



# DAYANANDA SAGAR COLLEGE OF ENGINEERING

(An Autonomous Institute Affiliated to VTU, Belagavi)

Shavige Malleshwara Hills, Kumaraswamy Layout, Bengaluru-560078

## DEPARTMENT OF MATHEMATICS

### COURSE : MATHEMATICS FOR COMPUTER ENGINEERS

COURSE CODE: 21MAT31A

MODULE – 1: Vector Spaces

### Question Bank

Q. No.	Questions
1	a) Explain vector space, subspace over the field F b) Show that the set of all 2x2 matrix with real elements is a vector space over the field of real numbers
2	a) Prove that the set $W = \{(x,y,z) / (x-3y+4z=0)\}$ of a vector space $V_3(R)$ is a subspace of $V_3(R)$ b) Let $V = R_3$ the vector space of all ordered triplets of real number over the field of real number show that the subset $W = \{(x,0,0) / x \in R\}$ is a subspace of $R_3$
3	a) Explain Linear Independent, Linear dependent of set V b) Show that the set $S = \{(1,0,1), (1,1,0), (-1,0,-1)\}$ is Linearly dependent in $V_3(R)$
4	a) Show that the set $S = \{(1,2,3), (3,-2,1), (1,-6,-5)\}$ is Linearly dependent b) Show that the vector $X_1 = (1,2,-3,4)$ , $X_2 = (3,-1,2,1)$ , $X_3 = (1,-5,8,-7)$ are Linearly dependent
5	a) Find the value of K for which the vectors $(1,-2,K)$ , $(2,-1,5)$ and $(3,-5, 7K)$ are LD b) Find whether or not the following set of vectors are LD or LI $\{(1,1,1,1), (0,1,1,1), (0,0,1,1), (0,0,0,1)\}$ in $V_4(R)$
6	a) Find whether or not the following set of vectors are LD or LI $\{(1,1,0), (1,0,1), (0,1,1)\}$ in $V_3(R)$ b) Find whether or not the following set of vectors are LD or LI $\{(1,-2), (2,1), (3,2)\}$ in $V_3(R)$
7	a) Find whether or not the following set of vectors are LD or LI $\{(-3,0,4), (5,-1,2), (1,1,3)\}$ in $V_3(R)$ b) Find whether or not the following set of vectors are LD or LI $\{(1,2,3), (3,2,1), (3,3,3)\}$ in $V_3(R)$
8	a) Find whether or not the following set of vectors are LD or LI $\{(-2,0,1), (3,2,5), (6,-1,1), (7,0,-2)\}$ in $V_4(R)$ b) Explain Basis and Dimension
9	a) Verify the following set of vectors are basis of $R_3$ , $S = \{(1,0,0), (2,2,0), (3,3,3)\}$ b) Verify the following set of vectors are basis of $R_3$ , $S = \{(1,-2,3), (5,6,-1), (3,2,1)\}$
10	a) Determine whether the set $\{(1,2,1), (3,4,-7), (3,1,5)\}$ is a Basis of $V_3(R)$



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	b) Show that $B = \{(1,1,0), (1,0,1), (0,1,1)\}$ is a Basis of $V_3(\mathbb{R})$
11	<p>a) Find the dimension and the basis of the subspace spanned by the vectors <math>(2,4,2)</math>, <math>(1,-1,0)</math>, <math>(1,2,1)</math> and <math>(0,3,1)</math> in <math>V_3(\mathbb{R})</math></p> <p>b) Find the dimension and the basis of the subspace spanned by the vectors <math>(2,4,2)</math>, <math>(1,-1,0)</math>, <math>(1,2,1)</math> and <math>(0,2,1)</math> in <math>V_3(\mathbb{R})</math></p>
12	<p>a) Determine whether the set <math>S = \{(1,2,3), (3,1,0), (-2, 1,3)\}</math> is a basis of <math>V_3(\mathbb{R})</math>. In case <math>S</math> is not a basis determine the dimension and the basis of the subspace spanned by <math>S</math></p> <p>b) Explain column space and null space</p>
13	<p>a) Explain Linear transformation</p> <p>b) If <math>F</math> is a mapping from <math>V_3(\mathbb{R})</math> is defined by <math>F(x,y,z) = (x+y, y+z)</math> show that <math>F</math> is a Linear transformation</p>
14	<p>a) Verify <math>F</math> is Linear transformation defined by <math>F(x,y) = xy</math></p> <p>b) Find the matrix of the linear transformation <math>T: V_2(\mathbb{R}) \rightarrow V_3(\mathbb{R})</math> defined by <math>T(x,y) = (x+y, x, 3x-y)</math> with respect to <math>B_1 = \{(1,1), (3,1)\}</math> and <math>B_2 = \{(1,1,1), (1,1,0), (1,0,0)\}</math></p>
15	<p>a) Find the matrix of the linear transformation <math>T: V_2(\mathbb{R}) \rightarrow V_3(\mathbb{R})</math> defined by <math>T(x,y) = (x+y, x, 3x-y)</math> with respect to <math>B_1 = \{(1,0), (0,1)\}</math> and <math>B_2 = \{(1,0,0), (0,1,0), (0,0,1)\}</math></p> <p>b) Find the matrix of the linear transformation <math>T: V_3(\mathbb{R}) \rightarrow V_2(\mathbb{R})</math> defined by <math>T(x,y) = (x+y, y+z)</math> with respect to <math>B_1 = \{(1,1,1), (1,0,0), (1,1,0)\}</math> and <math>B_2 = \{(1,0), (0,1)\}</math></p>
16	<p>a) Explain Rank of the linear transformation</p> <p>b) Find the rank of the linear transformation defined by <math>T(x,y,z) = (x+y, x-y, 2x+z)</math></p>
17	<p>a) Find the rank of the linear transformation defined by <math>T(x,y,z) = (y-x, y-z)</math></p> <p>b) if <math>T</math> is a linear transform from <math>V_3(\mathbb{R}) \rightarrow V_4(\mathbb{R})</math> defined by <math>T(1,0,0) = (0,1,0,2)</math>, <math>T(0,1,0) = (0,1,1,0)</math> and <math>T(0,0,1) = (0,1,-1,4)</math>, find rank of the linear transformation</p>
18	<p>a) Show that the set of all solution of the differential equation <math>y'' + p(x)y' + q(x)y = 0</math> form a vector space over <math>\mathbb{R}</math></p> <p>b) Show that the set of all solution of system of equation <math>a_1x + a_2y + a_3z = 0</math>, <math>b_1x + b_2y + b_3z = 0</math>, <math>c_1x + c_2y + c_3z = 0</math> form a vector space</p>
19	<p>a) Prove that the set <math>W = \{(x,y,z) / (x-4y+7z=0)\}</math> of a vector space <math>V_3(\mathbb{R})</math> is a subspace of <math>V_3(\mathbb{R})</math></p> <p>b) Prove that the set <math>W = \{(x,y,z) / (3x+5y+z=0)\}</math> of a vector space <math>V_3(\mathbb{R})</math> is a subspace of <math>V_3(\mathbb{R})</math></p>
20	<p>a) Verify the following set of vectors are basis of <math>\mathbb{R}_3</math>, <math>S = \{(2,3,0), (1,2,5), (1,9,4)\}</math></p> <p>b) Verify the following set of vectors are basis of <math>\mathbb{R}_3</math>, <math>S = \{(4,3,0), (3,2,-1), (4,3,-2)\}</math></p>