Pol Sci 630: Problem Set 1 - Probability Theory and Distributions - Solutions

Prepared by: Jan Vogler (jan.vogler@duke.edu)

Due Date: Tuesday, September 1st, 2015, 6 PM

R Programming

Problem 1

```
### a
factorial(10)

## [1] 3628800

factorial(8)

## [1] 40320

### b
factorial(15)/factorial(5)

## [1] 10897286400

factorial(10)/factorial(5)

## [1] 30240

### c
choose(12, 3)

## [1] 220

choose(9, 3)

## [1] 84
```

Problem 2

```r
### a
Multi = function(a, b, c) {
    if (-5 <= a & a <= 10 & -5 <= b & b <= 10 & -5 <= c & c <= 10) {
        print(a * b * c)
    } else {
        print("The values of the variables have to be between -5 and 10.")
    }
}

Multi(2, 3, 4)

## [1] 24

Multi(-6, 3, 4)

## [1] "The values of the variables have to be between -5 and 10."

### b
Permutation = function(n, k) {
    print(factorial(n)/factorial(n - k))
}

Permutation(n = 10, k = 8)

## [1] 1814400

### c
DiceAverage = function(rolls) {
    die = c(1, 2, 3, 4, 5, 6)
    print(mean(sample(die, size = rolls, replace = T)))
}

DiceAverage(1000)

## [1] 3.479
```

```
### Bonus
DiceAverage2 = function(rolls) {
    if (rolls%%1 == 0 & rolls >= 0) {
        die = c(1, 2, 3, 4, 5, 6)
        print(mean(sample(die, size = rolls, replace = T)))
    } else {
        print("The number of rolls must be a natural number")
    }
}

DiceAverage2(-10)

## [1] "The number of rolls must be a natural number"

# Returns the error message written above.

DiceAverage2(1.5)

## [1] "The number of rolls must be a natural number"

# Returns the error message written above.

### Alternative solution for bonus question
DiceAverage3 = function(rolls) {
    if (rolls == round(rolls) & rolls >= 0) {
        die = c(1, 2, 3, 4, 5, 6)
        print(mean(sample(die, size = rolls, replace = T)))
    } else {
        print("The number of rolls must be a natural number.")
    }
}

DiceAverage3(-10)

## [1] "The number of rolls must be a natural number."
```

```
# Returns the error message written above.

DiceAverage3(1.5)

## [1] "The number of rolls must be a natural number."

# Returns the error message written above.
```

Problem 3

```
### Let's write a function to represent the Monty Hall problem. How successful
### are you if you always switch to the other door when Monty reveals an empty
### one?

Switching = function(trials) {
    successes = rep(0, trials)
    # Create a null vector with the length of the number of trials
    for (i in 1:trials) {
        # For every trial, indexed by 'i', do the following
        prizeoptions = c(1, 2, 3)
        # The prize can be located behind door 1, 2, or 3
        prize = sample(prizeoptions, size = 1)
        # Draw a random location of the prize
        doorchosen = sample(prizeoptions, size = 1)
        # Choose a door at random
        if (doorchosen == prize) {
            # If the door you chose is the same door as the door of the prize
            doorshown = sample(prizeoptions[-prize], size = 1)
            # Monty will sample between the other two doors and show you one at random
        } else {
            # If the door you chose is not the door with the prize
            doorshown = prizeoptions[-c(prize, doorchosen)]
            # Monty has to show you the third door that is empty
        }
```

```
        switchtodoor = prizeoptions[-c(doorshown, doorchosen)]
        # You will always switch to the door that is NOT the door you chose
        # originally and NOT the door that Monty has revealed to be empty
        if (switchtodoor == prize) {
            # If you switched to the correct door
            successes[i] = 1
            # The entry at position 'i' of the successes vector will be recoded to 1
        }
    }
    print(sum(successes)/trials)
    # Eventually we will calculate the sum of successes and divide it by the
    # number of trials
}

Switching(1000)

## [1] 0.652

# Returns approximately 0.68.
```

Probability Theory

Problem 4

I will insert the solution here later.

a) If a and b are independent events, are the following true or false?

1. True
2. False.
3. True.

b) In general, $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ and $P(A \cap B) = P(A|B)P(B)$ which, when combined, yield: $P(A \cup B) = P(A) + P(B) - P(A|B)P(B)$. If the two events are independent then $P(A|B) = P(A)$, giving $P(A \cup B) = P(A) + P(B) - P(A)P(B) = P(B)(1 - P(A)) + P(A)$.

We solve for $P(B)$ to get $P(B) = \frac{P(A \cup B) - P(A)}{1 - P(A)} = \frac{0.5 - 0.3}{1 - 0.3} = \frac{2}{7}$.

c) A committee contains fifteen legislators with ten men and five women. Find the number of ways that a delegation of six:

1. This is the number of ways 6 elements can be chosen from 15, or $\binom{15}{6}$.

2. Now we have the joint probability of two independent events: choosing 3 women from 5 and 3 men from 10. This is: $\binom{10}{3}\binom{5}{3}$.

3. Finally, we have the joint probability of two independent events: choosing 2 women from 5 and 4 men from 10, since there are twice as many men as women in the full group. This is: $\binom{10}{4}\binom{5}{2}$.

d)

e) There are 36 possible outcomes for the dice rolls of player A and player B. All these outcomes are equally likely. There are several ways to compute the probability of player A having a strictly greater number than player B. One possible way is the following:

The probability of player B to roll any number from 1 to 6 is $\frac{1}{6}$, i.e. each possible number occurs with probability $\frac{1}{6}$.

If player B rolls a 1, player A can beat him with five different outcomes, i.e. the numbers from 2 to 6. This implies that the likelihood of beating him is $\frac{5}{6}$.

If player B rolls a 2, player A can beat him with four different outcomes, i.e. the numbers from 3 to 6. This implies that the likelihood of beating him is $\frac{4}{6}$.

Following this logic and applying it to all outcomes, player A will beat player B with the following probability:

$\sum_{i=1}^{6} Pr(B\ rolling\ i) * Pr(A\ beating\ B|B\ rolling\ i)$

$= \frac{1}{6} * \frac{5}{6} + \frac{1}{6} * \frac{4}{6} + \frac{1}{6} * \frac{3}{6} + \frac{1}{6} * \frac{2}{6} + \frac{1}{6} * \frac{1}{6} + \frac{1}{6} * \frac{0}{6} = \frac{15}{36} = \frac{5}{12}$

This is the probability for player A to win a single game against B.

We have to model the probability that player A will win four games against player B with a binomial distribution.

$\binom{5}{4}(\frac{5}{12})^4(\frac{7}{12})^1 = 0.088$

Additionally, player A could also win all five games against player B, so you have to add a second probability.

$\binom{5}{5}(\frac{5}{12})^5(\frac{7}{12})^0 = 0.012$

The total probability of both outcomes is approximately 0.1 or 10 percent.