

# Pol Sci 630: Problem Set 10: Functional Form, Endogeneity, Power

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Due Date: Nov 9 (Beginning of Class)

## 1 Functional specification

There's a famous dataset called the Anscombe quartet. You load it in R like so:

```
head(anscombe)

##   x1 x2 x3 x4   y1   y2   y3   y4
## 1 10 10 10 10   8 8.04 9.14   7.46 6.58
## 2   8  8  8  8   8 6.95 8.14   6.77 5.76
## 3 13 13 13 13   8 7.58 8.74  12.74 7.71
## 4   9  9  9  8   8 8.81 8.77   7.11 8.84
## 5 11 11 11 11   8 8.33 9.26   7.81 8.47
## 6 14 14 14 14   8 9.96 8.10   8.84 7.04
```

### 1.1 Explore Anscombe

There are 4 pairs of  $x$  and  $y$ . Run 4 regressions of  $y$  on  $x$ . Check out the regression result. A bit late for Halloween, but what spooky thing do you notice?

Then plot the data.

### 1.2 Ramsey RESET

Use Ramsey RESET on the 4 models. Which kind of functional misclassification can it catch? Can you think of why?

## 2 Endogeneity – Omitted Variable Bias

### 2.1 Sign the bias – math

Given the following data generating process (DGP)

Table 1: Signing the bias

|                | $\beta_2 > 0$ | $\beta_2 < 0$ |
|----------------|---------------|---------------|
| $\delta_1 > 0$ |               |               |
| $\delta_1 < 0$ |               |               |

$$x_2 = \delta_0 + \delta_1 x_1 + v \quad (1)$$

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + w \quad (2)$$

The following equation (lecture 09/26) shows what happens when we regress  $y$  on  $x_1$ , omitting  $x_2$ . **Prove this equation.**

$$y = (\beta_0 + \beta_2 \delta_0) + (\beta_1 + \beta_2 \delta_1) x_1 + (\beta_2 v + w)$$

## 2.2 Sign the bias - simulation

In the equation you proved above, the estimated coefficient for  $x_1$  is  $\beta_1 + \beta_2 \delta_1$ , different from its true value  $\beta_1$ . The bias is  $\beta_2 \delta_1$ . The sign of the bias thus depends on  $\beta_2$  and  $\delta_1$ , as discussed in the lecture and reproduced in Table 1.

Conduct 4 simulations with appropriate values of  $\beta_2$  and  $\delta_1$  corresponding to the 4 cells in the table. Show that the sign of the bias is as we learned in class.

## 3 Power calculation

In this exercise we practice power calculation for the simplest experiment setup.

Assume that our binary treatment has an effect size of 2 on the outcome, as follows:

$$y = 1 + 2 \times \text{Treatment} + u$$

$$u \sim \text{Normal}(\text{mean} = 1, \text{sd} = 10)$$

In our experiment, we randomly assigned  $n$  experimental units into 2 groups, treated and control, i.e. treatment = 1 and treatment = 0. Calculate the power of our experiment (i.e. the probability that we can reject the null of zero treatment effect) for different values of the sample size  $n$ .

The end product I want to see is a graph with  $n$  on the x-axis and power on the y-axis. How big must your sample size be to get a power of 0.8?