Stochastic Models for blockchain analysis

Decentralized finance and cryptopricing

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Types of Crypto Assets

Decentralized finance

- Cryptocurrencies
- Utility Token
- Security Token
- Non Fungible token

Cryptocurrency

Decentralized finance

Digital currency as a medium of exchange with three key characteristics

- Anonymity
- No central authority
- Protected against double spending attack

J. Lansky, "Possible state approaches to cryptocurrencies," *Journal of Systems Integration*, vol. 9, pp. 19–31, jan 2018.

How does it work ?

1. No central authority (Decentralized network)
2. Ledger to record all the transactions and coin ownership (blockchain)
3. A coin generation process (block finding reward)
   ↪ Incentive to the full nodes
4. Ownership can be proved cryptographically (wallet associated to a public/private key)
5. Transactions can be issued by an entity proving ownership of the cryptographic unit (through the private key)
6. The system cannot process more than one transaction associated to the same cryptographic unit (double spending)

More on anonymity

Decentralized finance

- Transparent account : The owner has revealed her identity in a credible manner
- Semi-transparent account : The owner identity is traceable by state authority
  - Exchange to fiat currency with an exchange office that abids by KYC rules
- Pseudo anonymous account : Owner identity is known by the owner's business partners (like a merchant who would remember the customer's face in the case of an extraordinary purchase).
- Anonymous account : Nobody knows the owner's identity, newly created account.

Purposes of cryptocurrencies

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- Micropayments : If the transaction fee is significantly lower than the amounts conveyed
  - ↪ $0.03 for DogeCoin
- Foreign payments : International payment without delay and bank fees
- Payments in countries with unstable local currencies : In some African and South American countries with high inflation rate
- Information retention : OP_RETURN transactions to add informations without transferring any amount of cryptographic unit.

Risk associated to cryptocurrencies

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- Low market capitalisation : If the number of users is limited and the market cap is low then one user's trade may have disproportionate consequences of the coin value
- Private key = ownership : Personal computers or server of wallet management services may be hacked. One solution is to resort to hardware to store the private key.
- Transaction irreversibility : If some funds are transfered by misstake, they are not recoverables
- Account anonymity : Whenever an account issue transactions, it becomes pseudo-anonymous. It is difficult to for the authority to find the identity of a pseudo anonymous account when funds are used for criminal activities (financial theft, tax evasions, extortions or bribery).

Cryptocurrency implementation

Decentralized finance

Blockchain parameters

- Consensus protocol (PoW or PoS)
  - ↪ Hash function (SHA-256 for Bitcoin and scrypt for LiteCoin)
  - ↪ Hybrid PoW/PoS (PeerCoin)
- Block generation time
  - ↪ every 10 minutes for Bitcoin
  - ↪ every 12 sec for Ethereum
- Block finding reward
  - ↪ Halved every 210,000 blocks in Bitcoin. It started at 50 BTC, is now 6.25 BTC `https://www.bitcoinblockhalf.com/`
- Total coin supply
  - ↪ 21,000,000 in total for Bitcoin
- Transaction fees
  - ↪ GAS in Ethereum

These choices lead to the creation of multiple cryptocurrencies

Examples

Bitcoin and AltCoins (Ethereum, LiteCoin, DogeCoin, Ripple...), see `https://en.wikipedia.org/wiki/List_of_cryptocurrencies`

Utility token

Digital asset that grant access to goods and services provided by the network.

- Digital coupon or digital casino chip
- Mainly powered by the Ethereum blockchain through smart contracts
- Crowdfunding means for blockchain based start up projects via Initial Coin Offerings (discussed later)

Examples

Funfair, Basic Attention Token, Golem token, FileCoin ...

Tokenized real-world assets

Decentralized finance

Tokenized version of a real-world, physical asset

- Increases the liquidity of certain type of assets
- Make certain classes of assets available to the many
- Can be used as store of value or collateral

These token can be backed by

- fiat currency ⇒ stablecoin
- commodities like gold `https://ekon.gold/`
- stocks (security token) that includes voting right and profit sharing mechanism
- Art
- Digital art (Non Fungible tokens on the Ethereum blockchain)

Central authority

This requires a custodian to ensure that the tokens are actually backed by these off-chain assets (except for NFTs).

OECD, "The tokenisation of assets and potential implications for financial markets," tech. rep., 2020.

Decentralized Finance applications

- Fundraising instruments
- Decentralized exchange platforms
  - Trades are settled on-chain (verifiable)
  - Exchange do not own the users' funds (non-custodial)
  - Automated Market Makers (AMM) to provide liquidity `https://uniswap.org/`
- DeFi lending protocols
  - Peer-to-peer lending
  - Borrow against a smart contract reserves made ofa pool of users deposit
  - *Overcollateralization*

Valuation models

- Cryptocurrencies are medium of exchange and may be priced via transaction cost model (Beaumol-Tobin and such)


W. J. Baumol, "The transactions demand for cash : An inventory theoretic approach," *The Quarterly Journal of Economics*, vol. 66, p. 545, nov 1952.


L. Schilling and H. Uhlig, "Some simple bitcoin economics," *Journal of Monetary Economics*, vol. 106, pp. 16–26, oct 2019.

- Tokenized asset depends on the real asset that backs the token


J. Hargrave, N. Sahdev, and O. Feldmeier, "How value is created in tokenized assets," in *Blockchain Economics : Implications of Distributed Ledgers*, pp. 125–143, WORLD SCIENTIFIC (EUROPE), jan 2019.

- Utility tokens


J. R. Gan, G. Tsoukalas, and S. Netessine, "Initial coin offerings, speculation, and asset tokenization," *Management Science*, vol. 67, pp. 914–931, feb 2021.


L. W. Cong, Y. Li, and N. Wang, "Tokenomics : Dynamic adoption and valuation," *The Review of Financial Studies*, vol. 34, pp. 1105–1155, aug 2020.

ICO tuning and timeline

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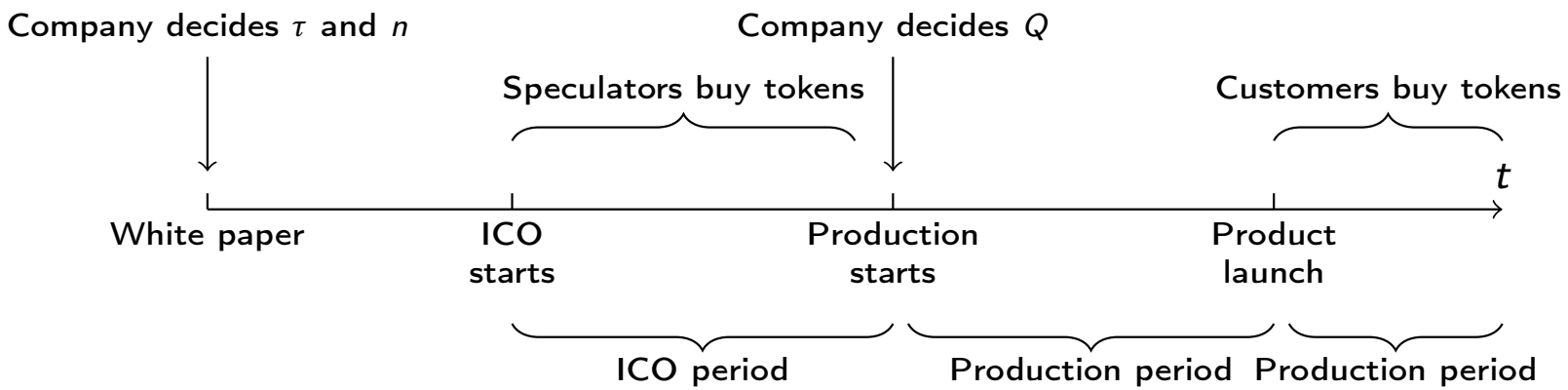
Game theoretic approach with three players : The firm, the speculators and the customers that interacts over three time period

1. ICO period
   - The firm publishes a white paper and set
     - The token price τ
     - The total number of token m
     - The number of token issued to the investors during the ICO $n \leq m$.
   - s among $z >> m$ investors buy token
2. Production period
   - The firm uses the funds raised $s\tau$ to finance the production of Q units of goods
3. Market period
   - Customers purchase token to meet their needs $D \sim F(.)$



The firm and customer side

Let

- c be the production cost of one unit of good ($ per unit)
- p be the value of the good in tokens per unit
- v How much the good is worth from the customers' point of view ($ per unit)
- τ_{eq} the token price at equilibrium

We have

$$\tau_{eq} = \frac{\min(Q, D) \cdot v}{m}$$

The speculator side

Denote by s the number of token bought during the ICO (one token = one investor).

- The participation condition reads as

$$\Delta = \mathbb{E}(\tau_{eq}) - \tau > 0$$

- The number of token sold is derived endogeneously in equilibrium.

Speculators and the firm then compete to sell the tokens to customers.

Firm's optimization problem

The firm aims at solving

$$\max_{\tau,n}\left\{s\cdot\tau+\max_{Q}\left\{[m-s]\mathbb{E}[\tau_{eq}(Q)]-c\cdot Q\right\}\right\}$$

subject to

- $s\cdot\tau\geq c\cdot Q$ (ICO funds cover the production cost)
- $\Delta\geq 0$ (speculators participation constraint)
- $\tau_{eq}=\frac{v}{m}\min(D,Q)$ (market clearing condition)

Equilibrium $\Rightarrow$ Backward induction

Moral Hazard

$Q=0$ is a feasible strategy due to the absence of regulation.

Optimal production quantity Q^*

Proposition

Given τ, n and s,

(i) if $0 < s < m\left(1 - \frac{c}{v}\right)$ then

$$Q^* = \min\left\{F^{-1}\left(1 - \frac{m}{m-s}\frac{c}{v}\right), \frac{\tau \cdot s}{c}\right\}$$

(ii) If $s = 0$ or $s \geq m\left(1 - \frac{c}{v}\right)$, then $Q^* = 0$

Interpretation

(i) $F^{-1}\left(1 - \frac{m}{m-s}\frac{c}{v}\right)$ is the unconstrained optimal production quantity and $\frac{\tau \cdot s}{c}$ is the firm budget constraint

(ii) $1 - \frac{c}{v}$ is the "misconduct" fraction if $s \geq m\left(1 - \frac{c}{v}\right)$ the start ups "divert" the funds to its own pocket (moral hazard).

On the number of investors

Proposition

Given τ and n,

(i) $s^*(\tau, n) \in [0, m(1 - \frac{c}{v})]$

(ii) $s^*(\tau, n) = n \cdot \mathbb{I}_{\Delta \geq 0}$

Interpretation

(i) The number of token sold must be positive without exceeding the misconduct threshold

(ii) If there is an expected profit to be made then all the ICO token are sold

Conditions for ICO success

Proposition (Condition for ICO success)

The ICO succeed if and only if

(i) $s \geq mc/v$

(ii) Customers have a high willingness to pay

$v > 2c$ (price-cost ratio requirement)

Interpretation

(i) A critical mass of token must be sold to finance production

(ii) Combines $n > mc/v$ and $n < m(1 - c/v)$

Equilibrium results

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Proposition

(i) If $v \leq 2c$, then the ICO fails

(ii) If $v > 2c$ then unique equilibrium

(a) $n^* \in (\frac{mc}{v}, \frac{m}{2})$ and satisfies

$$\frac{n^* \tau^*}{c} = Q^*$$

(b) $\tau^* = \frac{v}{m}\mathbb{E}[\min(D, Q^*)]$

(c) $Q^* = F^{-1}\left(1 - \frac{m}{m-n^*}\frac{c}{v}\right)$

(d) $s^* = n^*$

(e) $\tau_{eq} = \frac{v}{m}\min(D, Q^*)$

Intuitions

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Optimal number of tokens sold n^*

The more token the firm sells during the ICO

- The more money to invest in production
- The less tokens it has to sell in the secondary market
- The less "skin in the game"
- The less it wants to invest in production ex post

n^* resolves the trade off between money now and money later while controlling moral hazard.

Optimal number of tokens sold τ^*

- Price too low : Not enough funds raised
- Price too high : not enough upside for investors

Gerry Tsoukalas talk at
`https://www.youtube.com/watch?v=E_NT4t4ws8U`

Dynamic valuation model

Model the transactional demand of platform users through the adoption of the application.
A platform user i derives a utility flow given by

$$\mathrm{d}v_{i,t} = (x_{i,t})^{1-\alpha}\left(N_t A_t e^{u_i}\right)^{\alpha}\mathrm{d}t - \phi\mathrm{d}t - x_{i,t} r\mathrm{d}t,$$

where

- $x_{i,t}$ is the token holding of user i
- α is a parameter of the utility function
- N_t is the number of platform user (the more the merrier)
- A_t is the platform productivity
- u_i user i individual preference
- ϕ cost of platform participation
- r is the risk free rate

L. W. Cong, Y. Li, and N. Wang, "Tokenomics : Dynamic adoption and valuation," *The Review of Financial Studies*, vol. 34, pp. 1105–1155, aug 2020.

Tokenized economy

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Fact

User i looks for the best token holding $x_{i,t}$ to maximize its utility flow $\mathrm{d}v_{i,t}$.

Denote by P_t the token value, the user i's token holding is worth

$$x_{i,t} = P_t k_{i,t},$$

with $K = \sum_i k_{i,t}$ being the total number of token in circulation.

Theorem

The price appreciation of the token (due to platform adoption and a limited number of tokens)

- offsets the cost of holding a medium of exchange

$$\mathrm{d}v_{i,t} + x_{i,t}\frac{\mathrm{d}P_t}{P_t}$$

- speeds up platform adoptions (S-curve)

Proof

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The mathematical derivation relies on

- A platform productivity as a geometric brownian motion
  ↪ Ito's lemma and Girsanov theorem
- The introduction of a stochastic discount factor and a risk neutral probability measure
- Markov equilibrium

L. W. Cong, Y. Li, and N. Wang, "Tokenomics : Dynamic adoption and valuation," *The Review of Financial Studies*, vol. 34, pp. 1105–1155, aug 2020.

Conclusion and perspectives

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A lot of interesting research work for mathematical finance experts!