[Two-Sided Matching Model]

by

[Anh Le]

Department of [Department of Political Science]

Duke University

Date:
Approved:
**
[Michael Ward], Supervisor
[Eddy Malesky]
[Daniel Stegmueller]
[Peter Hoff]

$\underline{Abstract}$

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[Damer Stegmuener]
[Peter Hoff]

An abstract of a dissertation submitted in partial fulfillment of the requirements for the degree of Doctor of Philosophy in the Department of [Department of Political Science]

% in the Graduate School of Duke University 2018

Abstract

Write your abstract here. You should not include references or mathematical notation.

If you want to dedicate your thesis to anyone do so here

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List of Abbreviations and Symbols

Symbols

Put general notes about symbol usage in text here. Notice this text is double-spaced, as required.

- \mathbb{X} A blackboard bold X. Neat.
- \mathcal{X} A caligraphic X. Neat.
- \mathfrak{X} A fraktur X. Neat.
- \mathbf{X} A boldface X.
- X A sans-serif X. Bad notation.
- X = A roman X.

Abbreviations

Long lines in the symbollist environment are single spaced, like in the other front matter tables.

- AR Aqua Regia, also known as hydrocloric acid plus a splash of nitric acid.
- SHORT Notice the change in alignment caused by the label width between this list and the one above. Also notice that this multiline description is properly spaced.
- OMFGTXTMSG4ME Abbreviations/Symbols in the item are limited to about a quarter of the textwidth, so don't pack too much in there. You'll bust the margins and it looks really bad.

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1

Introduction

Much of our economic and political life is governed by a two-sided matching process, in which two sets of actors evaluate each other's characteristics and voluntary form a match if deemed satisfactory. Marriage is a prominent example of such process. Others include the matching of firms and workers, federal judges and law clerks, countries and multinational firms who want to make an investment, or the *formateur* of a government and other minority parties.

Two sided matching market is substantively consequential because it involves scarce, one-of-a-kind "good", such as a life commitment to a marital partner, or a political allegiance in a government formation—certainly not the commodity like loaves of bread. It is also intellectually interesting because its different structure means that our understanding of the normal, one-sided market is inadequate to explain and analyze.

A two-sided market means that the market involves two disjoint set of actors, and one side decision affect the other side (?) who evaluate each other's characteristics. For example, in a marriage market, both men and women evaluate one another's income, appearance, background, to consider a match. This stands in contrast with

one-sided market in which only one side makes the judgment whether a transaction exist, e.g. a grocery shopper evaluates the plumpness of the grocer's tomato, while the grocer does not evaluate the shopper. This means that whether a transaction occurs depend not only on an individual's demand, but also on the individual's opportunity. Indeed, while we typically assume that in a grocery market, all the milk and tomato are always available to buy, and the sole decision is to buy how much, in a marriage market, the no less important question is what is available to the individual.

A matching market means that once a match is being made, the transaction is exclusive between the two individuals being matched. Once a match has been made, both sides are no longer available on the market. This results in strategic behaviors, i.e. delaying accepting an offer, falsifying preference, that causes market failures. For example, in many matching markets, such as the American physician market 1945-51, the American law graduate market, the elite Japanese university graduate markets, all suffer from the problem of offers being made aggressively earlier in order to scoop the best candidates from the market.

The study of two sided matching markets start within the market design subfield of economics, where scholars are chiefly concerned with explaining the market failures in the market, and how to design a mechanism to resolve such failure. In such studies, the preferences of the actors are given, and the goal is to find a "stable" matching.

The first work in this tradition is Gale and Shapely (1962) and Shapely and Shubik (1972). Some key proofs: - From any set of preference, a stable matching always exist - From a random matching, we will converge to a stable matching with probability one.

1.0.1 History of the game theory

(?) was the first to study the one-to-one matching market, named the marriage

market. In such a market, there are two finite and disjoint set of actors, men and women. Each man has preference over each woman, and vice versa. To each man and woman the option of remaining single is always available.

A matching μ is a function that matches a man with a woman. For convenience, we can say that if a man or woman decides to remain single, they are matched with themselves. We refer to $\mu(x)$ as the *mate* of x.

A matching can be improved upon in two ways. First, an individual can prefer to remain single than to be matched with his or her mate $\mu(x)$ under the current matching μ . Second, a man and a woman can improve a matching if they prefer to be with one another rather than whom they are currently matched with. Therefore, we define that a matching μ is stable if it cannot be improved upon by any individual or any pair of agent.

The first result from (?) that is relevant to our purpose is that for any set of preference, there will always be a stable matching. This means that it is reasonable for us to assume that the process that generates the final match we observe in the data is one of agents maximizing their utilities. Crucially, this provides the justification for us modeling their choice using the familiar tools in the discrete choice literature.¹

(?) treats the case of many-to-one matching (which they call the "college admission" game) essentially the same. However, ? shows that a well defined many-to-one matching game is slightly different. However, hen generalize this model to many-to-

¹ (?) provides a constructive proof of stable matching. They describe the process, called deferred acceptance, that produces the stable matching. The process works as follows. In the first stage, every man proposes to his preferred mate. Every woman rejects all of her suitors except the one that she most prefers. However, she does not yet accept her favorite suitor (so far), but keep him along. In the second stage, every man that was rejected proposes to his second choice. Every woman then picks her favorite from the set of new proposers and the man she keeps along from the last round. The procedure continues until there is no longer any woman that is unmatched. The resulting match is stable because, throughout the process, every woman has received all the offers that would have been made to her, and she has chosen her favorite among all of those offers. If there is any other man that she would prefer to her current match, that man would not be available to her. Therefore, the final match cannot be further improved by any man or woman.

one matching setting of firm and workers, all the striking results remain, but we do have to make the assumption of substitutability, meaning that firms treat workers as substitute, not complements. Formally, this means that firm never regret hiring a worker if another worker does not accept the offer. This is certainly not universally true: a football team strategizing for passing play may want to hire both a good passer and a good running back, and in the case one does not accept, will go to another strategy altogether.

So we know that a stable match always exist, meaning it is possible for agents to reach a final match that maximize their utilities. A central coordinator employing the deferred acceptance algorithm is guaranteed to reach this stable match. However, in a decentralized market without a central coordinator, would agents be able to reach this outcome by themselves? How likely is that this will happen. This matters for our empirical approach because we need to know whether the final outcome we observe is indeed a stable match, i.e. which would inform our estimation strategy.(?) shows that, starting from an arbitrary matching, the matching can converge to a stable matching with probability 1 if we allow random blocking pairs, i.e. a woman and a man that are not matched to one another but prefer each other to their current match, to match. In addition, (?) shows that a random search process, in which man and woman randomly meet, evaluate, and decide to pair or not, will converge towards a stable match assuming that the search cost is negligible (specified as having a time discount = 1). This gives us confidence that the data we observed is close to a stable match, a fact that will guide us in our modeling strategy.

1.0.2 Empirical studies of two-sided matching market

Given the importance of many two sided matching markets, there have been many empirical studies that try to estimate the preference of participants in these markets. Below I discuss several examples, starting with the marriage and the labor market, where the two-sided matching literature started. Then I discuss the law graduate market and the FDI market, two markets that are relevant to politics, whose two-sided nature has not been fully appreciated and modeled accordingly.

Estimating preference is a difficult task, and throughout the discussion it will become apparent that most empirical approaches either 1) use survey to get the stated preference, not revealed preference, or 2) failing to consider the two-sided nature of the market in their estimation method, thus unable to distinguish between preference and opportunity. Our two-sided matching model aims to address both these shortcomings: it will estimate preference based on observed match, i.e. revealed preference, and will be able to distinguish between the effect of opportunity and choice.

Labor market

(?) is also estimating preference, disentangling the two-sided effect, but require salary data.

Law graduate market

In the United States, graduates at top law schools vie for the best federal clerkship. These temporary, one-to-two-year position are the launching pad for Supreme Court clerkship, prestigious teaching jobs or positions at top law firms. On the other hand, federal judges also compete for the best law graduates, who help reduce the judges' workload, from copy-editing, to research, and even drafting the opinions (??, 795). As clerkship having an outsized influence on law graduates, studying the law market has important implication for the polarization of the judicial branch.

This market has long been recognized as a classic case of a two-sided market. Clerks look for positions with not only prestige and connection but also comfortable living situation (?). Judges select law graduates based on not only academic creden-

tials but also, some afraid, ideology, gender, and race (?). This market also suffers from strategic behavior emblematic of a matching market, such as offers being made as early as two years before the clerkship start date, and with a short time to accept (??).

The two sided nature of the market makes it difficult to quantitatively study the preference of the market participants. One approach has been to survey clerks and judges directly (?). However, this only allows researchers to measure stated preference, which is unlikely to be accurate when it comes to dimensions that we care about most such as discrimination based on ideology, gender, or race.

Other quantitative approaches circumvent this problem by using observed hiring outcome to study preference. However, no study so far has properly taken into account the two-sided nature of the market, thus confusing the effects of opportunity and preference. For example, ? use political contribution data (DIME dataset) to measure political ideology, then find the correlation between the hiring judges' ideology and their clerks' ideology. This approach does not take into account the pool of clerk applicants, which leads to curious conclusion such as that conservative judges hire more liberal clerks than conservative clerks (?, 31). This curious finding has a potentially simple explanation: the pool of top law graduates tend to be overwhelmingly liberal, and conservative judges may not have much choice. Despite this drawback, the authors proceed to measure judges' ideology by taking the average of their clerks' ideology. Without taking the pool of applicant into account, we may wrongly conclude that conservative judges are more liberal than they are.

Similarly, ? models the process as a discrete choice problem, in which clerks are differentiated products that the Supreme Court justices pick in order to maximize their utilities. However, it doesn't consider what clerks think about the offer. ? can make this assumption because they focus on Supreme Court clerkship, whose unparalleled prestige ensures that any offer made will be accepted. However, if we

want to extend the model to the broader market of federal clerkship, such assumption is untenable.

Government formation

Besides election, government formation is probably the most consequential political process in determining the government that people are subject to. Most extant studies of government formation has either been game theoretic or thick description, "inside-the-Beltway" narrative of what happened. Potential advances be made when we consider government formation as a two-sided matching market, with the *formateur* on one side and all other minority parties on the other.² There are empirical studies of policy-seeking politicians on portfolio allocation, but it's broad statement like this agragrian party will take the agriculture ministry if the party is in the coalition and that there is such a party.

An empirical study of government formation as a two-sided matching market complements the game theory literature that model politicians as policy-seeking (as opposed to office seeking). When politicians are policy-seeking, they have positions that can be modeled as their characteristics, and parties choosing one another to form coalition based on their policy positions akin to men and women choosing one another to form a marriage based on height or income ?.³ As the game theory literature suggests, ideologically compact coalitions are more valuable because they entail fewer costs in policy compromises. With the empirical matching model, we can test if parties do indeed prefer others closer to themselves ideologically.

In addition, the two sided matching approach has the advantage of studying

² The *formateur* party could be the one with the procedural power to set up the coalition, e.g. the incumbent party, or the largest party in established coalitions.

³ In contrast, when politicians are office seeking, the only coin of the realm is the number of legislative seats that a party controls. It determines the inclusion of the party in the government, its portfolio allocation, etc. In this framework, concept like power indices and dominant parties is all about which coalition parties can join to turn it into a winning / losing coalition.

multidimensional policy space. It works quite naturally by considering a party's positions on various policies as their many characteristics.

The FDI market

Estimating the preference: democracy and FDI attraction, survey, comparing FDI and equity, type of FDI (greenfield vs brownfield).

Also demand for the type of FDI.

1.0.3 The two-sided matching approach

The labor market

The marriage market

The machine learning approach

Two-Sided Matching Model

This section describes the behavioral model of many-to-one matching. To aid the exposition, as an example for many-to-one matching I will use the labor market where many workers can be matched with one firm.

We assume that the matching process in the labor market happens in two stage. In the first stage, each firm evaluates each worker in the sample, then decide whether to hire that worker or not. At the end of this stage, each worker has a set of offers from firms, which we call his *opportunity set*. In the second stage, each worker evaluates the firms in his opportunity set and chooses the firm that he likes best. This constitutes a final, observed match between a worker and a firm. This is a many-to-one matching problem because a firm can make offers to multiple workers, none, some, or all of which can be accepted by workers.

If we assume that firms and workers are utility-maximizing agents, at the end of this process, no firm or worker would voluntarily change their final matches. This property is called *stability* in the game theoretic two-sided matching literature. We want the model to have this property because real life matching market tends to be stable as well. Indeed, (?) show that for any given set of preferences, a stable match

always exist. Furthermore, (?) and (?) show that a decentralized market with agents making independent, utility-maximizing decisions can also reach a stable match by itself.

This stability property does not implies that the matches will not change. Indeed, if actors' preference shift, their characteristics change, or new actors enter the market, the matches will also change as a result of actors' recalculating their utility and adjust their decisions. Therefore, since we are estimating the actors' preference using only a snapshot of matching market, we are making the assumption that on a systemic level, the average characteristics of the actors and their preference remain sufficiently static for our estimates to be meaningful.

2.0.1 The decision making of firms

A firm j's decision on whether to hire worker i rests on two utility functions. First, firm j's utility for hiring worker i is:

$$U_j(i) = \beta_j' X_i + \epsilon_{1ij} \tag{2.1}$$

where

 β_j is a vector of firm j's preference for worker characteristics x_i is a vector of worker i's measured values on those characteristics ϵ_{1ij} is the unobserved component that influences firm j's utility

On the other hand, the utility of not hiring worker i is:

$$U_j(\neg i) = b_j + \epsilon_{0ij} \tag{2.2}$$

where

 b_j is the baseline utility of firm j

 ϵ_{0ij} is the unobserved component that influences firm j's utility

For each worker i, firm j will make an offer to hire if $U_j(i) > U_j(\neg i)$. Relevant worker characteristics (i.e. X_i) that a firm may consider are age, education, or experience. The corresponding β 's represent the firm's preference for these characteristics.

We make several assumptions that are standard in the discrete choice literature. First, we assume a linear utility function. Second, we assume that the error terms ϵ_{1ij} , ϵ_{0ij} are uncorrelated. This assumption is most likely not true: unobserved factors in one firm's utility are likely to share some components with unobserved factors in another firm's utility, and thus correlated. The hope is that we have modeled the observed portion of firm's utility sufficiently well that the remaining unobserved factors are close to white noise. In any case, this issue afflicts any application of discrete choice models and is not unique to our case.¹

Third, we assume that the as error terms ϵ_{1ij} , ϵ_{0ij} follow the Gumbel distribution, which has a slightly fatter tail the normal distribution allowing for slightly more extreme variation in the unobserved utility. In practice, the difference between using Gumbel and independent normal error terms is small (?). The choice of the Gumbel distribution is largely motivated by its convenience since we can derive the probability of firm j making an offer to worker i as the familiar binomial logit form:

Also discuss IIA here

¹ The discrete choice model has developed solutions for correlated error structure, such as nested logit, probit, and mixed logit, that can be applied here if we suspect that firms' unobserved utility is strong correlated.

$$Pr(o_{ij} = 1) = Pr(U_j(i) > U_j(\neg i))$$
 (2.3)

$$= Pr(\epsilon_{0ij} - \epsilon_{1ij} < \beta_j' X_i - b_j) \tag{2.4}$$

$$= \frac{\exp(\beta_j' X_i)}{1 + \exp(\beta_j' X_i)} \tag{2.5}$$

$$p(O_i|\boldsymbol{\beta}) = \prod_{j \in O_i} p(o_{ij} = 1|\boldsymbol{\beta}) \prod_{j \notin O_i} p(o_{ij} = 0|\boldsymbol{\beta})$$
(2.6)

$$= \prod_{j \in O_i} \frac{\exp(\boldsymbol{\beta}_j' X_i)}{1 + \exp(\boldsymbol{\beta}_j' X_i)} \prod_{j \notin O_i} \frac{1}{1 + \exp(\boldsymbol{\beta}_j' X_i)}$$
(2.7)

In our observed data, since we only observe the final matching of firms and countries, this opportunity set is unobserved. As will discuss, we use the Metropolis-Hastings algorithm to approximate the posterior distribution of the opportunity set.

2.0.2 Firms' utility

On the other side, for firm i, the utility of investing in country j is:

$$V_i(j) = \alpha' W_i + v_{ij} \tag{2.8}$$

where

 α is a vector of firms' preference for relevant characteristics of countries W_j is a vector of country j measured values on those characteristics v_{ij} is the unobserved component that influences firm i's utility

Firm i evaluates all the countries that welcome it to invest and chooses the country that brings the highest utility. This choice of firms concludes the matching

process, resulting in the observed final match between a firm and a country in our data.

In our model, relevant country characteristics can be: labor quality, level of development, and market size. Since all firms are considered having homogeneous preferences, α does not have a subscript i. The model can be easily extended so that there is heterogeneous preference among firms.

If v_{ij} is modeled as having a Gumbel distribution, then the probability that firm i will accept the offer of official j out of all the offers in its opportunity set O_i takes the multinomial logit form (?):

$$p(A_i = a_i | O_i, \alpha_i) = \frac{\exp(\alpha' W_{a_i})}{\sum\limits_{j:j \in O_i} \exp(\alpha' W_j)}$$
(2.9)

2.1 Model Estimation

Because the opportunity set is unobserved, we have to use MCMC to estimate it.

$$U_j(i) = \beta'_j X_i + \epsilon_{1ij} U_j(\neg i) \qquad = b_j + \epsilon_{0ij} V_i(j) = \alpha' W_j + v_{ij}$$
 (2.10)

$$Pr(o_{ij} = 1) = Pr(U_j(i) > U_j(\neg i))$$
 (2.11)

$$= Pr(\epsilon_{0ij} - \epsilon_{1ij} < \beta'_j X_i - b_j)$$
 (2.12)

$$= \frac{\exp(\beta_j' X_i)}{1 + \exp(\beta_j' X_i)} \tag{2.13}$$

$$p(O_i|\boldsymbol{\beta}) = \prod_{j \in O_i} p(o_{ij} = 1|\boldsymbol{\beta}) \prod_{j \notin O_i} p(o_{ij} = 0|\boldsymbol{\beta})$$
(2.14)

$$= \prod_{j \in O_i} \frac{\exp(\boldsymbol{\beta}_j' X_i)}{1 + \exp(\boldsymbol{\beta}_j' X_i)} \prod_{j \notin O_i} \frac{1}{1 + \exp(\boldsymbol{\beta}_j' X_i)}$$
(2.15)

$$p(A_i = a_i | O_i, \alpha_i) = \frac{\exp(\alpha' W_{a_i})}{\sum\limits_{j:j \in O_i} \exp(\alpha' W_j)}$$
(2.16)

Joint likelihood:

$$p(O, A, \alpha, \beta, \mu_{\beta}, \tau_{\beta}) = p(A|O, \alpha)p(O|\beta)p(\alpha)p(\beta|\mu_{\beta}, \tau_{\beta})p(\mu_{\beta})p(\tau_{\beta})$$
(2.17)

2.1.1 Updating the opportunity set

Target distribution for a firm i

$$p(O_i|A_i,\alpha,\boldsymbol{\beta}) = \frac{p(O_i,A_i,\alpha,\boldsymbol{\beta})}{p(A_i,\alpha,\boldsymbol{\beta})}$$
(2.18)

$$MH_O = \frac{p(O_i^*|A_i, \alpha, \boldsymbol{\beta})}{p(O_i|A_i, \alpha, \boldsymbol{\beta})} = \frac{p(O_i^*, A_i, \alpha, \boldsymbol{\beta})}{p(A_i, \alpha, \boldsymbol{\beta})} \times \frac{p(A_i, \alpha, \boldsymbol{\beta})}{p(O_i, A_i, \alpha, \boldsymbol{\beta})}$$
(2.19)

$$= \frac{p(O_i^*, A_i, \alpha, \boldsymbol{\beta})}{p(O_i, A_i, \alpha, \boldsymbol{\beta})}$$
(2.20)

$$= \frac{p(A_i|O_i^*, \alpha)p(O_i^*|\boldsymbol{\beta})}{p(A_i|O_i, \alpha)p(O_i|\boldsymbol{\beta})}$$
(2.21)

(2.22)

where the factorization of the likelihood in (2.21) is due to the fact that the acceptance of firm i only depends on what is offered to it and what is its preference, $p(A_i|O_i^*,\alpha)$; what is offered to i depends on the preferences of all countries, $p(O_i^*|\beta)$. If we plug in (2.16) and (2.15)

$$\frac{p(O_i^*|A_i, \alpha, \boldsymbol{\beta})}{p(O_i|A_i, \alpha, \boldsymbol{\beta})} = \frac{\sum\limits_{j:j \in O_i} \exp(\alpha' W_j)}{\sum\limits_{j:j \in O_i} \exp(\alpha' W_j) + \exp(\alpha' W_{j*})} \times \exp(\boldsymbol{\beta}'_{j*} X_i)$$
(2.23)

where j^* is the index of the newly sampled job. This is the case when the newly proposed job is not already offered, so it's added to the opportunity set.

When the newly proposed job is already offered, so it's removed from the opportunity set, we have

$$\frac{p(O_i^*|A_i, \alpha, \boldsymbol{\beta})}{p(O_i|A_i, \alpha, \boldsymbol{\beta})} = \frac{\sum\limits_{j:j \in O_i} \exp(\alpha' W_j)}{\sum\limits_{j:j \in O_i} \exp(\alpha' W_j) - \exp(\alpha' W_{j*})} \times \exp(-\boldsymbol{\beta}'_{j*} X_i)$$
(2.24)

2.1.2 Updating firms' parameters, α

Target distribution:

$$p(\alpha|A, O, \boldsymbol{\beta}) = \frac{p(O, A, \alpha, \boldsymbol{\beta})}{p(A, O, \boldsymbol{\beta})}$$
(2.25)

We propose a new α^* using a symmetric proposal distribution that sample α^* in a box whose boundary is $\alpha^* \pm \epsilon_{\alpha}$

Metropolis-Hasting acceptance ratio:

$$MH_{\alpha} = \frac{p(\alpha^*|A, O, \boldsymbol{\beta})}{p(\alpha|A, O, \boldsymbol{\beta})} = \frac{p(A_i|O_i, \alpha^*)p(O_i|\boldsymbol{\beta})}{p(A_i|O_i, \alpha)p(O_i|\boldsymbol{\beta})}$$
(2.26)

$$= \frac{p(A_i|O_i, \alpha^*)}{p(A_i|O_i, \alpha)} \tag{2.27}$$

where (2.27) is due to the flat prior (so $\frac{p(\alpha^*)}{p(\alpha)} = 1$) and the symmetric proposal distribution (so $\frac{p(\alpha^*|\alpha)}{p(\alpha|\alpha^*)} = 1$)

If we plug in (2.16),

$$MH_{\alpha} = \prod_{i} \left[\frac{\exp(\alpha^{*\prime}W_{a_{i}})}{\exp(\alpha^{\prime}W_{a_{i}})} \times \frac{\sum_{j:j\in O_{i}} \exp(\alpha^{\prime}W_{j})}{\sum_{j:j\in O_{i}} \exp(\alpha^{*\prime}W_{j})} \right]$$
(2.28)

$$= \prod_{i} \left[\exp(\epsilon'_{\alpha} W_{a_{i}}) \times \frac{\sum\limits_{j:j \in O_{i}} \exp(\alpha' W_{j})}{\sum\limits_{j:j \in O_{i}} \exp(\alpha^{*} W_{j})} \right]$$
(2.29)

Finally, we log transform the MH acceptance ratio for numerical stability.

$$\log MH_{\alpha} = \sum_{i} \left[\epsilon_{\alpha}' W_{a_{i}} + \log \left(\sum_{j:j \in O_{i}} \exp(\alpha' W_{j}) \right) - \log \left(\sum_{j:j \in O_{i}} \exp(\alpha^{*\prime} W_{j}) \right) \right]$$
(2.30)

2.1.3 Updating countries' parameters, β

Target distribution:

$$p(\boldsymbol{\beta}|A, O, \alpha) = \frac{p(O, A, \alpha, \boldsymbol{\beta})}{p(A, O, \alpha)}$$
(2.31)

We propose a new β^* using a symmetric proposal distribution that sample β^* in a box with side length ϵ_{β}

Metropolis-Hasting acceptance ratio:

$$MH_{\beta} = \frac{p(\beta^*|A, O, \alpha)}{p(\beta|A, O, \alpha)} = \frac{p(A_i|O_i, \alpha)p(O_i|\beta^*)p(\beta^*|\mu_{\beta}, \tau_{\beta})}{p(A_i|O_i, \alpha)p(O_i|\beta)p(\beta|\mu_{\beta}, \tau_{\beta})}$$
(2.32)

$$= \frac{p(O_i|\boldsymbol{\beta}^*)p(\boldsymbol{\beta}^*|\mu_{\beta},\tau_{\beta})}{p(O_i|\boldsymbol{\beta})p(\boldsymbol{\beta}|\mu_{\beta},\tau_{\beta})}$$
(2.33)

where (2.32) is due to the flat prior on β and the symmetric proposal distribution. We plug in (2.15),

$$MH_{\beta} = \prod_{i} \left[\prod_{j \in O_{i}} \frac{\exp(\beta_{j}^{*\prime} X_{i})}{\exp(\beta_{j}^{\prime} X_{i})} \times \prod_{j} \frac{1 + \exp(\beta_{j}^{*\prime} X_{i})}{1 + \exp(\beta_{j}^{\prime} X_{i})} \right] \times \frac{MVN(\boldsymbol{\beta}^{*} | \mu_{\beta}, \tau_{\beta})}{MVN(\boldsymbol{\beta} | \mu_{\beta}, \tau_{\beta})}$$
(2.34)

$$\log MH_{\beta} = \sum_{i} \left[\sum_{j \in O_{i}} \beta_{j}^{*'} X_{i} - \beta_{j}' X_{i} + \sum_{j} \log(1 + \exp(\beta_{j}^{*'} X_{i})) - \log(1 + \exp(\beta_{j}' X_{i})) \right]$$
(2.35)

+ log
$$MVN(\boldsymbol{\beta}^*|\mu_{\beta}, \tau_{\beta})$$
 - log $MVN(\boldsymbol{\beta}|\mu_{\beta}, \tau_{\beta})$

In the MCMC implementation, since β is high dimensional, in each step, we randomly update several β 's at one time.

2.1.4 Update $\mu_{\beta}, \tau_{\beta}$

Similar to a multivariate normal model, where β is the "data".

$$p(\mu_{\beta}) \sim MVN(\mu_0, \Sigma_0) \tag{2.36}$$

$$p(\mu_{\beta}|\beta, \tau_{\beta}) \sim MVN(m, V)$$
 where (2.37)

$$V = (\Sigma_0^{-1} + n\tau_\beta)^{-1} \tag{2.38}$$

$$m = (\Sigma_0^{-1} + n\tau_\beta)^{-1} (\Sigma_0^{-1} \mu_0 + n\tau_\beta \bar{\beta})$$
 (2.39)

3

Basic Document Class Features

This chapter is an example of how to format normal material in the dissertation

style. Most of this information is standard to LATEX.

Intra-chapter divisions: Sections 3.1

Section headlines are \Large and in the standard font. Compare them to subsections

below.

3.1.1 Subsections: Wow! Italics!

Yes, italics. You may now dance. Isn't it funny that upright letters are called "ro-

man" while slanted letters are "italic". That's like Italian, and Romans are Italians

too. What gives?

Subsubsections: Smaller and smaller

Subsubsections are allowed, but are not numbered and don't appear in the table of

contents. Likewise, you can use the next level of sectioning.

18

Paragraphs These divisions are unnumbered and do not appear in the Table of Contents.

Subparagraphs This is the finest division possible. It's also unnumbered and omitted from the Table of Contents.

3.2 Let's do some math

Let's look at an equation:

$$\partial ft = f(t)$$
 subject to $f(0) = c$. (3.1)

We've used the \newcommand defined in the preamble of dissertation.tex to produce the derivative. You can get a second derivative like $\partial^2 f t^2$ by adding some sneaky superscripts. Fancy.

More advanced equation formatting is available in the AMS environments. See the guide amsmath user's guide. Here are some nice examples of cases people usually have trouble with.

An equation that's too long for one line — use multline:

$$a+b+c+d+e+f+g+h+i+j+k+l+m+n+o$$

$$= p+q+r+s+t+u+v+w+x+y+z. (3.2)$$

An equation with multiple parts and one number per line — use align:

$$a_1 = b_1 + c_1 \tag{3.3}$$

$$a_2 = b_2 + c_2. (3.4)$$

The same equation, set inside the subequations environment:

$$a_1 = b_1 + c_1 (3.5a)$$

$$a_2 = b_2 + c_2. (3.5b)$$

Notice that by clever placement of labels, I can reference the pair via (3.5), the first (3.5a), or the second (3.5b). One number for multiple equations can be accomplished using the split environment:

$$a = b + c - d$$

$$+ e - f$$

$$= g + h$$

$$= i.$$
(3.6)

People often struggle under the complicated and ugly 'eqnarray' environment. Don't do it! The AMS ones are easy. Other stumbling blocks are cases:

$$a = \begin{cases} b & \text{for } x > 0 \\ c & \text{otherwise,} \end{cases}$$
 (3.7)

matrices:

$$A = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}, \tag{3.8}$$

and evaluation bars:

$$a = \frac{\partial u}{\partial x} \bigg|_{x=0}.$$
 (3.9)

See the source file for details.

When we reference an equation with something like \eqref (3.1). If you click on the above references in the PDF, your viewer should scroll up to the above equation. It's handy. Labels and references may be attached to all sorts of objects. There is a \label attached to this chapter (it appears at the top of this file), and we may reference it by (Chapter~\ref{chap:example}), producing "Chapter 3". By default these ref's are hyperlinked as well. Later, we'll see labeled and referenced figures and tables. Particular pages may be labeled with standard \label commands in the text and referenced via \pageref.

You might also like the links from \cite commands to the corresponding bibliographic entry. Go look at this imaginary book by Stephen Colbert? If you're not a bibtex expert, look in mybib.bib at the @ARTICLE that generated this entry. It shows an example of accents on author names and how to preserve upper-case for letters in the title. Other entries show the use of the and keyword between author names. You may order a particular author's name as either "first last" or as "last, first". The actual format of the bibliography is controlled by the \bibliographystyle{} command in dissertation.tex.

3.3 Table of Contents Behavior

Now is a good time to look back at the Table of Contents. Notice that you may click on entries here to warp to the corresponding document location. In Adobe Acrobat and many other viewers, you can open a 'bookmarks' pane. This should be populated with named and numbered sections and subsections identical to the Table of Contents.

3.4 Figures and footnotes

Figures are set with very little space between the caption and the bottom of the included graphic. This is because most graphics programs pad the edges of images. If you find the spacing unsatisfactory, you may always add a bit manually. The text of the caption is single-spaced, and the word 'Figure' is set in small caps. See Fig. 3.1. Notice the use of the nonbreakable space "~" between the "Fig." and the reference. Figures (and tables) are examples of 'floats — objects that LATEX decides where to place for you. You may give LATEX some hints. Change the \begin{figure}[tbp] to a \begin{figure}[tbp] to restrict the placement. Inside the [], you can put the following

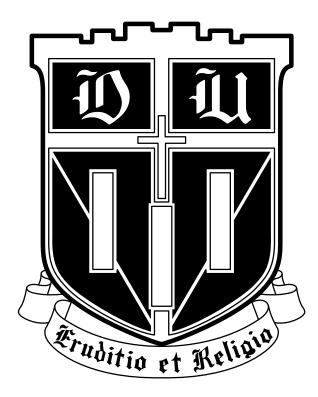


FIGURE 3.1: Longer caption for actual body of dissertation. Figure captions should be BELOW the figure.

- t Allow placement at the top of the page
- b Allow placement at the bottom of the page
- h Allow placement 'here', in the middle of the page close to the text that the figure environment appears next to.
- p Allow placement on a seperate 'floats page' that has no body text.
- ! Tighten the screws on the placement algorithm. This doesn't force things to happen as you say, but it makes it much more likely. Be careful: the bang option can cause figures to appear above the chapter title and in other bad locations.

Notice that each entry just changes what is allowed, but no preference among the

entries can be registered. The default is [tbp], which is a very good default for a document like this, since floats in the middle of a page trap too much whitespace for double-spaced text. There is also a prohibition against having a page with more than 75% float. Instead, long floats will get kicked over onto float pages. Float pages are often a bad idea, as the creation of one will often cause a domino effect, with all subsequent figures appearing on float pages themselves, and all these float pages appearing together at the end of the chapter. (This is more like sinking than floating.) Avoid this by physically moving where the figure environment appears in your source file to an earlier location. Don't be afraid to put the environment before the first spot you reference it! Many float problems can be solved by a combination of relocating the figure environment and a little fiddling with the [] options.

Also notice the order of the graphic, caption, and label. If you deviate from this, strange things can happen. The caption of this figure shows the use of short captions (inside []). These caption appear in the List of Tables, while the captions appear in the body. If you omit the [] short caption, the long caption will be used in its place.

Another technical note: since this style sheet is designed for processing by pdflatex, \includegraphics looks for PDFs, PNGs, and JPGs instead of the usual PS, EPS, and TIFF formats. You can convert existing graphics with a vareity of tools. PDF graphics are preferred, as they scale nicely. The open-source software Inkscape runs on Mac OSX, Windows, Linux, and some UNIX variants. Versions 0.46 and beyond have great support for creating and editing PDFs. It can even be used to convert other docs.

3.4.1 List of Figures

If you've put even one measly figure in your document, grad school rules say you need a List of Figures. It's automatically generated for you if you do a \listoffigures

Table 3.1: Long table caption appears on in the body text. See the short caption in the List of Tables. Table captions need to be ABOVE the table.

Numbers	Letters	Symbols
1	a	†
2	b	Ü
3	c	×
4	d	#

in the master file (heck, it's there right now). Go look at the list of figures now. You should be able to click on the figure number to warp to the figure. You'll also see the result of the 'short caption' used above.

3.5 Table example

Just to make sure tables are formatted correctly, here's an example of a table float, see Table 3.1. You should note that [b] formatting (\begin{table}[b]) can cause floats to appear under the footnotes. Try changing it here and see the ugliness. Tables are identical to figures, except that the word 'Table' appears in the caption and its entry is in the List of Tables instead of the List of Figures.

3.5.1 Footnotes

Footnotes are allowed.¹ They are numbered with arabic numerals inside each chapter and appear at the bottom of the page.² The little footnote numbers are also hyperlinks. Try clicking them. You should place the footnote command immediately following the period of the sentence it is attached to. Any spaces or newlines will result in strange spacing between the number and the sentence.

 $^{^{1}}$ But, you should probably just work them into the text since it's annoying to jump around when reading.

² ... rather than the end of the chapter or the thesis. Those would properly be endnotes, I guess.

3.6 Corner cases in formatting, such as very very long section titles. Man, this goes on forever.

Common corner-cases involve very long titles (like above). In these cases, the long titles are set single-spaced both here and in the Table of Contents.

3.6.1 Figure and Table caption cases are neat, and this is an absurdly long subsection heading

Consider the shield logo again with an absurd caption, as in Fig. 3.2. Also examine the new table, Table 3.2. Both of these have been forced onto a floats page so you can see what that looks like.



FIGURE 3.2: The Duke logo again, but now with a really long rambling caption. This caption should be set single-spaced in the LoF and in the body text. What do you think about having graphics in the main directory of a project? I'd prefer them in a folder, then put 'foldername/picturename' as the argument to includegraphics.

Table 3.2: The same silly table again, but with a really r

Numbers	Letters	Symbols
1	a	†
2	b	Ü
3	c	×
4	d	#

Appendix A

Populo Ornatus

Ut quando convenire scripserit mei, ut accusam noluisse eam. At scripta democritum quo, reque everti an qui, posidonium efficiendi ut mel. Pro an reque habemus, augue nemore conceptam in vim. Eu cibo ancillae takimata usu.

No vis albucius rationibus, eum doming ceteros constituto id. Ad suas zzril laudem cum. Natum mollis singulis vel te, ea elit imperdiet duo, odio inermis et eos. Nam ad vocibus tractatos, sit no vidisse diceret omnesque, mollis omnesque ea mea.

Ut est ridens principes scribentur, menandri interesset adversarium ius ut. Ut duo elit dissentias, at sea eleifend scripserit, eam nibh rebum definitiones an. Cum te quaeque epicuri mentitum, his elitr essent et, in sea habeo aliquid convenire. Quo euismod sadipscing definitionem an, ut duo iusto aliquando, graece appetere ne nec.

Consul imperdiet dignissim vis et, mei liber vidisse principes et, eu nam docendi voluptua democritum. Qui no dicat tamquam sanctus, saepe tincidunt no mel. Pro ignota albucius consetetur in, sint qualisque assueverit eam ut, vis graeco denique signiferumque ne. Sale appellantur contentiones eu his, pro magna ornatus ut, ad vidit omnesque euripidis pri. Sea congue moderatius in, his dicit suscipit no, mei

ei incorrupte assueverit. Nusquam nominavi et quo, idque delenit vim an, posse quaeque an mea.

Suas elitr lucilius sit an, aeterno persius vel eu. Mel at essent aperiam repudiare. Tale consul eum ne, eam no meis delenit iudicabit, an sint mutat pri. Nec no clita propriae pericula, duo explicari gubergren ei. Ne sit autem nominavi, te falli deserunt per. Quo tractatos suscipiantur ex, electram dignissim usu no, cu congue iriure vivendo vim.

At vero graeci fuisset his, quo similique persequeris ad. Ex est graece mandamus, antiopam voluptatum his ea. Assum appellantur mel an, ei mea veri commune efficiendi. Pri blandit urbanitas no. Nam ex enim reque, ut nec iusto regione ullamcorper, facer harum pertinacia mei ei. Erant veniam imperdiet an eam, veniam mucius equidem ius eu, at scripta labitur est.

Et quo soluta graecis accommodare. Tamquam mentitum menandri vim ut. Ut nec melius senserit, ut mei sale aeque. Prompta delectus mea te, fierent adipisci ad per, mei odio pertinax senserit et. Per ut persius singulis. Id qui malorum iracundia, semper conceptam cu sed.

Vis nominavi urbanitas intellegat an, ut numquam deseruisse sea. Et quo dico aeque adipiscing, ius ea commodo epicurei, eum cu nulla imperdiet efficiantur. Ea cum simul scripserit. Ius reque decore voluptaria ei, nec sensibus mediocrem eu, sit iriure vivendo ad. At munere maiestatis mel, ex persius honestatis nec. Nihil omnes definiebas duo cu, dicat ancillae no vix. No est prompta apeirian, mel ad quaestio theophrastus mediocritatem.

Persequeris intellegebat disputationi et nec, nam ne alia solum reque, ad pri clita appellantur reprehendunt. Clita iracundia ex cum, placerat invidunt dissentias ius id. Possit dictas recteque sed ne. At eam singulis recusabo intellegat, ius in probo clita posidonium, id atqui paulo rationibus pro. Ut elit mucius qui. Mel ea ubique nostrud takimata. Cu eos vituperatoribus temporibus feugait.

Per putent nusquam oportere cu, nullam discere te sea, an vix quot mutat. Cibo reque nostrum nec eu, justo mucius aeterno vis id. Facer tempor cu vix, ex saepe similique maiestatis qui, ne pro eripuit offendit. Id mel cetero efficiantur. Cum homero aeterno euismod an, vulputate definitiones ne quo.

Sed exerci incorrupte et, usu mundi molestiae reformidans in, at probo vocibus quo. Ex vel aliquip maluisset. Qui enim error an, molestie incorrupte an ius. Id maiestatis temporibus mei, tantas oporteat ocurreret id pri. Ei nam velit doming utroque.

Suas vituperata mel eu, ex veri omnes duo, an modo molestie ius. At vis moderatius dissentias scripserit, nullam aliquam usu no. Cibo diceret sed an. Sea cu ridens convenire.

Pro prima blandit no. His ut dicit iriure oblique, eos meis urbanitas abhorreant te. Usu cu perpetua principes. Mutat utinam insolens id cum. Quo tale iudicabit conclusionemque ex. Eos id harum accommodare.

Vel legere liberavisse ut, et aeque timeam usu. Vis eu dico sanctus appetere, id vix graecis repudiare, ad persecuti mnesarchum mei. No atqui nemore deseruisse eum. Meliore accumsan accommodare in qui, an tation rationibus has, ea nulla aliquip euismod his.

Utinam ridens cum eu. Duo aliquam omnesque cu, sea elitr appetere ea. Mea no quas discere apeirian, munere hendrerit conceptam duo an, nec ad habeo tritani. Vis exerci volumus no. Omittantur reprehendunt no has. Malis accusata necessitatibus no nam.

No solet assentior ius, an ferri dissentiet pro, vix ad tantas offendit. Pro tollit consequat gloriatur ne, eu vix amet posidonium. Errem utamur veritus vix ea. Sed laboramus omittantur id, ut sonet voluptatum has, cu doctus iriure menandri eos.

Regione iudicabit ei per. Cum ea aliquip voluptatibus. Sit in partem explicari. Ne probo labores placerat mei. Ullum pertinax ea his, per cu persius impedit adipisci.

Fabulas ancillae dignissim ei his, ius no nulla melius suscipiantur, ne vel laudem eripuit gubergren.

Ei qui equidem adolescens. Has ad accusata urbanitas voluptatum, no pri ferri dicit. Ne qui veritus omittam neglegentur, usu et lorem audiam mediocrem. Vim falli dictas labitur cu, dolores laboramus constituto id has, sit ea sint summo utroque.

Mei graecis definiebas eu, ad his brute omittam elaboraret. Ridens laoreet eos ne, diam mnesarchum ne sea. Idque everti ea pro, eruditi probatus patrioque eu has. Cum omnes gubergren ex, cum te noster offendit indoctum. Putant dissentiunt duo ex, dicat etiam cu quo. Duo esse probatus complectitur ex, vitae eripuit nostrum no sed, cum odio veri reformidans ex.

Vide ipsum ei vel, at diam nominavi his. Etiam assueverit nam eu, ut habeo nusquam eleifend mei. Pro eirmod perpetua id, minim urbanitas usu no. Vim elitr nominati definitionem ex. Ex tollit quaerendum has, nonumy inciderint eos ne. Vis posse munere honestatis ut.

Eu ornatus meliore usu, enim aeque possim eu cum. Pri ad everti fabellas, at pro omnium convenire repudiare, ut mel hinc minimum. Eam et puto reque mollis. Sed ad ponderum lobortis, cu pro viris vitae. Est et iriure inimicus, eos eu laoreet feugait voluptatum, agam aliquando voluptatibus pro eu.

Ex tale eirmod nec. Illud conclusionemque ad his. Sit augue error in, eu mea labitur voluptua, labores ullamcorper vis te. No usu enim aperiri facilisi. Ad vis brute soluta fastidii, meis mundi iuvaret his ea. Has eu cibo rebum. Mundi numquam repudiare ei cum, pri dicam tritani recusabo ea, pri id appareat qualisque.

Eu facete perfecto nec, te vel tale choro petentium. Mel in essent quodsi, ocurreret corrumpit in pri. Est in fabulas similique elaboraret, in viderer delenit vim. Eu vel paulo graeco viderer. Et ius elit debet latine, ad vel ferri voluptaria appellantur.

Vis ad docendi albucius, ne nam sale prima comprehensam. Adhuc inani accusam ex vis. Utamur labitur adipisci nec ei. No persius conceptam adversarium pro, ei dicunt officiis lucilius usu. Te pri petentium vituperata, vis at solum dicit quaeque, minimum delectus singulis ei vim. His te nibh patrioque dignissim, qui ne euismod argumentum. Quo lucilius sensibus cu.

Est ea nihil debitis deseruisse, mea ne malis nostrum. Vel cu doctus euismod disputationi. Eos ut harum habemus, minim verear maiestatis mei ut. Te nam mundi deseruisse sententiae, pri an nibh eros velit. Ne omnium torquatos ius. Sit id congue quaeque intellegebat, homero volutpat dissentiunt ne usu, ei elit vituperata reformidans eum.

Ut sed corpora accumsan, his cu vero iriure probatus. Ubique latine ea per, usu no erant facilisis. Augue diceret eruditi ea vel, in diam maiorum ullamcorper eum. Vel no iriure latine suscipiantur, cu nec omittam liberavisse disputationi.

Congue repudiandae delicatissimi ut duo, fastidii iudicabit ut sea, eum integre sadipscing an. Cu vel alii liber, ceteros nostrum expetendis per eu, ne congue gloriatur vulputate cum. Eius fierent pericula has cu. Ea has atqui perfecto. Pericula torquatos ius ei, convenire theophrastus id sea, in dicit facilis facilisis mel. Nam modo diam ocurreret an.

Ferri sensibus eloquentiam quo et, mel an nullam vituperata, mollis dignissim sententiae sit ne. An pro perpetua democritum, te eam feugiat delicata deterruisset. Per minim choro ad, prodesset voluptatum ea usu, ea tempor putent quo. Mazim facete scribentur ea sea. Ea pri doctus feugait, ius eu vituperatoribus menandri, dico munere ubique ne his.

Mel no assum nusquam intellegebat, ius platonem consulatu an. Populo ornatus in sea. Sea soleat salutatus ne. Quo error saepe adolescens at, id cum duis voluptatum. Per maiorum mentitum te. Quem iudicabit percipitur per ea. Qui aliquid eruditi ad, ne vix veritus scripserit.

An duo postea aliquip. Nusquam luptatum id vis. Vim no magna inani. Eos et agam aliquid ancillae, verear ponderum no qui.

Bibliography

Biography

Your biography is limited to one page and must contain

- 1. Full name
- 2. Date and place of birth
- 3. Every degree you've earned, including this one, and where you earned it from.

Mostly, that information is to narrow down which John Smith wrote that dissertation on the mating habits of sea cucumbers. Sexy!

You may also include

- 1. Any awards you've won related to your discipline since your undergraduate degree.
- 2. Any fellowships you've held
- 3. Anything you've published (papers, books, book chapters). Don't be afraid to cite it here, so that the full bibliographic record of your article appears in the bibliography!
- 4. Where your next job will be, if you know