Blockchain and applications

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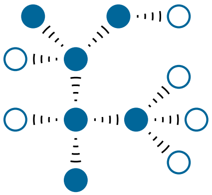
Blockchain

A data ledger made of a sequence of blocks maintained by a achieving consensus in a Peer-To-Peer network.

- Decentralized
- Public/private
- Permissionned/permissionless
- Immutable
- Incentive compatible



Consensus protocols

The mechanism to make all the nodes agree on a common data history.

The three dimensions of blockchain systems analysis

1. Efficiency (Queueing theory)
   - Throughputs
   - Transaction confirmation time
2. Decentralization (Entropy)
   - Fair distribution of the accounting right
3. Security (Insurance Risk Theory)
   - Resistance to attacks

Applications of blockchain : Cryptocurrency

S. Nakamoto, "Bitcoin : A peer-to-peer electronic cash system." Available at https://bitcoin.org/bitcoin.pdf, 2008.

- Transaction anonymity
- No need for a thrusted third party

Reading list I

Introduction

- Q.-L. Li, J.-Y. Ma, and Y.-X. Chang, *Blockchain Queue Theory*, pp. 25–40. Springer International Publishing, 2018.
- G. Angeris, A. Agrawal, A. Evans, T. Chitra, and S. Boyd, *Constant Function Market Makers : Multi-asset Trades via Convex Optimization*, pp. 415–444. Springer International Publishing, 2022.
- A. Cartea, F. Drissi, and M. Monga, "Execution and statistical arbitrage with signals in multiple automated market makers," in *2023 IEEE 43rd International Conference on Distributed Computing Systems Workshops (ICDCSW)*, pp. 37–42, 2023.
- H. Albrecher and P.-O. Goffard, "On the profitability of selfish blockchain mining under consideration of ruin," *Operations Research*, vol. 70, pp. 179–200, Jan. 2022.
- Z. Li, A. M. Reppen, and R. Sircar, "A mean field games model for cryptocurrency mining," *Management Science*, vol. 70, pp. 2188–2208, Apr. 2024.
- R. Bowden, H. P. Keeler, A. E. Krzesinski, and P. G. Taylor, "Modeling and analysis of block arrival times in the bitcoin blockchain," *Stochastic Models*, vol. 36, pp. 602–637, July 2020.

Reading list II

- A. K. Tiwari, S. Kumar, and R. Pathak, "Modelling the dynamics of bitcoin and litecoin : Garch versus stochastic volatility models," *Applied Economics*, vol. 51, pp. 4073–4082, Mar. 2019.
- A. Dembo, S. Kannan, E. N. Tas, D. Tse, P. Viswanath, X. Wang, and O. Zeitouni, "Everything is a race and nakamoto always wins," in *Proceedings of the 2020 ACM SIGSAC Conference on Computer and Communications Security*, CCS '20, ACM, Oct. 2020.
- A. Park, "The conceptual flaws of decentralized automated market making," *Management Science*, vol. 69, pp. 6731–6751, Nov. 2023.
- I. Roșu and F. Saleh, "Evolution of shares in a proof-of-stake cryptocurrency," *Management Science*, vol. 67, pp. 661–672, Feb. 2021.
- H. Adams, N. Zinsmeister, M. Salem, R. Keefer, and D. Robinson, "Uniswap v3 core," *Tech. rep., Uniswap, Tech. Rep.*, 2021.

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Consensus protocol

Definition

Algorithm to allows the full nodes to agree on a common data history

It must rely on the scarce resources of the network

- bandwidth
- computational power
- storage (disk space)

Types of consensus protocols

1. Voting based

   L. Lamport, R. Shostak, and M. Pease, "The byzantine generals problem," *ACM Transactions on Programming Languages and Systems*, pp. 382–401, July 1982.

   - ⚠ Communication overhead
   - ⚠ Denial of service

2. Leader based

   - Proof-of-Work (computational power)
   - Proof-of-Capacity and Proof-of-Spacetime (storage)
   - Proof-of-Interaction (bandwidth)
   - Proof-of-Stake (tokens)

Proof-of-Work

Consensus protocol

Objective

Elect a leader based on computational effort to append the next block.

What's inside a block ?

A block consists of

- a header
- a list of "transactions" that represents the information recorded through the blockchain.

The header usually includes

- the date and time of creation of the block,
- the block height which is the index inside the blockchain,
- the hash of the block
- the hash of the previous block.

Question

What is the hash of a block ?

Cryptographic Hash function

A function that maps data of arbitratry size (message) to a bit array of fixed size (hash value)

$$h : \{0,1\}^* \mapsto \{0,1\}^d.$$

A good hash function is

- deterministic
- quick to compute
- One way
  - $\hookrightarrow$ For a given hash value $\overline{h}$ it is hard to find a message m such that

$$h(m) = \overline{h}$$

- Colision resistant
  - $\hookrightarrow$ Impossible to find m_1 and m_2 such that

$$h(m_1) = h(m_2)$$

- Chaotic

$$m_1 \approx m_2 \Rightarrow h(m_1) \neq h(m_2)$$

SHA-256

The SHA-256 function which converts any message into a hash value of 256 bits.

Example

The hexadecimal digest of the message

`Is DeFi the future?`

is

`60a147c28568dc925c347bce20c910ef90f3774e2501ac63344f3411b6a6bf79`

Hidden prediction

Consensus protocol

Matt Levine
@matt_levine

Here is a SHA-256 hash of a prediction I am making:
64b70b0494580b278d7f1f551d482a3fb952a4b018b43090ffeb87b662d34847.

M. Levine, "The crypto story." Bloomberg business week, Oct. 2022.

Mining a block

Consensus protocol

```
Block Hash: 1fc23a429aa5aaf04d17e9057e03371f59ac8823b1441798940837fa2e318aaa
Block Height: 0
Time:2022-02-25 12:42:04.560217
Nonce:0
Block data: [{'sender': 'Coinbase', 'recipient': 'Satoshi', 'amount': 100, 'fee': 0}, {'sen
der': 'Satoshi', 'recipient': 'Pierre-O', 'amount': 5, 'fee': 2}]
Previous block hash: 0
Mined: False
--------------
```

Figure – A block that has not been mined yet.

Mining a block

Consensus protocol

The maximum value for a 256 bits number is

$$T_{\text{max}} = 2^{256} - 1 \approx 1.16e^{77}.$$

Mining consists in drawing at random a nonce

$$\text{Nonce} \sim \text{Unif}(\{0, \dots, 2^{32} - 1\}),$$

until

$$h(\text{Nonce}|\text{Block info}) < T,$$

where T is referred to as the target.

Difficulty of the cryptopuzzle

$$D = \frac{T_{\text{max}}}{T}.$$

https://laub.au/mine/

Mining a block

Consensus protocol

If we set the difficulty to $D = 2^4$ then the hexadecimal digest must start with at least 1 leading 0

```
Block Hash: 0869032ad6b3e5b86a53f9dded5f7b09ab93b24cd5a79c1d8c81b0b3e748d226
Block Height: 0
Time:2022-02-25 13:41:48.039980
Nonce:2931734429
Block data: [{'sender': 'Coinbase', 'recipient': 'Satoshi', 'amount': 100, 'fee': 0}, {'sen
der': 'Satoshi', 'recipient': 'Pierre-O', 'amount': 5, 'fee': 2}]
Previous block hash: 0
Mined: True
--------------
```

Figure – A mined block with a hash value having on leading zero.

The number of trial is geometrically distributed

- Exponential inter-block times
- Lenght of the blockchain = Poisson process

. block info

Conflict resolution in blockchain

Fork

A fork arises when there is a disagreement between the nodes resulting in several branches in the blockchain.

LCR

The *Longest Chain Rule* states that if there exist several branches of the blockchain then the longest should be trusted.

In practice

- A branch can be considered legitimate if it is $k \in \mathbb{N}$ blocks ahead of its pursuers.
- Fork can be avoided when

$$\text{block appending time} > \text{ propagation delay}$$

Bitcoin protocol

- One block every 10 minutes on average
- Depends on the hashrate of the network
- Difficulty adjustment every 2,016 blocks (≈ two weeks)
- Reward halving every 210,000 blocks

. https://www.bitcoinblockhalf.com/

Mining equipments

How it started

- CPU, GPU

How it is going

- Application Specific Integrated Chip (ASIC)
  - `Network electricity consumption`
  - `E-Waste`
  - `Centralization issue`

Proof of Stake

Consensus protocol

PoW is slow and ressource consuming. Let $\{1, \ldots, N\}$ be a set of miners and $\{\pi_1, \ldots, \pi_N\}$ be their share of cryptocoins.

PoS

1. Node $i \in \{1, \ldots, N\}$ is selected with probability π_i to append the next block

Nodes are chosen according to what they own.

- Nothing at stake problem
- Rich gets richer ?
- `https://www.peercoin.net/`

F. Saleh, "Blockchain without waste : Proof-of-stake," *The Review of Financial Studies*, vol. 34, pp. 1156–1190, jul 2020.

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TradFi Pain Points I

1. Access barrier
   - TradFi: Criteria ⇒ bank account
   - DeFi: No Barrier
2. Centralization
   - TradFi: Bank are record keepers ⇒ Cyber risk
   - DeFi: Decentralized Ledger
3. High costs and intermediation
   - TradFi: Transaction fees, account maintenance fees, wire transfer fees,...
   - DeFi: Intermediaries ⇒ Smart contracts (Gas fees)
4. Slow transaction settlement
   - TradFi: Cross border transactions ⇒ Takes day to be settled
   - DeFi: Near instant settlement (30 min on the bitcoin blockchain)
5. Transparency and auditability
   - TradFi: Difficulty to verify the accuracy of transactions and asset holding
   - DeFi: Publicly available ledger and open source code

TradFi Pain Points II

Cryptocurrencies and Decentralized Finance (and insurance)

6. Censorship and restrictions
   - TradFi: Asset in custody, censorhip by governments
   - DeFi: No interference of central authority
7. Global accessibility
   - TradFi: Respect the operating hours
   - DeFi: Operates 24/7
8. Fractional ownership
   - TradFi: Real estate and art work are not divisible
   - DeFi: Tokenized real worl assets
9. Innovation and interoperability
   - TradFi: Use outdated IT solutions
   - DeFi: Interoperability between platforms and project

A. Lipton and A. Treccani, *Blockchain and Distributed Ledgers*. WORLD SCIENTIFIC, apr 2021.

Cryptocurrencies

1. No central authority (Decentralized network)
2. Ledger to record all the transactions and coin ownership (blockchain)
3. A coin generation process (block finding reward)
   $\hookrightarrow$ Incentive to the full nodes
4. Ownership can be proved cryptographically (wallet associated to a public/private key)
5. Transactions can be issued by an entity proving ownership of the cryptographic unit (through the private key)
6. The system cannot process more than one transaction associated to the same cryptographic unit (double spending)

J. Lansky, "Possible state approaches to cryptocurrencies," *Journal of Systems Integration*, vol. 9, pp. 19–31, jan 2018.

Cryptocurrency implementation

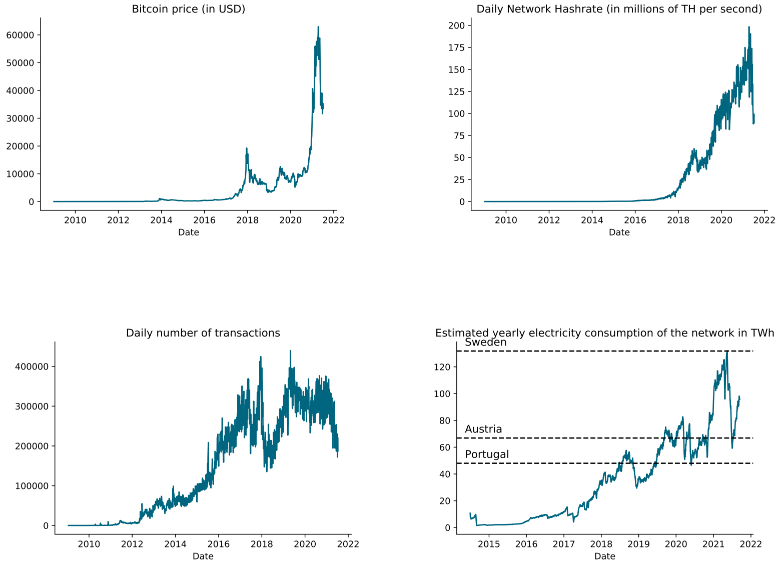
Blockchain parameters

- Consensus protocol (PoW or PoS)
  - ↪ Hash function (SHA-256 for Bitcoin and scrypt for LiteCoin)
  - ↪ Hybrid PoW/PoS (PeerCoin)
- Block generation time (https://txstreet.com/v/eth-btc)
  - ↪ every 10 minutes for Bitcoin
  - ↪ every 12 sec for Ethereum
- Block finding reward
  - ↪ Halved every 210,000 blocks in Bitcoin. It started at 50 BTC, is now 6.25 BTC https://www.bitcoinblockhalf.com/
- Total coin supply
  - ↪ 21,000,000 in total for Bitcoin
- Transaction fees
  - ↪ GAS in Ethereum

These choices lead to the creation of multiple cryptocurrencies

Examples

Bitcoin and AltCoins (Ethereum, LiteCoin, DogeCoin, Ripple...), see https://en.wikipedia.org/wiki/List_of_cryptocurrencies



. **Sources** : `https://www.blockchain.com/` & `https://cbeci.org/`

Decentralized finance

DeFi extends the Bitcoin promises to more complex financial operations

- Collateralized lending
- Decentralized Exchange Platform
- Tokenized assets

Thanks to Smart Contract on the Ethereum blockchain.

S. M. Werner, D. Perez, L. Gudgeon, A. Klages-Mundt, D. Harz, and W. J. Knottenbelt, "Sok : Decentralized finance (defi)," 2021.

. `https://uniswap.org/`

Decentralized Exchange platforms

Centralized Exchange

Binance, Coinbase, Kraken,...

$\Rightarrow$ Order book

Decentralized Exchange

Exchange where you trade one token for another through a smart contract on the Ethereum blockchain (ETH).

$\Rightarrow$ Automated Market Makers (AMM) and Liquidity Providers (LPs)

Order book

Table – Buy Orders

Price	Quantity	Total
9,950	10	99,500
9,900	5	49,500

Table – Sell Orders

Price	Quantity	Total
10,100	2	20,200
10,200	15	153,000

and a market maker in the middle to offer liquidity...

⚠ Lots of transaction must be issued

↪ Slow (10-15 txs/s)

↪ Expensive (gas fees)

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Automated Market Makers (AMM)

A blockchain requires an algorithm $\Rightarrow$ AMM

- Exchange one token against another, usually Crypto/Stable Coins
- Constant Function Market Makers
- Liquidity Providers

Stable Coin

A bridge from fiat to crypto currency

- Fiat-Collateralized Stablecoins (e.g. USDC backed by one USD)
- Crypto-Collateralized Stablecoins (e.g. DAI backed by crypto locked in a smart contract)

Constant Product Market Maker

Let k be a constant such that

$$x \cdot y = k,$$

- x is the amount of token X
- y is the amount of token Y

Example

ETH/DAI Constant Product Market Maker

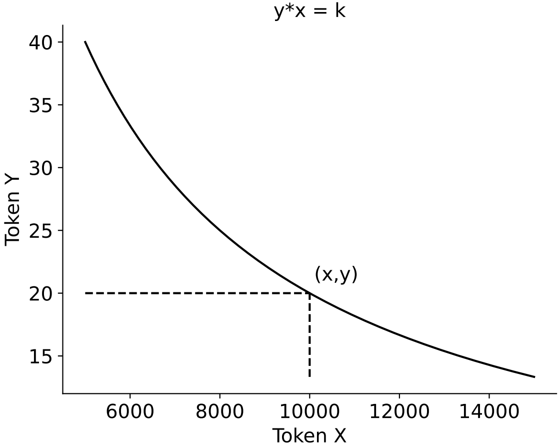
- Say that the price for one ETH is $P = \$500$
- A first LP provides 20 ETH plus 10,000 DAI which sets

$$k = 20,000$$

- The liquidity provided is measured by

$$L = \sqrt{x \cdot y} \approx 447 \text{ (geometric mean)}$$

Trade on a curve



. The pool never runs out of X nor Y

Swap X for Y

Acquire dy of token Y, then deposit dx that solves

$$(x+dx)(y-dy)=k \Leftrightarrow dx=\frac{x\cdot dy}{y-dy}$$

and pay a fee $\alpha\cdot dx$ to the LPs.

$\Rightarrow$ Price of Y rise in the pool

Example

Arbitrageur takes 2 ETH then deposits

$$dx=1,111$$

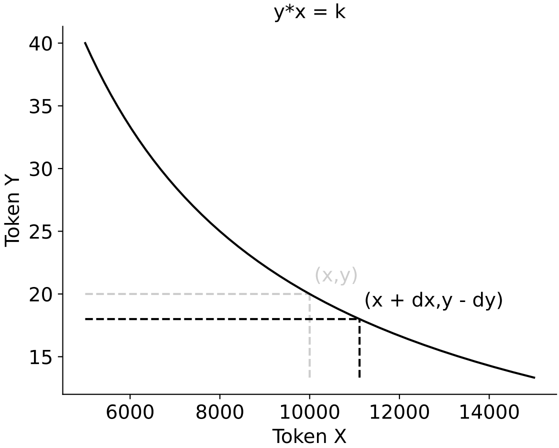
and give out $0.3\cdot 1,111=333$ worth of fees

- The price of ETH in the pool rises to

$$\frac{x+dx}{y-dy}=\$617.$$

Trade on a curve

Cryptocurrencies and Decentralized Finance (and insurance)



Add Liquidity

Another LP provides dx of token X and thus $dy = \frac{y}{x}dx$ (the price must not change)

- New level $k' = (x + dx)(y + dy)$
- Liquidity rises $L' = \sqrt{x \cdot y} + \sqrt{dx \cdot dy}$
- LPs are weighted according to the liquidity they provide
- Fees are distributed according to these weights

Example

New LP deposits \$5,000 worth of tokens to the pool then

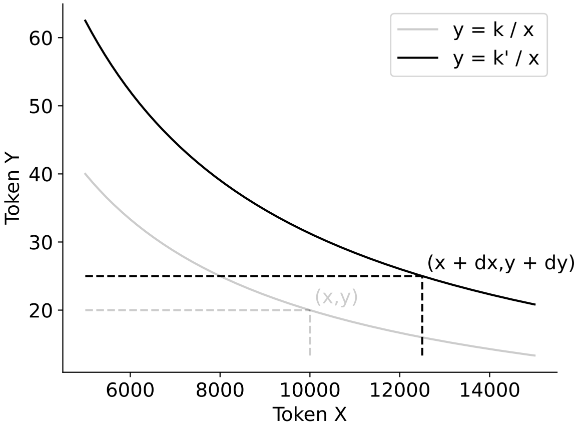
- $dx = 2,500$
- $dy = 5$
- $k' = 312,500$
- $L' \approx 559$

The weights are

$$\frac{L}{L'} = 0.8 \text{ and } \frac{L' - L}{L'} = 0.2.$$

Trade on a new curve

Cryptocurrencies and Decentralized Finance (and insurance)



Flaws of AMM based DEXs

- Price Slippage
- Divergent (or impermanent loss)

Slippage

Large trade when using CFMM yields price fluctuations

- Many transaction are submitted at the same time
  - $\hookrightarrow$ expected price $\neq$ spot

Sandwich attack

Validating nodes are aware of pending transaction

1. Find a transaction with high traded volume
2. Front run that transaction to buy low and/or sell high

A. Park, "The conceptual flaws of decentralized automated market making," *Management Science*, vol. 69, pp. 6731–6751, Nov. 2023.

Impermanent loss

Holding tokens VS Locking Tokens in a Smart Contract

1. Liquidity providers deposit pairs of assets
2. Trade occurs, the initial ratio of tokens change
3. LP withdraws $\Rightarrow$ less of the asset that appreciated in value
4. Divergent loss due to the spread between the initial and current ratio of token that set the price.

Impermanent Loss

The price of Y in the pool is given by

$$P = \frac{x}{y},$$

in terms of token X.

- Price of Y becomes $P' > P$ on another trading venue then arbitrageurs wish to find

$$\underset{0<dy<y}{\text{argmax}}\ dy \cdot P' - dx \cdot (1+\alpha) = \underset{0<dy<y}{\text{argmax}}\ dy \cdot P' - \frac{x \cdot dy}{y - dy}(1+\alpha)$$

- We have

$$dy^* = y - \sqrt{\frac{k(1+\alpha)}{P'}}$$

- The arbitrageurs profit is

$$dy^* \cdot P' - dx^*(1+\alpha)$$

- It coincides with the impermanent loss of the LPs

$$y \cdot P' + x - [(y - dy^*) \cdot P' + x + dx^*(1+\alpha)]$$

Impermanent Loss

The loss becomes permanent if

- The LPs withdraw their funds
- The price of the asset is not mean reverting

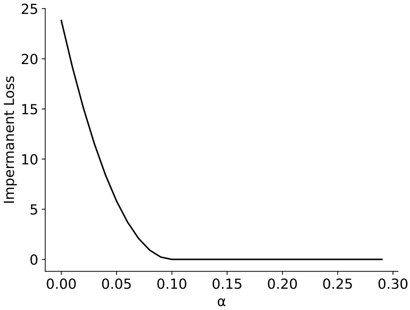
Example

Suppose that $P' = \$550$ and $\alpha = 0$ then

- $dy^* = 0.93$
- The impermanent loss is then $dy^* \cdot P' - \frac{xdy^*}{y - dy^*} = \$23$

Impermanent loss

The trading fee allows a pool to mitigate the impermanent loss



But high trading fees makes the DEX unattractive... UNISWAP V3 and concentrated liquidity .

. https://uniswap.org/whitepaper-v3.pdf

Decentralized insurance

Parametric insurance

Compensation if a measurable quantity reaches a threshold

- Example : Flight delay insurance
  - `https://etherscan.io/address/0xdc3d8fc2c41781b0259175bdc19516f7da11cba7`
- Use smart contract and off-chain data through oracles
- Transparent and automatic

Blockchain as a research topic

- Computer science
  - Peer-to-peer networks and consensus algorithm
  - Cryptography and security
- Economics
  - Game theory to study the incentive mechanism at play
  - Nature of the cryptoassets
- Operations research
  - Optimization of complex system
- Financial math
  - Valuation models for cryptoassets
- Machine learning and statistics
  - Open data
  - Interaction between blockchain users
  - (Social) network analysis
  - Clustering of public keys and addresses in the bitcoin blockchain.

Byzantine General problem

n generals must agree on a common battle plan, to either

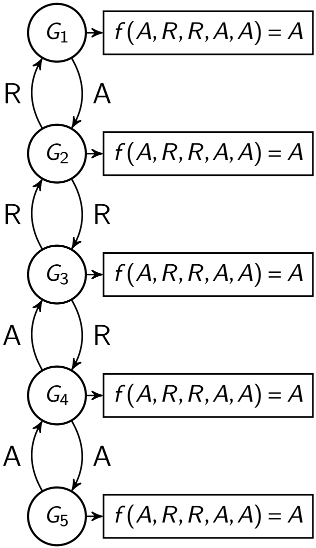
- Attack (A)
- Retreat (R)

Problem

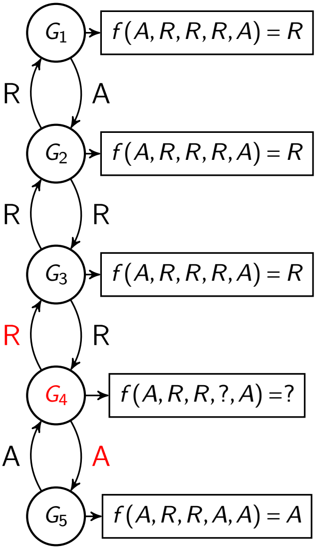
There are $m < n$ traitors among the generals

1. message $m(i,j)$ is sent to general j by general i
2. Consensus is reached as general j applies

$$f(\{m(i,j);\ i=1,\dots,n\}) = \begin{cases} A, & \text{if } \sum_{i=1}^{n} \mathbb{I}_{m(i,j)=A} > n/2, \\ R, & \text{else.} \end{cases}$$



(e) No traitor



(f) One traitor

Figure – Majority vote with or without a traitor

Commanders and Lieutenants

One general is the commander while the others are the lieutenants

Objective

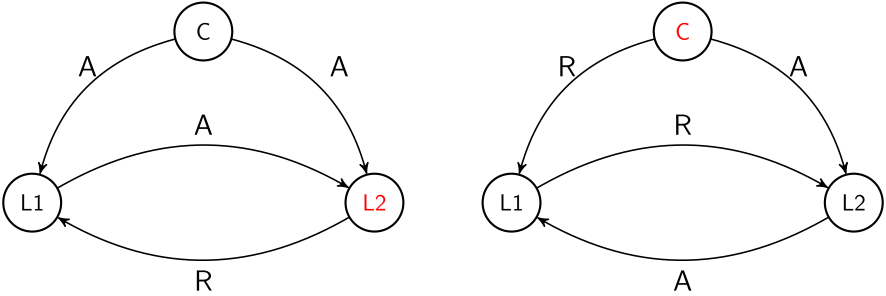
Design an algorithm so that the following conditions are met :

C1 All the loyal lieutenants obey the same order

C2 If the commanding general is loyal, then every loyal lieutenants obey the order he sends

Byzantine Fault Tolerance Theorem (Lamport et al.)

There are no solution to the Byzantine General problem for $n < 3m+1$ generals, where m is the number of traitors.



(a) Commander is loyal (b) Commander is a traitor

Figure – Majority vote with or without a traitor

$n = 4$ and $m = 1$: Step 1

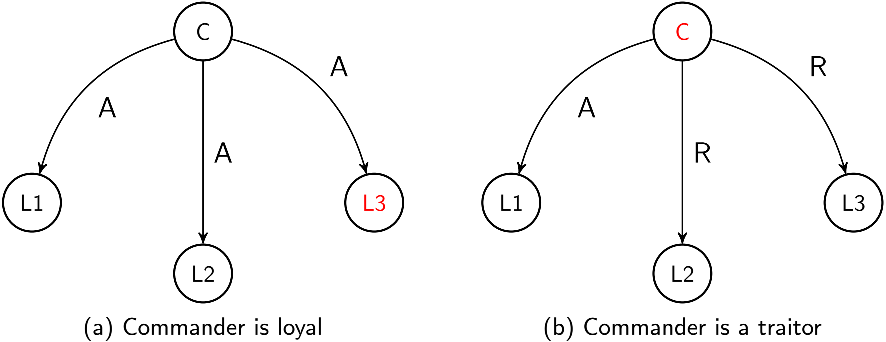
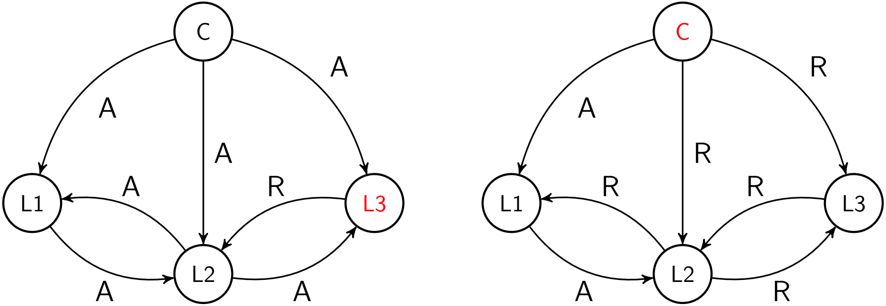


Figure – Illustration of the OM(m) algorithm in the case where $n = 4$ and $m = 1$.

$n = 4$ and $m = 1$: Step 2

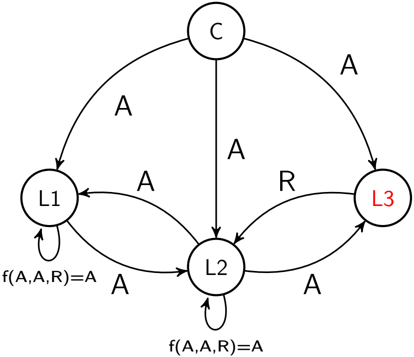


(a) Commander is loyal

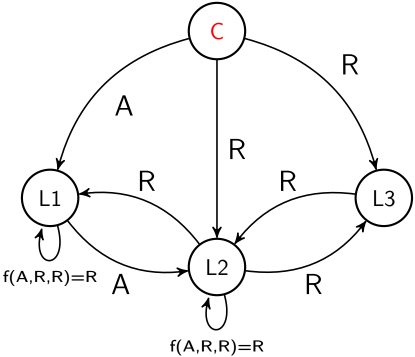
(b) Commander is a traitor

Figure – Illustration of the OM(m) algorithm in the case where $n = 4$ and $m = 1$.

$n = 4$ and $m = 1$: Step 3



(a) Commander is loyal, C1 and C2



(b) Commander is a traitor, C1

Figure – Illustration of the OM(m) algorithm in the case where $n = 4$ and $m = 1$.

The problem with majority vote

The OM algorithm requires to send n^{m+1}

- ⚠ Communication overhead
- ⚠ Denial of service

Solution

Leader based protocols !