

1 Expected value problem

2 Binomial problem

This problem is taken from Pitman (1993) Probability: Suppose a fair coin is tossed n times. Find a simple formula in terms of n and k for the following probability: $Pr(k \text{ heads} | k-1 \text{ heads or } k \text{ heads})$. Please pay close attention to the formula, particularly what event is conditioned on what events. Ch. 2.1, Problem 10 b) (p. 91)

Hint 1: Use the binomial distribution to model this.

Hint 2: Because those events are mutually exclusive, calculate the following:

$$\frac{Pr(k \text{ heads})}{Pr(k \text{ heads}) + Pr(k-1 \text{ heads})}$$

This is true because: $Pr(A|B) = \frac{Pr(A \cap B)}{Pr(B)}$

The intersection of events A and B in this case, $Pr(k \text{ heads} \cap (k \text{ heads} \cup k-1 \text{ heads}))$, reduces to $Pr(k \text{ heads})$ because the two events are mutually exclusive.

SOLUTION

$$\begin{aligned} & \frac{Pr(k \text{ heads})}{Pr(k \text{ heads}) + Pr(k-1 \text{ heads})} \\ &= \frac{\binom{n}{k} 0.5^k 0.5^{n-k}}{\binom{n}{k} 0.5^k 0.5^{n-k} + \binom{n}{k-1} 0.5^{k-1} 0.5^{n-(k-1)}} \\ &= \frac{\binom{n}{k} 0.5^n}{\binom{n}{k} 0.5^n + \binom{n}{k-1} 0.5^n} \\ &= \frac{\binom{n}{k}}{\binom{n}{k} + \binom{n}{k-1}} \\ &= \frac{\frac{n!}{(n-k)!k!}}{\frac{n!}{(n-k)!k!} + \frac{n!}{(n-(k-1))!(k-1)!}} \\ &= \frac{\frac{n!}{(n-k)!k!} * \frac{n-k+1}{n-k+1}}{\frac{n!}{(n-k)!k!} * \frac{n-k+1}{n-k+1} + \frac{n!}{(n-k+1)!(k-1)!} * \frac{k}{k}} \\ &= \frac{\frac{n!(n-k+1)}{(n-k+1)!k!}}{\frac{n!(n-k+1)}{(n-k+1)!k!} + \frac{n!}{(n-k+1)!k!}} \\ &= \frac{n!(n-k+1)}{n!(n-k+1) + n!k} \end{aligned}$$

$$\begin{aligned}
&= \frac{n-k+1}{n-k+1+k} \\
&= \frac{n-k+1}{n+1}
\end{aligned}$$

3 R programming problem