Metropolis-Hasting for two-sided logit

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1 Notation

Characteristics (observed data):

- $\underset{(njobs \times nw)}{WW}$ is a matrix of job characteristics, whose jth row $\underset{(nw \times 1)}{WW_j}$ is job j's characteristics
- $\underset{(nworkers \times nx)}{XX}$ is a matrix of worker characteristics, whose ith row $\underset{(nx \times 1)}{XX_i}$ is the characteristics of worker i

Preferences (parameters to be estimated):

- $\underset{(nw \times 1)}{\alpha}$: a vector of workers' preference for jobs' characteristics
- $\underset{(nx \times njobs)}{\beta}$: a matrix of employers' preference for workers' characteristics. Each employer has his own set of preference. β_j— β's jth column—is the preference of employer j.

Matching structure:

- Worker i's opportunity set is O_i, which includes unemployment and other jobs that are offered to worker i.

2 Updating the opportunity set

We update the opportunity set by sampling $J^*(i)$ as a new job to be considered for worker i. The M-H acceptance ratio is $rr1 \times rr2$. To calculate $rr1, rr2$ for worker i:

$$den_i = \sum_{j: j \in O_i} \exp(\alpha' WW_j) \qquad \text{line 81, } \texttt{den <- opp \%*\% avec} \tag{1}$$

$$denstar_i = \begin{cases} den_i - \exp(\alpha' WW_{J^*(i)}) & \text{if } J^*(i) \text{ is already offered} \\ den_i + \exp(\alpha' WW_{J^*(i)}) & \text{if } J^*(i) \text{ is not yet offered} \end{cases} \tag{2}$$

$$\text{line 125, } \texttt{denstar <- den+avec[new]*plusminus} \tag{3}$$

$$rr1 = \frac{den_i}{denstar_i} \tag{4}$$

line 126, `rr1 <- den/denstar`

$$rr2 = \begin{cases} -\exp(\beta'_{J^*(i)} XX_1) & \text{if } J^*(i) \text{ is already offered} \\ \exp(\beta'_{J^*(i)} XX_1) & \text{if } J^*(i) \text{ is not yet offered} \end{cases} \tag{5}$$

line 129, `rr2 <- exp(plusminus*xb)`

In sum, the MH acceptance ratio for the proposed new job $J^*(i)$ for worker i is:

$$rr1 \times rr2 = \frac{den_i}{denstar_i} \times rr2 \tag{6}$$

$$= \frac{\sum_{j:j \in O_i} \exp(\alpha' WW_j)}{\sum_{j:j \in O_i} \exp(\alpha' WW_j) \pm \exp(\alpha' WW_{J^*(i)})} \times \pm \exp(\beta'_{J^*(i)} XX_1) \tag{7}$$