
log-log-calibration

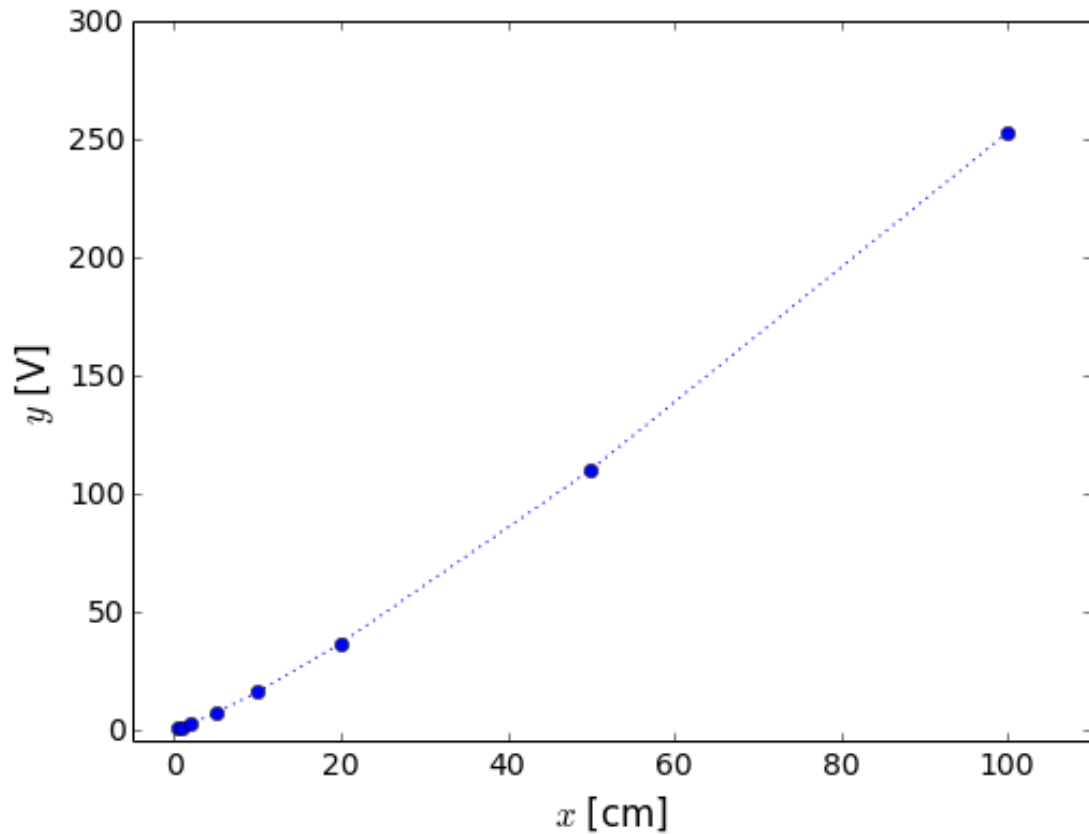
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1 Use of log-log plot in calibraton and regression

```
In [10]: # read the data
import numpy as np
x = np.array([0.5, 1.0, 2.0, 5.0, 10.0, 20.0, 50.0, 100.0]) # cm
y = np.array([0.4, 1.0, 2.3, 6.9, 15.8, 36.4, 110.1, 253.2]) # Volt

mpl.rc_context(rc={'font.size': 14, 'axes.labelsize': 'large', 'figure.figsize': (8,6)})
In [11]: <matplotlib.rc_context at 0x1062d4850>
Out [11]:
In [12]: fig = figure()
plot(x,y,'o:')
xlim(-5,110)
ylim(-5,300)
xlabel('$x$ [cm]')
ylabel('$y$ [V]')
<matplotlib.text.Text at 0x1062cc950>
Out [12]:
```



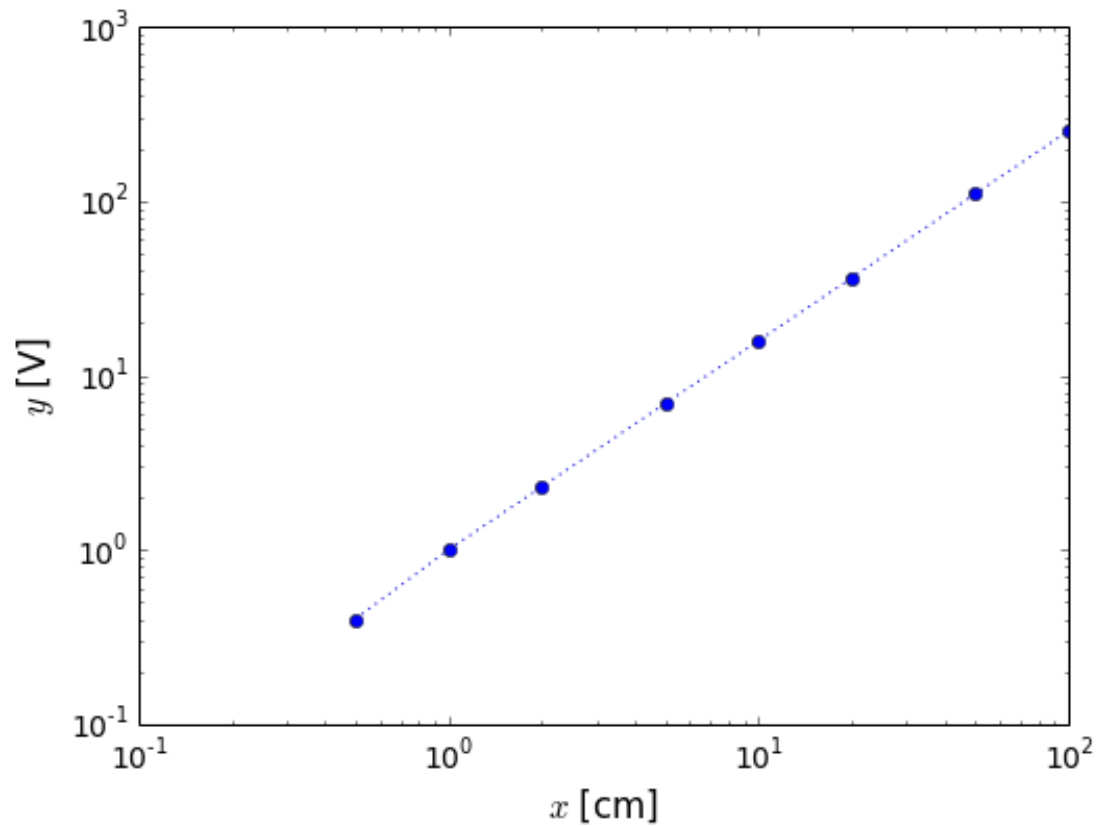
1.1 Conclusion:

1. Obviously the result is non-linear and the best sensitivity we can get is between 20 and 40 cm, but not for small distances
2. If the relationship is a known function and it's a physically relevant one, we could use it to show that distance to voltage should be related as a power law and not really linearly. Let's check the logarithmic scale, since we know that:

$$\log y = \log(bx^m) = \log b + m \log x$$

```
In [13]: fig = figure()
loglog(x,y,'o:')
# xlim(-5,110)
# ylim(-5,300)
xlabel('$x$ [cm]')
ylabel('$y$ [V]')
<matplotlib.text.Text at 0x1062e5e10>
```

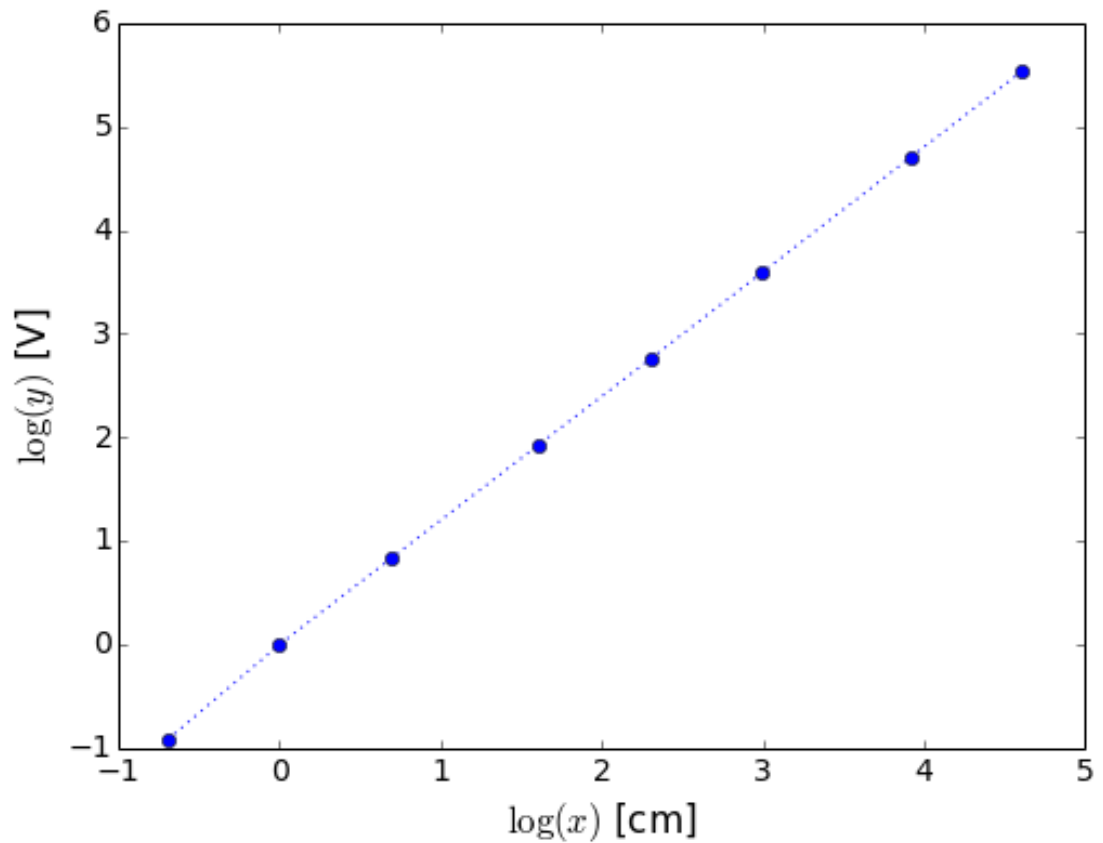
Out [13]:



This result shows that the result is close to linear in the log-log space. Let's do calibration in $\log(x)$ and $\log(y)$. It can be either \log or \log_{10}

```
In [14]: fig = figure()
plot(log(x), log(y), 'o:')
# xlim(-5, 110)
# ylim(-5, 300)
xlabel('$\log (x)$ [cm]')
ylabel('$\log (y)$ [V]')
<matplotlib.text.Text at 0x106e1df10>
```

Out [14]:

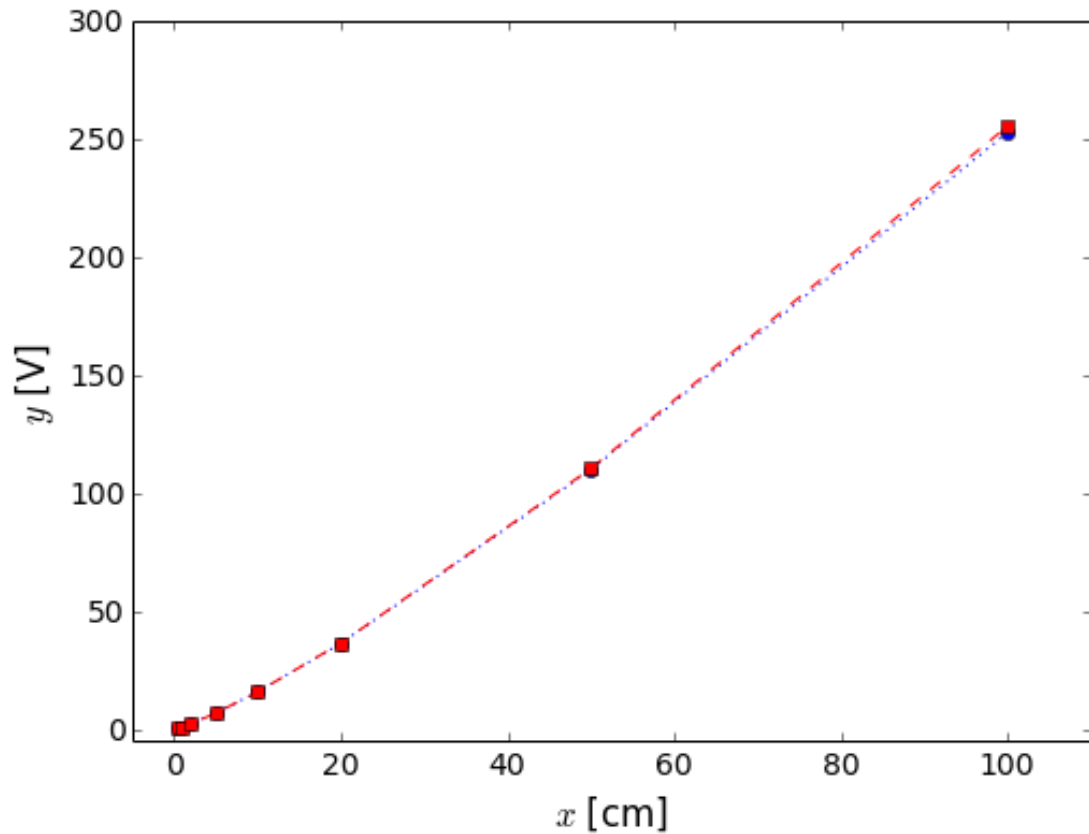


```
In [15]: # linear fit
p = polyfit(log10(x), log10(y), 1)
p
array([ 1.21031575, -0.01252781])
```

```
Out [15]: # let's check the fit:
```

```
In [16]: fig = figure()
yfit = 10**(p[0]*log10(x) + p[1])
plot(x, y, 'o:', x, yfit, 'rs--')
xlim(-5, 110)
ylim(-5, 300)
xlabel('$x$ [cm]')
ylabel('$y$ [V]')
<matplotlib.text.Text at 0x106e4bf90>
```

```
Out [16]:
```



1.2 how do we measure?

1. measure the y [V], take a $\log_{10}(y)$, estimate the $\log_{10}(x)$ using the calibration curve:

$$\log_{10}(x) = (\log_{10}(y) + 0.0125)/1.21$$

2. Note that we can use this function in the full input scale and full output scale, take:

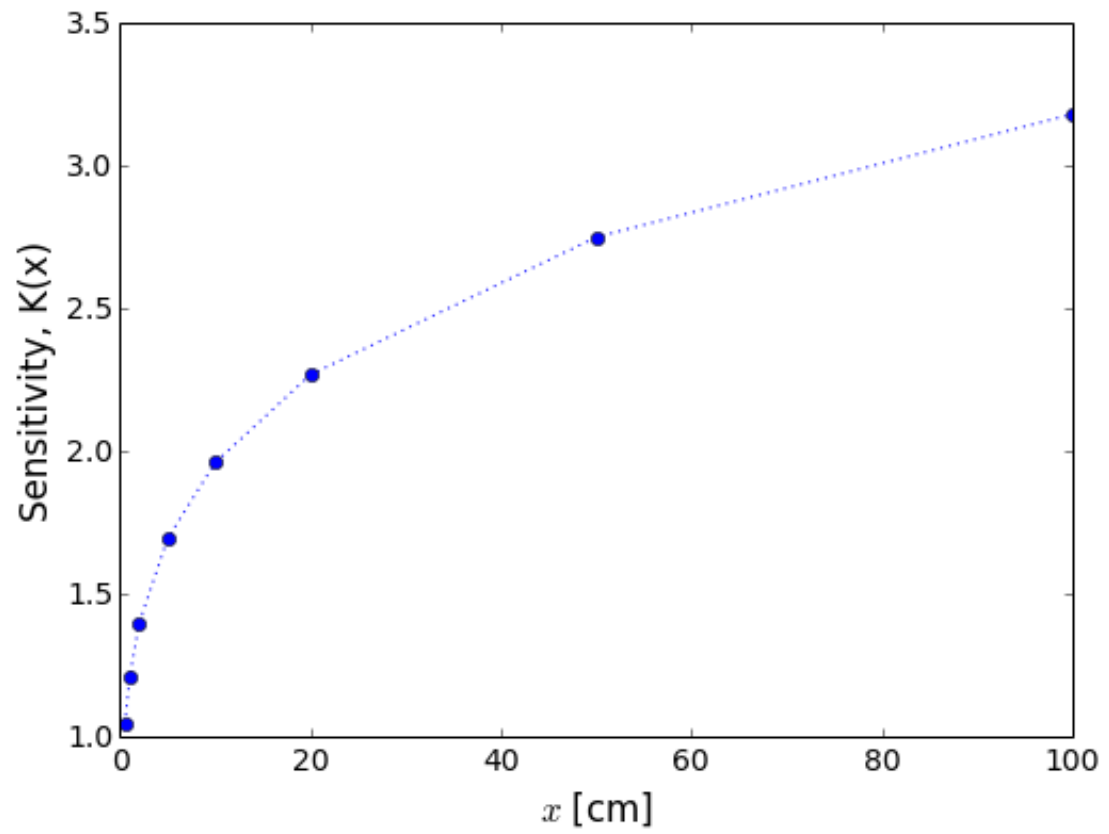
$$10^{\log_{10}(x)}$$

3. The sensitivity of this function is:

$$K(x) = dy/dx = d/dx(x^{1.2}) = 1.2x^{0.2}$$

```
In [22]: fig = figure()
K = 1.21*x**0.21
plot(x,K,'o:')
xlabel('$x$ [cm]')
ylabel('Sensitivity, K(x) ')
<matplotlib.text.Text at 0x106eeb750>
```

Out [22]:



In []: