

# Stochastic Models for blockchain analysis

## Introduction

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# Blockchain

A data ledger made of a sequence of blocks maintained by a achieving consensus in a Peer-To-Peer network.

- Decentralized
- Public/private
- Permissionned/permissionless
- Immutable
- Incentive compatible
  - The nodes are compensated for their hard work in cryptocurrencies

We will focus on public blockchain and their associated consensus protocol.

# Blocks

A block contains

- block height/ID
- Time stamp
- hash of the block
- hash of the previous block
- Set of transactions (data stored in the blockchain)

```
Block Height: 0
Block Hash: a52bea61a9f4131588cc101e8e1c731fa9f69f16934c5ab3a05a2134a42c13e0
Time:2021-07-12 10:03:04.812744
Block data: [{"sender": "Coinbase", "recipient": "Satoshi", "amount": 100, "fee": 1}]
Mined: False
Previous block hash: 0
-----
```

# Cryptographic Hash function

A function that maps data of arbitrary size (message) to a bit array of fixed size (hash value)

$$h : \{0,1\}^* \mapsto \{0,1\}^d.$$

A good hash function is

- deterministic
- quick to compute
- One way

→ For a given hash value  $\bar{h}$  it is hard to find a message  $m$  such that

$$h(m) = \bar{h}$$

- Collision resistant
  - Impossible to find  $m_1$  and  $m_2$  such that

$$h(m_1) = h(m_2)$$

- Chaotic

$$m_1 \approx m_2 \Rightarrow h(m_1) \neq h(m_2)$$

# Consensus protocols

The mechanism to make all the nodes agree on a common data history.

The three dimensions of blockchain systems analysis

**1** Efficiency (Queueing theory)

- Throughputs
- Transaction confirmation time

**2** Decentralization (Entropy)

- Fair distribution of the accounting right

**3** Security (Insurance Risk Theory)

- Resistance to attacks



X. Fu, H. Wang, and P. Shi, "A survey of blockchain consensus algorithms : mechanism, design and applications," *Science China Information Sciences*, vol. 64, nov 2020.

# Proof of Work

The nodes compete to solve a cryptographic problem by brute force search.

## PoW

- 1 Draw a random number (nonce)

$$X \sim \{1, \dots, 2^{32}\}.$$

- 2 While  $X > L$ , where  $L$  is the target then try again

Nodes are chosen according to their computing power



S. Nakamoto, "Bitcoin : A peer-to-peer electronic cash system." Available at <https://bitcoin.org/bitcoin.pdf>, 2008.

# Proof of Stake

PoW is slow and ressource consuming. Let  $\{1, \dots, N\}$  be a set of miner and  $\{\pi_1, \dots, \pi_N\}$  be their share of cryptocoins.

## PoS

Node  $i \in \{1, \dots, N\}$  is selected with probability  $\pi_i$ ; to append the next block

Nodes are chosen according to what they own.

- Nothing at stake problem
- Rich gets richer ? (To be discussed later on)



F. Saleh, "Blockchain without waste : Proof-of-stake," *The Review of Financial Studies*, vol. 34, pp. 1156–1190, jul 2020.

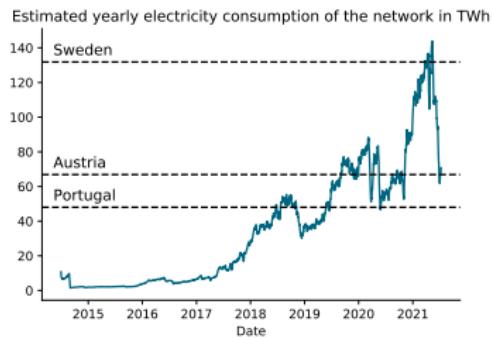
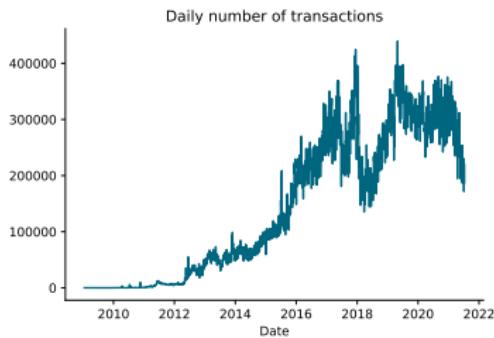
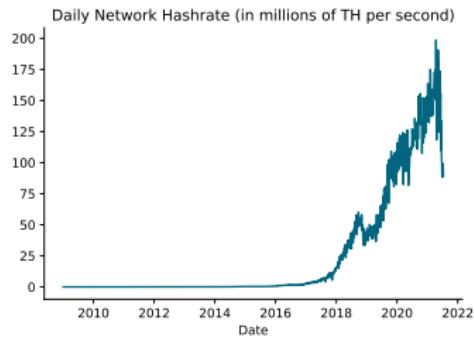
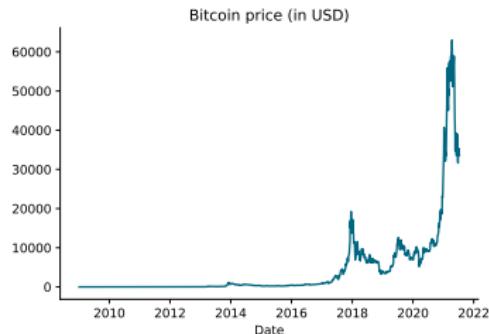
# Applications of blockchain : Cryptocurrency



S. Nakamoto, "Bitcoin : A peer-to-peer electronic cash system." Available at <https://bitcoin.org/bitcoin.pdf>, 2008.



- Transaction anonymity
- Banking and reliable currency in certain regions of the world
- Money Transfer worldwide (at low fare)
- No need for a trusted third party



## Decentralized application

The network provide ressources such as

- storage
- computing power

through a smart contract on the ethereum blockchain.

GOLEM (<https://www.golem.network/>)

Build a network of idle computers to do paralell computing.

Utility tokens are used to access the service and provision the network ressources.

Equation of Exchange (Fisher 1911)

$$MV = PQ$$

# Decentralized finance

DEFI creates new financial architecture

- + Non custodial
- + Anonymous
- + Permissionless
- + openly auditable
- Unregulated
- Tax evasion
- Fraud
- Money laundering

Extends the Bitcoin promises to more complex financial operations

- Collateralized lending
- Decentralized Exchange Platform
- Tokenized assets
- Fundraising vehicle (ICO, STO, ...)



S. M. Werner, D. Perez, L. Gudgeon, A. Klages-Mundt, D. Harz, and W. J. Knottenbelt,  
"Sok : Decentralized finance (defi)," 2021.

# Decentralized insurance

## Parametric insurance

Compensation if a measurable quantity reaches a threshold

- Example : Flight delay insurance
  - [https://etherscan.io/address/  
0xdc3d8fc2c41781b0259175bdc19516f7da11cba7](https://etherscan.io/address/0xdc3d8fc2c41781b0259175bdc19516f7da11cba7)
- Use smart contract and off-chain data through oracles
- Transparent and automatic

Another example is P2P insurance.