This problem is taken from Pitman (1993) Probability: Suppose a fair coin is tossed n times. Find a simple formula in terms of n and k for the following probability: $Pr(k\ heads|k-1\ heads\ or\ k\ heads)$. Please pay close attention to the formula, particularly what event is conditioned on what events. Ch. 2.1, Problem 10 b) (p. 91)

Hint 1: Use the binomial distribution to model this.

Hint 2: Because those events are mutually exclusive, calculate the following:
$$\frac{Pr(k\ heads)}{Pr(k\ heads) + Pr(k-1\ heads)}$$
This is true because: $Pr(A|B) = \dfrac{Pr(A \cap B)}{Pr(B)}$

The intersection of events A and B in this case, $Pr(k\ heads \cap (k\ heads \cup k-1\ heads))$, reduces to $Pr(k\ heads)$ because the two events are mutually exclusive.

SOLUTION

$$\frac{Pr(k\ heads)}{Pr(k\ heads) + Pr(k-1\ heads)}$$
$$= \frac{\binom{n}{k}0.5^k 0.5^{n-k}}{\binom{n}{k}0.5^k 0.5^{n-k} + \binom{n}{k-1}0.5^{k-1}0.5^{n-(k-1)}}$$
$$= \frac{\binom{n}{k}0.5^n}{\binom{n}{k}0.5^n + \binom{n}{k-1}0.5^n}$$
$$= \frac{\binom{n}{k}}{\binom{n}{k} + \binom{n}{k-1}}$$
$$= \frac{\dfrac{n!}{(n-k)!k!}}{\dfrac{n!}{(n-k)!k!} + \dfrac{n!}{(n-(k-1))!(k-1)!}}$$
$$= \frac{\dfrac{n!}{(n-k)!k!} * \dfrac{n-k+1}{n-k+1}}{\dfrac{n!}{(n-k)!k!} * \dfrac{n-k+1}{n-k+1} + \dfrac{n!}{(n-k+1)!(k-1)!} * \dfrac{k}{k}}$$
$$= \frac{\dfrac{n!(n-k+1)}{(n-k+1)!k!}}{\dfrac{n!(n-k+1)}{(n-k+1)!k!} + \dfrac{n!}{(n-k+1)!k!}}$$
$$= \frac{n!(n-k+1)}{n!(n-k+1) + n!k}$$
$$= \frac{n-k+1}{n-k+1+k}$$
$$= \frac{n-k+1}{n+1}$$