Metropolis-Hasting for two-sided logit

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April 6, 2016

1 Notation

Characteristics (observed data):

- $\underset{(njobs \times nw)}{WW}$ is a matrix of job characteristics, whose jth row $\underset{(nw \times 1)}{WW_j}$ is job j's characteristics
- $\underset{(nworkers \times nx)}{XX}$ is a matrix of worker characteristics, whose ith row $\underset{(nx \times 1)}{XX_i}$ is the characteristics of worker i

Preferences (parameters to be estimated):

- $\underset{(nw \times 1)}{\alpha}$: a vector of workers' preference for jobs' characteristics
- $\underset{(nx \times njobs)}{\beta}$: a matrix of employers' preference for workers' characteristics. Each employer has his own set of preference. β_j— β's jth column—is the preference of employer j.

Matching structure:

- Worker i's opportunity set is O_i, which includes unemployment and other jobs that are offered to worker i.

2 Updating the opportunity set

We update the opportunity set by sampling $J^*(i)$ as a new job to be considered for worker i. The M-H acceptance ratio is $rr1 \times rr2$, with $rr1$ coming from $P(A|O, \alpha)$ and $rr2$ coming from $P(O|\beta)$. To calculate $rr1, rr2$ for worker i:

$$den_i = \sum_{j: j \in O_i} \exp(\alpha' WW_j) \qquad \text{line 81, } \texttt{den <- opp \%*\% avec} \tag{1}$$

$$denstar_i = \begin{cases} den_i - \exp(\alpha' WW_{J^*(i)}) & \text{if } J^*(i) \text{ is already offered} \\ den_i + \exp(\alpha' WW_{J^*(i)}) & \text{if } J^*(i) \text{ is not yet offered} \end{cases} \tag{2}$$

$$\text{line 125, } \texttt{denstar <- den+avec[new]*plusminus} \tag{3}$$

$$rr1 = \frac{den_i}{denstar_i} \qquad \text{line 126, } \texttt{rr1 <- den/denstar} \tag{4}$$

$$rr2 = \begin{cases} -\exp(\beta'_{J^*(i)} XX_1) & \text{if } J^*(i) \text{ is already offered} \\ \exp(\beta'_{J^*(i)} XX_1) & \text{if } J^*(i) \text{ is not yet offered} \end{cases} \qquad \text{line 129, } \texttt{rr2 <- exp(plusminus*xb)} \tag{5}$$

In sum, the MH acceptance ratio for the proposed new job $J^*(i)$ for worker i is:

$$\begin{aligned} rr1 \times rr2 &= \frac{den_i}{denstar_i} \times rr2 && (6)\\ &= \frac{\sum_{j:j\in O_i} \exp(\alpha' WW_j)}{\sum_{j:j\in O_i} \exp(\alpha' WW_j) \pm \exp(\alpha' WW_{J^*(i)})} \times \pm \exp(\beta'_{J^*(i)} XX_1) && (7) \end{aligned}$$

3 Preliminaries

$$p(A_i|O_i, \alpha_i) = \frac{\exp(\alpha' W_{C(i)})}{\sum_{j:j\in O_i} \exp(\alpha' W_j)} \tag{8}$$

4 Update opportunity set

Target distribution for a firm i

$$p(O_i|A_i, \alpha_i, \boldsymbol{\beta}) = \frac{p(O_i, A_i, \alpha, \boldsymbol{\beta})}{p(A_i, \alpha_i, \boldsymbol{\beta})} \tag{9}$$

Metropolis-Hasting acceptance ratio:

$$\begin{aligned} \frac{p(O_i^*|A_i, \alpha_i, \boldsymbol{\beta})}{p(O_i|A_i, \alpha_i, \boldsymbol{\beta})} &= \frac{p(O_i^*, A_i, \alpha, \boldsymbol{\beta})}{p(A_i, \alpha_i, \boldsymbol{\beta})} \times \frac{p(A_i, \alpha_i, \boldsymbol{\beta})}{p(O_i, A_i, \alpha_i, \boldsymbol{\beta})} && (10)\\ &= \frac{p(O_i^*, A_i, \alpha_i, \boldsymbol{\beta})}{p(O_i, A_i, \alpha_i, \boldsymbol{\beta})} && (11)\\ &= \frac{p(A_i|O_i^*, \alpha_i) p(O_i^*|\boldsymbol{\beta})}{p(A_i|O_i, \alpha_i) p(O_i|\boldsymbol{\beta})} && (12)\\ &= && (13) \end{aligned}$$

where Equation (12) is due to the fact that the acceptance of firm i only depends on what is offered to it and what is its preference, $p(A_i|O_i^*, \alpha_i)$; what is offered to i depends on the preferences of all countries, $p(O_i^* \; \boldsymbol{\beta})$.