Introduction to Fluid Dynamics

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- https://josephmacmillan.github.io/IntroFluidDynamics (web)
- https://josephmacmillan.github.io/IntroFluidDynamics/IntroFluidDynamics.pdf (PDF)

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Chapter 1

Introduction

1.1 Opening Thoughts

What, exactly, is a fluid? Well, it's anything that *flows*, of course. But that can mean a surprising number of things.

One obvious example is water, which we'll use extensively as our go-to fluid for a number of applications. Other fluids might be highly viscous – syrup is a good example – in which case the behaviour might be very different. Later on in the book we'll take an introductory look at *aerodynamics*, where the fluid is of course air.

These are all good examples of what's called a *Newtonian fluid* – the fluid has certain, fairly simple properties and can be modelled well by a set of equations called the *Navier-Stokes* equations. Other fluids, though, are more complicated (and, for the most part, beyond the scope of this book).

For example, the behaviour of fluids with a net charge (e.g., a plasma) add interesting complications that must be dealt with by including the theory of electricity and magnetism. Combining Maxwell's equations with the Navier-Stokes equations leads to the theory of magnetohydrodynamics, or MHD for short.

Even something as commonplace as blood, though, can be beyond the Navier-Stokes equations to model. That's because blood is an example of a non-Newtonian fluid – it's nonhomogeneous and is a "shear-thinning" fluid, which means it becomes less viscous at high shear strain. Consider what happens to the blood during an anaphylactic shock – an extreme allergic reaction. The body's first response is to release histamine, which causes the blood vessels to widen. When this happens, the blood will slow down, for reasons we'll learn have to do with conservation of mass. But because blood is shear-thinning, it becomes more viscous as it goes slower. This leads to a feedback loop – increased viscosity causes the blood to slow further, causing it to be more viscous, which means the blood slows even more, and so on. This is why anaphylaxis is so severe and needs to be treated right away with adrenaline, which increases blood flow.

As a final, surprising, example, traffic can sometimes be modelled as a fluid – even though cars are large, discrete objects.

Although the study of fluid dynamics is centered around solving the Navier-Stokes equations, in this first chapter we'll begin with some preliminary ideas about the flow of fluids, in particular how we can mathematically describe and visualize it. But before we start with the heavy lifting of solving differential equation, we'll also need to be able to identify various properties of the fluid, such as its vorticity and viscosity, since in many cases understanding these can lead to significantly simpler methods of solving the equations of fluid dynamics.

1.2 Describing Fluid Flow

The main goal of fluid dynamics is to find the fluid velocity $\mathbf{u}$ at every point $\mathbf{x}$ at any time t:

$$\mathbf{u} = \mathbf{u}(\mathbf{x}, t). \tag{1.1}$$

This is a good time to discuss some of the notation we'll be using. First, note that I'm bolding any quantity that's a *vector*, like position and velocity. Scalars are represented by italicized letters. Secondly, I'm using $\mathbf{u}$ to denote the velocity rather than the more usual $\mathbf{v}$. In fact, the letter

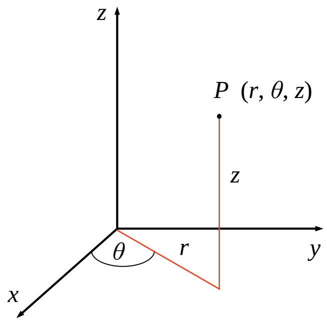

Figure 1.1: Cylindrical coordinates (r, θ, z).

v serves a different purpose – it's the y component of the fluid velocity. The x and z components are denoted by u and w, respectively, so we can write the full vector form of the velocity as

$$\mathbf{u} = u\,\hat{i} + v\,\hat{j} + w\,\hat{k} = [u, v, w]. \tag{1.2}$$

Keep in mind that u, v, and w are each functions of x, y, z, and time:

$$u = u(x, y, z, t), \quad v = v(x, y, z, t), \quad \text{and} \quad w = w(x, y, z, t),$$

and note that I'm using some vector short hand with the square brackets.

This can sometimes be confusing notation, so be careful with it. Also note that (to add to the confusion) u does double duty: it's both the x component of the flow, as well as the name of the entire velocity vector field.

So far, I've written everything down in Cartesian coordinates, but we'll use cylindrical coordinates (r, θ, z) frequently as well. Figure 1.1 shows the usual cylindrical coordinate setup (note that I'm using θ rather than the usual ϕ; this is mostly because all my notes come from a math book), and it's clear that

$$x = r\cos\theta, \quad y = r\sin\theta, \quad \text{and} \quad z = z.$$

We can write the fluid velocity in cylindrical coordinates as

$$\mathbf{u} = u_r(r, \theta, z, t)\hat{r} + u_\theta(r, \theta, z, t)\hat{\theta} + u_z(r, \theta, z, t)\hat{z}. \tag{1.3}$$

I'll avoid using the "bracket" shorthand if we're using cylindrical coordinates, though.

In much of our examination of fluid dynamics, we'll deal with special cases and symmetries which will make our job (slightly) easier. The below examples discuss two of these special cases.

Example 1.1 – Steady Flow.

A *steady flow* has no explicit time dependence, so that

$$\frac{\partial \mathbf{u}}{\partial t} = 0. \tag{1.4}$$

This means that, at any point, the speed and direction of the flow are constant. We'll be dealing with this case quite frequently, especially at the beginning of the book.

Example 1.2 – Two Dimensional Flow.

A *two dimensional* flow has the form

$$\mathbf{u} = [u(x, y, t), v(x, y, t), 0]. \tag{1.5}$$

Note that not only is there no z component to the velocity field, but there is furthermore no z dependence on the x and y components, either.
