Mystery #1: Chris says slide 9/15, Tues significance

Interpretation

$$P_{ij} = \frac{\exp(x_i'\beta_j)}{\sum_\ell \exp(x_i'\beta_\ell)}$$

low d high - high d low sq $\frac{\partial}{\partial x_i}\exp(x_i'\beta)$

$$\frac{\partial P_{ij}}{\partial x_i} = \frac{\sum_\ell \exp(x_i'\beta_\ell)\exp(x_i'\beta_j)\cdot\beta_j}{(\sum_\ell \exp(x_i'\beta_\ell))^2} - \frac{\exp(x_i'\beta_j)\sum \exp(x_i'\beta_\ell)\cdot\beta_\ell}{(\sum_\ell \exp(x_i'\beta_\ell))^2}$$

$$= \frac{\exp(x_i'\beta_j)\cdot\beta_j}{(\sum_\ell \exp(x_i'\beta_\ell))^2} - \frac{\exp(x_i'\beta_j)}{(\sum_\ell \exp(x_i'\beta_\ell))^2} \times \frac{\sum \exp(x_i'\beta_\ell)\cdot\beta_\ell}{\sum \exp(x_i'\beta_\ell)}$$

$$= P_{ij}\cdot\beta_j - P_{ij}\sum_\ell P_{i\ell}\cdot\beta_\ell$$

$$= P_{ij}\left(\beta_j - \sum_\ell P_{i\ell}\cdot\beta_\ell\right)$$

$\sum_\ell P_{i\ell}\cdot\beta_\ell$: β weighted by prob

$$= P_{ij}(\beta_j - \bar{\beta}_i)$$

Sign of change is not necessarily given by the sign of β_j, unless $\beta_j > \beta_k \; \forall k \neq j$

Ex: $\beta_j > 0$
$\beta_j < 0$

MYSTERY

→ Does not make sense to test for significance $\beta \neq 0$

compute aug response $\frac{1}{N}\sum \frac{\partial P_{ij}}{\partial x_i} = \frac{1}{N}\sum P_{ij}(\beta_j - \bar{\beta}_i)$

Mystery #2: MNL can be rewritten as binary logit.

Comparison to base category 8/15

Why do we normalize addendum

$$Pr[y=j \mid y=j \text{ or } k] = \frac{P_j}{P_j + P_k} = \frac{\frac{\exp}{\sum}}{\frac{\exp}{\sum} + \frac{\exp}{\sum}}$$

$$= \frac{\exp(x'\beta_j)}{\exp(x'\beta_j) + \exp(x'\beta_k)}$$

$$= \frac{\exp(x'(\beta_j - \beta_k))}{1 + \exp(x'(\beta_{jk} - k))}$$ divide by $\exp(x'\beta_k)$

Let's say we set $\beta_1 = 0$

$$Pr[y_i = j \mid y_i = j \text{ or } 1] = \frac{\exp(x_i'\beta_j)}{1 + \exp(x_i'\beta_j)}$$

odd of choosing j rather than 1 is

$$\frac{Pr[y_i = j]}{Pr[y_i = 1]} = \exp(x_i'\beta_k)$$

$\Downarrow$ $\log(\cdot) = x_i'\beta_j$ → MYSTERY

Proportionate change $\frac{Pr(y_i=j) - Pr(y_i=1)}{Pr(y_i=1)} - 1$

IIA: MNL reduces to a binary choice logit model between any pair. The odd does not depend on other choices