## Two-Sided Matching Model

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#### 1 Using Adaptive Metropolis

 $\alpha$  (length-2 vector) is the employee's preference for the employer's prestige and autonomy.  $\beta$  (4 × 18 matrix) is the employer's preference for the employee's intercept, age, education, race. There are 18 employers.

I update  $\alpha$  and  $\beta$  together, using the Adaptive Metropolis algorithm (Haario et al. 2001).

The sampling for  $\alpha$  and  $\beta$  has poor mixing, but does converge towards the same values (given the same starting points). The values of  $\beta$  also make substantive sense, e.g. professional employer has a positive preference for education, while farm employer has a negative preference for education.

However, when we start from random, over-dispersed starting points, the mixing is both extremely poor, and the chains don't converge to any values.

Below are three MCMC chains from the same starting points (0 for  $\alpha$ , and one-sided logit estimates for  $\beta$ ).

- 1.1 3 MCMC chains for  $\alpha$
- 1.2 3 MCMC chains for  $\beta$
- 1.3 Correlation between  $\alpha$  and  $\beta$

To check if the correlation is the cause of slow mixing, I plot the sample correlation matrix between  $\alpha$  and  $\beta$ . There doesn't seem to be a lot of correlation

#### 2 Replicating Adaptive Metropolis algorithm

Replicating the result of (Haario et al. 1999, 2001), showing that the Adaptive Metropolis algorithm outperforms the Metropolis-Hastings (MH) and the Adaptive Proposal (AP) algorithm. I run each algorithm 100 times, and each time, record the percentage of the resulting sample that falls within the 68.3% confidence region of the target distribution. (Ideally, 68.3% of the resulting sample should fall within this region.) The boxplot shows the distribution of this 100 runs. The boxplot for the Adaptive Metropolis (AM) algorithm is both closer to the theoretical idea (i.e. horizontal line at 68.3%) and has a smaller spread.

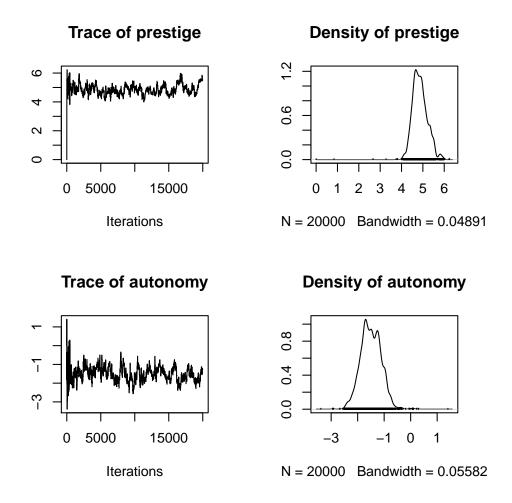


Figure 1: Traceplot of employees' preference for firms' characteristic

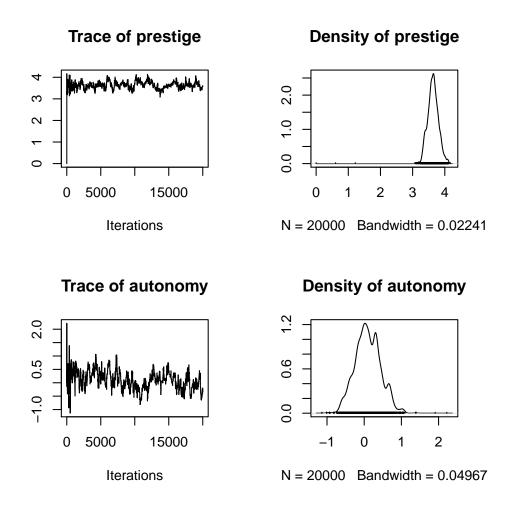


Figure 2: Traceplot of employees' preference for firms' characteristic

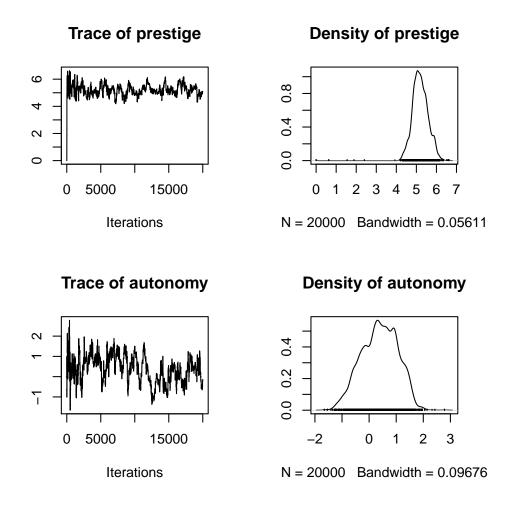
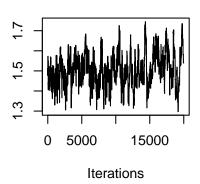
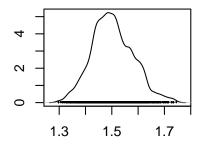


Figure 3: Traceplot of employees' preference for firms' characteristic

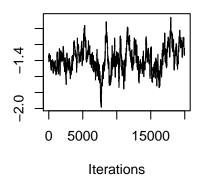
#### Trace of Professionals, Salaric Density of Professionals, Salar



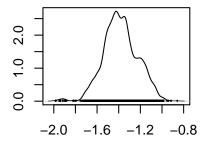


N = 20000 Bandwidth = 0.01154

#### **Trace of Farm laborers**



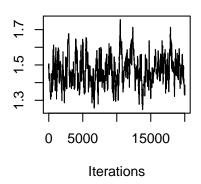
## **Density of Farm laborers**

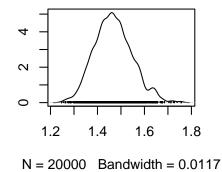


N = 20000 Bandwidth = 0.02207

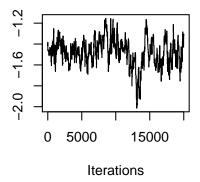
Figure 4: Traceplot of employers' preference

#### Trace of Professionals, Salaric Density of Professionals, Salar

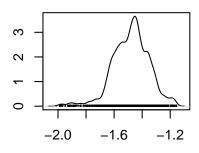




#### **Trace of Farm laborers**



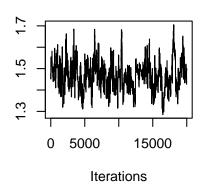
## **Density of Farm laborers**

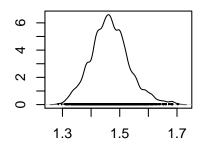


N = 20000 Bandwidth = 0.01807

Figure 5: Traceplot of employers' preference

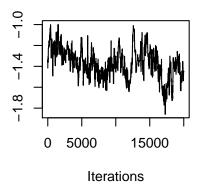
#### Trace of Professionals, Salaric Density of Professionals, Salar



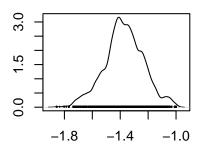


N = 20000 Bandwidth = 0.009066

#### **Trace of Farm laborers**



## **Density of Farm laborers**



N = 20000 Bandwidth = 0.01896

Figure 6: Traceplot of employers' preference

## Correlation across 2 alpha and 72 beta estimates: 18 employers, 4 beta each

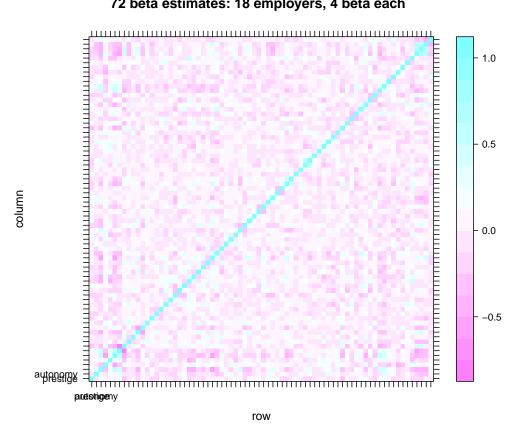


Figure 7: Traceplot of employers' preference

# Comparing Adaptive Proposal (AP), Adaptive Metropolis (AM), and Metropolis-Hastings (MH)

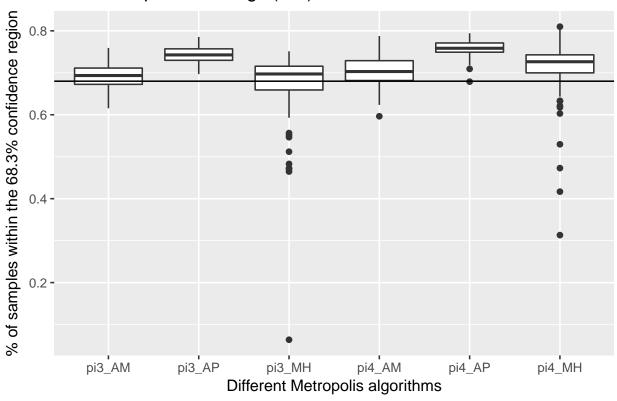


Figure 8: Traceplot of employees' preference for firms' characteristic

## References

Haario, H., E. Saksman, and J. Tamminen (1999). Adaptive proposal distribution for random walk Metropolis algorithm. *Computational Statistics* 14(3), 375.

Haario, H., E. Saksman, and J. Tamminen (2001). An Adaptive Metropolis Algorithm. Bernoulli 7(2), 223.