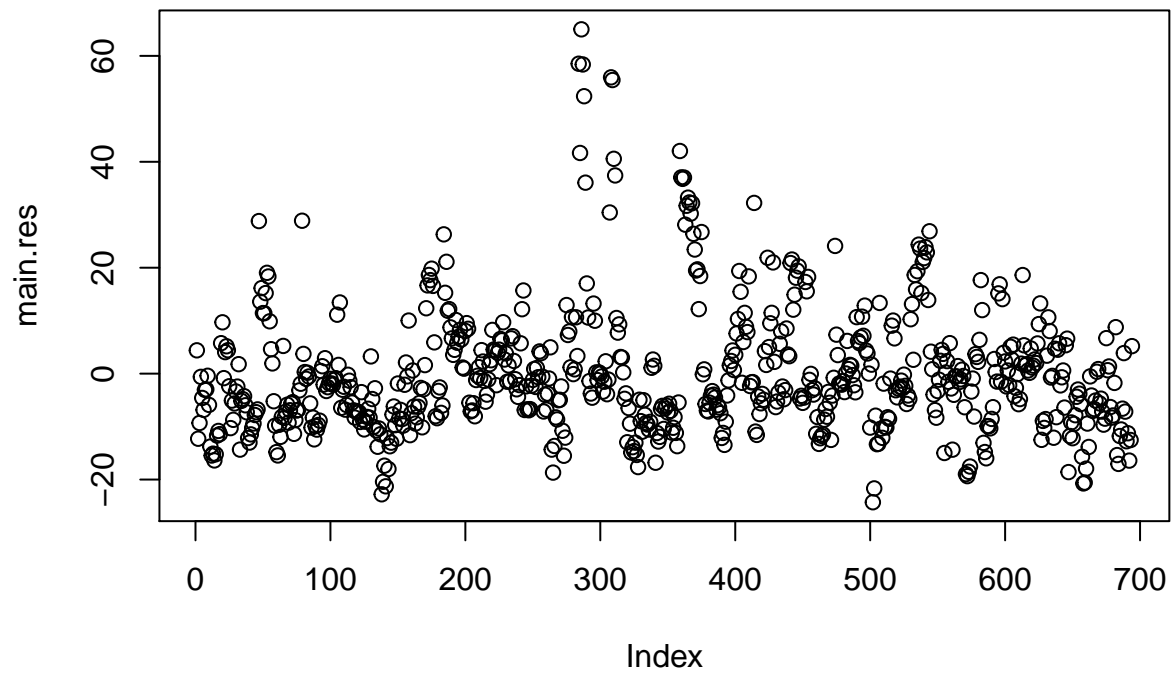
```
##  [1] "coefficients"  "residuals"     "effects"       "rank"
##  [5] "fitted.values" "assign"        "qr"            "df.residual"
##  [9] "na.action"     "xlevels"       "call"          "terms"
## [13] "model"
```

4. Graphical representation

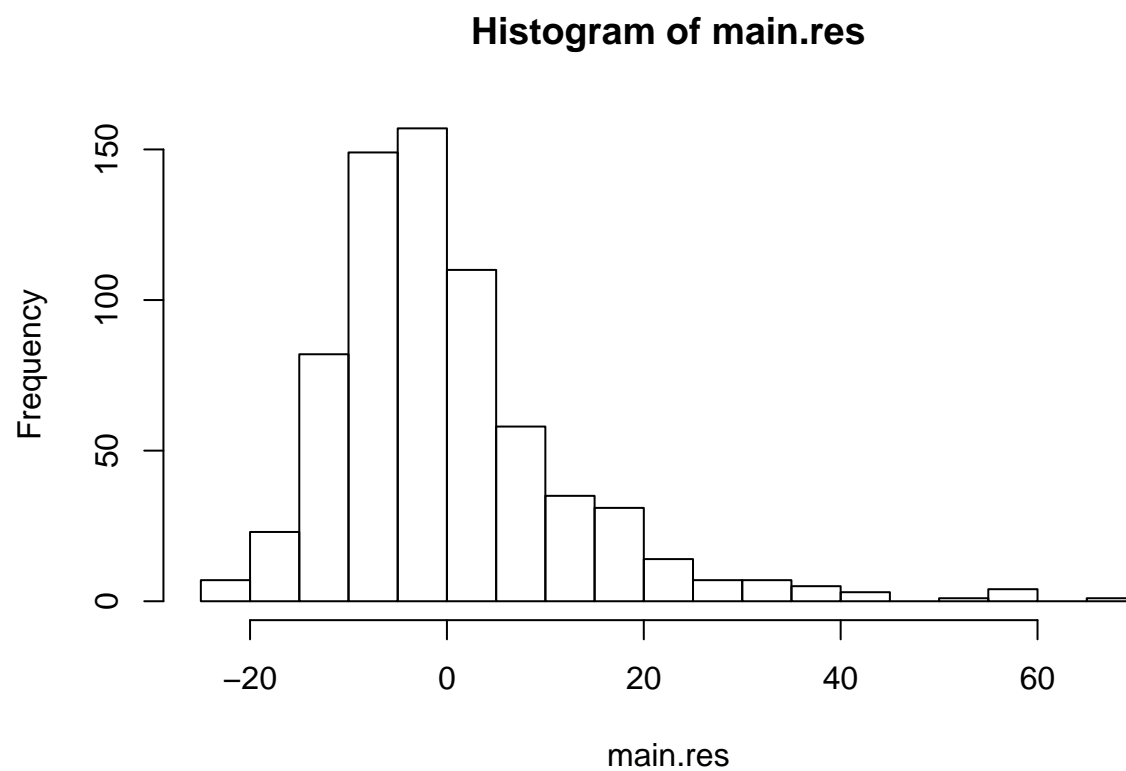Let us first have a look at the distribution of errors in our model.

```
main.res = resid(main)
plot(main.res, main="Values of the Error Term")
```
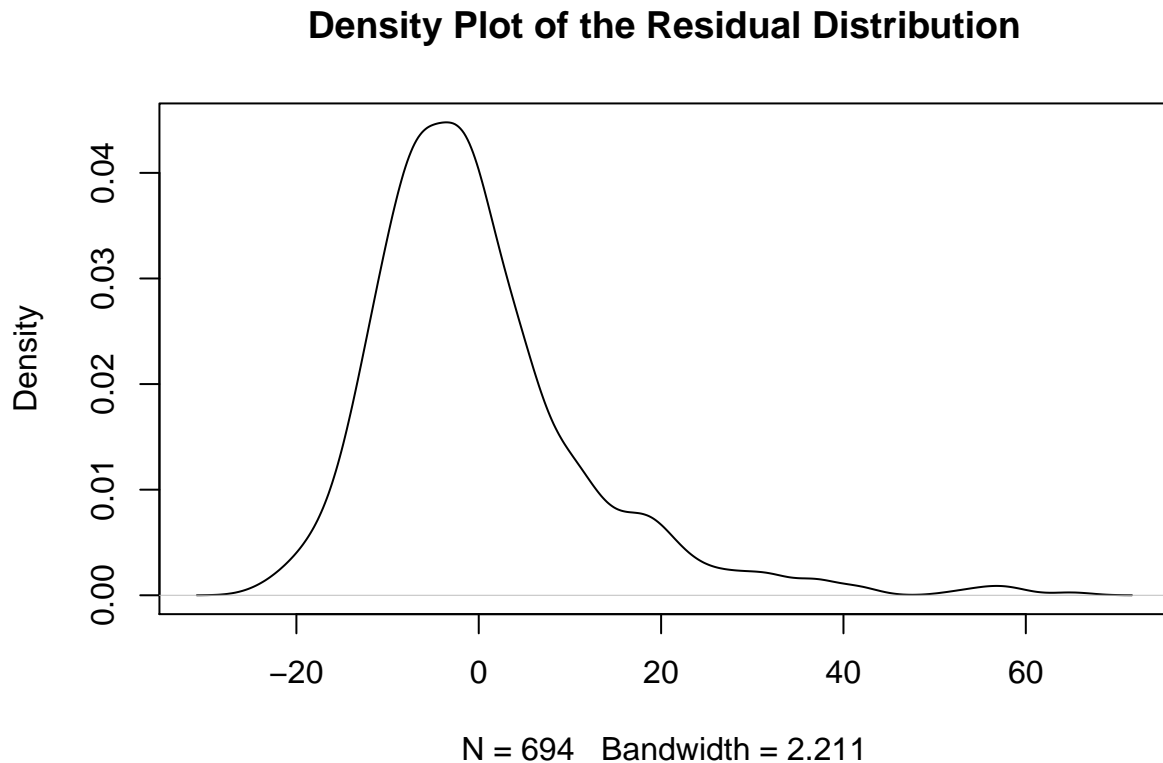
Values of the Error Term

Let us look at the distribution of the error term:

```
hist(main.res, breaks=20)
```

Histogram of main.res


```
res.density=density(main.res)
plot(res.density, main="Density Plot of the Residual Distribution")
```

Density Plot of the Residual Distribution

N = 694   Bandwidth = 2.211

The distribution of the errors is approximately normal. If this condition is met, then more precise statements about the distribution of the coefficients can be made (they're also normal). ALso, under these conditions, OLS is equivalent to a maximum likelihood approach.

Plotting predicted values

Let us plot some predicted values with confidence intervals for our multiple regression.

In order to do that we first create a dataframe that contains different values for our main predictor variable and the average values for all variables.

```
nd <- data.frame(l1polity=seq(-10,10,by=1), l1signed=rep(0.1511,21), l1office=rep(8.431,21), l1gdp_pc=re
```

Next we use the model we estimated to predict values based on this new dataframe.

```
pred.p1 <- predict(main, type="response", se.fit=TRUE, newdata=nd)

pred.table <- cbind(pred.p1$fit, pred.p1$se.fit)
pred.table
```

```
##        [,1]      [,2]
## 1  14.19185 1.3422851
## 2  13.98655 1.2793558
## 3  13.78125 1.2188955
## 4  13.57595 1.1612899
## 5  13.37065 1.1069847
## 6  13.16535 1.0564892
```

```
## 7   12.96005 1.0103744
## 8   12.75475 0.9692661
## 9   12.54945 0.9338254
## 10 12.34415 0.9047189
## 11 12.13885 0.8825733
## 12 11.93355 0.8679216
## 13 11.72825 0.8611465
## 14 11.52295 0.8624336
## 15 11.31765 0.8717473
## 16 11.11235 0.8888351
## 17 10.90705 0.9132609
## 18 10.70175 0.9444554
## 19 10.49645 0.9817738
## 20 10.29115 1.0245470
## 21 10.08585 1.0721223
```

Finally, we create the plot:

```
fit <- pred.p1$fit
low <- pred.p1$fit - 2*pred.p1$se.fit
high <- pred.p1$fit + 2*pred.p1$se.fit
cis <- cbind(fit, low, high)

cis ### To extract the values
```

```
##          fit       low     high
## 1   14.19185 11.507283 16.87642
## 2   13.98655 11.427842 16.54526
## 3   13.78125 11.343462 16.21904
## 4   13.57595 11.253373 15.89853
## 5   13.37065 11.156683 15.58462
## 6   13.16535 11.052373 15.27833
## 7   12.96005 10.939303 14.98080
## 8   12.75475 10.816219 14.69328
## 9   12.54945 10.681800 14.41710
## 10  12.34415 10.534713 14.15359
## 11  12.13885 10.373703 13.90400
## 12  11.93355 10.197706 13.66939
## 13  11.72825 10.005956 13.45054
## 14  11.52295  9.798082 13.24782
## 15  11.31765  9.574154 13.06114
## 16  11.11235  9.334678 12.89002
## 17  10.90705  9.080526 12.73357
## 18  10.70175  8.812837 12.59066
## 19  10.49645  8.532900 12.45999
## 20  10.29115  8.242053 12.34024
## 21  10.08585  7.941602 12.23009
```

```
plot(pred.p1$fit, type="l", ylim=c(5,20), main="Polity IV Score and Tariff Level",
     xlab="Polity IV Score", ylab="Tariff Level", axes=FALSE)
axis(1, at=seq(1,21), labels=seq(-10,10,1))
axis(2, at=seq(5,20), labels=seq(5,20))
matlines(cis[,c(2,3)], lty=2, col="black")
```

Polity IV Score and Tariff Level
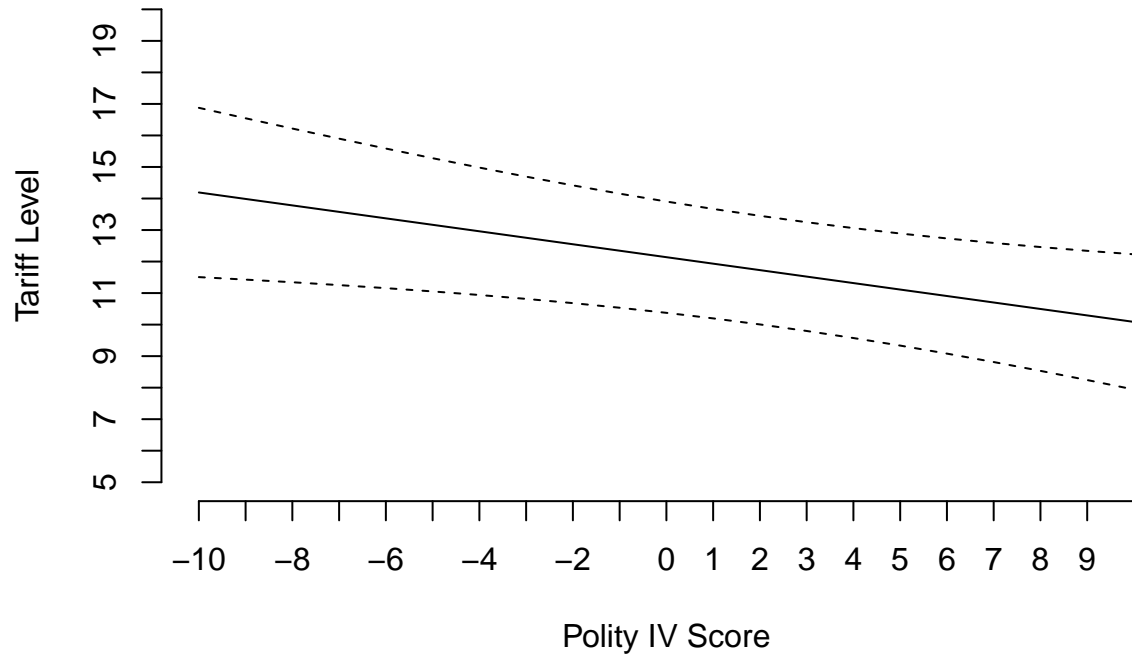
5. Tips for your final paper

1. Start working on it early.
2. Consult with your professors and TAs.
3. Try to find a comprehensive dataset in your area of interest.
4. Work on it throughout the semester and try to include new things that you've learned.
5. Make sure that you use all the tools you've learned: interpret your findings carefully and visualize them.
6. Annotate your code extensively and explain what you did.