module 1.

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Vector Space.

Linear Dependent and linearly independent.

A set of vectors $\{V_1, V_2, V_3 \ldots V_n\}$ is said to be L·D or L·I if there are scalars $c_1, c_2, c_3 \ldots c_n$ such that

$$c_1V_1 + c_2V_2 + c_3V_3 \cdots c_nV_n = \vec{0} \text{ (null vector)}$$

1) if all c_i or $c_1, c_2 \ldots c_n$ are zero $\longrightarrow$ L·I.

2) if atleast one c_i is not zero, then $\longrightarrow$ LD.

let $V_1 = (x_1, y_1, z_1)$ $V_2(x_2, y_2, z_2)$ $V_3(x_3, y_3, z_3)$

then $c_1V_1 + c_2V_3 + c_3V_3 = \vec{0}$

$$c_1(x_1, y_1, z_1) + c_2(x_2\ y_2\ z_2) + c_3(x_3\ y_3\ z_3) = (0,0,0).$$

$$(c_1x_1 + c_2x_2 + c_3x_3,\ c_1y_1 + c_2y_2 + c_3y_3,\ c_1z_1 + c_2z_2 + c_3z_3) = (0,0,0)$$

$$\Rightarrow c_1x_1 + c_2x_2 + c_3x_3 = \vec{0}.$$

$$c_1y_1 + c_2y_2 + c_3y_3 = \vec{0}$$

$$c_1z_1 + c_2z_2 + c_3z_3 = \vec{0}.$$

$$\underbrace{\begin{bmatrix} x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \\ z_1 & z_2 & z_3 \end{bmatrix}}_{\Downarrow\ \text{matrix } A} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}.$$

then $\begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} A^{-1}.$

1) If A^{-1} exists then $|A| \neq 0$. then vectors are LI
if $|A| = 0$ then vectors are LD.] only for square matrix.

Q) show that set $S = \{(1,2,3), (3,-2,1), (1,-6,-5)\}$ is L·D.

let $c_1\ c_2\ c_3$ be scalar.

$$c_1(1,2,3) + c_2(3,-2,1) + c_3(1,-6,-5) = (0,0,0).$$

$$A \begin{bmatrix} 1 & 2 & 3 \\ 3 & -2 & 1 \\ 1 & -6 & -5 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}.$$

for LD $|A| = 0$. $|A| = 0 \Rightarrow$ L·D

9) $x_1(1, 2, -3, 4)$, $x_2(3, -1, 2, 1)$, $x_3(1, -5, 8, -7)$ are LD.

$$c_1(1, 2, -3, 4) + c_2(3, -1, 2, 1) + c_3(1, -5, 8, -7) = 0, 0, 0.$$

$$\begin{bmatrix} 1 & 2 & -3 & 4 \\ 3 & -1 & 2 & 1 \\ 1 & -5 & 8 & -7 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ c_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

here $A \neq$ square matrix.

(or).

$$\underset{4\times3}{\begin{bmatrix} 1 & 3 & 1 \\ 2 & -1 & -5 \\ -3 & 2 & 8 \\ 4 & 1 & -7 \end{bmatrix}} \underset{3\times1}{\begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix}} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}.$$

$$(4\times1) \begin{bmatrix} c_1 + 3c_2 + c_3 \\ 2c_1 - c_2 - 5c_3 \\ -3c_1 + 2c_2 + 8c_3 \\ 4c_1 + c_2 - 7c_3 \end{bmatrix}. = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}.$$

$$c_1 + 3c_2 + c_3 = 0$$

$$c_1 = -c_3 - 3c_2.$$

$$2(-c_3 - 3c_2) - c_2 - 5c_3 = 0.$$

$$-7c_3 - 7c_2 = 0.$$

$$c_3 + c_2 = 0$$

$$\Rightarrow c_3 = -c_2.$$

$$4(+c_2 - 3c_2) + c_2 + 7c_2 = 0$$

$$4c_2 - 12c_2 + c_2 + 7c_2 = 0.$$

$$12c_2 - 12c_2 = 0.$$

$$c_2 = 0.$$

$$c_3 = 0.$$

$$c_1 = -c_3 - 3c_2$$

$$c_1 = 0.$$

Linearly independent

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$$\begin{bmatrix} 1 & 3 & 1 \\ 2 & -1 & -5 \\ -3 & 2 & 8 \\ 4 & 1 & -7 \end{bmatrix} \quad \begin{matrix} R_2 = R_2 - 2R_1. \\ R_3 = R_3 + 3R_1. \\ R_4 = R_4 - 4R_1 \end{matrix} = \begin{bmatrix} 1 & 3 & 1 \\ 0 & -7 & -7 \\ 0 & 11 & 11 \\ 0 & -11 & -11 \end{bmatrix}.$$

$= R_2 = R_2/7 \quad R_3 = R_3/11 \quad R_4 = R_4/-11$

$$\begin{bmatrix} 1 & 3 & 1 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix} \sim \begin{bmatrix} 1 & 3 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$\Rightarrow$ Rank(number of non zero rows) = 2.

rank can be within $(4, 3) = 3$.
as $2 < 3$ so they are LD.

WRITE COLUMN WISE

Q) $C_1(1, -2, k) + C_2(2, -1, 5) + C_3(3, -5, 7k)$ are LD or LI

$$\begin{bmatrix} 1 & -2 & k \\ 2 & -1 & 5 \\ 3 & -5 & 7k \end{bmatrix} \begin{bmatrix} C_1 \\ C_2 \\ C_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \text{ but } \begin{bmatrix} 1 & 2 & 3 \\ -2 & -1 & -5 \\ k & 5 & 7k \end{bmatrix}.$$

square matrix

$$|A| = 1(-7k + 25) + 2(14k - 15) + k(-10 + 3)$$

$$= -7k + 25 + 28k - 30 - 10k + 3k \cancel{= 0}.$$

$$= 14k - 5$$

$LD \Rightarrow |A| = 0$ $\qquad$ $LI \Rightarrow |A| \neq 0$

$\Rightarrow 14k - 5 = 0$ $\qquad$ $14k - 5 \neq 0$

$k = 5/14.$ $\qquad$ $k \neq 5/14.$

same as Rank!

Dimension: The maximum number of LI vectors in vector space V is called the dimension of V and is denoted as dim V.

Basis: A LI set in vector space V consisting of a maximum possible number of vectors in V is called as a basis for V. Thus any largest possible set of independent vectors in V terms. forms basis for V. It implies that, if we add one / more vector to set - the set will be LD.

[In row echelon form - traverse each row - the first non zero element of each row -- LI element]

Linear span: Any vector V is said to be linear span to vectors $\{V_1, V_2, V_3 \dots V_n\}$ if we can write V as linear combination of $V_1, V_2 \dots V_n$ ie.

$$V = V_1 C_1 + C_2 V_2 \dots + C_n V_n$$

Q) Verify that the following set of vectors are basis of R^3

$$S = \{(1,0,0)\ (2,2,0)\ (3,3,3)\}$$

$R^3 \Rightarrow$ 3 dimensional vector space.

1) Prove above vectors are LI.

row/column wise determinant is true

but usually write column vector

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 2 & 3 \\ 0 & 0 & 3 \end{bmatrix} \qquad 1(6) - 2(0) + 3(0) = |A|$$

$$|A| = 6.$$

$$|A| \neq 0 \Rightarrow LI.$$

Let any vector in R^3 space be (x, y, z).

Now

$$(x, y, z) = c_1(1,0,0) + c_2(2,2,0) + c_3(3,3,3)$$

$$x, y, z = (c_1 + 2c_2 + 3c_3), (2c_2 + 3c_3), 3c_3.$$

$$c_1 + 2c_2 + 3c_3 = x. \qquad 3c_3 = z.$$

$$2c_2 + 3c_3 = y. \qquad \underline{c_3 = z/3}$$

$$2c_2 + z = y$$

$$c_2 = \frac{y - z}{2} \qquad c_1 + y - z + z = x.$$

$$\underline{c_1 = x - y.}$$

$$(x, y, z) = (x-y)(1,0,0) + \left(\frac{y-z}{2}\right)(2,2,0) + \frac{z}{3}(3,3,3) \text{ --- } (1)$$

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∴ Using Eq ① – any vector in 3d space can be found – hence, it is called a basis of R^3 vector space.

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Q) Determine whether set $S = \{(1,2,3)\ (3,1,0)\ (-2,1,3)\}$ is basis of $V_3(R)$. In case S is not basis, determine the dimension & the basis of the subspace spanned by S.

1) LI / not
$$\begin{bmatrix} 1 & 3 & -2 \\ 2 & 1 & 1 \\ 3 & 0 & 3 \end{bmatrix} \quad 1(3) - 3(6-3) - 2(-3)$$
$$3 - 9 + 6 = -9 + 9 = 0$$

$|A| = 0$

⇒ Linearly dependent vectors.

⇒ not basis of R_3.

2) DO / Reduce to Row echelon.

$$\begin{bmatrix} 1 & 3 & -2 \\ 0 & -5 & 5 \\ 0 & -9 & 9 \end{bmatrix} = \begin{bmatrix} 1 & 3 & -2 \\ 0 & -1 & 1 \\ 0 & -9 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 3 & -2 \\ 0 & -1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

dimension = number of non zero rows = 2.

⇒ 2 vectors are linearly independent

pivot element

$$\begin{bmatrix} \boxed{1} & 3 & -2 \\ 0 & \boxed{-1} & 1 \\ 0 & 0 & 0 \end{bmatrix} \Rightarrow (1\ 3\ -2) \text{ and } (0, -1, 1)$$

are linearly idependent

/or $(1, 3, -2)$ and $(2, 1, 1)$

⇒ $(1, 3, -2)\ (2, 1, 1)$ – form basis of vector $(3\,0\,3)$.

Vector space :

F – field V = vector space $\alpha, \beta \rightarrow$ vectors of vector space.

Addition can be defined for sub.

1) Closure laws.

$\left.\begin{matrix}\alpha \in V \\ \beta \in V\end{matrix}\right]$ then $\alpha + \beta \in V$

2) Associative.

$(\alpha+\beta)+\gamma = \alpha+(\beta+\gamma)$

3) Identity

$\alpha + \bar{0} = \bar{0} + \alpha = \alpha.$

4) commutative

$\alpha + \beta = \beta + \alpha.$

5) Inverse

$\alpha + (-\alpha) = (-\alpha) + \alpha = \bar{0}$

6) $a(\alpha+\beta) = a\alpha + a\beta.$

7) $(a+b)\alpha = a\alpha + b\alpha$

8) $\alpha \cdot 1 = 1 \cdot \alpha = \alpha.$

9) $(ab)\alpha = a(b\alpha).$

If vector satisfies all these 9 laws – then vector belongs to vector space (V)

ie vector $\in V$.

Q) Prove all 2x2 matrix belongs to vector space.

$\alpha = [a_{ij}]_{2\times2} \quad \beta = [b_{ij}]_{2\times2}.$

closure

$\alpha + \beta \in V.$

$[a_{ij}]_{2\times2} + [\beta_{ij}]_{2\times2} \in V$ if $\alpha \in V$ and $\beta \in V$.

Commutative

$[a_{ij}]_{2\times2} + [b_{ij}]_{2\times2} = [b_{ij}]_{2\times2} + [a_{ij}]_{2\times2}.$

Associative

$[[a_{ij}]_{2\times2} + [b_{ij}]_{2\times2}] + [c_{ij}]_{2\times2} = [a_{ij}] + [[b_{ij}] + [c_{ij}]].$

Identity

$[a_{ij}]_{2\times2} + [\text{null matrix}]_{2\times2} = [a_{ij}]_{2\times2}.$

Inverse

$[a_{ij}] + [-a_{ij}] = [\text{null}]_{2\times2}.$

- $c(\alpha+\beta) = c[a_{ij} + b_{ij}] = c[a_{ij}] + c[b_{ij}].$
- $(c+d)(a_{ij}) = c[a_{ij}] + d[a_{ij}].$
- $a_{ij} \cdot 1 = a_{ij}$
- $(cd)[a_{ij}] = c[d(a_{ij})].$

Q) Singular matrix is not a part of linear space [2×2].
Singular => |Matrix| = 0.

Let A be $\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$ B $\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$.

A + B ∈ non singular but should be singular.

$A+B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ $|A+B| = 1$
$\neq 0 \Rightarrow A+B \notin V$.

Q) Non singular matrix (n×n) - not part of vector space // find answer.

Subspace.
W is a subspace of V if
1) Closure law ✓ [$\alpha \in W$ $\beta \in W \Rightarrow \alpha+\beta \in W$].
2) $c \cdot \alpha \in W$. [for $c \in F$ $\alpha \in W$]

Q) $W = \{x, 0, 0\}$ $x \in R$.
• $\alpha = (x_1, 0, 0)$ $\beta = (x_2, 0, 0)$ for $\alpha, \beta \in W$.
$\alpha+\beta = (x_1+x_2, 0, 0) \in W$.
• $\left.\begin{matrix} c \in R. \\ \alpha \in W \end{matrix}\right]$ $c\alpha = c(x_1, 0, 0) = (cx_1, 0, 0) \in W$.
$\Rightarrow$ W is a subspace of V(R).

Q) $W = \underbrace{(x, y, z)}_{\text{general form}} / (x - 3y + 4z = 0)$ is subspace.
substitute and prove for this eqn.

a) $\alpha = (x_1, y_1, z_1) \simeq (x_1 - 3y_1 + 4z_1) = 0$
$\beta = (x_2, y_2, z_2)$. $(x_2 - 3y_2 + 4z_2) = 0$.
$\alpha+\beta = (x_1+x_2, y_1+y_2, z_1+z_2)$.
$= (x_1+x_2, -3(y_1+y_2) + 4(z_1+z_2))$
$= (x_1 - 3y_1 + 4z_1) + (x_2 - 3y_2 + 4z_2)$
$= 0 + 0 = 0$
$\alpha+\beta \in W$.

b) $c\alpha = c(x_1, y_1, z_1) = c(x_1 - 3y_1 + 4z_1)$
$= c(0)$

or $c(x_1, y_1, z_1)$
$= (cx_1, cy_1, cz_1)$
$= cx_1 - 3cy_1 + 4cz_1$
$= c(x_1 - 3y_1 + 4z_1)$
$= c(0) = 0$.

Consistent system $Rank(A) = Rank(A:B)$ – then solution present

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Q) span (3, 5, 2)

$\Rightarrow (3,5,2) = c_1(1,1,0) + c_2(2,3,0) + c_3(0,0,1)$

$(3,5,2) = (c_1 + 2c_2 + 0,\ c_1 + 3c_2,\ c_3)$

$\Rightarrow c_3 = 2.$

$$\begin{aligned} c_1 + 3c_2 &= 5 \\ c_1 + 2c_2 &= 3 \\ \hline c_2 &= 2 \end{aligned} \qquad \begin{aligned} c_1 &= -1 \\ c_2 &= 2 \\ c_3 &= 2. \end{aligned}$$

(or)

$$\begin{bmatrix} 1 & 2 & 0 \\ 1 & 3 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ 5 \\ 2 \end{bmatrix} = \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix}.$$ = the kind solution.

Q) $(3,-7,6) = c_1(1,-3,2) + c_2(2,4,1) + c_3(1,1,1).$

$$\Rightarrow \left.\begin{aligned} c_1 + 2c_2 + c_3 &= 3. \\ -3c_1 + 4c_2 + c_3 &= -7. \\ 2c_1 + c_2 + c_3 &= 6. \end{aligned}\right] \qquad \begin{aligned} c_1 &= 2 \\ c_2 &= -1 \\ c_3 &= 3. \end{aligned}$$

[if on calci ⇒ can't be solved ⇒ say infinite solution (or no solution) ⇒ inconsistent]

$\Rightarrow (3,-7,6) = 2(1,-3,2) + (-1)(2,4,1) + 3(1,1,1).$

Q) $(2,-5,3)$ is not span of $(1,-3,2)$ $(2,-4,-1)$ $(1,-5,7)$

$$\Rightarrow \left.\begin{aligned} c_1 + 2c_2 + c_3 &= 2. \\ -3c_1 - 4c_2 - 5c_3 &= -5. \\ 2c_1 + c_2 + 7c_3 &= 3. \end{aligned}\right]$$ calci – says no solution ⇒ inconsistant or prove

$$\Rightarrow \left[\begin{array}{ccc:c} 1 & 2 & 1 & 2 \\ -3 & -4 & -5 & -5. \\ 2 & 1 & 7 & 3 \end{array}\right]$$ → Bring to echelon and prove. $Rank(A) \neq Rank(A:B)$

linear dependence $\det|vector| = 0$ – atleast c not 0.

linear independent $\det|vector| \neq 0$ $c_1 = c_2 \dots = c_n = 0.$

LD also if $R(Vector) <$ given number of vectors.

Dimension = Rank.

Basis – 1) LI 2) Span.

II) If not LI → then basis of some other space.

that space given by rows with non zero elements [in echelon].

Q) find the basis & dimension of subspace spanned by
$(2,4,2)$ $(1,-1,0)$ $(1,2,1)$ $(0,3,1)$.

$$\begin{bmatrix} 2 & 4 & 2 \\ 1 & -1 & 0 \\ 1 & 2 & 1 \\ 0 & 3 & 1 \end{bmatrix} \text{ or } \begin{bmatrix} 2 & 1 & 1 & 0 \\ 4 & -1 & 2 & 3 \\ 2 & 0 & 1 & 1 \end{bmatrix}.$$

$$R_1 = R_1/2 \begin{bmatrix} 1 & 2 & 1 \\ 1 & -1 & 0 \\ 1 & 2 & 1 \\ 0 & 3 & 1 \end{bmatrix} \simeq \begin{bmatrix} 1 & 2 & 1 \\ 0 & -3 & -1 \\ 0 & 0 & 0 \\ 0 & 3 & 1 \end{bmatrix} \simeq \begin{bmatrix} \boxed{1 \quad 2 \quad 1} \\ \boxed{0 \quad -3 \quad -1} \\ 0 \quad 0 \quad 0 \\ 0 \quad 0 \quad 0 \end{bmatrix}.$$

$\Rightarrow$ dimension $= 2$. $\because$ {because Rank $= 2$}.

basis $= \{(2,4,2)\ (1,-1,0)\}$.

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Linear Transformation :

U, V are vector spaces. over the same field F.
T : V → U is called linear mapping / linear transformation, is satisfied if .

a) $T(V+U) = T(V) + T(U)$

b) $T(CV) = CT(V)$ [for $C \in F$].

Transforming one matrix of m dimension to another matrix of n dimension

Q) $F(x, y) = xy$. Prove the transformation : Do it as so ↓.

~~$\alpha = x_1y_1$ $\beta = x_2y_2$.~~

~~$F(\alpha+\beta) = F(x_1y_1 + x_2y_2)$.~~

~~$= F(x_1y_1) + F(x_2y_2)$~~

~~$= F(\alpha) + F(\beta)$.~~

~~$F(C\alpha) = F(Cx_1y_1)$~~

~~$= CF(x_1y_1) = CF(\alpha)$.~~

$\alpha = (x_1, y_1)$ $\beta = (x_2, y_2)$

$\alpha + \beta = x_1 + y_1, x_2 + y_2$

$F(\alpha+\beta) =$

$(x_1+y_1)(x_2+y_2)$

$x_1x_2 + x_1y_2 + x_2y_1 + y_1y_2$.

$\neq F(\alpha) + F(\beta)$.

so the above mapping is not a linear transformation

Q) If T is mapping from $V_3(R)$ as $T(x, y, z) = (x+y, y+z)$ show that T is L.T.

$T(\alpha) = (x_1+y_1, y_1+z_1)$.

$T(\beta) = (x_2+y_2, y_2+z_2)$.

$\alpha = x_1, y_1, z_1$

$\beta = x_2, y_2, z_2$

$$T(\alpha+\beta) = T(x_1+x_2, y_1+y_2, z_1+z_2).$$
$$= [x_1+x_2+y_1+y_2, y_1+y_2+z_1+z_2].$$
$$= [(x_1+y_1)+(x_2+y_2), (y_1+z_1)+(y_2+z_2)]$$
$$= [(x_1+y_1), (y_1+z_1)] + [(x_2+y_2), (y_2+z_2)]$$
$$= T(\alpha) + T(\beta).$$

$T(C\alpha)$.

$C\alpha = Cx_1, Cy_1, Cz_1$.

$$T(C\alpha) = [Cx_1 + Cy_1, Cy_1 + Cz_1].$$
$$= [C(x_1+y_1), C(y_1+z_1)]$$
$$= CT(\alpha).$$

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Q) $F(x,y) = (x+y, x)$ is linear.

$\alpha = x_1, y_1 \Rightarrow F(\alpha) = (x_1+y_1, x_1)$.

$\beta = x_2, y_2 \Rightarrow F(\beta) = (x_2+y_2, x_2)$.

$\Rightarrow F(\alpha+\beta) = F(x_1+x_2, y_1+y_2)$

$= F(x_1+y_1+x_2+y_2, x_1+x_2)$.

$= F((x_1+y_1)+(x_2+y_2), x_1+x_2)$.

$= F((x_1+y_1), x_1) + F((x_2+y_2), x_2)$

$= F(\alpha) + F(\beta)$.

$\Rightarrow F(c\alpha) = F(cx_1, cy_1)$

$= F(cx_1+cy_1 \cancel{*}, cx_1)$

$= Fc(x_1+y_1, x_1) = cF(\alpha)$.

Q) 1) $F(x,y) = (xy, x)$.

2) $F(x,y) = (x+3, 2y, x+y)$.

3) $F(x,y,z) = (|x|, y+z)$.

Matrix of linear Transformation.

let V be the vector space of dimension m and n respectively over the same field F

$\beta_1 = \{\alpha_1, \alpha_2 \ldots \alpha_n\}$, $\beta_2 = \{\beta_1, \beta_2 \ldots \beta_n\}$ β_1, β_2 - are ordered basis of V and W.

$T: V \to W$ be a LT defined as

$T(\alpha_1) = a_{11}\beta_1 + a_{21}\beta_2 + a_{31}\beta_3 \ldots a_{n1}\beta_n$.

$T(\alpha_2) = a_{12}\beta_1 + a_{22}\beta_2 \ldots a_{n2}\beta_n$.

$T(\alpha_m) = a_{1m}\beta_1 + a_{2m}\beta_2 \ldots a_{nm}\beta_n$

The transpose of..

$$T_T = \begin{bmatrix} a_{11} & a_{12} & \ldots & a_{1m} \\ a_{n1} & a_{n2} & & a_{nm} \end{bmatrix}. \quad \text{or } A_T B_n = T(\alpha_m).$$

Q) $T: R^2 \to R^3$. $T\begin{bmatrix} 3 \\ 2 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$. and $T\begin{bmatrix} 4 \\ 3 \end{bmatrix} = \begin{bmatrix} 0 \\ -5 \\ 1 \end{bmatrix}$.

Transformation $= 3 \times 2$

$$A = \begin{bmatrix} a & b \\ c & d \\ e & f \end{bmatrix} \quad \Rightarrow \underset{3\times 2}{A} \odot \underset{\times 2}{B_n} = \underset{\times 3}{T(\alpha_m)}.$$

$$\Rightarrow A \begin{bmatrix} a & b \\ c & d \\ e & f \end{bmatrix} \begin{bmatrix} 3 \\ 2 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}.$$

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$\Rightarrow 3a+2b=1.$
$3c+2d=2$
$3e+3f=3.$

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case ②. $\begin{bmatrix} a & b \\ c & d \\ e & f \end{bmatrix} \begin{bmatrix} 4 \\ 3 \end{bmatrix} = \begin{bmatrix} 0 \\ -5 \\ 1 \end{bmatrix}.$ solve 6 eqs. find a,b,c,d,e,f. substitute in transformation matrix.

$4a+3b=0.$
$4c+3d=-5.$
$4e+3f=1.$

Standard basis = identity matrix.

Q) matrix of LT $-$ $T(V_2 R) \rightarrow V_3(R)$ where

$T(x,y) = (x+y, y, 3x-y)$

R^2 standard for $2\times 2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

R^3 standard $= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$

$T(1,0) = (1, 0, 3).$
$T(0,1) = (1, 1, -1)$

$(1 0 3)$ - spans R^3 $(1,1,-1)$ also spans R^3:

$\Rightarrow (1 0 3) = c_1(1 0 0) + c_2(0 1 0) + c_3(0 0 1).$

$\Rightarrow \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 3 \end{bmatrix}.$ $\Rightarrow c_1 = 0 \quad c_2 = 0 \quad c_3 = 3.$

$\Rightarrow (1, 1, -1) = c_1(100) + c_2(010) + c_3(001).$

$\Rightarrow c_1 = 1 ; c_2 = 1 ; c_3 = -1.$

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Q) $\beta_1 = \{(1,1),(3,1)\}$ and $\beta_2 = \{(1,1,1),(1\,1\,0),(1\,0\,0)\}$.

$V_2(R) \longrightarrow V_3(R)$.

$T(x,y) = (x+y,\ y,\ 3x-y)$

$T(1,1) = (2,1,2)$.

$T(3,1) = (4,3,8)$.

$(2,1,2) = c_1(1,1,1) + c_2(1\,1\,0) + c_3(1\,0\,0)$

$c_1 + c_2 + c_3 = 2$.

$c_1 + c_2 = 1$

$c_1 = 2$.

$c_2 = 1-2 = -1$.

$\cancel{2} - 1 + c_3 = \cancel{2}$

$\Rightarrow c_3 = 1$.

column vector I

$$\begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}.$$

$(4\,1\,8) = c_1(1\,1\,1) + c_2(1\,1\,0) + c_3(1\,0\,0)$.

$c_1 + c_2 + c_3 = 4$

$c_1 + c_2 = 1$

$c_1 = 8$.

$c_2 = 1-8 = -7$.

$8 - 7 + c_3 = 4$.

$c_3 = 4-1 \Rightarrow c_3 = 3$.

column vector II

$$\begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix} = \begin{bmatrix} 8 \\ -7 \\ 3 \end{bmatrix}.$$

Transformation

$$\begin{bmatrix} c_{1\cdot1} & c_{1\cdot2} \\ c_{2\cdot1} & c_{2\cdot2} \\ c_{3\cdot1} & c_{3\cdot2} \end{bmatrix} = \begin{bmatrix} 2 & 8 \\ -1 & -7 \\ 1 & 3 \end{bmatrix}.$$

Q) $T(x,y,z) = (x+y,\ x-y,\ 2x+z)$ find rank of L·T.
taking standard basis of 3 dimension

$\beta_1 = \{(1\,0\,0), (0\,1\,0), (0\,0\,1)\}$.

$T(1\,0\,0) = (1, 1, 2)$
$T(0\,1\,0) = (1, -1, 0)$
$T(0\,0\,1) = (0, 0, 1)$.

$$\text{matrix} = \begin{bmatrix} 1 & 1 & 2 \\ 1 & -1 & 0 \\ 2 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 \\ 0 & -2 & 0 \\ 0 & -2 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

rank = 3

Q) $T(x,y,z) = (y-x,\ y-z)$
$\beta\ (1\,0\,0), (0\,1\,0), (0\,0\,1)$
$T(1\,0\,0) = (-1, 0)$
$T(0\,1\,0) = (1, 1)$
$T(0\,0\,1) = (0, -1)$.

$$\text{matrix} \begin{bmatrix} -1 & 1 & 0 \\ 0 & 1 & -1 \end{bmatrix}.$$

Q) $R^4 \to R^3$ $T(x,y,z,t) = (x-y+z+t,\ 2x-2y+3z+4t,\ 3x-3y+4z+5t)$
1) find basis & dimension of image T.
2) find the rank of linear mapping.

$$\text{standard } R^4 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

$T(1\,0\,0\,0) = (1, 2, 3)$
$T(0\,1\,0\,0) = (-1, -2, -3)$
$T(0\,0\,1\,0) = (1, 3, 4)$.
$T(0\,0\,0\,1) = (1, 4, 5)$.

$$\begin{bmatrix} 1 & -1 & 1 & 1 \\ 2 & -2 & 3 & 4 \\ 3 & -3 & 4 & 5 \end{bmatrix}.$$

↓ bring to echelon.
Rank = 2.

⇒ dimension = 2.

Basis = $\{(1,-1,1,1), (0,0,1,2)\}$.