Multinomial model's coefficient

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We have

$$p_{ij} = \frac{\exp(x_i \beta_j)}{\sum_l \exp(x_i \beta_l)} \tag{1}$$

$$\tag{2}$$

We want to derive the marginal effect of x_i on p_{ij} , so we take the derivative of p_{ij} with regards to x_{ij}

$$\begin{align}
\frac{\partial p_{ij}}{\partial x_i} &= \frac{[\sum_l \exp(x_i \beta_l)] \exp(x_i \beta_j) \beta_j - \exp(x_i \beta_j) [\sum_l \exp(x_i \beta_l) \beta_l]}{[\sum_l \exp(x_i \beta_l)]^2} \tag{3} \\
&= \frac{\exp(x_i \beta_j) \beta_j}{\sum_l \exp(x_i \beta_l)} - \frac{\exp(x_i \beta_j)}{\sum_l \exp(x_i \beta_l)} \times \frac{\sum_l \exp(x_i \beta_l) \beta_l}{\sum_l \exp(x_i \beta_l)} \tag{4} \\
&= \frac{\exp(x_i \beta_j)}{\sum_l \exp(x_i \beta_l)} \beta_j - \frac{\exp(x_i \beta_j)}{\sum_l \exp(x_i \beta_l)} \times \sum_l \left[\frac{\exp(x_i \beta_l)}{\sum_l \exp(x_i \beta_l)} \beta_l \right] \tag{5} \\
&= p_{ij} \beta_j - p_{ij} \times \sum_l p_{il} \beta_l \tag{6} \\
&= p_{ij} (\beta_j - \sum_l p_{il} \beta_l) \tag{7}
\end{align}$$

The key point here is that even if we know the sign of β_j, we won't be able to deduce the sign of $\frac{\partial p_{ij}}{\partial x_i}$, i.e. the marginal effect of x_j.

Indeed, since $p_{ij} > 0$, for $\frac{\partial p_{ij}}{\partial x_i}$ to be positive, $\beta_j - \sum_l p_{il} \beta_l$ must be positive, i.e. β_j needs to be larger than all the other β. That is not guaranteed even if $\beta_j > 0$. So even if we know $\beta_j > 0$, we know nothing about the sign of $\frac{\partial p_{ij}}{\partial x_i}$.