Tutorial 5: Regression Model Interpretation

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Today's Agenda

1. Marginal effects and intercepts
2. Hypothesis testing
3. Multiple Regression
4. Graphical Representation
5. Tips for your final paper

1. Marginal effects and intercepts

An essential aspect of all linear models are the marginal effects that predictor variables are estimated to have on the response variable.

Note that the word "effect" may be problematic because it implies causality. However, without any additional assumptions or additional model features, linear models allow us to make statements with respect to correlation only. This means we can't say anything about causality when just having a linear model. So let us be very cautious when we use the word "marginal effect".

Every linear model has one response variable (dependent variable) and at least one predictor variable (independent variable) plus an intercept.

Let's assume that Y is our response variable and X is our only predictor variable. The model may look like this:

Y = 5 + 2X + error

How would we interpet the marginal effect of X?

The interpretation would be: For a 1-point increase in X we expect a 2-point increase in Y.

How would we interpret the intercept?

The intercept is the expected value of Y when X is at a value of 0.

Illustration of the marginal effect interpretation

Let's load another R dataset that can illustrate the interpretation of marginal effects. The "airquality" dataset. According to the documentation, this is "Daily air quality measurements in New York, May to September 1973."

More details can be found here:

```r
data(airquality)
summary(airquality)
```

```
##      Ozone           Solar.R           Wind             Temp
##  Min.   :  1.00   Min.   :  7.0   Min.   : 1.700   Min.   :56.00
##  1st Qu.: 18.00   1st Qu.:115.8   1st Qu.: 7.400   1st Qu.:72.00
##  Median : 31.50   Median :205.0   Median : 9.700   Median :79.00
```

```
##  Mean   : 42.13   Mean   :185.9   Mean   : 9.958   Mean   :77.88
##  3rd Qu.: 63.25   3rd Qu.:258.8   3rd Qu.:11.500   3rd Qu.:85.00
##  Max.   :168.00   Max.   :334.0   Max.   :20.700   Max.   :97.00
##  NA's   :37       NA's   :7
##      Month            Day
##  Min.   :5.000   Min.   : 1.0
##  1st Qu.:6.000   1st Qu.: 8.0
##  Median :7.000   Median :16.0
##  Mean   :6.993   Mean   :15.8
##  3rd Qu.:8.000   3rd Qu.:23.0
##  Max.   :9.000   Max.   :31.0
##
```

Our question is: is there a linear relationship between the Ozone measures and the Solar.R measures?

Let us use linear regression to answer this question:

```r
lm1=lm(Ozone ~ Solar.R, data=airquality)
```

The summary of this linear regression will return a t-value and a p-value for the intercept and all coefficients.

```r
summary(lm1)
```

```
##
## Call:
## lm(formula = Ozone ~ Solar.R, data = airquality)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -48.292 -21.361  -8.864  16.373 119.136
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept) 18.59873    6.74790   2.756 0.006856 **
## Solar.R      0.12717    0.03278   3.880 0.000179 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 31.33 on 109 degrees of freedom
##   (42 observations deleted due to missingness)
## Multiple R-squared:  0.1213, Adjusted R-squared:  0.1133
## F-statistic: 15.05 on 1 and 109 DF,  p-value: 0.0001793
```

How would we interpret the finding with respect to the linear relationship between the two variables? The interpreation would look like this:

There is a positive linear relationship between Ozone and Solar.R. For a 1-point increase in Solar.R, we would expect a 0.13 increase in Ozone (in a multivariate model we would have to add: "holding all other variables constant").

Furthermore (already going into the next topic): The associated t-value is 3.880. This t-value imples a p-value of 0.0002. This $p < 0.001$ corresponds to a type-1 error rate of alpha < 0.001, meaning that the relationship is significant at all common levels of statistical significance.

How do we interpret the R-squared statistic? Our model explains a proportion of the total variation in the dependent variable. The R-squared statistic returns this proportion. How well does our model do?

2. Hypothesis testing

Let us use another dataset to conduct some hypothesis tests.

We will look at data from an article that was published in the journal "International Organization", the leading journal in the field of international relations. The article was written by Helen Milner and Keiko Kubota.

The article deals with the effect that democratization has on trade barriers. The authors believe that democratization has a negative effect on trade barriers in developing countries (that are scarce in capital). Their theory is based on the Stolper Samuelson theorem and the selectorate model by Bueno de Mesquita et al.

Let us try to emulate their test. In order to load their dataset you need to use the following command: install.packages("foreign")

```
setwd('C:/Users/Jan/OneDrive/Documents/GitHub/ps630_lab/W5')
library(foreign)
LDC=read.dta("LDC_IO_replication.dta")
summary(LDC)
```

```
##     country        ctylabel              date       gatt_wto_new
##  Min.   :186.0   Length:5370        Min.   :1970   Min.   :0.0000
##  1st Qu.:423.0   Class :character   1st Qu.:1977   1st Qu.:0.0000
##  Median :628.0   Mode  :character   Median :1984   Median :0.0000
##  Mean   :605.9                      Mean   :1984   Mean   :0.4747
##  3rd Qu.:816.0                      3rd Qu.:1992   3rd Qu.:1.0000
##  Max.   :968.0                      Max.   :1999   Max.   :1.0000
##                                                    NA's   :698
##      aclpn             bpc1        dopen_wacz2        ecris2
##  Min.   :0.0000   Min.   :0.000   Min.   :0.0000   Min.   :0.0000
##  1st Qu.:0.0000   1st Qu.:0.000   1st Qu.:0.0000   1st Qu.:0.0000
##  Median :0.0000   Median :1.000   Median :0.0000   Median :0.0000
##  Mean   :0.3002   Mean   :0.591   Mean   :0.3097   Mean   :0.0641
##  3rd Qu.:1.0000   3rd Qu.:1.000   3rd Qu.:1.0000   3rd Qu.:0.0000
##  Max.   :1.0000   Max.   :1.000   Max.   :1.0000   Max.   :1.0000
##  NA's   :1183     NA's   :2734    NA's   :2580     NA's   :1967
##      fdignp           gdp_pc_95d         l1aclpn          l1bpc1
##  Min.   :-27.2356   Min.   :    0.0   Min.   :0.0000   Min.   :0.0000
##  1st Qu.:  0.0361   1st Qu.:  442.9   1st Qu.:0.0000   1st Qu.:0.0000
##  Median :  0.6644   Median : 1266.5   Median :0.0000   Median :1.0000
##  Mean   :  1.8962   Mean   : 2885.5   Mean   :0.2924   Mean   :0.5909
##  3rd Qu.:  2.0829   3rd Qu.: 3002.4   3rd Qu.:1.0000   3rd Qu.:1.0000
##  Max.   :184.5647   Max.   :44164.5   Max.   :1.0000   Max.   :1.0000
##  NA's   :2294       NA's   :1679      NA's   :1341     NA's   :2735
##     l1ecris2          newtar       polityiv_update2      signed
##  Min.   :0.0000   Min.   :  0.00   Min.   :-10.000   Min.   :0.0000
##  1st Qu.:0.0000   1st Qu.: 10.95   1st Qu.: -7.000   1st Qu.:0.0000
##  Median :0.0000   Median : 17.00   Median : -6.000   Median :0.0000
##  Mean   :0.0641   Mean   : 20.54   Mean   : -2.074   Mean   :0.1465
##  3rd Qu.:0.0000   3rd Qu.: 27.00   3rd Qu.:  6.000   3rd Qu.:0.0000
##  Max.   :1.0000   Max.   :102.20   Max.   : 10.000   Max.   :1.0000
##  NA's   :1967     NA's   :4463     NA's   :2003      NA's   :1362
##     yrsoffic          usheg           l1usheg          l1fiveop
##  Min.   : 0.000   Min.   :0.2434   Min.   :0.2434   Min.   :10.20
```

```
##  1st Qu.: 2.000   1st Qu.:0.2574   1st Qu.:0.2574   1st Qu.:10.90
##  Median : 5.000   Median :0.2663   Median :0.2655   Median :12.35
##  Mean   : 8.431   Mean   :0.2696   Mean   :0.2683   Mean   :12.03
##  3rd Qu.:12.000   3rd Qu.:0.2785   3rd Qu.:0.2784   3rd Qu.:12.72
##  Max.   :44.000   Max.   :0.3083   Max.   :0.2988   Max.   :13.20
##  NA's   :2361                      NA's   :179      NA's   :358
##     l1gdp_pc          avsw           avnewtar         l1avsw
##  Min.   :    0   Min.   :0.1398   Min.   : 0.00   Min.   :0.1398
##  1st Qu.:  442   1st Qu.:0.1505   1st Qu.: 0.00   1st Qu.:0.1505
##  Median : 1266   Median :0.1720   Median :17.43   Median :0.1613
##  Mean   : 2888   Mean   :0.3097   Mean   :14.91   Mean   :0.2974
##  3rd Qu.: 2999   3rd Qu.:0.5269   3rd Qu.:24.37   3rd Qu.:0.5054
##  Max.   :44165   Max.   :0.6667   Max.   :30.52   Max.   :0.6559
##  NA's   :1823                                     NA's   :179
##    l1avnewtar         lnpop          l1lnpop         l1office
##  Min.   : 0.00   Min.   :10.57   Min.   :10.62   Min.   : 0.000
##  1st Qu.: 0.00   1st Qu.:13.86   1st Qu.:13.86   1st Qu.: 2.000
##  Median :18.73   Median :15.32   Median :15.31   Median : 5.000
##  Mean   :15.01   Mean   :15.11   Mean   :15.10   Mean   : 8.431
##  3rd Qu.:24.37   3rd Qu.:16.40   3rd Qu.:16.39   3rd Qu.:12.000
##  Max.   :30.52   Max.   :20.95   Max.   :20.94   Max.   :44.000
##  NA's   :179     NA's   :490     NA's   :661     NA's   :2361
##  l1partyage2000       l1fdi             l1polity          l2polity
##  Min.   :  0.00   Min.   :-27.2356   Min.   :-10.000   Min.   :-10.00
##  1st Qu.: 10.00   1st Qu.:  0.0269   1st Qu.: -7.000   1st Qu.: -7.00
##  Median : 19.50   Median :  0.6382   Median : -6.000   Median : -7.00
##  Mean   : 24.18   Mean   :  1.7931   Mean   : -2.215   Mean   : -2.36
##  3rd Qu.: 32.00   3rd Qu.:  1.9904   3rd Qu.:  6.000   3rd Qu.:  5.00
##  Max.   :183.00   Max.   :184.5647   Max.   : 10.000   Max.   : 10.00
##  NA's   :3284     NA's   :2423       NA's   :2124      NA's   :2246
##     l3polity         l1signed          milit2             sp2
##  Min.   :-10.000   Min.   :0.0000   Min.   :0.0000   Min.   :0.0000
##  1st Qu.: -7.000   1st Qu.:0.0000   1st Qu.:0.0000   1st Qu.:0.0000
##  Median : -7.000   Median :0.0000   Median :0.0000   Median :0.0000
##  Mean   : -2.512   Mean   :0.1511   Mean   :0.1119   Mean   :0.1959
##  3rd Qu.:  5.000   3rd Qu.:0.0000   3rd Qu.:0.0000   3rd Qu.:0.0000
##  Max.   : 10.000   Max.   :1.0000   Max.   :1.0000   Max.   :1.0000
##  NA's   :2371      NA's   :1517
##      pers2           l1milit2           l1sp2          dictator1
##  Min.   :0.0000   Min.   :0.0000   Min.   :0.0000   Min.   :1.000
##  1st Qu.:0.0000   1st Qu.:0.0000   1st Qu.:0.0000   1st Qu.:2.000
##  Median :0.0000   Median :0.0000   Median :0.0000   Median :5.000
##  Mean   :0.1665   Mean   :0.1135   Mean   :0.1986   Mean   :4.737
##  3rd Qu.:0.0000   3rd Qu.:0.0000   3rd Qu.:0.0000   3rd Qu.:8.000
##  Max.   :1.0000   Max.   :1.0000   Max.   :1.0000   Max.   :8.000
##                   NA's   :179      NA's   :179      NA's   :1157
##   l1dictator1        yr70             yr80            l1ssch
##  Min.   :1.000   Min.   :0.0000   Min.   :0.0000   Min.   :0.0140
##  1st Qu.:2.000   1st Qu.:0.0000   1st Qu.:0.0000   1st Qu.:0.4562
##  Median :5.000   Median :0.0000   Median :0.0000   Median :0.8519
##  Mean   :4.708   Mean   :0.3333   Mean   :0.3333   Mean   :1.0411
##  3rd Qu.:8.000   3rd Qu.:1.0000   3rd Qu.:1.0000   3rd Qu.:1.4652
##  Max.   :8.000   Max.   :1.0000   Max.   :1.0000   Max.   :4.4422
##  NA's   :1315                                      NA's   :3140
```

```
##     closedyr         _spline1         _spline2          _spline3
##  Min.   : 0.000   Min.   :-24389   Min.   :-7854.0   Min.   :-9030.0
##  1st Qu.: 0.000   1st Qu.: -3375   1st Qu.:-2048.3   1st Qu.:-1629.3
##  Median : 7.000   Median :  -343   Median : -260.2   Median : -165.6
##  Mean   : 8.691   Mean   : -3075   Mean   :-1388.8   Mean   :-1340.9
##  3rd Qu.:15.000   3rd Qu.:     0   3rd Qu.:    0.0   3rd Qu.:    0.0
##  Max.   :29.000   Max.   :     0   Max.   :    0.0   Max.   :    0.0
##  NA's   :2580     NA's   :2580     NA's   :2580      NA's   :2580
##  l1gatt_wto_new
##  Min.   :0.000
##  1st Qu.:0.000
##  Median :0.000
##  Mean   :0.468
##  3rd Qu.:1.000
##  Max.   :1.000
##  NA's   :868
```

For information on the meaning of the variables see “LDCcodebook.pdf”.

An important condition for using OLS is that the predictor variables are not subject to multicollinearity. Let’s check multicollinearity. We need another package for this: install.packages(“car”)

```
library(car)

LDC2=as.data.frame(LDC[,c("l1polity","l1signed","l1office","l1gdp_pc","l1lnpop","l1ecris2","l1bpc1","l1a
cor(LDC2)
```

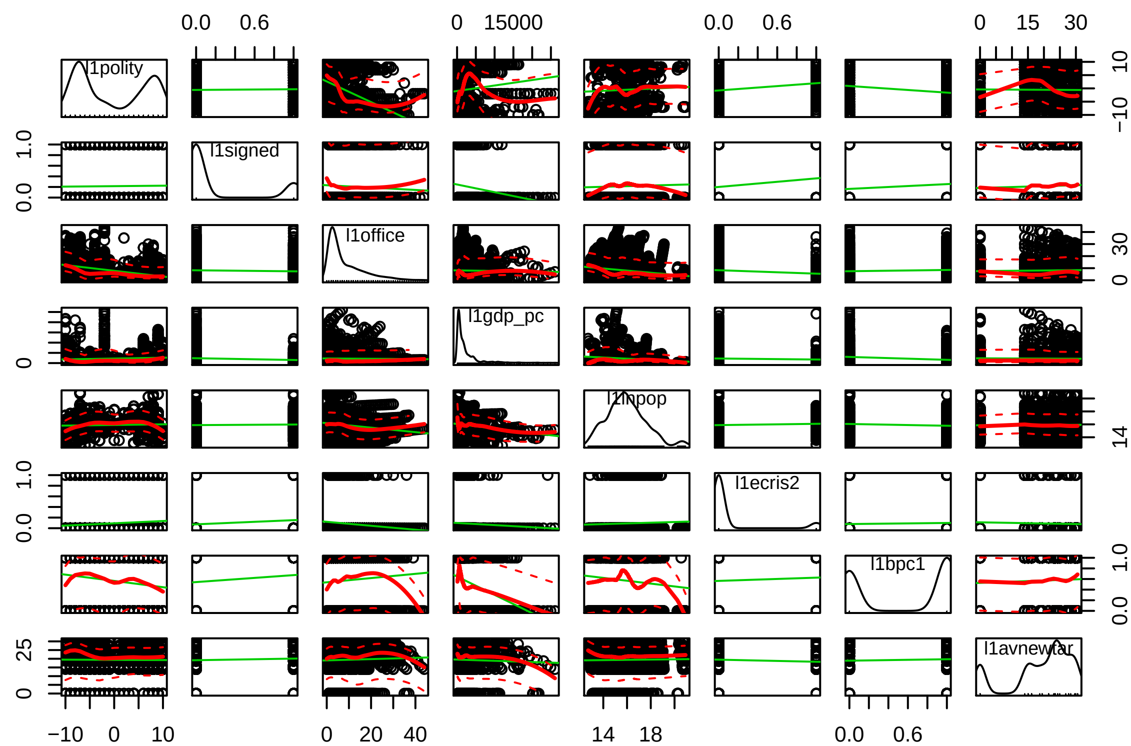
```
##            l1polity l1signed l1office l1gdp_pc l1lnpop l1ecris2 l1bpc1
## l1polity          1       NA       NA       NA      NA       NA     NA
## l1signed         NA        1       NA       NA      NA       NA     NA
## l1office         NA       NA        1       NA      NA       NA     NA
## l1gdp_pc         NA       NA       NA        1      NA       NA     NA
## l1lnpop          NA       NA       NA       NA       1       NA     NA
## l1ecris2         NA       NA       NA       NA      NA        1     NA
## l1bpc1           NA       NA       NA       NA      NA       NA      1
## l1avnewtar       NA       NA       NA       NA      NA       NA     NA
##            l1avnewtar
## l1polity           NA
## l1signed           NA
## l1office           NA
## l1gdp_pc           NA
## l1lnpop            NA
## l1ecris2           NA
## l1bpc1             NA
## l1avnewtar          1
```

```
LDC3=na.omit(LDC2)
cor(LDC3)
```

```
##                l1polity    l1signed    l1office    l1gdp_pc     l1lnpop
## l1polity    1.000000000  0.01499208 -0.42901753  0.09129002  0.04404210
## l1signed    0.014992081  1.00000000 -0.04303356 -0.11240587  0.02321607
## l1office   -0.429017535 -0.04303356  1.00000000 -0.01936245 -0.17457461
```

```
## l1gdp_pc     0.091290020 -0.11240587 -0.01936245  1.00000000 -0.14082411
## l1lnpop      0.044042096  0.02321607 -0.17457461 -0.14082411  1.00000000
## l1ecris2     0.108433533  0.11269712 -0.10331828 -0.04446845  0.03580805
## l1bpc1      -0.171762855  0.10895350  0.06396247 -0.22176992 -0.08570695
## l1avnewtar -0.008790383  0.03967319  0.03040646 -0.02357807  0.01262803
##               l1ecris2      l1bpc1    l1avnewtar
## l1polity     0.10843353 -0.17176286 -0.008790383
## l1signed     0.11269712  0.10895350  0.039673186
## l1office    -0.10331828  0.06396247  0.030406457
## l1gdp_pc    -0.04446845 -0.22176992 -0.023578065
## l1lnpop      0.03580805 -0.08570695  0.012628033
## l1ecris2     1.00000000  0.03517381 -0.036064786
## l1bpc1       0.03517381  1.00000000  0.042851289
## l1avnewtar -0.03606479  0.04285129  1.000000000
```

```r
scatterplotMatrix(~ l1polity + l1signed + l1office + l1gdp_pc + l1lnpop + l1ecris2 + l1bpc1 + l1avnewta
```



The results above indicate that there generally is a low level of multicollinearity among our variables.

Let us start with a simple model that is easy to interpet:

```r
simple=lm(newtar~l1polity, data=LDC)
summary(simple)
```

```
##
## Call:
```

```
## lm(formula = newtar ~ l1polity, data = LDC)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -23.425  -9.200  -3.425   5.275  80.475
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept) 21.92495    0.52865  41.474  < 2e-16 ***
## l1polity    -0.30001    0.07293  -4.113  4.3e-05 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 14.9 on 804 degrees of freedom
##   (4564 observations deleted due to missingness)
## Multiple R-squared:  0.02061,    Adjusted R-squared:  0.01939
## F-statistic: 16.92 on 1 and 804 DF,  p-value: 4.298e-05
```

What can we conclude from these statistics? What can we say about the hypothesis that there is a linear relationship between "l1polity" and "newtar"? What is the total variation that is explained by our model?

If there's too much information in this type of summary, try another one. We need another package: install.packages("arm")

```r
library(arm)
```

```
## Loading required package: MASS
## Loading required package: Matrix
## Loading required package: lme4
##
## arm (Version 1.8-6, built: 2015-7-7)
##
## Working directory is C:/Users/Jan/OneDrive/Documents/GitHub/ps630_lab/W5
##
##
## Attaching package: 'arm'
##
## The following object is masked from 'package:car':
##
##     logit
```

```r
display(simple)
```

```
## lm(formula = newtar ~ l1polity, data = LDC)
##             coef.est coef.se
## (Intercept) 21.92    0.53
## l1polity    -0.30    0.07
## ---
## n = 806, k = 2
## residual sd = 14.90, R-Squared = 0.02
```

As you can see, this is narrowed down to just a few pieces of information. Sometimes reducing the amount of information that is displayed can be very useful.

3. Multiple linear regression

In the vast majority of cases there are good reasons to include multiple predictor variables.

The most important reasons to do so are:

1. Omitted Variable Bias
2. Reviewers that demand you to include them

```
main=lm(newtar ~ l1polity + l1signed + l1office + l1gdp_pc + l1lnpop + l1ecris2 + l1bpc1 + l1avnewtar, d
summary(main)
```

```
##
## Call:
## lm(formula = newtar ~ l1polity + l1signed + l1office + l1gdp_pc +
##     l1lnpop + l1ecris2 + l1bpc1 + l1avnewtar, data = LDC)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -24.286  -7.694  -2.175   4.490  65.008
##
## Coefficients:
##               Estimate Std. Error t value Pr(>|t|)
## (Intercept) -4.901e+01  5.912e+00  -8.289 6.03e-16 ***
## l1polity    -2.053e-01  8.347e-02  -2.460 0.014151 *
## l1signed     4.758e-01  1.099e+00   0.433 0.665332
## l1office    -1.759e-01  6.989e-02  -2.516 0.012083 *
## l1gdp_pc    -1.281e-03  1.495e-04  -8.564  < 2e-16 ***
## l1lnpop      3.693e+00  3.217e-01  11.478  < 2e-16 ***
## l1ecris2    -5.736e+00  1.517e+00  -3.780 0.000171 ***
## l1bpc1       4.564e-01  9.681e-01   0.471 0.637462
## l1avnewtar   7.103e-01  8.413e-02   8.442  < 2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 12.16 on 685 degrees of freedom
##   (4676 observations deleted due to missingness)
## Multiple R-squared:  0.3781, Adjusted R-squared:  0.3708
## F-statistic: 52.05 on 8 and 685 DF,  p-value: < 2.2e-16
```

How well does our model do compared to the simple linear regression? Do we observe an improvement in the total variation that is explained by our model?

Again, it would be possible to reduce the amount of information with another command:

```
display(main)
```

```
## lm(formula = newtar ~ l1polity + l1signed + l1office + l1gdp_pc +
##     l1lnpop + l1ecris2 + l1bpc1 + l1avnewtar, data = LDC)
##             coef.est coef.se
## (Intercept) -49.01     5.91
## l1polity     -0.21     0.08
## l1signed      0.48     1.10
```

```
## l1office     -0.18     0.07
## l1gdp_pc      0.00     0.00
## l1lnpop       3.69     0.32
## l1ecris2     -5.74     1.52
## l1bpc1        0.46     0.97
## l1avnewtar    0.71     0.08
## ---
## n = 694, k = 9
## residual sd = 12.16, R-Squared = 0.38
```

We can access different elements of our model. Let's have a look at what those are:

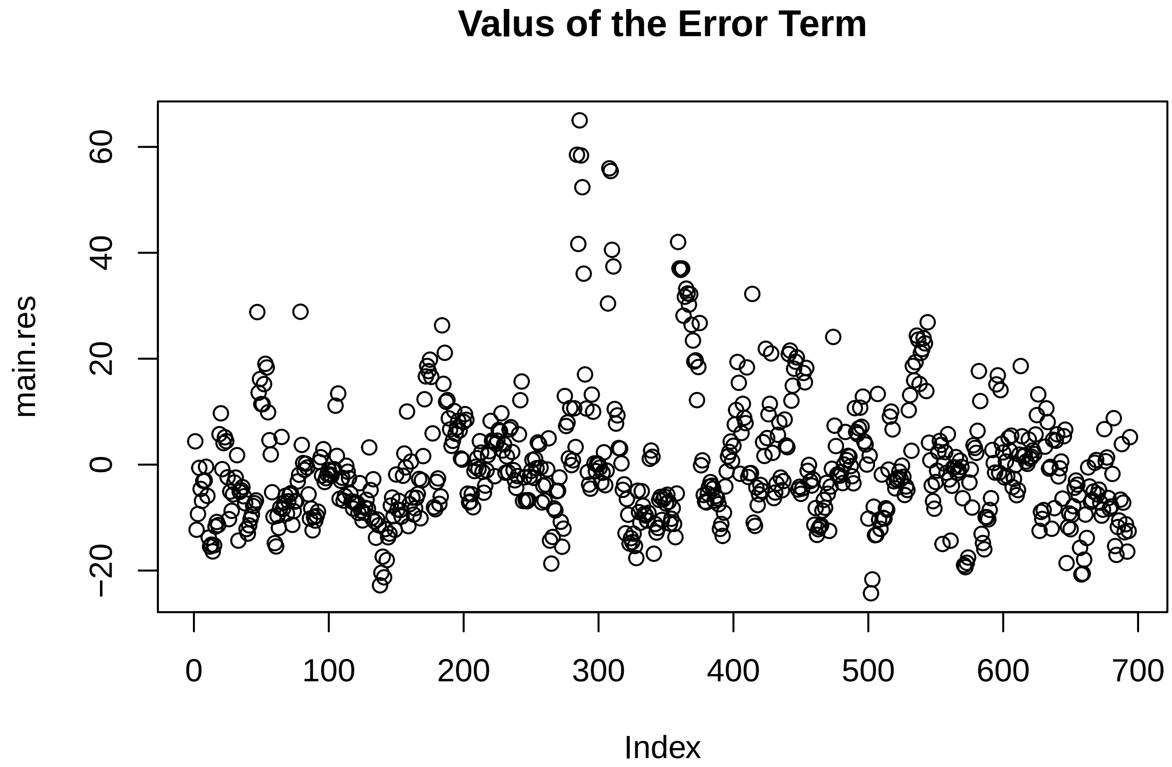
```r
names(main)
```

```
##  [1] "coefficients"  "residuals"     "effects"       "rank"
##  [5] "fitted.values" "assign"        "qr"            "df.residual"
##  [9] "na.action"     "xlevels"       "call"          "terms"
## [13] "model"
```

4. Graphical representation

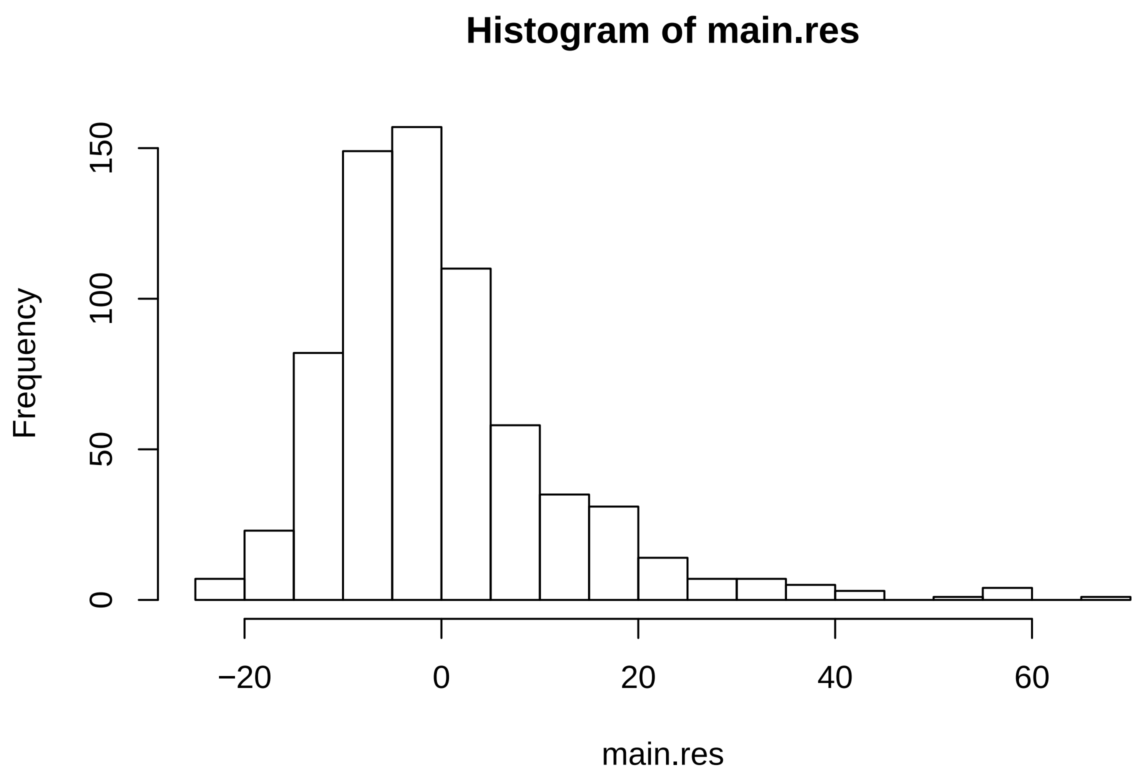
Let us first have a look at the distribution of errors in our model.

```r
main.res = resid(main)
plot(main.res, main="Valus of the Error Term")
```

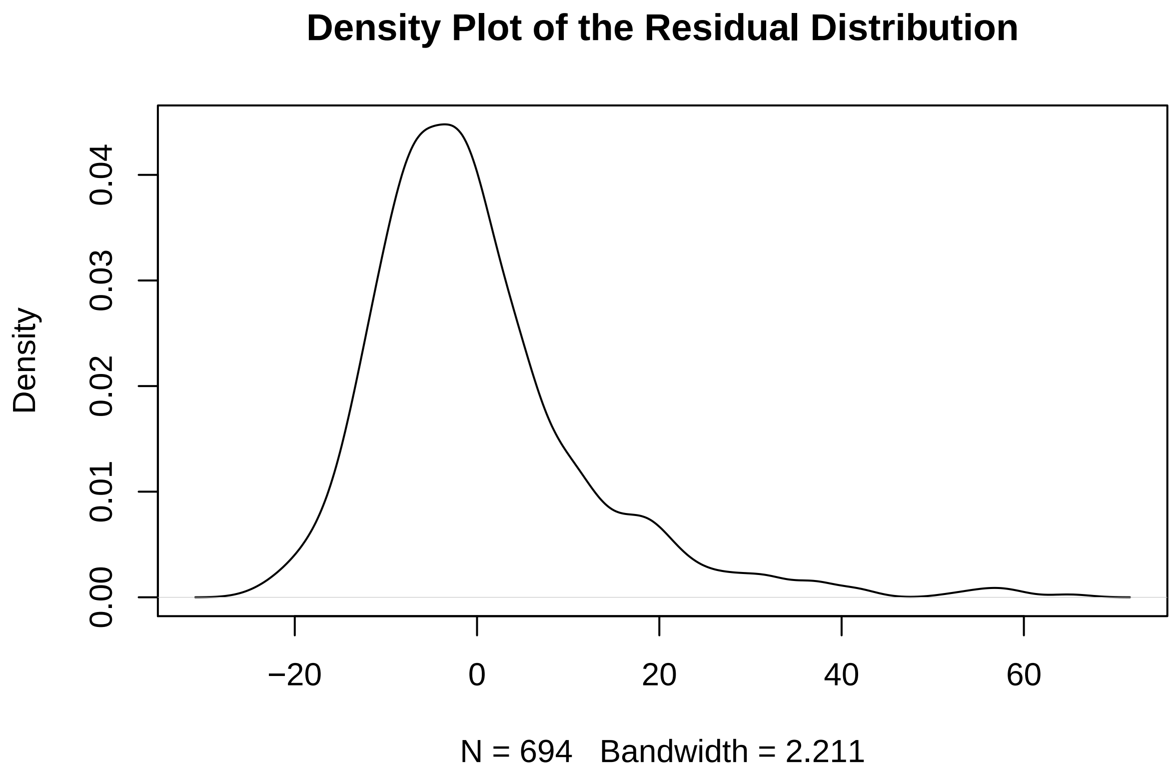


Let us look at the distribution of the error term:

```
hist(main.res, breaks=20)
```




```
res.density=density(main.res)
plot(res.density, main="Density Plot of the Residual Distribution")
```



The distribution of the errors is approximately normal, so this condition of OLS is met.

Let us plot some predicted values with confidence intervals for our multiple regression.

In order to do that we first create a dataframe that contains different values for our main predictor variable and the average values for all variables.

```
nd <- data.frame(l1polity=seq(-10,10,by=1), l1signed=rep(0.1511,21), l1office=rep(8.431,21), l1gdp_pc=r
```

Next we use the model we estimated to predict values based on this new dataframe.

```
pred.p1 <- predict(main, type="response", se.fit=TRUE, newdata=nd)

pred.table <- cbind(pred.p1$fit, pred.p1$se.fit)
pred.table
```

```
##         [,1]      [,2]
## 1  14.19185 1.3422851
## 2  13.98655 1.2793558
## 3  13.78125 1.2188955
## 4  13.57595 1.1612899
## 5  13.37065 1.1069847
## 6  13.16535 1.0564892
## 7  12.96005 1.0103744
## 8  12.75475 0.9692661
## 9  12.54945 0.9338254
```

```
## 10 12.34415 0.9047189
## 11 12.13885 0.8825733
## 12 11.93355 0.8679216
## 13 11.72825 0.8611465
## 14 11.52295 0.8624336
## 15 11.31765 0.8717473
## 16 11.11235 0.8888351
## 17 10.90705 0.9132609
## 18 10.70175 0.9444554
## 19 10.49645 0.9817738
## 20 10.29115 1.0245470
## 21 10.08585 1.0721223
```

Finally, we create the plot:

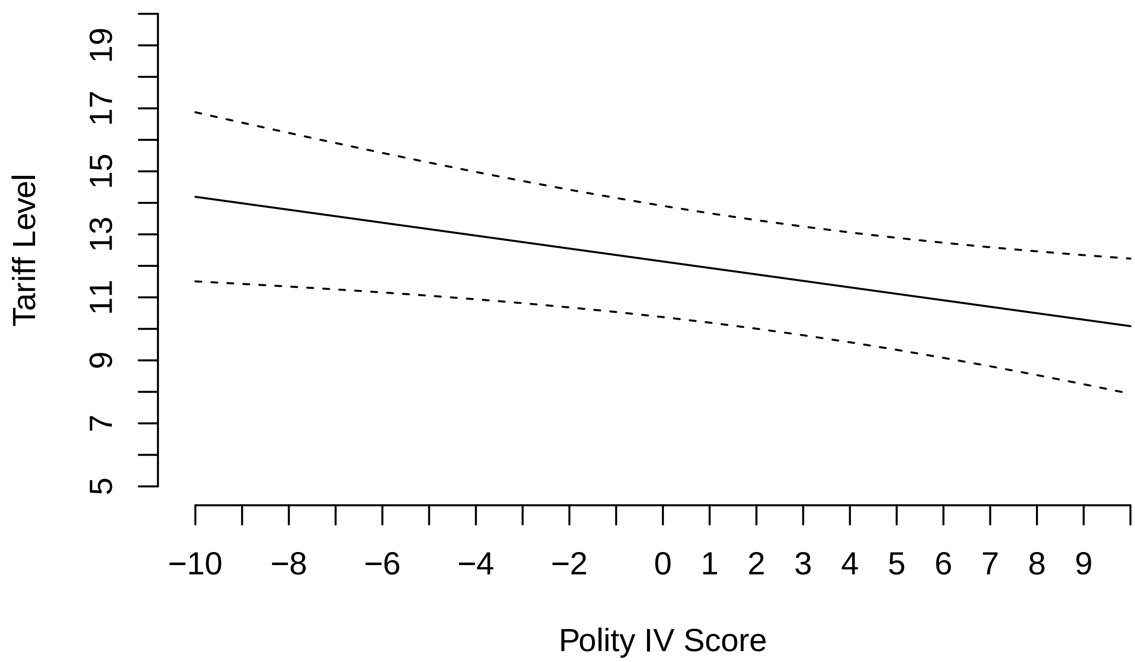
```r
fit <- pred.p1$fit
low <- pred.p1$fit - 2*pred.p1$se.fit
high <- pred.p1$fit + 2*pred.p1$se.fit
cis <- cbind(fit, low, high)

cis ### To extract the values
```

```
##         fit       low     high
## 1  14.19185 11.507283 16.87642
## 2  13.98655 11.427842 16.54526
## 3  13.78125 11.343462 16.21904
## 4  13.57595 11.253373 15.89853
## 5  13.37065 11.156683 15.58462
## 6  13.16535 11.052373 15.27833
## 7  12.96005 10.939303 14.98080
## 8  12.75475 10.816219 14.69328
## 9  12.54945 10.681800 14.41710
## 10 12.34415 10.534713 14.15359
## 11 12.13885 10.373703 13.90400
## 12 11.93355 10.197706 13.66939
## 13 11.72825 10.005956 13.45054
## 14 11.52295  9.798082 13.24782
## 15 11.31765  9.574154 13.06114
## 16 11.11235  9.334678 12.89002
## 17 10.90705  9.080526 12.73357
## 18 10.70175  8.812837 12.59066
## 19 10.49645  8.532900 12.45999
## 20 10.29115  8.242053 12.34024
## 21 10.08585  7.941602 12.23009
```

```r
plot(pred.p1$fit, type="l", ylim=c(5,20), main="Polity IV Score and Tariff Level",
     xlab="Polity IV Score", ylab="Tariff Level", axes=FALSE)
axis(1, at=seq(1,21), labels=seq(-10,10,1))
axis(2, at=seq(5,20), labels=seq(5,20))
matlines(cis[,c(2,3)], lty=2, col="black")
```

Polity IV Score and Tariff Level



5. Tips for your final paper

1. Start working on it early.
2. Consult with your professors and TAs.
3. Try to find a comprehensive dataset in your area of interest.
4. Work on it throughout the semester and try to include new things that you've learned.
5. Make sure that you use all the tools you've learned: interpret your findings carefully and visualize them.