calibration_error_analysis

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0.1 Hysteresis example

Given calibration of an instrument for an increasing and decreasing input x [mV] and output of the instrument y [mV]

In [35]:
```
import numpy as np
import pylab as pl
```

In [36]:
```
from IPython.core.display import Image
Image(filename='../Lectures/Lecture3/statics/hysteresis_example.png')
```

Out [36]:

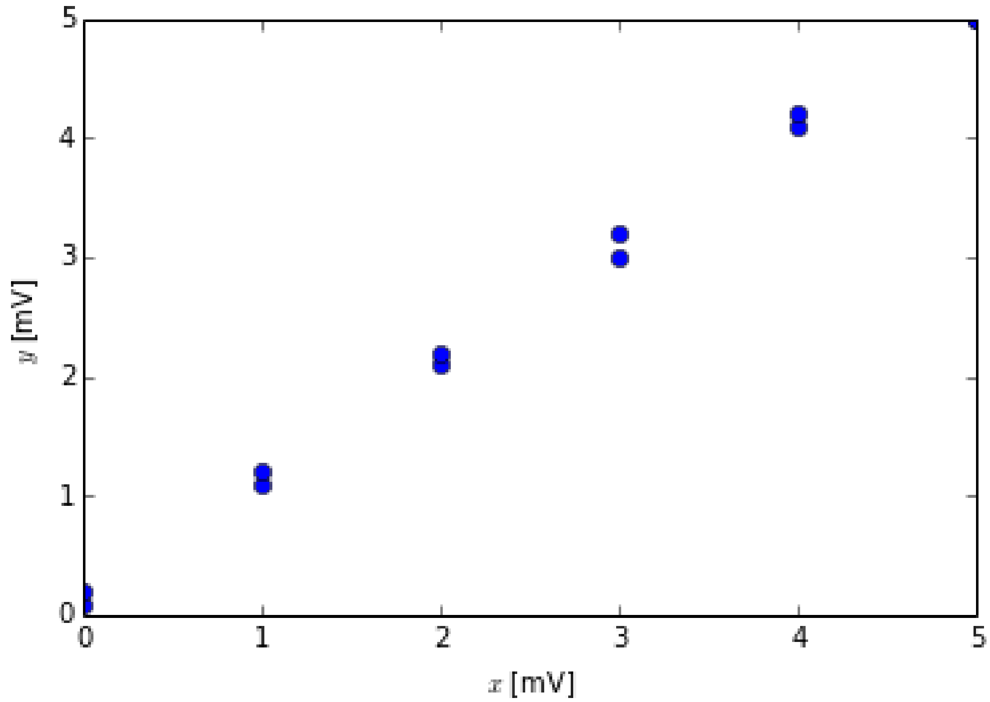
Table 1.6 Voltmeter Calibration Data

Increasing Input [mV]		Decreasing Input [mV]	
X	*Y*	*X*	*Y*
0.0	0.1	5.0	5.0
1.0	1.1	4.0	4.2
2.0	2.1	3.0	3.2
3.0	3.0	2.0	2.2
4.0	4.1	1.0	1.2
5.0	5.0	0.0	0.2

In [37]:
```
x = np.array([0.0, 1.0, 2.0, 3.0, 4.0, 5.0, 5.0, 4.0, 3.0, 2.0, 1.0, 0.0])
y = np.array([0.1, 1.1, 2.1, 3.0, 4.1, 5.0, 5.0, 4.2, 3.2, 2.2, 1.2, 0.2])
```

In [38]:
```
pl.plot(x,y,'o')
pl.xlabel('$x$ [mV]')
pl.ylabel('$y$ [mV]')
```

Out [38]:
```
<matplotlib.text.Text at 0x1088ea150>
```

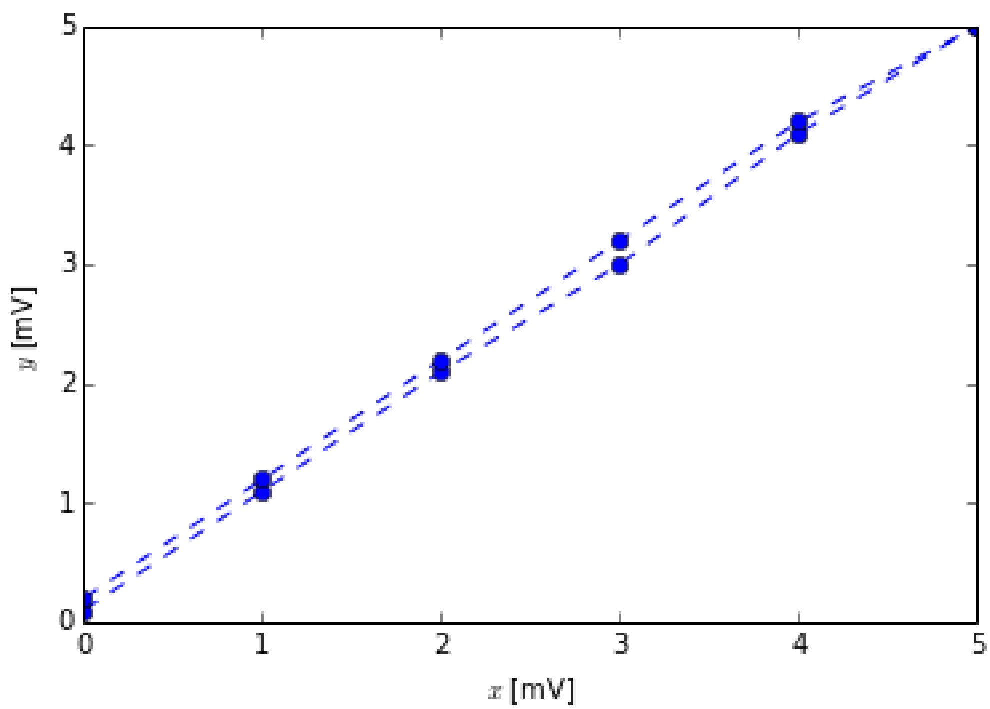


1. We see the error, but we do not know if it is a random or not
2. In order to see the hysteresis, we have to set the plot with the lines connecting points:

In [39]:
```
pl.plot(x,y,'--o')
pl.xlabel('$x$ [mV]')
pl.ylabel('$y$ [mV]')
```

```
<matplotlib.text.Text at 0x108b36e10>
```

Out [39]:



Estimate the hysteresis error:

$e_h = y_{up} - y_{down}$

$e_{h_{max}} = max(|e_h|)$

$e_{h_{max}}\% = 100\% \cdot \frac{e_{h_{max}}}{y_{max} - y_{min}}$

In [40]:
```
e_h = y[:6]-np.flipud(y[6:])
print "e_h =", e_h,"[mV]"
```
```
e_h = [-0.1 -0.1 -0.1 -0.2 -0.1  0. ] [mV]
```

In [41]:
```
e_hmax = np.max(np.abs(e_h))
print "e_hmax= %3.2f %s" % (e_hmax,"[mV]")
```
```
e_hmax= 0.20 [mV]
```

In [42]:
```
e_hmax_p = 100*e_hmax/(np.max(y) - np.min(y))
print "Relative error = %3.2f%s FSO" % (e_hmax_p,"%")
```
```
Relative error = 4.08% FSO
```

1 Sensitivity error example

In [43]:
```
from IPython.core.display import Image
Image(filename='../Lectures/Lecture3/statics/sensitivity_error_example.png')
```

Out [43]:

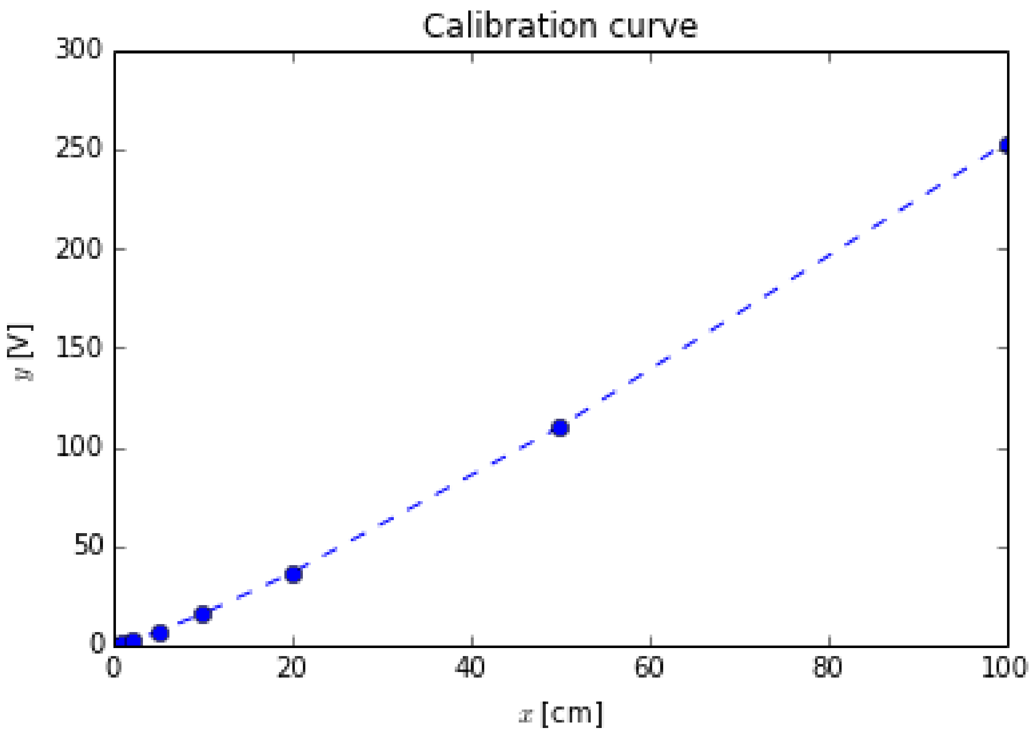
Table 1.5 Calibration Data

X [cm]	Y [V]	X[cm]	Y [V]
0.5	0.4	10.0	15.8
1.0	1.0	20.0	36.4
2.0	2.3	50.0	110.1
5.0	6.9	100.0	253.2

In [44]:
```
x = np.array([0.5, 1.0, 2.0, 5.0, 10.0, 20.0, 50.0, 100.0])
y = np.array([0.4, 1.0, 2.3, 6.9, 15.8, 36.4, 110.1, 253.2])
```

In [45]:
```
pl.plot(x,y,'--o')
pl.xlabel('$x$ [cm]')
pl.ylabel('$y$ [V]')
pl.title('Calibration curve')
```
```
<matplotlib.text.Text at 0x108be78d0>
```

Out [45]:



Sensitivity, K is:

$K_i = \left(\frac{\partial y}{\partial x}\right)_{x_i}$

In [46]:
```
K = np.diff(y)/np.diff(x)
print K
```

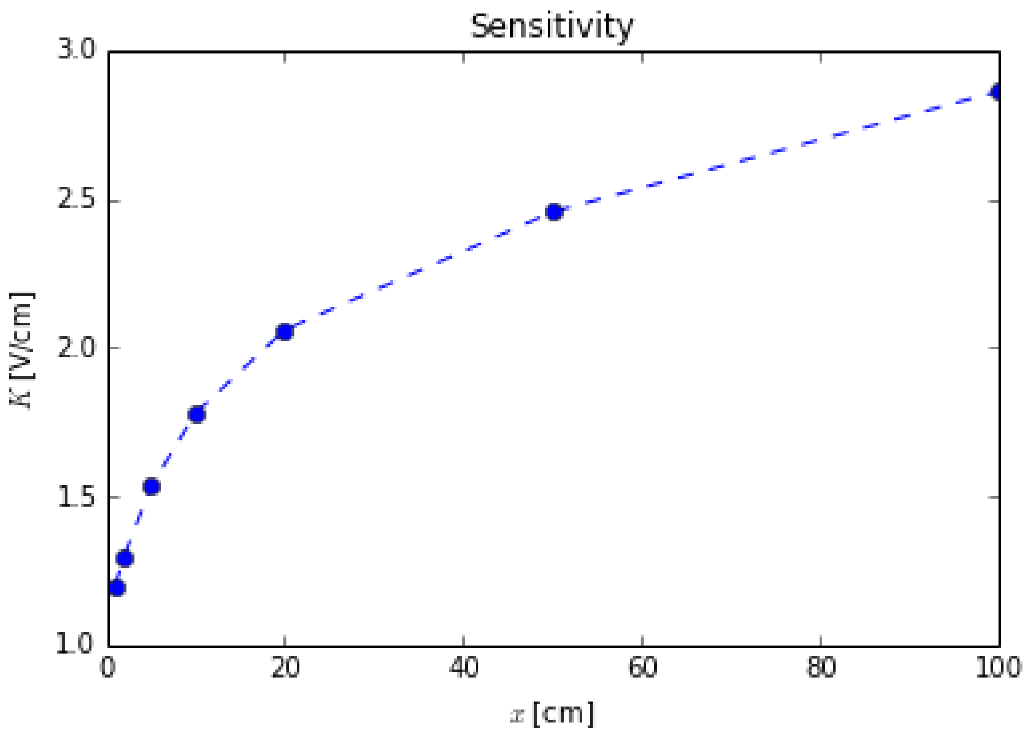
```
[ 1.2         1.3         1.53333333  1.78        2.06
2.45666667
  2.862     ]
```

In [46]:

In [46]:

In [47]:
```
pl.plot(x[1:],K,'--o')
pl.xlabel('$x$ [cm]')
pl.ylabel('$K$ [V/cm]')
pl.title('Sensitivity')
```

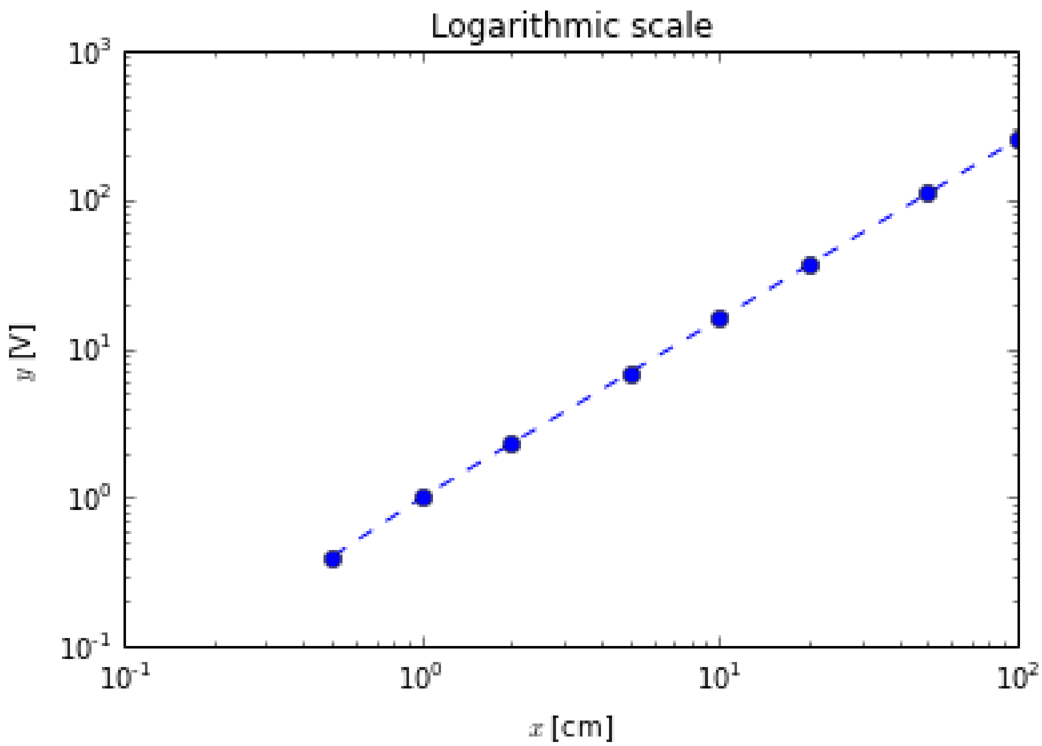
Out [47]:
```
<matplotlib.text.Text at 0x108e99310>
```



Instead of working with non-linear curve of sensitivity we can use the usual trick: the logarithmic scale

In [48]:
```
pl.loglog(x,y,'--o')
pl.xlabel('$x$ [cm]')
pl.ylabel('$y$ [V]')
pl.title('Logarithmic scale')
```

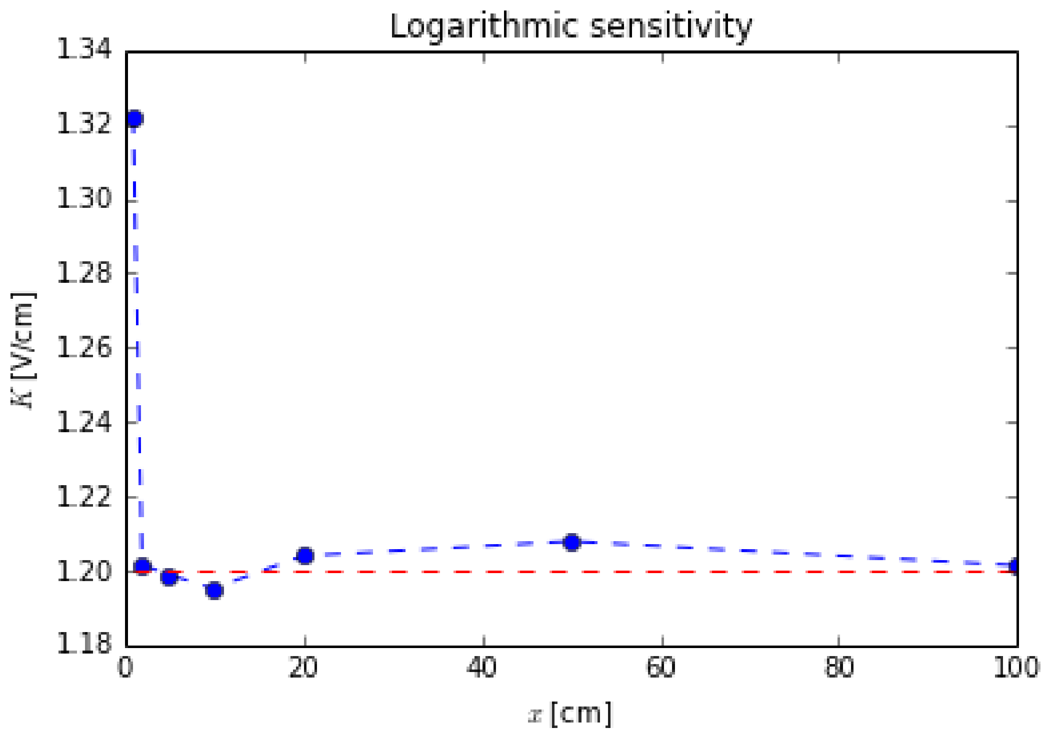
Out [48]:
```
<matplotlib.text.Text at 0x108f59c90>
```



In [49]:
```
logK = np.diff(np.log(y))/np.diff(np.log(x))
print logK
pl.plot(x[1:],logK,'--o')
pl.xlabel('$x$ [cm]')
pl.ylabel('$K$ [V/cm]')
pl.title('Logarithmic sensitivity')
pl.hold(True)
pl.plot([x[1],x[-1]],[1.2,1.2],'r--')
```

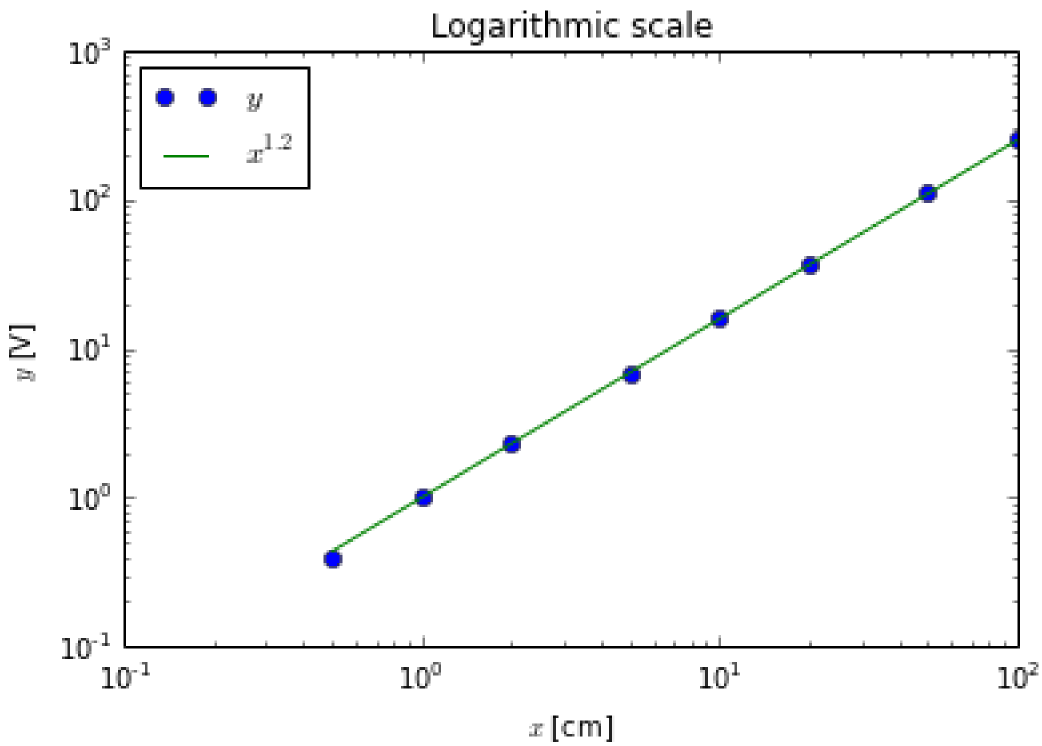
```
[ 1.32192809  1.20163386  1.19897785  1.19525629  1.20401389
1.20793568
  1.20146294]
[<matplotlib.lines.Line2D at 0x108f85690>]
```

Out [49]:



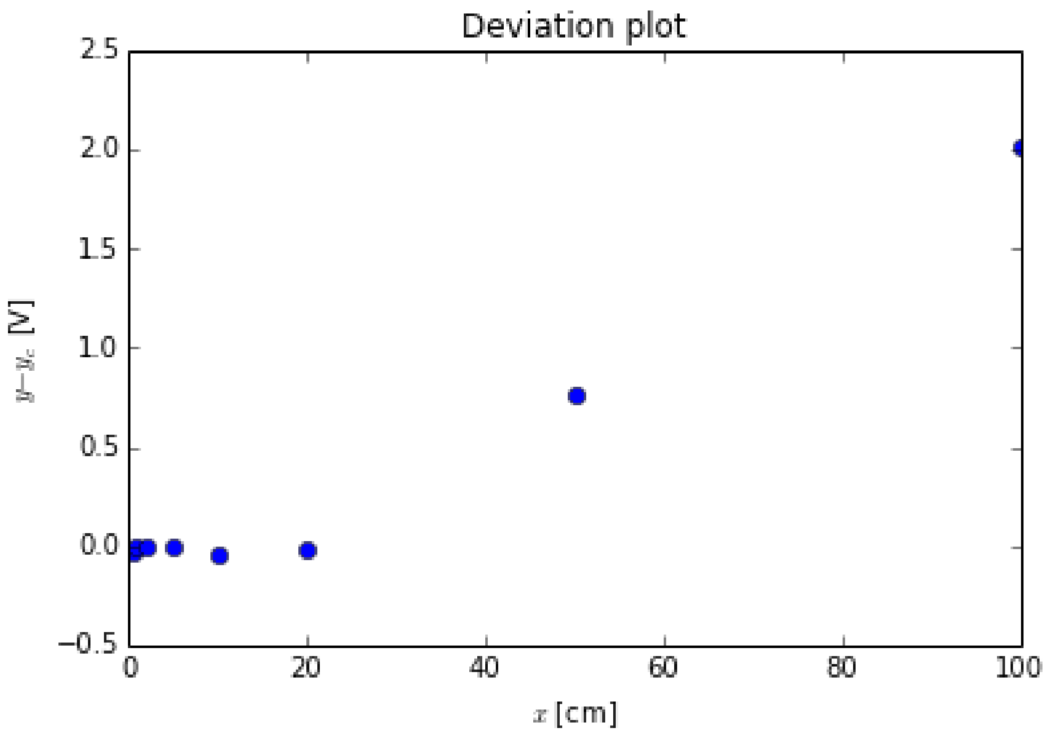
In [50]:
```
pl.loglog(x,y,'o',x,x**(1.2))
pl.xlabel('$x$ [cm]')
pl.ylabel('$y$ [V]')
pl.title('Logarithmic scale')
pl.legend(('$y$','$x^{1.2}$'),loc='best')
```

Out [50]:
```
<matplotlib.legend.Legend at 0x10937f350>
```



In [51]:
```
pl.plot(x,y-x**(1.2),'o')
pl.xlabel('$x$ [cm]')
pl.ylabel('$y - y_c$ [V]')
pl.title('Deviation plot')
# pl.legend(('$y$','$x^{1.2}$'),loc='best')
```

Out [51]:
```
<matplotlib.text.Text at 0x108f808d0>
```



1.1 Regression analysis

Following the recipe of http://www.answermysearches.com/how-to-do-a-simple-linear-regression-in-python/124/

In [52]:
```
from math import sqrt
def linreg(X, Y):
    """
    Summary
        Linear regression of y = ax + b
    Usage
        real, real, real = linreg(list, list)
    Returns coefficients to the regression line "y=ax+b" from x[] and y[], and R^2 Val
    """
    if len(X) != len(Y):  raise ValueError, 'unequal length'
    N = len(X)
    Sx = Sy = Sxx = Syy = Sxy = 0.0
    for x, y in map(None, X, Y):
        Sx = Sx + x
        Sy = Sy + y
        Sxx = Sxx + x*x
        Syy = Syy + y*y
        Sxy = Sxy + x*y
    det = Sxx * N - Sx * Sx
    a, b = (Sxy * N - Sy * Sx)/det, (Sxx * Sy - Sx * Sxy)/det
    meanerror = residual = 0.0
    for x, y in map(None, X, Y):
        meanerror = meanerror + (y - Sy/N)**2
        residual = residual + (y - a * x - b)**2
    RR = 1 - residual/meanerror
    ss = residual / (N-2)
    Var_a, Var_b = ss * N / det, ss * Sxx / det
    #print "y=ax+b"
    #print "N= %d" % N
    #print "a= %g \\pm t_{%d;\\alpha/2} %g" % (a, N-2, sqrt(Var_a))
```

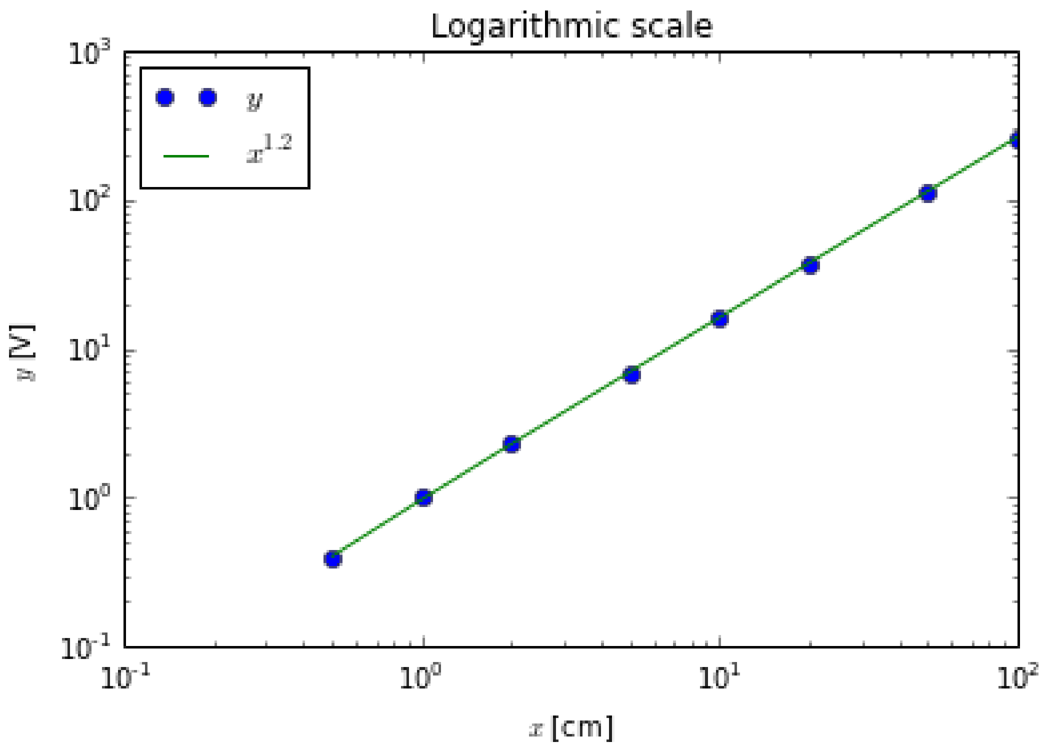
```
    #print "b= %g \\pm t_{%d;\\alpha/2} %g" % (b, N-2, sqrt(Var_b))
    #print "R^2= %g" % RR
    #print "s^2= %g" % ss
    return a, b, RR
print linreg(np.log(x),np.log(y))
```

In [53]: (1.2103157469888082, -0.028846347359456247, 0.99988824734217 92)

In [54]:
```
pl.loglog(x,y,'o',x,x**(1.21)-0.0288)
pl.xlabel('$x$ [cm]')
pl.ylabel('$y$ [V]')
pl.title('Logarithmic scale')
pl.legend(('$y$','$x^{1.2}$'),loc='best')
```

<matplotlib.legend.Legend at 0x109a5cb90>

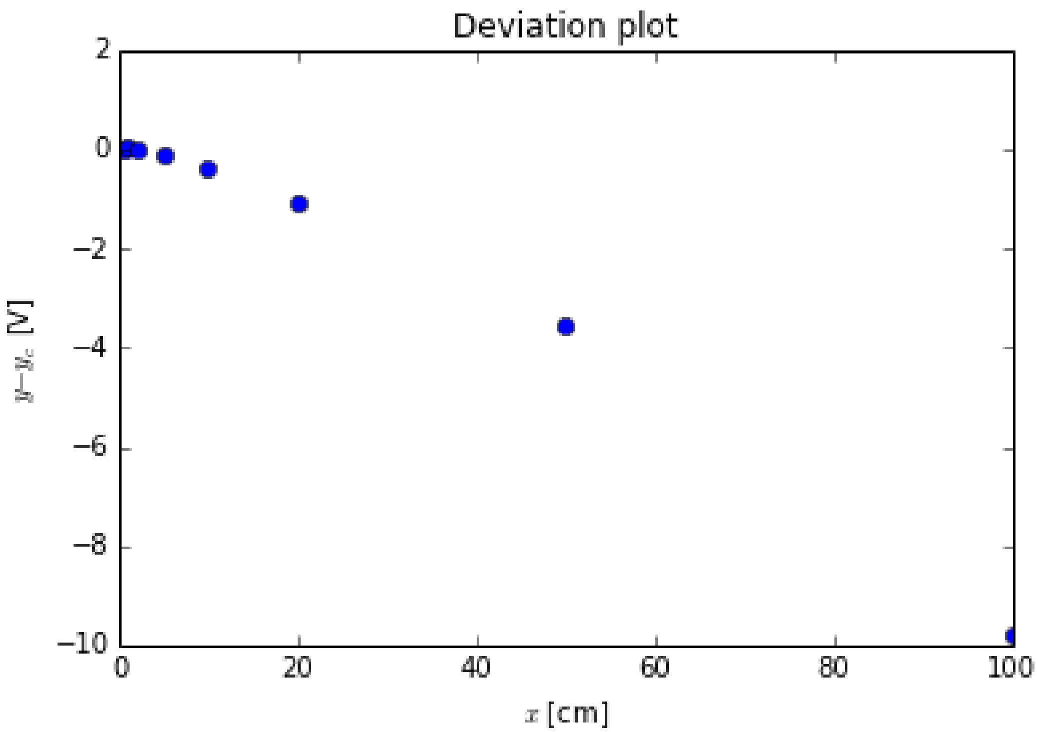
Out [54]:



In [55]:
```
pl.plot(x,y-(x**(1.21)-0.0288),'o')
pl.xlabel('$x$ [cm]')
pl.ylabel('$y - y_c$ [V]')
pl.title('Deviation plot')
# pl.legend(('$y$','$x^{1.2}$'),loc='best')
```

<matplotlib.text.Text at 0x109a41f90>

Out [55]:



In [55]: