Tutorial 5: Regression Model Interpretation

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Today's Agenda

- 1. Marginal effects and intercepts
- 2. Hypothesis testing
- 3. Multiple linear regression
- 4. Graphical representation
- 5. Tips for your final paper

1. Marginal effects and intercepts

An essential aspect of all linear models are the marginal effects that predictor variables (independent variables) are estimated to have on the response variable (dependent variable).

Note that the word "effect" may be problematic because it implies causality. However, without any additional assumptions or additional model features, linear models allow us to make statements with respect to correlation only. This means we can't say anything about causality when just having a linear model. So let us be very cautious when we use the word "marginal effect".

Every linear model has one response variable (dependent variable) and at least one predictor variable (independent variable) plus an intercept.

Let's assume that Y is our response variable and X is our only predictor variable. The model may look like this:

$$Y = 5 + 2X + error$$

How would we interpet the marginal effect of X?

The interpretation would be: For a 1-point increase in X we expect a 2-point increase in Y.

How would we interpret the intercept?

The intercept is the expected value of Y when X is at a value of 0.

Illustration of the marginal effect interpretation

Let's load another R dataset that can illustrate the interpretation of marginal effects. The "airquality" dataset. According to the documentation, this is "Daily air quality measurements in New York, May to September 1973."

More details can be found here:

```
data(airquality)
summary(airquality)
```

##	Ozone			Solar.R			Wind			Temp	
##	Min.	:	1.00	Min.	:	7.0	Min.	:	1.700	Min.	:56.00
##	1st Qu.	:	18.00	1st Qu	:1	15.8	1st Qu	. :	7.400	1st Qu	:72.00
##	Median	:	31.50	Median	:2	05.0	Median	:	9.700	Median	:79.00
##	Mean	:	42.13	Mean	:1	85.9	Mean	:	9.958	Mean	:77.88

```
3rd Qu.: 63.25
                      3rd Qu.:258.8
                                        3rd Qu.:11.500
                                                          3rd Qu.:85.00
##
            :168.00
                              :334.0
                                               :20.700
                                                                  :97.00
##
    Max.
                      Max.
                                       Max.
                                                          Max.
##
    NA's
            :37
                      NA's
                              :7
##
        Month
                           Day
##
    Min.
            :5.000
                     Min.
                             : 1.0
##
    1st Qu.:6.000
                     1st Qu.: 8.0
##
    Median :7.000
                     Median:16.0
##
    Mean
            :6.993
                     Mean
                             :15.8
##
    3rd Qu.:8.000
                     3rd Qu.:23.0
##
    Max.
           :9.000
                     Max.
                             :31.0
##
```

Our question is: is there a linear relationship between the Ozone measures and the Solar.R measures?

```
Let us use linear regression to answer this question:
```

 $lm1 = lm(Ozone \sim Solar.R, data = airquality)$

The summary of this linear regression will return a t-value and a p-value for the intercept and all coefficients.

summary(lm1)

```
##
## Call:
## lm(formula = Ozone ~ Solar.R, data = airquality)
##
## Residuals:
##
       Min
                1Q Median
                                3Q
                                       Max
##
   -48.292 -21.361
                    -8.864
                            16.373 119.136
##
## Coefficients:
##
               Estimate Std. Error t value Pr(>|t|)
## (Intercept) 18.59873
                           6.74790
                                     2.756 0.006856 **
## Solar.R
                           0.03278
                                     3.880 0.000179 ***
                0.12717
## ---
                   0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
## Signif. codes:
## Residual standard error: 31.33 on 109 degrees of freedom
     (42 observations deleted due to missingness)
## Multiple R-squared: 0.1213, Adjusted R-squared: 0.1133
## F-statistic: 15.05 on 1 and 109 DF, p-value: 0.0001793
```

How would we interpret the finding with respect to the linear relationship between the two variables? The interpretation would look like this:

There is a positive linear relationship between Ozone and Solar.R. For a 1-point increase in Solar.R, we would expect a 0.13 increase in Ozone (in a multivariate model we would have to add: "holding all other variables constant").

Furthermore (already going into the next topic): The associated t-value is 3.880. This t-value implies a p-value of 0.0002. This p < 0.001 corresponds to a type-1 error rate of alpha < 0.001, meaning that the relationship is significant at all common levels of statistical significance.

How do we interpret the R-squared statistic? Our model explains a proportion of the total variation in the dependent variable. The R-squared statistic returns this proportion. How well does our model do?

2. Hypothesis testing

Let us use another dataset to conduct some hypothesis tests.

We will look at data from an article that was published in the journal "International Organization", the leading journal in the field of international relations. The article was written by Helen Milner and Keiko Kubota.

The article deals with the effect that democratization has on trade barriers. The authors believe that democratization has a negative effect on trade barriers in developing countries (that are scarce in capital). Their theory is based on the Stolper Samuelson theorem and the selectorate model by Bueno de Mesquita et al

Let us try to emulate their test. In order to load their dataset you need to use the following command: install.packages("foreign")

Note that the working directory you set depends on where you have the file on your computer.

```
setwd("C:/Users/Jan/OneDrive/Documents/GitHub/ps630_lab/ps630_f16/")
# Sets the working directory
library(foreign)
# Allows you to read more data formats
LDC = read.dta("LDC_IO_replication.dta")
summary(LDC)
```

```
##
       country
                        ctylabel
                                                date
                                                           gatt_wto_new
##
    Min.
            :186.0
                     Length:5370
                                                  :1970
                                                                  :0.0000
                                          Min.
                                                          Min.
##
    1st Qu.:423.0
                     Class : character
                                          1st Qu.:1977
                                                          1st Qu.:0.0000
##
    Median :628.0
                     Mode
                           :character
                                          Median:1984
                                                          Median :0.0000
##
    Mean
            :605.9
                                          Mean
                                                  :1984
                                                          Mean
                                                                  :0.4747
##
    3rd Qu.:816.0
                                          3rd Qu.:1992
                                                          3rd Qu.:1.0000
##
    Max.
            :968.0
                                          Max.
                                                  :1999
                                                                  :1.0000
                                                          Max.
##
                                                          NA's
                                                                  :698
##
                                         dopen_wacz2
                                                               ecris2
        aclpn
                            bpc1
##
    Min.
            :0.0000
                      Min.
                              :0.000
                                                :0.0000
                                                          Min.
                                                                  :0.0000
##
    1st Qu.:0.0000
                      1st Qu.:0.000
                                        1st Qu.:0.0000
                                                          1st Qu.:0.0000
                                        Median :0.0000
##
    Median :0.0000
                      Median :1.000
                                                          Median :0.0000
##
    Mean
            :0.3002
                              :0.591
                                                :0.3097
                                                                  :0.0641
                      Mean
                                        Mean
                                                          Mean
##
    3rd Qu.:1.0000
                      3rd Qu.:1.000
                                        3rd Qu.:1.0000
                                                          3rd Qu.:0.0000
##
    Max.
            :1.0000
                              :1.000
                                                :1.0000
                                                                  :1.0000
                      Max.
                                        Max.
                                                          Max.
##
    NA's
            :1183
                      NA's
                              :2734
                                        NA's
                                                :2580
                                                          NA's
                                                                  :1967
##
                           gdp_pc_95d
                                                                   l1bpc1
        fdignp
                                                l1aclpn
##
    Min.
            :-27.2356
                         Min.
                                      0.0
                                            Min.
                                                    :0.0000
                                                               Min.
                                                                       :0.0000
##
    1st Qu.: 0.0361
                         1st Qu.:
                                            1st Qu.:0.0000
                                                               1st Qu.:0.0000
                                   442.9
              0.6644
                         Median: 1266.5
                                            Median :0.0000
##
    Median :
                                                               Median :1.0000
##
    Mean
            :
               1.8962
                         Mean
                                : 2885.5
                                            Mean
                                                    :0.2924
                                                               Mean
                                                                       :0.5909
##
    3rd Qu.:
               2.0829
                         3rd Qu.: 3002.4
                                            3rd Qu.:1.0000
                                                               3rd Qu.:1.0000
##
    Max.
            :184.5647
                         Max.
                                 :44164.5
                                            Max.
                                                    :1.0000
                                                               Max.
                                                                       :1.0000
##
    NA's
            :2294
                         NA's
                                 :1679
                                            NA's
                                                    :1341
                                                               NA's
                                                                       :2735
##
       11ecris2
                           newtar
                                         polityiv_update2
                                                                 signed
##
            :0.0000
                              : 0.00
                                                 :-10.000
                                                                    :0.0000
    Min.
                      Min.
                                         Min.
                                                             Min.
                      1st Qu.: 10.95
##
    1st Qu.:0.0000
                                         1st Qu.: -7.000
                                                             1st Qu.:0.0000
##
    Median :0.0000
                      Median: 17.00
                                         Median : -6.000
                                                             Median :0.0000
##
    Mean
            :0.0641
                      Mean
                              : 20.54
                                         Mean
                                                 : -2.074
                                                             Mean
                                                                    :0.1465
    3rd Qu.:0.0000
                      3rd Qu.: 27.00
                                         3rd Qu.: 6.000
                                                             3rd Qu.:0.0000
```

```
Max.
          :1.0000
                    Max.
                           :102.20
                                     Max.
                                           : 10.000
                                                      Max.
                                                             :1.0000
##
   NA's
          :1967
                    NA's
                           :4463
                                     NA's
                                           :2003
                                                      NA's
                                                           :1362
##
      yrsoffic
                        usheg
                                        11usheg
                                                        11fiveop
   Min. : 0.000
##
                           :0.2434
                                     Min.
                                           :0.2434
                                                     Min. :10.20
                    Min.
##
   1st Qu.: 2.000
                    1st Qu.:0.2574
                                     1st Qu.:0.2574
                                                     1st Qu.:10.90
##
   Median : 5.000
                    Median :0.2663
                                     Median :0.2655
                                                     Median :12.35
   Mean : 8.431
                    Mean :0.2696
                                     Mean :0.2683
                                                     Mean :12.03
                                                     3rd Qu.:12.72
   3rd Qu.:12.000
                    3rd Qu.:0.2785
                                     3rd Qu.:0.2784
##
##
   Max.
          :44.000
                    Max. :0.3083
                                     Max.
                                           :0.2988
                                                     Max. :13.20
##
   NA's
          :2361
                                     NA's :179
                                                     NA's :358
      l1gdp_pc
                                      avnewtar
                                                       l1avsw
                       avsw
   Min. : 0
                                    Min. : 0.00
##
                   Min. :0.1398
                                                   Min. :0.1398
                                    1st Qu.: 0.00
   1st Qu.: 442
                   1st Qu.:0.1505
                                                   1st Qu.:0.1505
   Median: 1266
                   Median :0.1720
                                    Median :17.43
                                                   Median : 0.1613
##
   Mean : 2888
                   Mean :0.3097
                                    Mean :14.91
                                                   Mean
                                                         :0.2974
##
   3rd Qu.: 2999
                   3rd Qu.:0.5269
                                    3rd Qu.:24.37
                                                   3rd Qu.:0.5054
##
   Max.
         :44165
                   Max. :0.6667
                                    Max. :30.52
                                                          :0.6559
                                                   Max.
##
   NA's
         :1823
                                                   NA's
                                                          :179
##
     11avnewtar
                                                     11office
                       lnpop
                                      111npop
##
   Min. : 0.00
                   Min. :10.57
                                   Min. :10.62
                                                  Min. : 0.000
##
   1st Qu.: 0.00
                   1st Qu.:13.86
                                   1st Qu.:13.86
                                                  1st Qu.: 2.000
   Median :18.73
                   Median :15.32
                                   Median :15.31
                                                  Median : 5.000
   Mean :15.01
                                   Mean :15.10
##
                   Mean :15.11
                                                  Mean : 8.431
   3rd Qu.:24.37
                   3rd Qu.:16.40
                                   3rd Qu.:16.39
                                                  3rd Qu.:12.000
##
                                   Max. :20.94
##
   Max. :30.52
                         :20.95
                   Max.
                                                  Max.
                                                         :44.000
   NA's
         :179
                   NA's :490
                                   NA's
                                        :661
                                                  NA's
                                                         :2361
##
   l1partyage2000
                        l1fdi
                                         11polity
                                                         12polity
   Min. : 0.00
                    Min. :-27.2356
                                      Min. :-10.000
                                                        Min. :-10.00
##
##
   1st Qu.: 10.00
                    1st Qu.: 0.0269
                                       1st Qu.: -7.000
                                                        1st Qu.: -7.00
   Median: 19.50
                    Median: 0.6382
                                       Median : -6.000
                                                        Median : -7.00
   Mean : 24.18
                    Mean : 1.7931
                                       Mean : -2.215
                                                        Mean : -2.36
##
##
   3rd Qu.: 32.00
                    3rd Qu.: 1.9904
                                       3rd Qu.: 6.000
                                                        3rd Qu.: 5.00
   Max. :183.00
                    Max. :184.5647
                                       Max. : 10.000
##
                                                        Max. : 10.00
                                                               :2246
##
   NA's
         :3284
                    NA's
                          :2423
                                       NA's
                                            :2124
                                                        NA's
##
      13polity
                        11signed
                                         milit2
                                                           sp2
                     Min. :0.0000
                                                      Min. :0.0000
##
   Min.
         :-10.000
                                      Min. :0.0000
   1st Qu.: -7.000
                     1st Qu.:0.0000
                                      1st Qu.:0.0000
                                                      1st Qu.:0.0000
##
   Median : -7.000
                     Median :0.0000
                                      Median :0.0000
                                                      Median :0.0000
   Mean : -2.512
##
                     Mean :0.1511
                                      Mean :0.1119
                                                      Mean :0.1959
                     3rd Qu.:0.0000
##
   3rd Qu.: 5.000
                                      3rd Qu.:0.0000
                                                      3rd Qu.:0.0000
   Max. : 10.000
                     Max. :1.0000
                                      Max. :1.0000
                                                      Max. :1.0000
   NA's :2371
                     NA's :1517
##
                       l1milit2
                                                       dictator1
##
       pers2
                                        l1sp2
##
         :0.0000
                                     Min. :0.0000
                                                     Min. :1.000
                    Min. :0.0000
   Min.
   1st Qu.:0.0000
                    1st Qu.:0.0000
                                     1st Qu.:0.0000
                                                     1st Qu.:2.000
   Median :0.0000
                    Median :0.0000
                                     Median :0.0000
                                                     Median :5.000
##
   Mean :0.1665
##
                    Mean :0.1135
                                     Mean :0.1986
                                                     Mean :4.737
##
   3rd Qu.:0.0000
                    3rd Qu.:0.0000
                                     3rd Qu.:0.0000
                                                     3rd Qu.:8.000
##
   Max. :1.0000
                    Max. :1.0000
                                     Max.
                                           :1.0000
                                                     Max. :8.000
##
                    NA's
                                     NA's
                          :179
                                           :179
                                                     NA's
                                                            :1157
##
    l1dictator1
                                                        11ssch
                        yr70
                                        yr80
##
   Min. :1.000
                   Min. :0.0000
                                    Min. :0.0000
                                                    Min. :0.0140
   1st Qu.:2.000
                   1st Qu.:0.0000
                                    1st Qu.:0.0000
                                                    1st Qu.:0.4562
   Median :5.000
                                    Median :0.0000
                   Median :0.0000
                                                    Median: 0.8519
```

```
##
    Mean
            :4.708
                     Mean
                             :0.3333
                                       Mean
                                               :0.3333
                                                          Mean
                                                                 :1.0411
##
    3rd Qu.:8.000
                     3rd Qu.:1.0000
                                       3rd Qu.:1.0000
                                                         3rd Qu.:1.4652
##
    Max.
            :8.000
                     Max.
                             :1.0000
                                       Max.
                                               :1.0000
                                                         Max.
                                                                 :4.4422
    NA's
            :1315
                                                                 :3140
##
                                                         NA's
                         _spline1
                                            _spline2
                                                               _spline3
##
       closedyr
##
            : 0.000
                              :-24389
                                                :-7854.0
                                                                   :-9030.0
    Min.
                                                            Min.
                      Min.
                                        Min.
    1st Qu.: 0.000
                      1st Qu.: -3375
                                        1st Qu.:-2048.3
                                                            1st Qu.:-1629.3
##
    Median : 7.000
                      Median :
                                -343
                                        Median : -260.2
                                                            Median : -165.6
##
           : 8.691
##
    Mean
                      Mean
                              : -3075
                                        Mean
                                                :-1388.8
                                                            Mean
                                                                   :-1340.9
##
    3rd Qu.:15.000
                      3rd Qu.:
                                        3rd Qu.:
                                                            3rd Qu.:
                                                                         0.0
                                    0
                                                     0.0
   Max.
            :29.000
                      Max.
                                        Max.
                                                     0.0
                                                            Max.
                                                                         0.0
                                        NA's
                                                            NA's
##
    NA's
            :2580
                      NA's
                              :2580
                                                :2580
                                                                   :2580
##
    l1gatt_wto_new
##
   Min.
            :0.000
##
    1st Qu.:0.000
##
    Median :0.000
##
    Mean
            :0.468
##
    3rd Qu.:1.000
##
   Max.
            :1.000
    NA's
##
            :868
```

For information on the meaning of the variables see "LDCcodebook.pdf".

Let's have a look at our data.

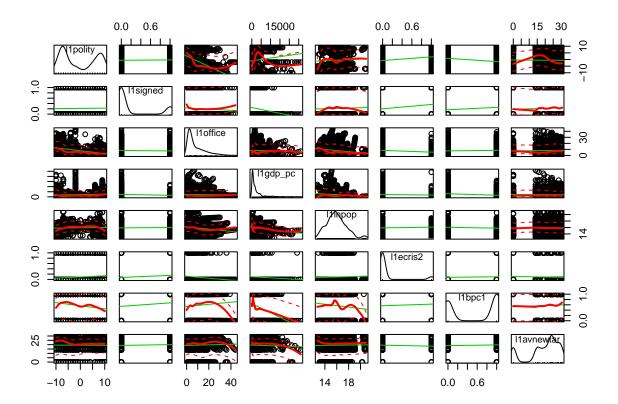
We need a package for generating a scatterplot matrix that allows us to see relationships in our matrix. install.packages("car")

```
##
               l1polity l1signed l1office l1gdp_pc l1lnpop l1ecris2 l1bpc1
## l1polity
                               NA
                                                            NA
                                                                      NA
                                                                              NA
                                          NA
                                                    NA
                       1
## l1signed
                      NA
                                          NA
                                                   NA
                                                            NA
                                                                      NA
                                                                              NA
                                 1
## l1office
                      NA
                                           1
                                                   NA
                                                            NA
                                                                      NA
                                                                              NA
                               NA
## l1gdp_pc
                                          NA
                                                    1
                                                            NA
                                                                              NA
                      NA
                                NA
                                                                      NA
## lllnpop
                      NA
                               NA
                                         NA
                                                   NA
                                                             1
                                                                      NA
                                                                              NA
## l1ecris2
                      NA
                                NA
                                         NA
                                                   NA
                                                            NA
                                                                       1
                                                                              NA
## 11bpc1
                      NA
                               NA
                                         NA
                                                   NA
                                                            NA
                                                                      NA
                                                                               1
## llavnewtar
                      NA
                                NA
                                         NA
                                                   NA
                                                            NA
                                                                      NA
                                                                              NA
##
               11avnewtar
## l1polity
                        NA
## l1signed
                        NA
## l1office
                        NA
## 11gdp_pc
                        NA
## lllnpop
                        NA
## l1ecris2
                        NA
## 11bpc1
                        NA
## llavnewtar
                         1
```

LDC3=na.omit(LDC2) cor(LDC3)

```
##
                 11polity
                             11signed
                                        11office
                                                    l1gdp_pc
                                                                 111npop
## l1polity
              1.000000000 0.01499208 -0.42901753
                                                  0.09129002
                                                              0.04404210
## l1signed
                          1.00000000 -0.04303356 -0.11240587
                                                              0.02321607
## l1office
             -0.429017535 -0.04303356 1.00000000 -0.01936245 -0.17457461
## l1gdp_pc
              0.091290020 -0.11240587 -0.01936245 1.00000000 -0.14082411
## lllnpop
              1.00000000
## l1ecris2
              0.108433533
                          0.11269712 -0.10331828 -0.04446845
                                                              0.03580805
                                      0.06396247 -0.22176992 -0.08570695
## 11bpc1
             -0.171762855
                          0.10895350
## l1avnewtar -0.008790383
                           0.03967319
                                      0.03040646 -0.02357807
                                                              0.01262803
##
                l1ecris2
                                      11avnewtar
                              11bpc1
## l1polity
              0.10843353 -0.17176286 -0.008790383
## l1signed
              0.11269712
                         0.10895350
                                     0.039673186
## l1office
             -0.10331828
                         0.06396247
                                     0.030406457
## l1gdp_pc
             -0.04446845 -0.22176992 -0.023578065
## lllnpop
              0.03580805 -0.08570695
                                     0.012628033
## 11ecris2
              1.00000000
                         0.03517381 -0.036064786
              0.03517381
## 11bpc1
                          1.00000000
                                     0.042851289
## l1avnewtar -0.03606479
                         0.04285129
                                     1.000000000
```

scatterplotMatrix(~ l1polity + l1signed + l1office + l1gdp_pc + l1lnpop + l1ecris2 + l1bpc1 + l1avnewta



The results above indicate that there generally is a low level of multicollinearity among our variables.

Let us start with a simple model that is easy to interpet:

```
simple = lm(newtar ~ l1polity, data = LDC)
summary(simple)
```

```
##
## Call:
## lm(formula = newtar ~ l1polity, data = LDC)
##
## Residuals:
##
      Min
                1Q Median
                                3Q
                                       Max
  -23.425 -9.200 -3.425
                             5.275
                                    80.475
##
## Coefficients:
##
              Estimate Std. Error t value Pr(>|t|)
                           0.52865 41.474 < 2e-16 ***
## (Intercept) 21.92495
               -0.30001
                           0.07293 -4.113 4.3e-05 ***
## l1polity
## Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
## Residual standard error: 14.9 on 804 degrees of freedom
     (4564 observations deleted due to missingness)
## Multiple R-squared: 0.02061,
                                    Adjusted R-squared: 0.01939
## F-statistic: 16.92 on 1 and 804 DF, p-value: 4.298e-05
```

What can we conclude from these statistics? What can we say about the hypothesis that there is a linear relationship between "l1polity" and "newtar"? What is the total variation that is explained by our model?

If there's too much information in this type of summary, try another one. We need another package: install.packages("arm")

library(arm)

```
## Loading required package: MASS

## Loading required package: Matrix

## Loading required package: lme4

## ## arm (Version 1.9-1, built: 2016-8-21)

## Working directory is C:/Users/Jan/OneDrive/Documents/GitHub/ps630_lab/ps630_f16/W5

## ## Attaching package: 'arm'

## The following object is masked from 'package:car':

## logit
```

display(simple)

```
## lm(formula = newtar ~ l1polity, data = LDC)
## coef.est coef.se
## (Intercept) 21.92     0.53
## l1polity     -0.30      0.07
## ---
## n = 806, k = 2
## residual sd = 14.90, R-Squared = 0.02
```

As you can see, this is narrowed down to just a few pieces of information. Sometimes reducing the amount of information that is displayed can be very useful.

3. Multiple linear regression

In the vast majority of cases there are good reasons to include multiple predictor variables.

The most important reasons to do so are:

- 1. Potential omitted variable bias
- 2. Theoretical reasons
- 3. Reviewers that demand you to include them

```
##
## Call:
## lm(formula = newtar ~ l1polity + l1signed + l1office + l1gdp_pc +
      11lnpop + l1ecris2 + l1bpc1 + l1avnewtar, data = LDC)
##
##
## Residuals:
      Min
##
               1Q Median
                               3Q
                                      Max
## -24.286 -7.694 -2.175
                            4.490
                                   65.008
##
## Coefficients:
##
                Estimate Std. Error t value Pr(>|t|)
## (Intercept) -4.901e+01 5.912e+00 -8.289 6.03e-16 ***
## l1polity
              -2.053e-01 8.347e-02 -2.460 0.014151 *
               4.758e-01 1.099e+00
## l1signed
                                      0.433 0.665332
## l1office
              -1.759e-01
                          6.989e-02
                                     -2.516 0.012083 *
              -1.281e-03 1.495e-04
## l1gdp_pc
                                     -8.564
                                            < 2e-16 ***
## lllnpop
               3.693e+00 3.217e-01
                                     11.478 < 2e-16 ***
              -5.736e+00 1.517e+00
                                     -3.780 0.000171 ***
## 11ecris2
## 11bpc1
               4.564e-01 9.681e-01
                                      0.471 0.637462
## llavnewtar
              7.103e-01 8.413e-02
                                      8.442 < 2e-16 ***
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
## Residual standard error: 12.16 on 685 degrees of freedom
```

```
## (4676 observations deleted due to missingness)
## Multiple R-squared: 0.3781, Adjusted R-squared: 0.3708
## F-statistic: 52.05 on 8 and 685 DF, p-value: < 2.2e-16</pre>
```

In the multiple linear regression, how would our expectation for the average tariff level change if our Polity Score increased from -10 to 10 and we had an economic crisis?

How well does our model do compared to the simple linear regression? Do we observe an improvement in the total variation that is explained by our model?

Again, it would be possible to reduce the amount of information with another command:

display(main)

```
## lm(formula = newtar ~ l1polity + l1signed + l1office + l1gdp_pc +
       111npop + 11ecris2 + 11bpc1 + 11avnewtar, data = LDC)
##
               coef.est coef.se
##
## (Intercept) -49.01
                          5.91
## l1polity
                -0.21
                          0.08
## l1signed
                 0.48
                          1.10
                          0.07
## l1office
                -0.18
## l1gdp_pc
                 0.00
                          0.00
## l1lnpop
                 3.69
                          0.32
## 11ecris2
                -5.74
                          1.52
## l1bpc1
                 0.46
                          0.97
                 0.71
                          0.08
## llavnewtar
## n = 694, k = 9
## residual sd = 12.16, R-Squared = 0.38
```

We can access different elements of our model. Let's have a look at what those are:

names(main)

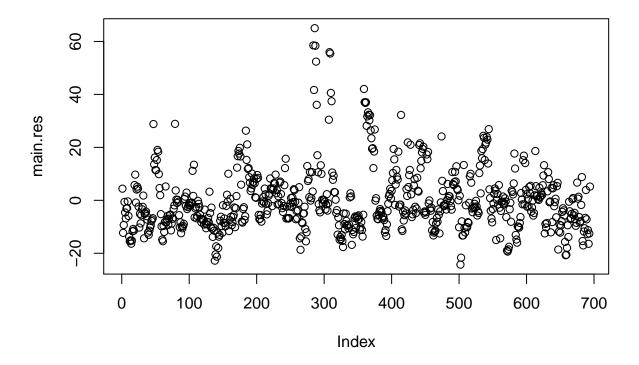
```
## [1] "coefficients" "residuals" "effects" "rank"
## [5] "fitted.values" "assign" "qr" "df.residual"
## [9] "na.action" "xlevels" "call" "terms"
## [13] "model"
```

4. Graphical representation

Let us first have a look at the distribution of errors in our model.

```
main.res = resid(main)
plot(main.res, main = "Values of the Error Term")
```

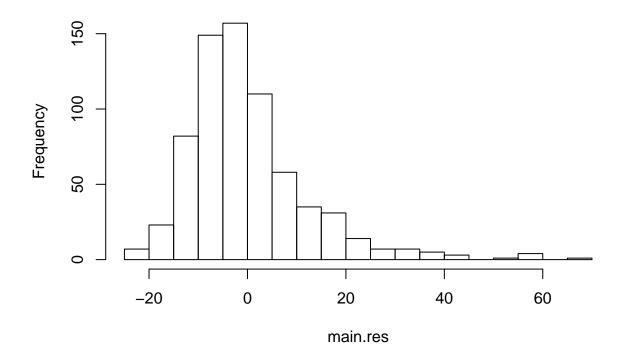
Values of the Error Term



Let us look at the distribution of the error term:

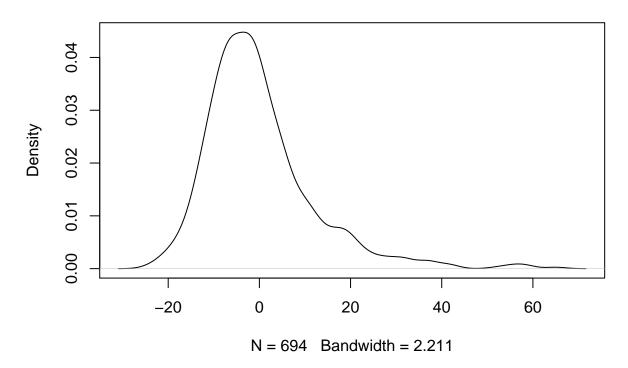
hist(main.res, breaks = 20)

Histogram of main.res



```
res.density = density(main.res)
plot(res.density, main = "Density Plot of the Residual Distribution")
```

Density Plot of the Residual Distribution



The distribution of the errors is approximately normal. If this condition is met, then more precise statements about the distribution of the coefficients can be made (they're also normal). Also, under these conditions, OLS is equivalent to a maximum likelihood approach.

Plotting predicted values

Let us plot some predicted values with confidence intervals for our multiple regression.

In order to do that we first create a dataframe that contains different values for our main predictor variable and the average values for all variables.

Note: Alternatively, we could also use the following code:

```
## l1polity l1signed l1office l1gdp_pc l1lnpop l1ecris2 l1bpc1
## 1 -10 0.1510511 8.431373 2887.762 15.10395 0.06406112 0.5908918
```

```
-9 0.1510511 8.431373 2887.762 15.10395 0.06406112 0.5908918
## 2
## 3
            -8 0.1510511 8.431373 2887.762 15.10395 0.06406112 0.5908918
## 4
            -7 0.1510511 8.431373 2887.762 15.10395 0.06406112 0.5908918
            -6 0.1510511 8.431373 2887.762 15.10395 0.06406112 0.5908918
## 5
## 6
            -5 0.1510511 8.431373 2887.762 15.10395 0.06406112 0.5908918
## 7
            -4 0.1510511 8.431373 2887.762 15.10395 0.06406112 0.5908918
## 8
            -3 0.1510511 8.431373 2887.762 15.10395 0.06406112 0.5908918
            -2 0.1510511 8.431373 2887.762 15.10395 0.06406112 0.5908918
## 9
## 10
            -1 0.1510511 8.431373 2887.762 15.10395 0.06406112 0.5908918
             0 0.1510511 8.431373 2887.762 15.10395 0.06406112 0.5908918
## 11
## 12
             1 0.1510511 8.431373 2887.762 15.10395 0.06406112 0.5908918
             2 0.1510511 8.431373 2887.762 15.10395 0.06406112 0.5908918
## 13
## 14
             3 0.1510511 8.431373 2887.762 15.10395 0.06406112 0.5908918
## 15
             4 0.1510511 8.431373 2887.762 15.10395 0.06406112 0.5908918
## 16
             5 0.1510511 8.431373 2887.762 15.10395 0.06406112 0.5908918
## 17
             6 0.1510511 8.431373 2887.762 15.10395 0.06406112 0.5908918
## 18
             7 0.1510511 8.431373 2887.762 15.10395 0.06406112 0.5908918
## 19
             8 0.1510511 8.431373 2887.762 15.10395 0.06406112 0.5908918
## 20
             9 0.1510511 8.431373 2887.762 15.10395 0.06406112 0.5908918
## 21
            10 0.1510511 8.431373 2887.762 15.10395 0.06406112 0.5908918
##
      11avnewtar
## 1
        15.01104
## 2
        15.01104
## 3
        15.01104
## 4
        15.01104
## 5
        15.01104
## 6
        15.01104
  7
##
        15.01104
## 8
        15.01104
## 9
        15.01104
## 10
        15.01104
## 11
        15.01104
## 12
        15.01104
## 13
        15.01104
## 14
        15.01104
## 15
        15.01104
## 16
        15.01104
## 17
        15.01104
## 18
        15.01104
## 19
        15.01104
## 20
        15.01104
## 21
        15.01104
```

Next we use the model we estimated to predict values based on this new dataframe.

```
pred.p1 = predict(main, type = "response", se.fit = TRUE, newdata = nd)
pred.table = cbind(pred.p1$fit, pred.p1$se.fit)
pred.table
```

```
## [,1] [,2]
## 1 14.27863 1.3372462
## 2 14.07333 1.2741682
```

```
## 3 13.86803 1.2135547
## 4 13.66272 1.1557934
## 5 13.45742 1.1013331
## 6 13.25212 1.0506874
## 7
     13.04682 1.0044334
## 8 12.84152 0.9632040
## 9 12.63622 0.9276694
## 10 12.43092 0.8985055
## 11 12.22562 0.8763485
## 12 12.02032 0.8617391
## 13 11.81502 0.8550644
## 14 11.60972 0.8565097
## 15 11.40442 0.8660344
## 16 11.19912 0.8833773
## 17 10.99382 0.9080905
## 18 10.78852 0.9395926
## 19 10.58322 0.9772273
## 20 10.37792 1.0203162
## 21 10.17262 1.0681995
```

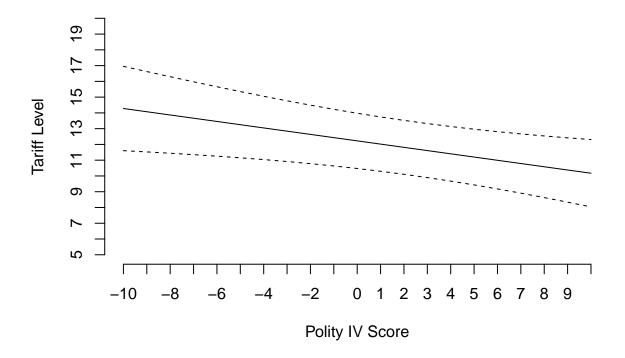
Finally, we create the plot:

```
fit = pred.p1$fit
low = pred.p1$fit - 2 * pred.p1$se.fit
high = pred.p1$fit + 2 * pred.p1$se.fit
cis = cbind(fit, low, high)

cis ### To extract the values
```

```
##
          fit
                    low
                            high
## 1 14.27863 11.604133 16.95312
## 2 14.07333 11.524989 16.62166
## 3 13.86803 11.440916 16.29513
## 4 13.66272 11.351138 15.97431
     13.45742 11.254758 15.66009
## 6
    13.25212 11.150749 15.35350
## 7 13.04682 11.037957 15.05569
## 8 12.84152 10.915115 14.76793
     12.63622 10.780884 14.49156
## 10 12.43092 10.633912 14.22793
## 11 12.22562 10.472925 13.97832
## 12 12.02032 10.296844 13.74380
## 13 11.81502 10.104893 13.52515
## 14 11.60972 9.896702 13.32274
## 15 11.40442 9.672352 13.13649
## 16 11.19912 9.432366 12.96588
## 17 10.99382 9.177639 12.81000
## 18 10.78852 8.909335 12.66770
## 19 10.58322 8.628765 12.53767
## 20 10.37792 8.337287 12.41855
## 21 10.17262 8.036220 12.30902
```

Polity IV Score and Tariff Level



5. Tips for your final paper

- 1. Start working on it early.
- 2. Consult with your professors and TAs.
- 3. Try to find a comprehensive dataset in your area of interest.
- 4. Work on it throughout the semester and try to include new things that you've learned.
- 5. Make sure that you use all the tools you've learned: interpret your findings carefully and visualize them.
- 6. Annotate your code extensively and explain what you did.