

Tutorial 5: Regression Model Interpretation

Jan Vogler (jan.vogler@duke.edu)

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Today's Agenda

1. Marginal effects and intercepts
2. Hypothesis testing
3. Multiple linear regression
4. Graphical representation
5. Tips for your final paper

1. Marginal effects and intercepts

An essential aspect of all linear models are the marginal effects that predictor variables (independent variables) are estimated to have on the response variable (dependent variable).

Note that the word “effect” may be problematic because it implies causality. However, without any additional assumptions or additional model features, linear models allow us to make statements with respect to correlation only. This means we can't say anything about causality when just having a linear model. So let us be very cautious when we use the word “marginal effect”.

Every linear model has one response variable (dependent variable) and at least one predictor variable (independent variable) plus an intercept.

Let's assume that Y is our response variable and X is our only predictor variable. The model may look like this:

$$Y = 5 + 2X + \text{error}$$

How would we interpret the marginal effect of X?

The interpretation would be: For a 1-point increase in X we expect a 2-point increase in Y.

How would we interpret the intercept?

The intercept is the expected value of Y when X is at a value of 0.

Illustration of the marginal effect interpretation

Let's load another R dataset that can illustrate the interpretation of marginal effects. The “airquality” dataset. According to the documentation, this is “Daily air quality measurements in New York, May to September 1973.”

More details can be found here:

```
data(airquality)
summary(airquality)
```

##	Ozone	Solar.R	Wind	Temp
##	Min. : 1.00	Min. : 7.0	Min. : 1.700	Min. : 56.00
##	1st Qu.: 18.00	1st Qu.: 115.8	1st Qu.: 7.400	1st Qu.: 72.00
##	Median : 31.50	Median : 205.0	Median : 9.700	Median : 79.00

```
## Mean : 42.13 Mean :185.9 Mean : 9.958 Mean :77.88
## 3rd Qu.: 63.25 3rd Qu.:258.8 3rd Qu.:11.500 3rd Qu.:85.00
## Max. :168.00 Max. :334.0 Max. :20.700 Max. :97.00
## NA's :37 NA's :7
## Month Day
## Min. :5.000 Min. : 1.0
## 1st Qu.:6.000 1st Qu.: 8.0
## Median :7.000 Median :16.0
## Mean :6.993 Mean :15.8
## 3rd Qu.:8.000 3rd Qu.:23.0
## Max. :9.000 Max. :31.0
##
```

Our question is: is there a linear relationship between the Ozone measures and the Solar.R measures?

Let us use linear regression to answer this question:

```
lm1=lm(Ozone ~ Solar.R, data=airquality)
```

The summary of this linear regression will return a t-value and a p-value for the intercept and all coefficients.

```
summary(lm1)
```

```
##
## Call:
## lm(formula = Ozone ~ Solar.R, data = airquality)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -48.292 -21.361  -8.864  16.373 119.136
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept) 18.59873     6.74790   2.756 0.006856 **
## Solar.R      0.12717     0.03278   3.880 0.000179 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 31.33 on 109 degrees of freedom
## (42 observations deleted due to missingness)
## Multiple R-squared:  0.1213, Adjusted R-squared:  0.1133
## F-statistic: 15.05 on 1 and 109 DF, p-value: 0.0001793
```

How would we interpret the finding with respect to the linear relationship between the two variables? The interpretation would look like this:

There is a positive linear relationship between Ozone and Solar.R. For a 1-point increase in Solar.R, we would expect a 0.13 increase in Ozone (in a multivariate model we would have to add: “holding all other variables constant”).

Furthermore (already going into the next topic): The associated t-value is 3.880. This t-value implies a p-value of 0.0002. This $p < 0.001$ corresponds to a type-1 error rate of $\alpha < 0.001$, meaning that the relationship is significant at all common levels of statistical significance.

How do we interpret the R-squared statistic? Our model explains a proportion of the total variation in the dependent variable. The R-squared statistic returns this proportion. How well does our model do?

2. Hypothesis testing

Let us use another dataset to conduct some hypothesis tests.

We will look at data from an article that was published in the journal “International Organization”, the leading journal in the field of international relations. The article was written by Helen Milner and Keiko Kubota.

The article deals with the effect that democratization has on trade barriers. The authors believe that democratization has a negative effect on trade barriers in developing countries (that are scarce in capital). Their theory is based on the Stolper Samuelson theorem and the selectorate model by Bueno de Mesquita et al.

Let us try to emulate their test. In order to load their dataset you need to use the following command: `install.packages(“foreign”)`

Note that the working directory you set depends on where you have the file on your computer.

```
setwd('C:/Users/Jan/OneDrive/Documents/GitHub/ps630_lab/W5')
# Sets the working directory
library(foreign)
# Allows you to read more data formats
LDC=read.dta("LDC_IO_replication.dta")
summary(LDC)
```

```
##      country      ctylabel      date      gatt_wto_new
## Min.   :186.0   Length:5370   Min.    :1970   Min.    :0.0000
## 1st Qu.:423.0   Class :character   1st Qu.:1977   1st Qu.:0.0000
## Median :628.0   Mode  :character   Median :1984   Median :0.0000
## Mean   :605.9                      Mean   :1984   Mean   :0.4747
## 3rd Qu.:816.0                      3rd Qu.:1992   3rd Qu.:1.0000
## Max.   :968.0                      Max.    :1999   Max.    :1.0000
##                                     NA's    :698
##      aclpn      bpc1      dopen_wacz2      ecris2
## Min.   :0.0000   Min.   :0.000   Min.   :0.0000   Min.   :0.0000
## 1st Qu.:0.0000   1st Qu.:0.000   1st Qu.:0.0000   1st Qu.:0.0000
## Median :0.0000   Median :1.000   Median :0.0000   Median :0.0000
## Mean   :0.3002   Mean   :0.591   Mean   :0.3097   Mean   :0.0641
## 3rd Qu.:1.0000   3rd Qu.:1.000   3rd Qu.:1.0000   3rd Qu.:0.0000
## Max.   :1.0000   Max.   :1.000   Max.   :1.0000   Max.   :1.0000
## NA's    :1183   NA's    :2734   NA's    :2580   NA's    :1967
##      fdignp      gdp_pc_95d      l1aclpn      l1bpc1
## Min.   : -27.2356   Min.    :  0.0   Min.   :0.0000   Min.   :0.0000
## 1st Qu.:  0.0361   1st Qu.: 442.9   1st Qu.:0.0000   1st Qu.:0.0000
## Median :  0.6644   Median :1266.5   Median :0.0000   Median :1.0000
## Mean   :  1.8962   Mean   :2885.5   Mean   :0.2924   Mean   :0.5909
## 3rd Qu.:  2.0829   3rd Qu.:3002.4   3rd Qu.:1.0000   3rd Qu.:1.0000
## Max.   :184.5647   Max.   :44164.5   Max.   :1.0000   Max.   :1.0000
## NA's    :2294   NA's    :1679   NA's    :1341   NA's    :2735
##      l1ecris2      newtar      polityiv_update2      signed
## Min.   :0.0000   Min.    :  0.0   Min.   : -10.000   Min.   :0.0000
## 1st Qu.:0.0000   1st Qu.:10.95   1st Qu.: -7.000   1st Qu.:0.0000
## Median :0.0000   Median :17.00   Median : -6.000   Median :0.0000
## Mean   :0.0641   Mean   :20.54   Mean   : -2.074   Mean   :0.1465
## 3rd Qu.:0.0000   3rd Qu.:27.00   3rd Qu.:  6.000   3rd Qu.:0.0000
```

##	Max.	:1.0000	Max.	:102.20	Max.	: 10.000	Max.	:1.0000
##	NA's	:1967	NA's	:4463	NA's	:2003	NA's	:1362
##	yrsoffic		usheg		l1usheg		l1fiveop	
##	Min.	: 0.000	Min.	:0.2434	Min.	:0.2434	Min.	:10.20
##	1st Qu.:	2.000	1st Qu.:	0.2574	1st Qu.:	0.2574	1st Qu.:	10.90
##	Median :	5.000	Median :	0.2663	Median :	0.2655	Median :	12.35
##	Mean :	8.431	Mean :	0.2696	Mean :	0.2683	Mean :	12.03
##	3rd Qu.:	12.000	3rd Qu.:	0.2785	3rd Qu.:	0.2784	3rd Qu.:	12.72
##	Max.	:44.000	Max.	:0.3083	Max.	:0.2988	Max.	:13.20
##	NA's	:2361			NA's	:179	NA's	:358
##	l1gdp_pc		avsw		avnewtar		l1avsw	
##	Min.	: 0	Min.	:0.1398	Min.	: 0.00	Min.	:0.1398
##	1st Qu.:	442	1st Qu.:	0.1505	1st Qu.:	0.00	1st Qu.:	0.1505
##	Median :	1266	Median :	0.1720	Median :	17.43	Median :	0.1613
##	Mean :	2888	Mean :	0.3097	Mean :	14.91	Mean :	0.2974
##	3rd Qu.:	2999	3rd Qu.:	0.5269	3rd Qu.:	24.37	3rd Qu.:	0.5054
##	Max.	:44165	Max.	:0.6667	Max.	:30.52	Max.	:0.6559
##	NA's	:1823					NA's	:179
##	l1avnewtar		lnpop		l1lnpop		l1office	
##	Min.	: 0.00	Min.	:10.57	Min.	:10.62	Min.	: 0.000
##	1st Qu.:	0.00	1st Qu.:	13.86	1st Qu.:	13.86	1st Qu.:	2.000
##	Median :	18.73	Median :	15.32	Median :	15.31	Median :	5.000
##	Mean :	15.01	Mean :	15.11	Mean :	15.10	Mean :	8.431
##	3rd Qu.:	24.37	3rd Qu.:	16.40	3rd Qu.:	16.39	3rd Qu.:	12.000
##	Max.	:30.52	Max.	:20.95	Max.	:20.94	Max.	:44.000
##	NA's	:179	NA's	:490	NA's	:661	NA's	:2361
##	l1partyage2000		l1fdi		l1polity		l2polity	
##	Min.	: 0.00	Min.	:-27.2356	Min.	:-10.000	Min.	:-10.00
##	1st Qu.:	10.00	1st Qu.:	0.0269	1st Qu.:	-7.000	1st Qu.:	-7.00
##	Median :	19.50	Median :	0.6382	Median :	-6.000	Median :	-7.00
##	Mean :	24.18	Mean :	1.7931	Mean :	-2.215	Mean :	-2.36
##	3rd Qu.:	32.00	3rd Qu.:	1.9904	3rd Qu.:	6.000	3rd Qu.:	5.00
##	Max.	:183.00	Max.	:184.5647	Max.	: 10.000	Max.	: 10.00
##	NA's	:3284	NA's	:2423	NA's	:2124	NA's	:2246
##	l3polity		l1signed		milit2		sp2	
##	Min.	:-10.000	Min.	:0.0000	Min.	:0.0000	Min.	:0.0000
##	1st Qu.:	-7.000	1st Qu.:	0.0000	1st Qu.:	0.0000	1st Qu.:	0.0000
##	Median :	-7.000	Median :	0.0000	Median :	0.0000	Median :	0.0000
##	Mean :	-2.512	Mean :	0.1511	Mean :	0.1119	Mean :	0.1959
##	3rd Qu.:	5.000	3rd Qu.:	0.0000	3rd Qu.:	0.0000	3rd Qu.:	0.0000
##	Max.	: 10.000	Max.	:1.0000	Max.	:1.0000	Max.	:1.0000
##	NA's	:2371	NA's	:1517				
##	pers2		l1milit2		l1sp2		dictator1	
##	Min.	:0.0000	Min.	:0.0000	Min.	:0.0000	Min.	:1.000
##	1st Qu.:	0.0000	1st Qu.:	0.0000	1st Qu.:	0.0000	1st Qu.:	2.000
##	Median :	0.0000	Median :	0.0000	Median :	0.0000	Median :	5.000
##	Mean :	0.1665	Mean :	0.1135	Mean :	0.1986	Mean :	4.737
##	3rd Qu.:	0.0000	3rd Qu.:	0.0000	3rd Qu.:	0.0000	3rd Qu.:	8.000
##	Max.	:1.0000	Max.	:1.0000	Max.	:1.0000	Max.	:8.000
##			NA's	:179	NA's	:179	NA's	:1157
##	l1dictator1		yr70		yr80		l1ssch	
##	Min.	:1.000	Min.	:0.0000	Min.	:0.0000	Min.	:0.0140
##	1st Qu.:	2.000	1st Qu.:	0.0000	1st Qu.:	0.0000	1st Qu.:	0.4562
##	Median :	5.000	Median :	0.0000	Median :	0.0000	Median :	0.8519

```
## Mean :4.708 Mean :0.3333 Mean :0.3333 Mean :1.0411
## 3rd Qu.:8.000 3rd Qu.:1.0000 3rd Qu.:1.0000 3rd Qu.:1.4652
## Max. :8.000 Max. :1.0000 Max. :1.0000 Max. :4.4422
## NA's :1315 NA's :3140
## closedyr _spline1 _spline2 _spline3
## Min. : 0.000 Min. : -24389 Min. : -7854.0 Min. : -9030.0
## 1st Qu.: 0.000 1st Qu.: -3375 1st Qu.: -2048.3 1st Qu.: -1629.3
## Median : 7.000 Median : -343 Median : -260.2 Median : -165.6
## Mean : 8.691 Mean : -3075 Mean : -1388.8 Mean : -1340.9
## 3rd Qu.:15.000 3rd Qu.: 0 3rd Qu.: 0.0 3rd Qu.: 0.0
## Max. :29.000 Max. : 0 Max. : 0.0 Max. : 0.0
## NA's :2580 NA's :2580 NA's :2580 NA's :2580
## l1gatt_wto_new
## Min. :0.000
## 1st Qu.:0.000
## Median :0.000
## Mean :0.468
## 3rd Qu.:1.000
## Max. :1.000
## NA's :868
```

For information on the meaning of the variables see “LDCcodebook.pdf”.

Let’s have a look at our data.

We need a package for generating a scatterplot matrix that allows us to see relationships in our matrix.

```
install.packages(“car”)
```

```
library(car)
```

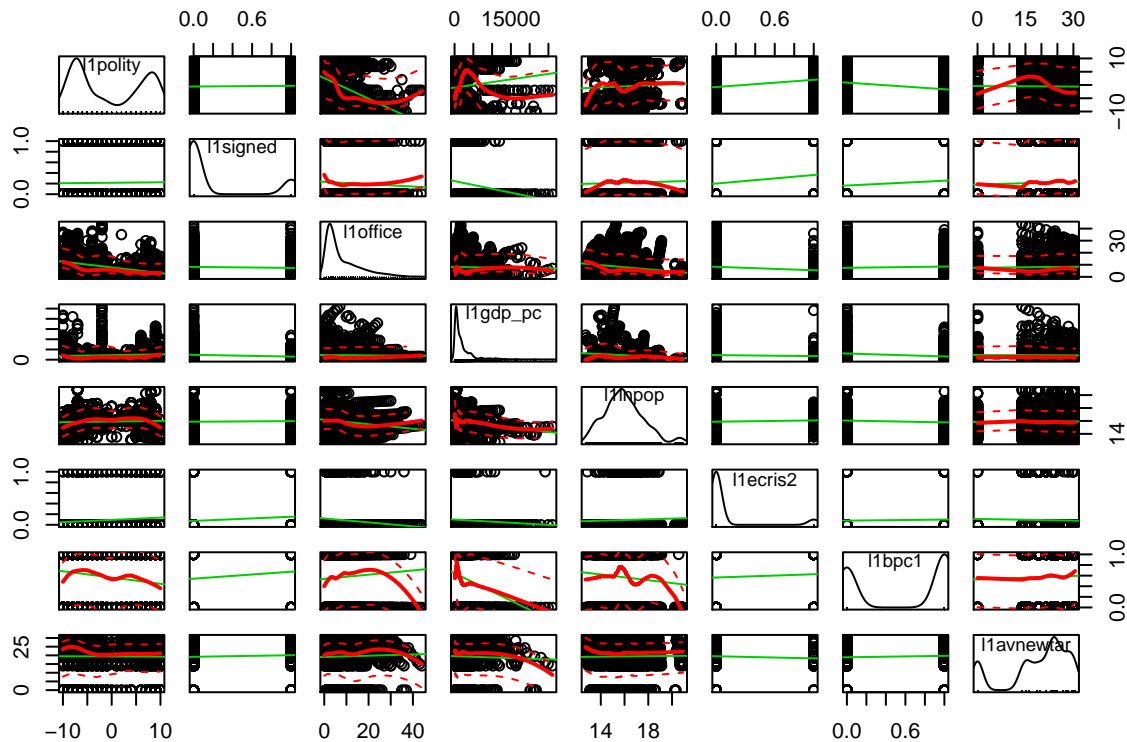
```
LDC2=as.data.frame(LDC[,c(“l1polity”, “l1signed”, “l1office”, “l1gdp_pc”, “l1lnpop”, “l1ecris2”, “l1bpc1”, “l1avnewtar”)])
cor(LDC2)
```

```
##          l1polity l1signed l1office l1gdp_pc l1lnpop l1ecris2 l1bpc1
## l1polity          1        NA        NA        NA        NA        NA
## l1signed          NA         1        NA        NA        NA        NA
## l1office          NA        NA         1        NA        NA        NA
## l1gdp_pc          NA        NA        NA         1        NA        NA
## l1lnpop           NA        NA        NA        NA         1        NA
## l1ecris2          NA        NA        NA        NA        NA         1
## l1bpc1            NA        NA        NA        NA        NA         1
## l1avnewtar        NA        NA        NA        NA        NA        NA
##          l1avnewtar
## l1polity          NA
## l1signed          NA
## l1office          NA
## l1gdp_pc          NA
## l1lnpop           NA
## l1ecris2          NA
## l1bpc1            NA
## l1avnewtar         1
```

```
LDC3=na.omit(LDC2)
cor(LDC3)
```

```
##          l1polity    l1signed    l1office    l1gdp_pc    l1lnpop
## l1polity    1.00000000    0.01499208   -0.42901753    0.09129002    0.04404210
## l1signed    0.01499208    1.00000000   -0.04303356   -0.11240587    0.02321607
## l1office   -0.42901753   -0.04303356    1.00000000   -0.01936245   -0.17457461
## l1gdp_pc    0.09129002   -0.11240587   -0.01936245    1.00000000   -0.14082411
## l1lnpop     0.04404209    0.02321607   -0.17457461   -0.14082411    1.00000000
## l1ecris2    0.10843353    0.11269712   -0.10331828   -0.04446845    0.03580805
## l1bpc1     -0.17176285    0.10895350    0.06396247   -0.22176992   -0.08570695
## l1avnewtar -0.008790383    0.03967319    0.03040646   -0.02357807    0.01262803
##          l1ecris2    l1bpc1    l1avnewtar
## l1polity    0.10843353   -0.17176286   -0.008790383
## l1signed    0.11269712    0.10895350    0.039673186
## l1office   -0.10331828    0.06396247    0.030406457
## l1gdp_pc   -0.04446845   -0.22176992   -0.023578065
## l1lnpop     0.03580805   -0.08570695    0.012628033
## l1ecris2    1.00000000    0.03517381   -0.036064786
## l1bpc1      0.03517381    1.00000000    0.042851289
## l1avnewtar -0.03606479    0.04285129    1.000000000
```

```
scatterplotMatrix(~ l1polity + l1signed + l1office + l1gdp_pc + l1lnpop + l1ecris2 + l1bpc1 + l1avnewtar
```



The results above indicate that there generally is a low level of multicollinearity among our variables.

Let us start with a simple model that is easy to interpret:

```
simple=lm(newtar~l1polity, data=LDC)
summary(simple)

##
## Call:
## lm(formula = newtar ~ l1polity, data = LDC)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -23.425  -9.200  -3.425   5.275  80.475
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)  21.92495    0.52865  41.474 < 2e-16 ***
## l1polity     -0.30001    0.07293  -4.113  4.3e-05 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 14.9 on 804 degrees of freedom
## (4564 observations deleted due to missingness)
## Multiple R-squared:  0.02061,    Adjusted R-squared:  0.01939
## F-statistic: 16.92 on 1 and 804 DF,  p-value: 4.298e-05
```

What can we conclude from these statistics? What can we say about the hypothesis that there is a linear relationship between “l1polity” and “newtar”? What is the total variation that is explained by our model?

If there’s too much information in this type of summary, try another one. We need another package: `install.packages(“arm”)`

```
library(arm)

## Warning: package 'arm' was built under R version 3.2.2

## Loading required package: MASS
## Loading required package: Matrix
## Loading required package: lme4

## Warning: package 'lme4' was built under R version 3.2.2

##
## arm (Version 1.8-6, built: 2015-7-7)
##
## Working directory is C:/Users/Jan/OneDrive/Documents/GitHub/ps630_lab/W5
##
##
## Attaching package: 'arm'
##
## The following object is masked from 'package:car':
##
##      logit
```

```
display(simple)
```

```
## lm(formula = newtar ~ l1polity, data = LDC)
##           coef.est coef.se
## (Intercept) 21.92      0.53
## l1polity    -0.30      0.07
## ---
## n = 806, k = 2
## residual sd = 14.90, R-Squared = 0.02
```

As you can see, this is narrowed down to just a few pieces of information. Sometimes reducing the amount of information that is displayed can be very useful.

3. Multiple linear regression

In the vast majority of cases there are good reasons to include multiple predictor variables.

The most important reasons to do so are:

1. Potential omitted variable bias
2. Theoretical reasons
3. Reviewers that demand you to include them

```
main=lm(newtar ~ l1polity + l1signed + l1office + l1gdp_pc + l1lnpop + l1ecris2 + l1bpc1 + l1avnewtar, data = LDC)
summary(main)
```

```
##
## Call:
## lm(formula = newtar ~ l1polity + l1signed + l1office + l1gdp_pc +
##      l1lnpop + l1ecris2 + l1bpc1 + l1avnewtar, data = LDC)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -24.286  -7.694  -2.175   4.490  65.008
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept) -4.901e+01  5.912e+00  -8.289 6.03e-16 ***
## l1polity     -2.053e-01  8.347e-02  -2.460 0.014151 *
## l1signed      4.758e-01  1.099e+00   0.433 0.665332
## l1office     -1.759e-01  6.989e-02  -2.516 0.012083 *
## l1gdp_pc     -1.281e-03  1.495e-04  -8.564 < 2e-16 ***
## l1lnpop       3.693e+00  3.217e-01  11.478 < 2e-16 ***
## l1ecris2     -5.736e+00  1.517e+00  -3.780 0.000171 ***
## l1bpc1       4.564e-01  9.681e-01   0.471 0.637462
## l1avnewtar    7.103e-01  8.413e-02   8.442 < 2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 12.16 on 685 degrees of freedom
```



```
## (4676 observations deleted due to missingness)
## Multiple R-squared: 0.3781, Adjusted R-squared: 0.3708
## F-statistic: 52.05 on 8 and 685 DF, p-value: < 2.2e-16
```

In the multiple linear regression, how would our expectation for the average tariff level change if our Polity Score increased from -10 to 10 and we had an economic crisis?

How well does our model do compared to the simple linear regression? Do we observe an improvement in the total variation that is explained by our model?

Again, it would be possible to reduce the amount of information with another command:

```
display(main)
```

```
## lm(formula = newtar ~ l1polity + l1signed + l1office + l1gdp_pc +
##      l1lnpop + l1ecris2 + l1bpc1 + l1avnewtar, data = LDC)
##               coef.est coef.se
## (Intercept) -49.01      5.91
## l1polity      -0.21      0.08
## l1signed       0.48      1.10
## l1office      -0.18      0.07
## l1gdp_pc       0.00      0.00
## l1lnpop        3.69      0.32
## l1ecris2      -5.74      1.52
## l1bpc1         0.46      0.97
## l1avnewtar     0.71      0.08
## ---
## n = 694, k = 9
## residual sd = 12.16, R-Squared = 0.38
```

We can access different elements of our model. Let's have a look at what those are:

```
names(main)
```

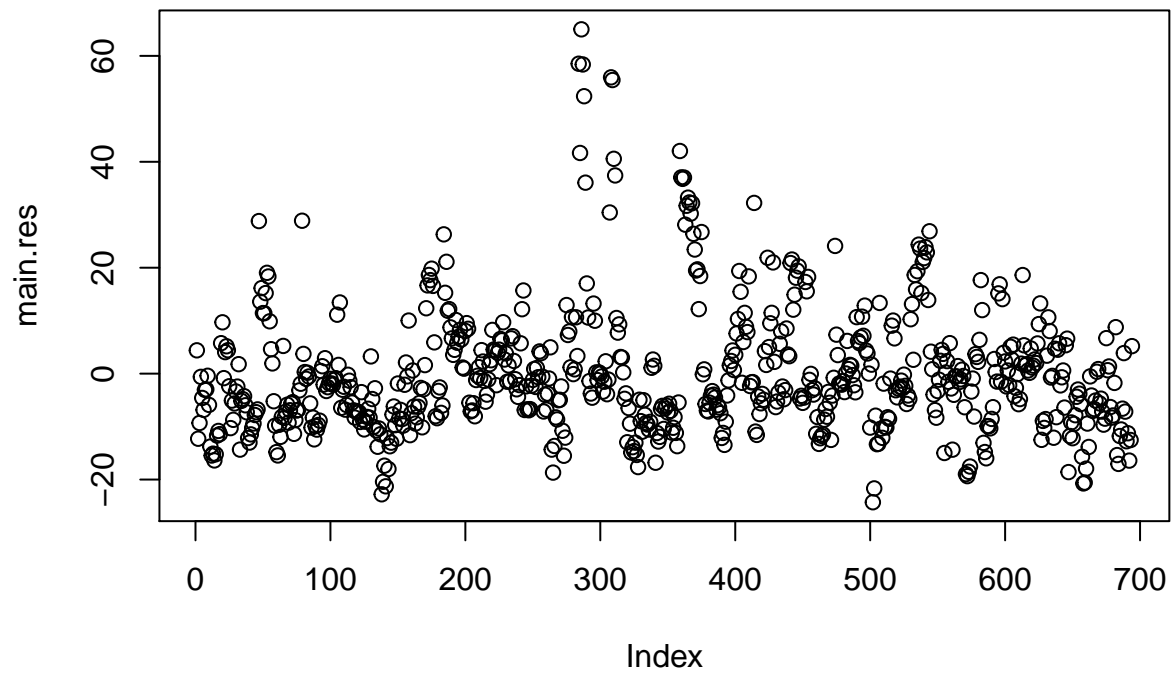
```
## [1] "coefficients" "residuals"      "effects"        "rank"
## [5] "fitted.values" "assign"          "qr"             "df.residual"
## [9] "na.action"     "xlevels"         "call"           "terms"
## [13] "model"
```

4. Graphical representation

Let us first have a look at the distribution of errors in our model.

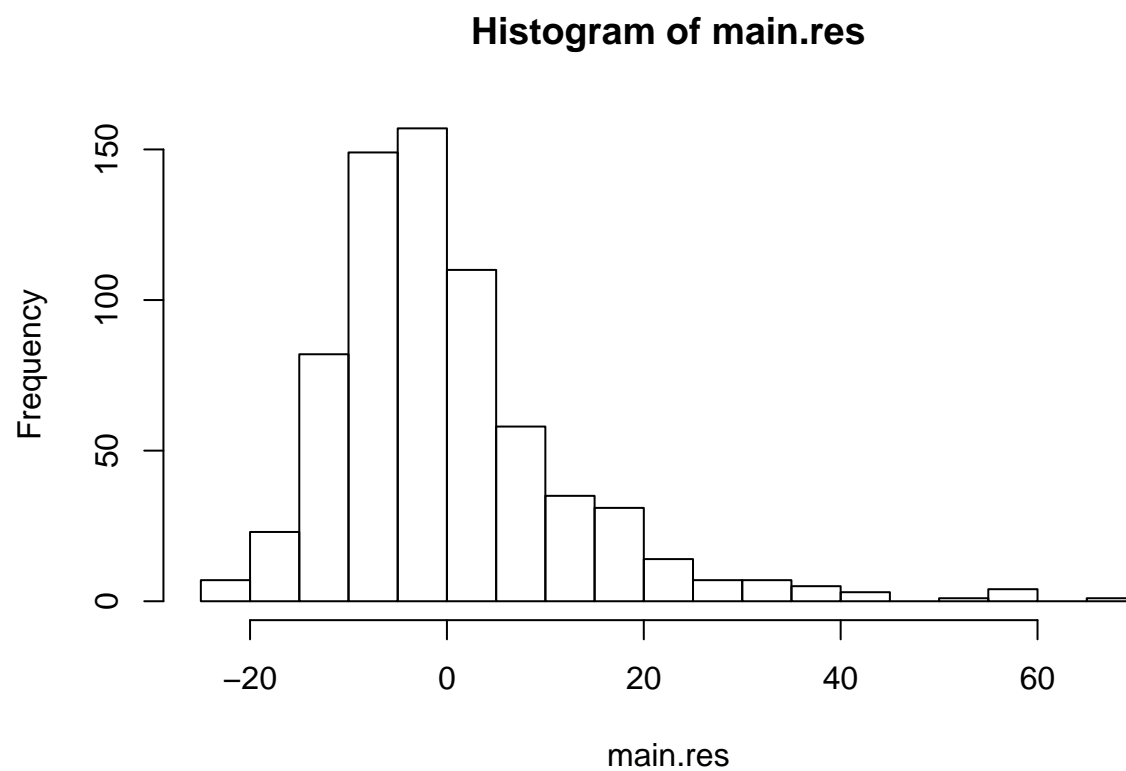
```
main.res = resid(main)
plot(main.res, main="Values of the Error Term")
```

Values of the Error Term

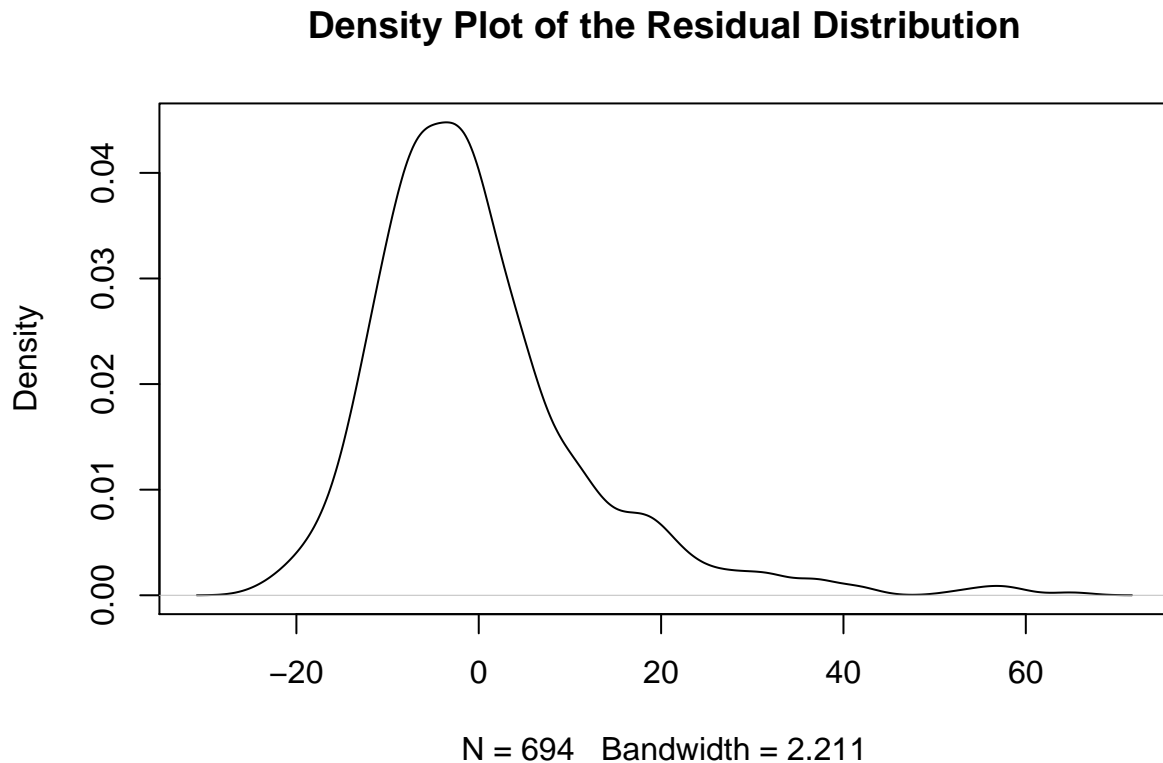


Let us look at the distribution of the error term:

```
hist(main.res, breaks=20)
```



```
res.density=density(main.res)  
plot(res.density, main="Density Plot of the Residual Distribution")
```



The distribution of the errors is approximately normal. If this condition is met, then more precise statements about the distribution of the coefficients can be made (they're also normal). Also, under these conditions, OLS is equivalent to a maximum likelihood approach.

Plotting predicted values

Let us plot some predicted values with confidence intervals for our multiple regression.

In order to do that we first create a dataframe that contains different values for our main predictor variable and the average values for all variables.

```
nd <- data.frame(l1polity=seq(-10,10,by=1), l1signed=rep(0.1511,21), l1office=rep(8.431,21), l1gdp_pc=r
```

Next we use the model we estimated to predict values based on this new dataframe.

```
pred.p1 <- predict(main, type="response", se.fit=TRUE, newdata=nd)
```

```
pred.table <- cbind(pred.p1$fit, pred.p1$se.fit)
pred.table
```

```
##      [,1]      [,2]
## 1  14.19185  1.3422851
## 2  13.98655  1.2793558
## 3  13.78125  1.2188955
## 4  13.57595  1.1612899
## 5  13.37065  1.1069847
## 6  13.16535  1.0564892
```

```
## 7  12.96005  1.0103744
## 8  12.75475  0.9692661
## 9  12.54945  0.9338254
## 10 12.34415  0.9047189
## 11 12.13885  0.8825733
## 12 11.93355  0.8679216
## 13 11.72825  0.8611465
## 14 11.52295  0.8624336
## 15 11.31765  0.8717473
## 16 11.11235  0.8888351
## 17 10.90705  0.9132609
## 18 10.70175  0.9444554
## 19 10.49645  0.9817738
## 20 10.29115  1.0245470
## 21 10.08585  1.0721223
```

Finally, we create the plot:

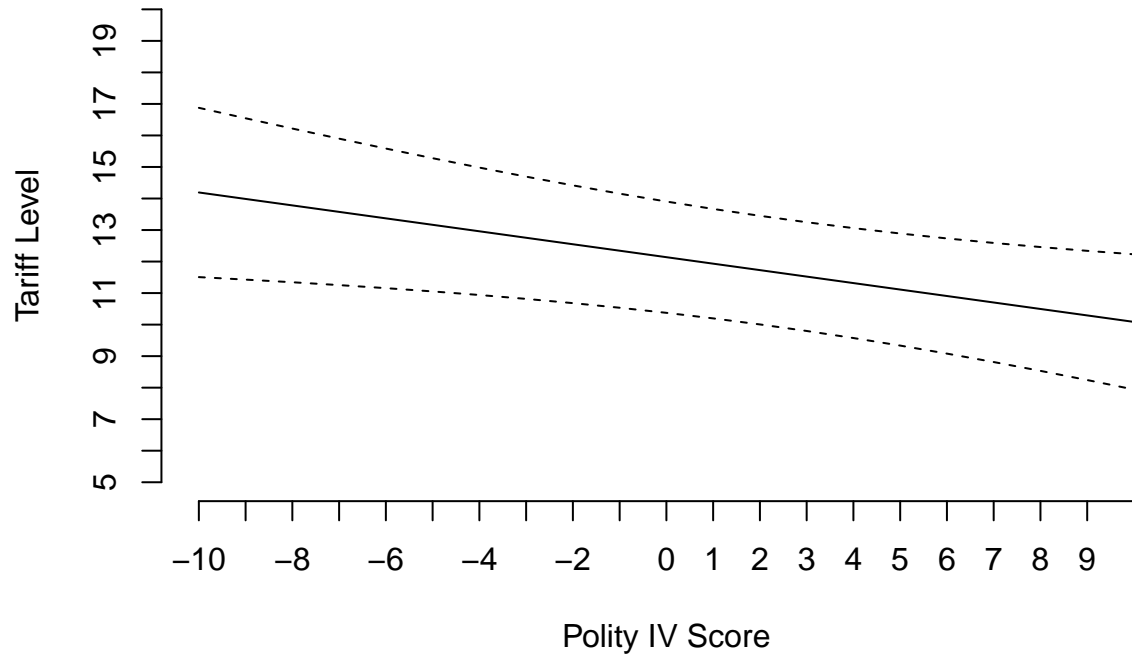
```
fit <- pred.p1$fit
low <- pred.p1$fit - 2*pred.p1$se.fit
high <- pred.p1$fit + 2*pred.p1$se.fit
cis <- cbind(fit, low, high)

cis ### To extract the values
```

```
##      fit      low      high
## 1  14.19185 11.507283 16.87642
## 2  13.98655 11.427842 16.54526
## 3  13.78125 11.343462 16.21904
## 4  13.57595 11.253373 15.89853
## 5  13.37065 11.156683 15.58462
## 6  13.16535 11.052373 15.27833
## 7  12.96005 10.939303 14.98080
## 8  12.75475 10.816219 14.69328
## 9  12.54945 10.681800 14.41710
## 10 12.34415 10.534713 14.15359
## 11 12.13885 10.373703 13.90400
## 12 11.93355 10.197706 13.66939
## 13 11.72825 10.005956 13.45054
## 14 11.52295  9.798082 13.24782
## 15 11.31765  9.574154 13.06114
## 16 11.11235  9.334678 12.89002
## 17 10.90705  9.080526 12.73357
## 18 10.70175  8.812837 12.59066
## 19 10.49645  8.532900 12.45999
## 20 10.29115  8.242053 12.34024
## 21 10.08585  7.941602 12.23009
```

```
plot(pred.p1$fit, type="l", ylim=c(5,20), main="Polity IV Score and Tariff Level",
      xlab="Polity IV Score", ylab="Tariff Level", axes=FALSE)
axis(1, at=seq(1,21), labels=seq(-10,10,1))
axis(2, at=seq(5,20), labels=seq(5,20))
matlines(cis[,c(2,3)], lty=2, col="black")
```

Polity IV Score and Tariff Level



5. Tips for your final paper

1. Start working on it early.
2. Consult with your professors and TAs.
3. Try to find a comprehensive dataset in your area of interest.
4. Work on it throughout the semester and try to include new things that you've learned.
5. Make sure that you use all the tools you've learned: interpret your findings carefully and visualize them.
6. Annotate your code extensively and explain what you did.