	DAYANANDA SAGAR COLLEGE OF ENGINEERING *(An Autonomous Institute Affiliated to VTU, Belagavi)* Shavige Malleshwara Hills, Kumaraswamy Layout, Bengaluru-560078 **DEPARTMENT OF MATHEMATICS**

Course Material

COURSE	**MATHEMATICS FOR COMPUTER ENGINEERS**
COURSE CODE	**21MAT31A**
MODULE	**4**
MODULE NAME	**PROBABILITY DISTRIBUTIONS**
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Objective:

At the end of this Module, student will be able

- Understand the concept of Random variables and types of randoms variables.
- Probability distributions- Discrete and Continuous
- Geometric distribution
- Poisson distribution
- Exponential distribution
- Normal distribution

Random variables

Introduction: In a random experiment, the outcomes (results) are governed by chance mechanism and the sample space S of such a random experiment consists of all outcomes of the experiment. When the outcomes of the sample space are non-numeric, they can be quantified by assigning a real number to every event of the sample space. This assignment rule, known as the random variable or stochastic variable. In other words a random variable is a function that assigns a real number to every sample point in the sample space of a random experiment. Random variables are usually denoted by X,Y,Z…….The set of all real number of a random variable X is called the range of X.

Example-1 While tossing a coin, suppose that the value 1 is associated for the outcome' head' and 0 for the outcome 'tail'. The sample space S=$\{H,T\}$and if X is the random variable then
X (H) =1and X(T)=0, Range of X = $\{0,1\}$

Example-2 A pair of fair dice is tossed. The sample space S consists of the 36 ordered pair (a,b)where a and b can be any integers between 1 and 6, that is S={(1,1),(1,2),………(6,6)}
Let X assign to each point (a,b) the maximum of its numbers, that is, X(a,b)=max(a,b). For example(1,1)=1,X(3,4)=4,X(5,2)=5

Then x is a random variable where any number between 1 and 6 could occur, and no other number can occur. Thus the range space of X = {1, 2, 3, 4, 5, 6}

Let Y assign to each point (a,b) the sum of its numbers, that is Y(a,b)=a+b.For example Y(1,1)=2, Y(3,4)=7, Y(6,3)=9. Then Y is a random variable where any number between 2 and 12 could occur, and no other number can occur. Thus the range space ={2,3,4,5,6,7,8,9,10,11,12}

Discrete random variables

Definition: a random variable is said to be discrete random variable if it's set of possible outcomes, the sample space S, is countable (finite or an unending sequence with a many elements as there are whole numbers).

Example:
1) Tossing a coin and observing the outcome.
2) Tossing a coin and observing the number of heads turning up.
3) Throwing a 'die' and observing the number of the face.

Continuous random variables

Definition: A random variable is said to be continuous random variables if sample space S contains infinite number of values.

Example:
1) Weight of articles.
2) Length of nails produced by a machine.
3) Observing the pointer on a speedometer/voltmeter.
4) Conducting a survey on the life of electric bulbs.

Generally counting problems corresponds to discrete random variables and measuring problems lead to continuous random variables.

PROBABILITY DISTRIBUTIONS
Probability distribution is the theoretical counterpart of frequency distribution, and plays an important role in the theoretical study of populations.

Discrete probability distribution:

Definition: If for each value x_i of a discrete random variable X, we assign number p(x_i) such that
i) p$(x_i) \geq 0$, ii)$\sum_i \mathrm{p}(x_i) = 1$ then the function p(x) is called a probability function. If the probability that X takes the values x_i isp_i, then P(X=x_i) $= p_i$or p(x_i).The set of values [x_i, p(x_i)] is called a discrete probability distribution of the discrete random variable X. The function P(X) is called the probability density function (p.d.f) or the probability mass function (p.m.f)

Cumulative distribution function

The distribution function f(x) is defined by f$(x) = P(X \leq x) = \sum_{i=1}^{x} \mathrm{p}(x_i)$, x being an integer is called the cumulative distribution function(c.d.f)

The mean and variance of the discrete probability distribution

Mean (μ) $or\ expectation E(X) \ = \sum_i x_i\, p(x_i)$

Variance (V) = $\sum_i (x_i - \mu)^2 \ p(x_i)$= $\sum_i x_i{}^2\, p(x_i) - [x_i p(x_i)]^2$=$\sum_i x_i{}^2\, p(x_i) - \mu^2$

Standard deviation (σ) $= \sqrt{V}$

Problems1: Determinethediscrete probability distribution, expectation, variance, standard deviation of a discrete random variable X which denotes the minimum of the two numbers that appear when a pair of fair dice is thrown once.

Solution: The totalnumber of cases are 6x6=36. The minimum number could be 1,2,3,4,5,6i.e.,X(s)=X(a,b)=min$\{a, b\}$. the number 6 will appear only in one case (6,6),so P(6)=P(X=6)=P($\{6,6\}$=1/36

For minimum 5, favorable cases are (5,5),(5,6),(6,5) so P(5)=P(X=5)=3/36 For minimum 4, favorable cases are(4,4,(4,5),(4,6),(5,4)(6,4) so P(4)=P(X=4)=5/36

For minimum 3, favorable cases are(3,3),(3,4),(3,5),(3,6),(4,3),(5,3),(6,3) so P(3)=P(X=3)=7/36

For minimum 2, favorable cases are (2,2),(2,3),(2,4),(2,5),(2,6),(3,2),(4,2),(5,2),(6,2) so P(2)=P(X=2)=9/36

For minimum 1, favorable cases are (1,1)(!,2),(1,3),(1,4),(1,5),(1,6),(2,1),(3,1),(4,1),(5,1),(6,1) so P(1)=P(X=1)=11/36

Thus required discrete probability distribution

X=x_i	1	2	3	4	5	6
p(x_i)	11/36	9/36	7/36	5/36	3/36	1/36

Mean = $\sum_i x_i\, p(x_i)$=1x11/36+2x9/36+3x7/36+4x5/36+5x3/36+6x1/36=2.5

Variance (V)=$\sum_i x_i{}^2\, p(x_i) - \mu^2$

=1x11/36+4x9/36+9x7/36+16x5/36+25x3/36+36x1/36$-(2.5)^2$

=1.9745

Standard deviation=$\sqrt{V}$=1.4

Example 2: The random variable X has the following probability mass function

X	0	1	2	3	4	5
P(X)	K	3K	5K	7K	9K	11K

i) find K ii) find P(X<3) iii) find P(3<X$\leq$ 5)(June-July2011)

Solution: If X is a discrete random variable then $\sum_i P(x_i) = 1$

$\Rightarrow$ K+3K+5K+7K+9K+11K =1

$\Rightarrow$ 36K=1

$\Rightarrow$ K = 1/36

ii) P(X<3) = P(X=0)+P(X=1)+P(X=2)

= K+3K+5K = 9K = 9/36=1/4

iii) P(3<X$\leq$ 5) = P(X=4)+P(X=5) =9K+11K= 20K= 20/36=5/9

Example 3: The probability distribution of a finite random variable X is given by the following table:

X_i	-2	-1	0	1	2	3
p(X_i)	0.1	k	0.2	2k	0.3	k

i) Find the value of K and calculate the mean and variance.

ii) EvaluateP(X<1). (July2006)

Solution: If X is a discrete random variable then $\sum_i P(x_i) = 1$

$\Rightarrow$0.1+K+0.2+2k+0.3+k =1

$\Rightarrow$ 0.6+4K=1

$\Rightarrow$ 4k=1-0.6=0.4

$\Rightarrow$ K = 0.1

Mean $(\mu) = \sum_i x_i\, p(x_i)$ =-2x0.1+-1x0.1+0x0.2+1x0.2+2x0.3+3x0.1

=0.8

Variance (V) = $\sum_i {x_i}^2\, p(x_i) - \mu^2$

= 4x0.1+1x0.1+0x0.2+1x0.2+4x0.3+9x0.1$-(0.8)^2$

=2.16

ii)P(X<1) = P(X=-2)+P(X=-1)+P(X=0)

=0.1+0.1+0.2

=0.4

Example 4:A random variable X has the following probability function for various values of x

x	0	1	2	3	4	5	6	7
P(x)	0	k	2k	2k	3k	k^2	$2k^2$	$7k^2$+k

i) Findk ii) Evaluate P(x<6) and P(3<x≤ 6) Also find the probability distribution and the cumulative distribution function of X

Solution: If X is a discrete random variable then $\sum_i P(x_i) = 1$and P(X)≥ 0

$\Rightarrow$ 0+k+2k+2k+3k+k^2+2k^2+7k^2+k=1

$\Rightarrow$10k^2+9k−1=0

$\Rightarrow$ k=1/10 andk=-1

Ifk=-1 the second condition fails and hence k$\neq -1$ ∴ k=1/10

Hence the probability distribution is as follows.

x	0	1	2	3	4	5	6	7
P(x)	0	0.1	0.2	0.2	0.3	0.01	0.02	0.17

P(x<6)=P(0)+P(1)+P(2)+P(3)+P(4)+P(5)

=0+0.1+0.2+0.2+0.3+0.01=0.81

P(3<x≤ 6)=P(4)+P(5)+P(6)

=0.3+0.01+0.02=0.33

Cumulative distribution function of X is as follows.

x	0	1	2	3	4	5	6	7
f(x)	0	0+0.1 =0.1	0.1+0.2 =0.3	0.3+0.2 0.5	0.5+0.3 0.8	0.8+0.01 0.81	0.81+0.02 0.83	0.83+0.17 1

In discrete probability distribution we are going to study Geometric distribution & Poisson distribution.

GEOMETRIC DISTRIBUTION

Consider a sequence of trials, where each trial has only two possible outcomes (designated failure and success). The probability of success is assumed to be the same for each trial. In such a sequence of trials, the geometric distribution is useful to model the number of failures before the first success. The distribution gives the probability that there are zero failures before the first success, one failure before the first success, two failures before the first success, and so on.

Examples:

1.A newly-wed couple plans to have children, and will continue until the first girl. What is the probability that there are zero boys before the first girl, one boy before the first girl, two boys before the first girl, and so on?

2.A doctor is seeking an anti-depressant for a newly diagnosed patient. Suppose that, of the available anti-depressant drugs, the probability that any particular drug will be effective for a particular patient is p=0.6. What is the probability that the first drug found to be effective for this patient is the first drug tried, the second drug tried, and so on? What is the expected number of drugs that will be tried to find one that is effective?

3.A patient is waiting for a suitable matching kidney donor for a transplant. If the probability that a randomly selected donor is a suitable match is p=0.1, what is the expected number of donors who will be tested before a matching donor is found?

Definition: If p be the probability of success and x be the number of failures preceding the first success then this distribution is
$p(x) = q^x p$, x =0,1,2,3….., q = 1-p

Obviously $\sum_{x=0}^{\infty} P(x) = p\sum_{x=0}^{\infty} q^x = p.\frac{1}{1-q} = 1$
Hence P(x) is a probability function.

Mean and standard deviation of the Geometric distribution

Mean(μ)=$\sum_{x=0}^{\infty} xP(x)$

$=\sum_{x=0}^{\infty} x\, q^x p$

$= p\sum_{x=1}^{\infty} x\, q^{x-1} q$

$= pq\sum_{x=1}^{\infty} \frac{d(q^x)}{dq}$

$=pq\frac{d}{dq}\sum_{x=1}^{\infty} q^x$

$=pq\ \frac{d}{dq}\left[\frac{q}{1-q}\right]$

$=pq\left[\frac{1}{(1-q)^2}\right]$

$\mu=\frac{q}{p}$

Variance(V) $=\sum_{x=0}^{\infty} x^2P(x)-\mu^2=\frac{q}{p^2}$

Problem 1 : What is the probability that the marketing representative must select more than 6 people before he finds one who attended the last home c.d.f of a Geometric R V with 1-p=0.08 and x=0.6.

Solution: $\mathrm{P(x>6)=1-p(x\le 6)=1-(1-0.8^6)=0.262}$

There are 26% chance.

Problem 2: The probability that a person succeded in finding is equal to 0.20 and let X denote the number of person to select until his first success.What is the probability that the marketing representative must select 4 person?

Solution: $\mathrm{p=0.20,1-p=0.8\ x=4}$

$$\mathrm{P(x=4)=0.80^3\times 0.20=0.1024}$$

POISSON DISTRIBUTION

Poisson distribution is the discrete probability distribution of discrete random variable X, which has no upper bound. It is defined for non-negative values of x as follows: P(x)= $\frac{m^xe^{-m}}{x!}$ for x=0,1,2,3…… Here m>0 is called the parameter of the distribution. In binomial distribution the number successes out of total definite number of n trials is determined, whereas in Poisson distribution the number of successes at a random point time and space is determined.

Poisson distribution is suitable for 'rare' events for which the probability of occurrence p is very small and the of trials n is very large. Also binomial distribution can be approximated by Poisson distribution when n→ ∞ and p→ 0 such that m= np= constant.

Example of rare events:

i) Number of printing mistake per page.
ii) Number of accidents on a highway.
iii) Number of bad cheques at a bank.
iv) Number of defectives in a production center.
We have in case of binomial distribution, the probability of x successes out of n trials,
$P(x) = n_{c_x} p^x q^{n-x}$

$$= \frac{n(n-1)(n-2)\ldots\ldots n-(x-1)}{x!} p^x q^{n-x}$$

$$= \frac{n.n\left(1-\frac{1}{n}\right)n\left(1-\frac{2}{n}\right)\ldots\ldots n.\left(1-\frac{x-1}{n}\right)}{x!} p^x q^{n-x}$$

$$= \frac{n^x\left(1-\frac{1}{n}\right)\left(1-\frac{2}{n}\right)\ldots\ldots\left(1-\frac{x-1}{n}\right)}{x!} p^x q^{n-x}$$

$$= \frac{(np)^x\left(1-\frac{1}{n}\right)\left(1-\frac{2}{n}\right)\ldots\ldots\left(1-\frac{x-1}{n}\right)}{x!q^x} q^n$$

But np =m; $q^n = (1-p)^n = (1-\frac{m}{n})^n = \left\{(1-\frac{m}{n})^{-n/m}\right\}^{-m}$ denoting $-\frac{m}{n} = k$
We have, $q^n = \left\{(1+k)^{1/k}\right\}^{-m} \to e^{-m}$ as n$\to \infty$ or $k \to 0$
Further $q^x = (1-p)^x \to 1$ for a fixed x as p$\to 0$.
Also the factor $\left(1-\frac{1}{n}\right)\left(1-\frac{2}{n}\right)\ldots\ldots\left(1-\frac{x-1}{n}\right)$ will also tend to 1 as n$\to \infty$
Thus P(x)= $\frac{m^x e^{-m}}{x!}$
This is known as the poisson distribution of the random variable.P(x) is called Poisson probability function and x is called a Poisson variate.
The distribution of probabilities for x=0,1,2,3….. is as follows.

x	0	1	2	3	…..
P(x)	e^{-m}	$\frac{me^{-m}}{1!}$	$\frac{m^2e^{-m}}{2!}$	$\frac{m^3e^{-m}}{3!}$	

We have P(x)≥ 0 and

$$\sum_{x=0}^{\infty} P(x) = e^{-m} + \frac{me^{-m}}{1!} + \frac{m^2e^{-m}}{2!} + \frac{m^3e^{-m}}{3!} + \cdots \ldots\ldots$$

$$= e^{-m}\left\{1 + \frac{m}{1!} + \frac{m^2}{2!} + \frac{m^3}{3!} + \cdots\right\}$$

$$= e^{-m}e^{m} = 1$$

Hence P(x) is a probability function.

Mean and standard deviation of the Poisson distribution

Mean(μ)=$\sum_{x=0}^{\infty} xP(x)$

$=\sum_{x=0}^{\infty} x\frac{m^x e^{-m}}{x!}$

$=\sum_{x=1}^{\infty} \frac{m^x e^{-m}}{(x-1)!}$

$=me^{-m}\sum_{x=1}^{\infty} \frac{m^{x-1}}{(x-1)!}$

$= me^{-m}\left\{1+\frac{m}{1!}+\frac{m^2}{2!}+\frac{m^3}{3!}+\cdots\right\}$

$= me^{-m}e^{m}$

Mean(μ) =m

Standard deviation(σ)=$\sqrt{V}$

Variance(V)=$\sum_{x=0}^{\infty} x^2P(x)-\mu^2$………(1)

Consider$\sum_{x=0}^{\infty} x^2P(x)$ = $\sum_{x=0}^{\infty}[x(x-1)+x]\frac{m^x e^{-m}}{x!}$

$= \sum_{x=2}^{\infty} \frac{m^x e^{-m}}{(x-2)!}+\sum_{x=1}^{\infty} \frac{m^x e^{-m}}{(x-1)!}$

$=m^2e^{-m}\sum_{x=2}^{\infty} \frac{m^{x-2}}{(x-2)!}+m$

$=m^2e^{-m}\left\{1+\frac{m}{1!}+\frac{m^2}{2!}+\frac{m^3}{3!}+\cdots\right\}+m$

$=m^2e^{-m}e^{m}+m$

$\sum_{x=0}^{\infty} x^2P(x)$ $=m^2+m$

Equation (1) implies

Variance(V)=m^2+m-m^2

=m

∴ Standard deviation(σ)=$\sqrt{m}$

Mean and variance are equal in Poisson distribution.

Problem1: The probabilitythatindividual suffers a bad reaction from an injection is 0.001. Find the probability that out of 2000 individuals i) more than 2 ii) exactly 3 will get bad reaction

Solution: As the probability of occurrence is very small, this follows Poisson distribution and we have

P(x)= $\frac{m^x e^{-m}}{x!}$

Mean=m=np=2000x0.001=2

i)P(x>2)=1-P(x≤ 2)

=1 – [P(x=0)+P(x=1)+P(x=2)]

= 1- $e^{-m}\left\{1+\frac{m}{1!}+\frac{m^2}{2!}\right\}$

=1 - $e^{-2}[1+2+2]$=0.3233

ii)P(x=3)=$\frac{2^3e^{-2}}{3!}$= 0.1804

Problem2:2% of the fuses manufactured by a firm are found to be defective. Find the probability that a box containing 200 fuses contains i)no defective fuse ii)3 or more defective fuses.(July-2007)

Solution: By data probability of defective fuse=2/100=0.02

Mean=m=np=200x0.02=4

Poisson distribution P(x)= $\frac{m^xe^{-m}}{x!}$

=$\frac{4^xe^{-4}}{x!}$

i)P(x=0)= e^{-4}=0.0183

ii)P(x$\geq 3) = 1 - P(x<3)$

=1- [P(X=0)+P(x=1)+P(x=2)]

=1 $-e^{-4}$ [1+$\frac{4}{1!}+\frac{4^2}{2!}$]

=0.7621

Problem3: There **is** a chance that 5% of the pages of a book contain typographical errors. If 100 pages of the book are chosen at random, find the probability that 2 of the pages contain typographical errors, using i)Binomial distribution ii)Poisson distribution.

Solution:i) Binomial distribution

The probability that a chosen page contains typographical errors is given as p=5%=0.05,

q =1-0.05=0.95, n=100

$P(x) = n_{c_x}p^xq^{n-x} \quad = 100_{C_x}(0.05)^x(0.95)^{100-x}$

P(x=2)= $100_{C_2}(0.05)^2(0.95)^{98}$= 0.081

ii) Poisson distribution.

Mean=m=np=100x0.05=5

P(x)= $\frac{m^xe^{-m}}{x!}$

P(x=2)=$\frac{5^2e^{-5}}{2!}$=0.084

Problem4: If x is a Poisson variate such that P(x=1)=0.2P(x=2).find the mean & evaluate P(x=0)

Solution:For the Poisson distribution, the p.d.f
P(x)= $\frac{m^x e^{-m}}{x!}$
P(x=1)=$e^{-m}m$
P(x=2)=$\frac{m^2e^{-m}}{2!}$
By data P(x=1)=0.2P(x=2)
=0.2$\frac{m^2e^{-m}}{2!}$
This implies m=10
∴p.d.f P(x) =$\frac{10^x e^{-10}}{x!}$
P(x=0)=e^{-10}

Continuous probability distribution:

The number of events are infinitely large the probability that a specific event will occur is practically zero for this reason continuous probability statement must be worded somewhat differently from discrete ones. Instead of finding the probability that x equals some value, we find the probability of x falling in a small interval.In this context we need a continuous probability function which is defined as follows.

Definition: If for every x belonging to the range of a continuous random variable X, we assign a real number P(x) satisfying the conditions
i) P(x) ≥ 0
ii) $\int_{-\infty}^{\infty} \mathrm{P}(x)\mathrm{dx} = 1$ then P(x) is called a Continuous probability function or probability density function(p.d.f).If (a,b) is a subinterval of the range space of X then the probability that x lies in the (a,b) is defined to be the interval of P(x) between a and b. i.e., P(a≤x≤b) = $\int_a^b \mathrm{P}(x)\mathrm{dx}$

Cumulative distribution function

If X is a continuous random variable with probability density function $\mathrm{P}(x)$ then the function $\mathrm{f}(x)$ is defined by $\mathrm{f}(x) = P(X \leq x) = \int_{-\infty}^{x} P(x)\mathrm{dx}$ is called the cumulative distribution function(c.d.f)of X

The mean and variance of the continuous probability distribution

Mean (μ) or Expectation E(X) $=\int_{-\infty}^{\infty} x.\,p(x)dx$

Variance (V)= $\int_{-\infty}^{\infty}(x_i-\mu)^2.\,p(x)dx$=$\int_{-\infty}^{\infty}(x_i)^2.\,p(x)dx-\mu^2$

Example 1:A random variable X has the density function P(x)=$\begin{cases} kx^2 & for \quad -3 \leq x \leq 3 \\ 0 & elsewhere \end{cases}$ find k. Also find P(x≤ 2) and P(x>1)

Solution: If X is a continuous random variable then i) $\mathrm{P}(x) \geq 0$

ii) $\int_{-\infty}^{\infty} \mathrm{P}(x)\mathrm{dx} = 1$

That is $\int_{-3}^{3} kx^2\mathrm{dx} = 1$

$$\Rightarrow \left[\frac{kx^3}{3}\right]_{-3}^{3} = 1$$

⇒ k=1/18

P(x≤ 2)=$\int_{-3}^{2} \frac{x^2}{18}\mathrm{dx} = \frac{1}{18}\left[\frac{x^3}{3}\right]_{-3}^{2} = \frac{35}{54}$

P(x>1) =$\int_{1}^{3} \frac{x^2}{18}\mathrm{dx} = \frac{1}{18}\left[\frac{x^3}{3}\right]_{1}^{3} = \frac{26}{54} = \frac{13}{27}$

Example 2: Thedaily consumption of electric power (in millions of kW-hours) is a random variable having the p.d.f P(x)= $\begin{cases} \frac{1}{9}xe^{-\frac{x}{3}} & x > 0 \\ 0 & x \leq 0 \end{cases}$ if the total production is 12million kW-hours, determine the probability that there is power cut (shortage) on any given day.

Solution: Probability that the power consumed is between 0to12 is P(0$\leq x \leq 12) =$ $\int_{0}^{12} \mathrm{P}(x)\mathrm{dx} = \int_{0}^{12} \frac{1}{9}xe^{-\frac{x}{3}}\mathrm{dx} = \left[-\frac{x}{3}e^{-\frac{x}{3}} - e^{-\frac{x}{3}}\right]_{0}^{12} = 1 - 5e^{-4}$

Power supply is inadequate if daily consumption exceeds 12million kW,i.e.,

P(x>12)=1− P(0≤ $x \leq 12$)= 1−$[1-5e^{-4}]$=$5e^{-4}$=0.0915781

Example 3: Find the mean and variance of p.d.f. f(x)= $\begin{cases} \frac{1}{4}e^{-\frac{x}{4}} & for \quad x > 0 \\ 0 & elsewhere \end{cases}$

Solution: Mean $=\int_{-\infty}^{\infty} x.f(x)dx = \int_0^{\infty} \mathrm{x}f(x)\mathrm{dx} = \int_0^{\infty} \frac{1}{4}\, xe^{-\frac{x}{4}}\mathrm{dx}$

$=\frac{1}{4}\left[\frac{xe^{-\frac{x}{4}}}{-\frac{1}{4}} - \frac{e^{-\frac{x}{4}}}{\frac{1}{16}}\right]_0^{\infty} = -4(0-1) = 4$

Variance(V) = $\int_{-\infty}^{\infty}(x_i)^2.f(x)dx - \mu^2 = \int_0^{\infty} \mathrm{x}^2 \frac{1}{4}\, xe^{-\frac{x}{4}}\mathrm{dx} - 16$

$= \frac{1}{4}\left[\frac{\mathrm{x}^2 e^{-\frac{x}{4}}}{-\frac{1}{4}} - 2x\frac{e^{-\frac{x}{4}}}{\frac{1}{16}} + 2\frac{e^{-\frac{x}{4}}}{-\frac{1}{64}}\right]_0^{\infty} - 16$

=32-16=16

In continuous probability distribution we study Normal & Exponential distribution.

EXPONENTIAL DISTRIBUTION

Many scientific experiments involve the measurement of the duration of time X between an initial point of time and the occurrence of some phenomenon of interest. For Example X is the life time of a light bulb which is turned on left until it burns out. The continuous random variable X having the probability density function f(x) = $\begin{cases} \boldsymbol{\alpha}e^{-\alpha x} & for \quad x > 0 \\ 0 & elsewhere \end{cases}$, where $\alpha > 0$ is known as the exponential distribution. Here the only parameter of the distribution is α

Example of random variables modeled as exponential are

i) Duration of telephone calls
ii) Time require for repair of a component
iii) Service time at a server in a queue

Mean and standard deviation of the exponential distribution

Mean $(\mu) = \int_{-\infty}^{\infty} x.f(x)dx = \int_0^{\infty} x.\boldsymbol{\alpha}e^{-\alpha x}dx = \boldsymbol{\alpha}\int_0^{\infty} x.e^{-\alpha x}dx$

$= \boldsymbol{\alpha}\left[x.\frac{e^{-\alpha x}}{-\boldsymbol{\alpha}} - 1\frac{e^{-\alpha x}}{\boldsymbol{\alpha}^2}\right]_0^{\infty}$

$= \boldsymbol{\alpha}\left[0 - \frac{1}{\boldsymbol{\alpha}^2}(0-1)\right] = \frac{1}{\boldsymbol{\alpha}}$

Mean $(\mu) = \frac{1}{\boldsymbol{\alpha}}$

Standard deviation (σ)=$\sqrt{V}$

Variance (V) = $\int_{-\infty}^{\infty}(x-\mu)^2 f(x)dx$

$= \boldsymbol{\alpha} \int_0^{\infty}(x-\mu)^2 e^{-\alpha x}dx$

$= \boldsymbol{\alpha}\left[(x-\mu)^2.\frac{e^{-\alpha x}}{-\boldsymbol{\alpha}} - 2(x-\mu)\frac{e^{-\alpha x}}{\boldsymbol{\alpha}^2} + 2\frac{e^{-\alpha x}}{-\boldsymbol{\alpha}^3}\right]_0^{\infty}$

$=\boldsymbol{\alpha}\left[(0-\mu^2)\frac{1}{-\boldsymbol{\alpha}} - 2([0-(-\mu)]\frac{1}{\boldsymbol{\alpha}^2} - 2\frac{1}{\boldsymbol{\alpha}^3}(0-1)\right]$

$= \boldsymbol{\alpha}\left[\frac{\mu^2}{\boldsymbol{\alpha}} - \frac{2\mu}{\boldsymbol{\alpha}^2} + \frac{2}{\boldsymbol{\alpha}^3}\right]$

$= \boldsymbol{\alpha}\left[\frac{1}{\boldsymbol{\alpha}^3} - \frac{2}{\boldsymbol{\alpha}^3} + \frac{2}{\boldsymbol{\alpha}^3}\right] \quad =\frac{1}{\boldsymbol{\alpha}^2}$

Standard deviation (σ)=$\sqrt{V}$=$\sqrt{\frac{1}{\boldsymbol{\alpha}^2}}$

Mean Standard deviation is equal in exponential transformation

Problem1: In a certain town the duration of a shower is exponentially distributed with mean 5 minutes. What is the probability that a shower will last for i) less than 10 minutes ii) 10 minutes or more iii) between 10minutes and 12 minutes (Dec.06/jan07)

Solution: The p.d.f of the exponential distribution is given by

f(x)=$\alpha e^{-\alpha x}, x > 0$ and mean = 1/ α

By data 1/ $\alpha = 5$ $\therefore$ $\alpha = 5$ and hence f(x) =$\frac{1}{5}e^{-\frac{x}{5}}$

i) P(x<10)=$\int_0^{10} \frac{1}{5}e^{-\frac{x}{5}}dx = -\left[e^{-\frac{x}{5}}\right]_0^{10}$ =1- e^2=0.8647

ii) P(x$\geq 10) = \int_{10}^{\infty} \frac{1}{5}e^{-\frac{x}{5}}dx$=$-\left[e^{-\frac{x}{5}}\right]_{10}^{\infty} = e^2 = 0.1353$

iii) P(10<x<12)=$\int_{10}^{12} \frac{1}{5}e^{-\frac{x}{5}}dx$=$-\left[e^{-\frac{x}{5}}\right]_{10}^{12}$=-($e^{\frac{12}{5}} - e^{-2}$)=0.0446

Problem2: Thesale per day in a shop is exponentially distributed with average sale amounting to Rs.100 and net profit is 8%. Find the probability that the net profit exceeds Rs.30 on a day.

Solution: Let x be the random variable of the sale in the shop. Since x is an exponential variate the p.d.f f(x)= $\alpha e^{-\alpha x}, x > 0$ mean = 1/ α=100

$\Rightarrow \alpha = 0.01$ hence f(x)= $0.01e^{-0.01x}, x > 0$

Let A be the amount for which profit is 8%

$\Rightarrow$ A. $\frac{8}{100} = 30 \therefore A = 375$

Probability of profit exceeding Rs.30 = 1- Prod(profit$\leq Rs. 30$)

=1-Prob(sales $\leq Rs. 375$)

$=1 - \int_0^{375} (0.01)e^{-0.01x}dx$

$=1+[e^{-0.01x}]_0^{375} = e^{-3.75}$

The probability that the net profit exceeds Rs.30 on a day is $e^{-3.75}$

Problem3: Let the mileage (in thousands of miles) of a particular tyre be random variable x having p.d.f $\begin{cases} \frac{1}{20}e^{-x/20} & for \quad x > 0 \\ 0 & elsewhere \end{cases}$ find the probability that i) at most 10,000miles ii)any where from 16,000 to 24,000miles iii) at least 30,000miles.iv)Find the mean and the variance of the given p.d.f.

Solution: By data $\alpha = 1/20$

i)P(x$\leq 10) = \int_0^{10} f(x)dx = \int_0^{10} \frac{1}{20}e^{-x/20}dx.$=$\left[\frac{1}{20}e^{-x/20}\frac{-20}{1}\right]_0^{10}$=1-$e^{-1/2}$=0.3934

ii)P(16$\leq x \leq 24) = \int_{16}^{24} f(x)dx = \int_{16}^{24} \frac{1}{20}e^{-x/20}dx.$ =-$\left[\frac{1}{20}e^{-x/20}\frac{-20}{1}\right]_{16}^{24}$

=$e^{-4/5}$-$e^{-6/5}$ =0.148

iii) P(x$\geq 30) = \int_{30}^{\infty} f(x)dx = \int_{30}^{\infty} \frac{1}{20}e^{-x/20}dx = \left[\frac{1}{20}e^{-x/20}\frac{-20}{1}\right]_{30}^{\infty} = e^{-3/2}$=0.2231

iv) Mean(μ) $= \frac{1}{\boldsymbol{\alpha}} = 20$

Variance(V)=$\frac{1}{\boldsymbol{\alpha}^2} = 20^2$

NORMAL DISTRIBUTION

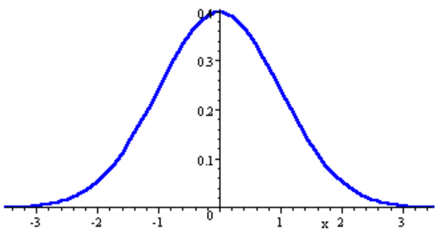
Normal distribution is the probability distribution of continuous random variable X, known as normal random variable or normal variate it is given by f(x)=$\frac{1}{\sigma\sqrt{2\pi}}e^{-(x-\mu)^2}/2\sigma^2$ where -$\infty < x < \infty$, -$\infty < \mu < \infty$ & σ>0. Is known as normal distribution.

$\int_{-\infty}^{\infty} f(x)dx$ =$\frac{1}{\sigma\sqrt{2\pi}}\int_{-\infty}^{\infty} e^{-(x-\mu)^2}/2\sigma^2$dx

μ & σ are the two parameters of the normal distribution is also known as Gaussian distribution.
This distribution is most important , simple, useful & is the corner stone of modern statistics because sampling distribution ' t ', F, χ^2 tend to be normal for large samples & it is applicable in statistical quality control in industry.

PROPERTIES OF NORMAL DISTRIBUTION

(i)The graph of the normal distribution y=f(x) in the XY-plane is known as normal curve. Normal curve is symmetric about y axis, it is bell shaped the mean, median,& mode coincide & therefore normal curve is unimodal.
Normal curve is asymptotic to both positive & negative x-axis.



(ii)Area under the curve is unity.

(iii)Probability that the continuous random variable lies between a&b is denoted by P(a$\leq x \leq b$)& is given by $\int_a^b \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-\mu)^2}/2\sigma^2 \ dx$………(i)
Since (1) depends on the two parameters μ & σ we get different normal curves for different values of μ & σ & it is impracticable task to plot all such normal curves.Instead by introducing Z=$\frac{(x-\mu)}{\sigma}$.The RHS integral in(1) becomes independent of the two parameters μ & σhere Z is known standard variate.

(iv) Change of scale from x-axis to z-axis
P(a$\leq x \leq b$)=$\int_a^b \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-\mu)^2}/2\sigma^2 \ dx$
P($z_1 \leq z \leq z_2$)=$\int_{z_1}^{z_2} \frac{1}{\sigma\sqrt{2\pi}} e^{-z^2}/2 \ \sigma dz$
=$\int_{z_1}^{z_2} \frac{1}{\sqrt{2\pi}} e^{-z^2}/2 \ dz$……………………(2)

Where $z_1=\frac{a-\mu}{\sigma}$, $z_2=\frac{b-\mu}{\sigma}$

(v) Error function or probability integral is defined as P(Z)=$\frac{1}{\sqrt{2\pi}}\int_0^z e^{-z^2}/2\ dz$.....(3)

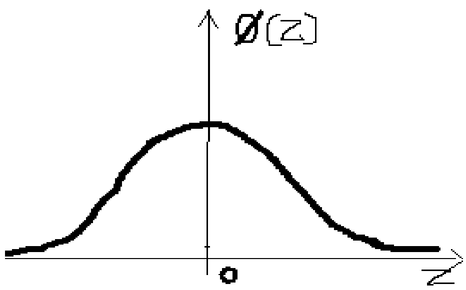
Now (2) can be written by using (3) as

P(a$\le x \le b$)= P$(z_1 \le z \le z_2)$=$\int_{z_1}^{z_2}\frac{1}{\sqrt{2\pi}}e^{-z^2}/2\ dz$............(4)

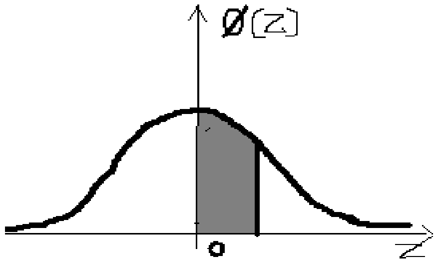
=P$(z_1) - P(z_2)$

Normal distribution f(x) transformed by the standard variable Z is given by F(Z) =$\frac{1}{\sqrt{2\pi}}e^{-z^2}/2$ with mean 0 & standard deviation 1 is known as standard normal distribution & its normal curve as standard normal curve. The probability integral (3) is tabulated for various values of Z varying from 0 to 3.9 & is known as normal table. Thus the entries in the normal table gives the area under the normal curve between the ordinates z=0 to z. Since normal curve is symmetric about y-axis the area from 0 to –z is same as the area from 0 to z. For this reason, normal table is tabulated only for positive values of z.

The integral in the RHS of (4) geometrically represents the area bounded by the standard normal curve F(Z) between z=z_1& z=z_2. Further in particular if $z_1 = 0\ we\ have\ \emptyset(Z) = \frac{1}{\sqrt{2\pi}}\int_0^z e^{-z^2}$/2dz. This represents the area under the



standard normal curve z=0 to z.



Normal probability table

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0753
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.2190	0.2224
0.6	0.2257	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2517	0.2549
0.7	0.2580	0.2611	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823	0.2852
0.8	0.2881	0.2910	0.2939	0.2967	0.2995	0.3023	0.3051	0.3078	0.3106	0.3133
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3304	0.3365	0.3389
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4761	0.4767
2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
2.5	0.4938	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964
2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974
2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4979	0.4980	0.4981
2.9	0.4981	0.4982	0.4982	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
3.0	0.4987	0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4989	0.4990	0.4990
3.1	0.4990	0.4991	0.4991	0.4991	0.4992	0.4992	0.4992	0.4992	0.4993	0.4993
3.2	0.4993	0.4993	0.4994	0.4994	0.4994	0.4994	0.4994	0.4995	0.4995	0.4995

3.3	0.4995	0.4995	0.4995	0.4996	0.4996	0.4996	0.4996	0.4996	0.4996	0.4997
3.4	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4998
3.5	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998
3.6	0.4998	0.4998	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999
3.7	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999
3.8	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999

Note:

1.$\int_{-\infty}^{\infty} \emptyset(Z)dz = 1 \Rightarrow$P(-∞ $< z \leq \infty) = 1$

2. $\int_{-\infty}^{0} \emptyset(Z)dz = \int_{0}^{\infty} \emptyset(Z)dz = 1/2 \Rightarrow$P(-∞ $\leq z \leq 0) = P(0 \leq z \leq \infty) = 1/2$

3.P(-∞ $< z < z_1$)=P(-∞ $< z \leq 0$)+P(0$\leq z < z_1$)=0.5+$\emptyset(z_1)$

4. P(z>z_2)=0.5-$\emptyset(z_2)$

MEAN & STANDARD DEVIATION OF THE NORMAL DISTRIBUTION

Mean $(\mu) = \int_{-\infty}^{\infty}$ x f(x) dx

$=\frac{1}{\sigma\sqrt{2\pi}}\int_{-\infty}^{\infty} x\, e^{-(x-\mu)^2}/2\sigma^2\; dx$

Putting t=$\frac{(x-\mu)}{\sigma\sqrt{2}}$ or x= $\mu + \sigma t\sqrt{2}$, we have dx= $\sigma\sqrt{2}$dt

t also varies from -∞ to ∞

mean=$\frac{1}{\sigma\sqrt{2\pi}}\int_{-\infty}^{\infty} \mu + \sigma t\sqrt{2}e^{-t^2}\; \sigma\sqrt{2}$dt

$=\frac{\mu}{\sqrt{\pi}}\int_{-\infty}^{\infty} e^{-t^2}\, dt + \sigma\frac{\sqrt{2}}{\sqrt{\pi}}\int_{-\infty}^{\infty} te^{-t^2}\, dt$

$=\frac{2\,\mu}{\sqrt{\pi}}\int_{0}^{\infty} e^{-t^2}\, dt + \sigma\frac{\sqrt{2}}{\sqrt{\pi}}\int_{-\infty}^{\infty} te^{-t^2}\, dt$

The second integral is 0 by standard property since $te^{-t^2}$is an odd function.

By gamma function $\int_0^{\infty} e^{-t^2}\, dt = \frac{\sqrt{\pi}}{2}$

Hence mean=$\frac{2\,\mu}{\sqrt{\pi}}\frac{\sqrt{\pi}}{2} + 0 = \mu$

Hence the mean of the normal distribution is equal to mean of the given distribution.

Standard deviation $\sigma = \sqrt{V}$

Variance (V)= $\int_{-\infty}^{\infty}(x-\mu)^2 f(x)dx$

$=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}(x-\mu)^2 e^{-(x-\mu)^2}/2\sigma^2\; dx$

Substituting t=$\frac{x-\mu}{\sqrt{2\pi}}$, x= $\mu + \sigma t\sqrt{2}$, we have dx= $\sigma\sqrt{2}$dt

t also varies from -∞ to ∞

Variance (V)=$\frac{1}{\sigma\sqrt{2\pi}}\int_{-\infty}^{\infty} 2\sigma^2 t^2 e^{-t^2}\; \sigma\sqrt{2}$dt

$=\frac{2\sigma^2}{\sqrt{\pi}}\int_{-\infty}^{\infty} t^2 e^{-t^2}dt$

$=\frac{2\sigma^2}{\sqrt{\pi}}2\int_{0}^{\infty} t^2 e^{-t^2}dt$

$=\frac{2\sigma^2}{\sqrt{\pi}}\int_{0}^{\infty} t\,(2te^{-t^2})dt$

Taking u=t, v=2te^{-t^2}

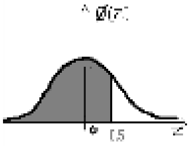
$\int uv$dt=u$\int v$dt-$\iint v$dt . $u'dt$

Variance (V)=$\frac{2\sigma^2}{\sqrt{\pi}}\{[te^{-t^2}]_0^{\infty}-\int_0^{\infty}-e^{-t^2}dt\}$

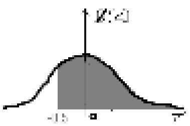
Variance (V)=$\frac{2\sigma^2}{\sqrt{\pi}}$[0+$\int_0^{\infty} e^{-t^2}dt]$=$\frac{2\sigma^2}{\sqrt{\pi}}\frac{\sqrt{\pi}}{2}$=$\sigma^2$

The variance/standard deviation of the normal distribution is equals to the variance of the given distribution.

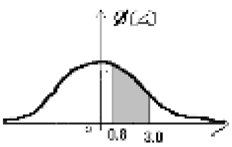
Area under standard normal curve to the left of z=1.5



Area under standard normal curve to the right of z=-0.5



Area under standard normal curve between z=0.8 to 3



Problem1: Find the following probabilities for the standard normal distribution with the help of normal probability table

a) P(-0.5$\leq z \leq 1.1)$ b) P(z $\geq 0.60)$ c) P(z $\leq 0.75)$ d)P(0.2$\leq z \leq 1.4)$

Solution:

a) P(-0.5$\leq z \leq 1.1)$ = P(-0.5$\leq z \leq 0) + \mathrm{P}(0 \leq z \leq 1.1)$

= P(0$\le z \le 0.5$)+ $\mathrm{P}(0 \le z \le 1.1)$
=$\emptyset(0.5) + \emptyset(1.1)$
=0.1915+0.3643
=0.5558

b) P(z ≥ 0.60) = P(z$\ge$ $0) - P(z \le 0.60)$
= 0.5 $-\emptyset(0.60)$
=0.5 -0.2258 =0.2742

c) P(z ≤ 0.75) = P(z ≤ 0) + P(0≤z ≤ 0.75)
=0.5 $+ \emptyset(0.75)$
=0.5 + 0.2422=0.7422

d)P(0.2$\le z \le 1.4$) = P(0$\le z \le 1.4) - \mathrm{d})\mathrm{P}(0 \le z \le 0.2)$
=$\emptyset(1.4) - \emptyset(0.2)$
=0.4192 -0.0793
=0.3399

Problem2: Assuming thatthe diameters of 1000 brass plugs taken consecutively from a machine form a normal distribution with mean 0.7515cm & standard deviation 0.002cm How many of the plugs are likely to rejected if the approved number is 0.752$\pm$0.004cm?

Solution: Let x represent the bras plugs, by data mean μ= 0.7515cm &
S.D $\sigma = 0.002$
We have standard normal variate(s.n.v) z= $\frac{x-\mu}{\sigma}$ =$\frac{x-0.7515}{0.002}$
Now 0.752+0.004=0.756
⇒ x =0.756 so z = $\frac{0.756-0.7515}{0.002}$=2.25
P(z>2.25)=P(0$\le z \le \infty) - P(0 \le z \le 2.25)$=0.5 $-\emptyset(2.25)$=0.5$-$0.4878=0.0122………….(1)
Now 0.752-0.004=0.748
⇒ x =0.748so z = $\frac{0.748-0.7515}{0.002}$=-1.75
P(z<1.75)=0.5$-\emptyset(1.75)$=0.5 – 0.4599=0.0401………(2)

Equation(1)+ Equation(2)
P(2.25<z<1.75)=0.0122+0.041=0.0523
For 1000 brass plugs 1000x0.0523=52.3=52
⇒52 plugs are rejected

Problem3:x isnormal random variable with 30 as mean & S.D 5. Find the probabilities that i)26$\le x \le 40$ ii) x≥ 45 iii)$|x-30| \le 5$ (May-June 2010)

Solution: We have s.n.v z= $\frac{x-\mu}{\sigma}$ =$\frac{x-30}{5}$

i)26$\leq x \leq 40$

x=26, z = $\frac{26-30}{5}$ =-0.8, x=40, z= $\frac{40-30}{5}$ =2

i.e., we shall find P(-0.8$\leq z \leq 2)$=P(-0.8$\leq z \leq 0) +$ P(0$\leq z \leq 2)$

= $\emptyset(0.8) + \emptyset(2)$=0.2881+0.4772=0.7653

ii) x≥ 45

z = $\frac{45-30}{5} = 3$

i.e., we shall find P(z$\geq 3) = P(0 \leq z \leq \infty) - P(0 \leq z \leq 3)$

=0.5 - $\emptyset(3)$=0.5 – 0.4987=0.0013

iii)$|x-30| \leq 5$⇒-5 $\leq x-30 \leq 5$⇒ 25$\leq x \leq 35$

x=25, z = $\frac{25-30}{5}$=-1, x=35, z = $\frac{35-30}{5}$=1

P(-1$\leq z \leq 1)$=P(-1$\leq z \leq 0) + P(0 \leq z \leq 1)$=$\emptyset(1) + \emptyset(1)$=2$\emptyset(1)$ = 2x0.3413=0.6826

Problem4: Find the mean and standard deviation of an examination in which grades 70 and 88 corresponding to standard scores of -0.6 and 1.4 respectively.

Solution: s.n.v z= $\frac{x-\mu}{\sigma}$

Hence -0.6 =$\frac{70-\mu}{\sigma}$ so $\mu - 0.6\sigma = 70$

1.4=$\frac{88-\mu}{\sigma}$ so $\mu + 1.4\sigma = 88$

By solving $\mu = 75.4, \sigma = 9$ are the mean and standard deviation.

Problem5: In a test of electric bulbs , it was found that the life time of a particular brand is distributed normally with an average life of 2000 hours and S.D of 60 hours. If a firm purchases 2500 bulbs find the number of bulbs that are likely to last for i) more than 2100 hours, ii) less than 1950 hours iii) between 1900 to 2100 hours.

Solution: By data $\mu = 2000, \sigma = 60$

We have s.n.v z= $\frac{x-\mu}{\sigma} = \text{z} = \frac{x-2000}{60}$

i)To find P(x>2100)=

If x=2100, z=$\frac{2100-2000}{60} = 1.67$

P(z>1.67)=P(z$\geq 0) - P(0 < z < 1.67)$=0.5 - $\emptyset(1.67)$=0.5 – 0.4525=0.0475

∴ number of bulbs that are likely to last for more than 2100 hours is

2500x0.0475 = 118.75 ≈ 119

ii) To find P(x< 1950)

If x =1950, z=$\frac{1950-2000}{60} = -0.83$

P(z<-0.83)=P(z>0.83)=P(z$\geq 0) - P(0 < z < 0.83)$= 0.5 - $\emptyset(0.83)$=0.5 – 0.2967 = 0.2033

∴ number of bulbs that are likely to last for less than 1950 hours is

2500x0.2033= 508.25≈ 508

iii) To find P(1900<x<2100)

If x =1900, z=$\frac{1900-2000}{60} = -1.67$ and if x= 2100, z=$\frac{2100-2000}{60} = 1.67$

P(-1.67<Z<1.67)=2P(0<z<1.67)

= 2$\emptyset(1.67)$= 2x 0.4525= 0.905

∴ number of bulbs that are likely to last between 1900and 2100 hours is

2500 x 0.905=2262.5≈ 2263