DAYANANDA SAGAR COLLEGE OF ENGINEERING

(An Autonomous Institute Affiliated to VTU, Belagavi)

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DEPARTMENT OF MATHEMATICS

COURSE : MATHEMATICS FOR COMPUTER ENGINEERS

COURSE CODE : 21MAT31A

MODULE – 2 : EIGEN VALUES & EIGEN VECTORS

Question Bank

Q.No	Questions
1.	a) Find the characteristic polynomial and the eigenvalue of the matrix $A=\begin{bmatrix}3 & 2\\ 3 & 8\end{bmatrix}$. b) Find the characteristic polynomial and the eigenvalue and corresponding eigenvectors of the matrix $A=\begin{bmatrix}2 & 1\\ 1 & 4\end{bmatrix}$.
2.	a) Find the characteristic polynomial and the eigenvalue of the matrix $A=\begin{bmatrix}6 & -3 & 1\\ 3 & 0 & 5\\ 2 & 2 & 6\end{bmatrix}$. b) Find the characteristic polynomial and the eigenvalue and corresponding eigenvectors of the matrix $A=\begin{bmatrix}3 & 6 & 7\\ 3 & 3 & 7\\ 5 & 6 & 5\end{bmatrix}$.
3.	a) Is $\lambda=4$ an eigenvalue of matrix $A=\begin{bmatrix}3 & 0 & -1\\ 2 & 3 & 1\\ -3 & 4 & 5\end{bmatrix}$? If so, find one corresponding eigenvector. b) Is $\lambda=3$ an eigenvalue of matrix $A=\begin{bmatrix}1 & 2 & 2\\ 3 & -2 & 1\\ 0 & 1 & 1\end{bmatrix}$? If so, find one corresponding eigenvector.
4.	a) Find a basis for the eigenspace corresponding to eigenvalue $\lambda=1,5$ for the matrix $A=\begin{bmatrix}5 & 0\\ 2 & 1\end{bmatrix}$. b) Find a basis for the eigenspace corresponding to eigenvalue $\lambda=2$ for the matrix $A=\begin{bmatrix}4 & -1 & 6\\ 2 & 1 & 6\\ 2 & -1 & 8\end{bmatrix}$.
5.	a) Prove that if $v_1, v_2, \ldots\ldots\ldots, v_r$ are eigenvectors that correspond to distinct eigenvalues $\lambda_1, \lambda_2, \ldots\ldots\ldots, \lambda_r$ of an $n\times n$ matrix A, then the set $\{v_1, v_2, \ldots\ldots\ldots, v_r\}$ is linearly independent. b) Show that λ^{-1} is an eigenvalue of A^{-1}, If λ be an eigenvalue of an invertible matrix A.
6.	a) Find the characteristic polynomial and the eigenvalue of the matrix $A=\begin{bmatrix}1 & 2 & 0\\ -1 & 1 & 2\\ 2 & 2 & 1\end{bmatrix}$. Also, verify that eigenvalues of A^2 are squares of those of eigenvalues of matrix A. b) Find the characteristic polynomial and the eigenvalue of the matrix $A=\begin{bmatrix}1 & 1\\ -1 & 1\end{bmatrix}$. Also, verify that eigenvalues of A^2 are squares of those of eigenvalues of matrix .

7.	a) Find the eigenspace of the matrix $A=\begin{bmatrix}16 & -4 & -2\\ 3 & 3 & -6\\ 2 & -8 & 11\end{bmatrix}$ for $\lambda=15$. b) Find the eigenspace of the matrix $A=\begin{bmatrix}0 & -6 & 3\\ 2 & -13 & 6\\ 4 & -24 & 11\end{bmatrix}$ for $\lambda=-1$.
8.	a) Find the eigenspace of the matrix $A=\begin{bmatrix}16 & -4 & -2\\ 3 & 3 & -6\\ 2 & -8 & 11\end{bmatrix}$ for $\lambda=5$. b) Find the value of h in the matrix $A=\begin{bmatrix}5 & -2 & 6 & -1\\ 0 & 3 & h & 0\\ 0 & 0 & 5 & 4\\ 0 & 0 & 0 & 1\end{bmatrix}$ such that the eigenspace for $\lambda=5$ is two dimensional.
9.	a) Define diagonalizable and diagonalize the matrix $A=\begin{bmatrix}1 & 0\\ 6 & -1\end{bmatrix}$, if possible. b) Define diagonalizable and diagonalize the matrix $A=\begin{bmatrix}3 & -1\\ 1 & 5\end{bmatrix}$, if possible.
10.	a) Show that the matrix $A=\begin{bmatrix}3 & 1 & -1\\ -2 & 1 & 2\\ 0 & 1 & 2\end{bmatrix}$ is diagonalizable. Hence , find P such that $P^{-1}AP$ is a diagonal matrix. b) Show that the matrix $A=\begin{bmatrix}-2 & 2 & -3\\ 2 & 1 & -6\\ -1 & -2 & 0\end{bmatrix}$ is diagonalizable. Hence , find P such that $P^{-1}AP$ is a diagonal matrix.
11.	a) Show that the matrix $A=\begin{bmatrix}1 & 1 & i\\ 1 & 0 & i\\ -i & -i & 1\end{bmatrix}$ is diagonalizable. Also, find the eigenvectors of A . b) Show that the matrix $A=\begin{bmatrix}0 & i & i\\ i & 0 & i\\ i & i & 0\end{bmatrix}$ is diagonalizable. Also, find the eigenvectors of A .
12.	a) Find the matrix A, if the eigenvectors of a 3×3 matrix A corresponding to eigenvalues 1,1,3 are $[1,0,-1]^T$, $[0,1,-1]^T$ and $[1,1,0]^T$ respectively. b) Find the matrix A, whose eigenvalues are 1,1,1 and corresponding eigenvectors are $[-1,1,1]^T$, $[1,-1,1]^T$ and $[1,1,-1]^T$ respectively.
13.	a) Find a formula for A^n, given that $A=PDP^{-1}$, where $A=\begin{bmatrix}7 & 2\\ -4 & 1\end{bmatrix}$, $P=\begin{bmatrix}1 & 1\\ -1 & -2\end{bmatrix}$ and $D=\begin{bmatrix}5 & 0\\ 0 & 3\end{bmatrix}$. b) Compute A^n, given that $A=PDP^{-1}$, where $A=\begin{bmatrix}-2 & 12\\ -1 & 5\end{bmatrix}$, $P=\begin{bmatrix}3 & 4\\ 1 & 1\end{bmatrix}$ & $D=\begin{bmatrix}2 & 0\\ 0 & 1\end{bmatrix}$.
14.	a) Compute A^8, where $A=\begin{bmatrix}4 & -3\\ 2 & -1\end{bmatrix}$. b) Compute A^4, where $A=\begin{bmatrix}-3 & 12\\ -2 & 7\end{bmatrix}$.
15.	a) Given that A is symmetric matrix and $D=P^{-1}AP$, then show that P is an orthogonal matrix. b) Show that product of two orthogonal matrix of the same order is also an orthogonal matrix.

16.	a) Find the conditions that a matrix $A = \begin{bmatrix} l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \\ l_3 & m_3 & n_3 \end{bmatrix}$ is an orthogonal matrix. b) Show that $\lvert A\rvert = \pm 1$, if A is an orthogonal matrix.
17.	a) Find the symmetric matrix B for the quadratic form $Q = 2{x_1}^2 + {x_2}^2 + 3x_1x_2$. b) Find the symmetric matrix B for the quadratic form $Q = {x_1}^2 - 5{x_2}^2 + 4{x_3}^2 + 2x_1x_2 - 4x_1x_3 + 6x_2x_3$.
18.	a) Find the orthogonal transform which transforms the quadratic form $Q = {x_1}^2 + 3{x_2}^2 + 3{x_3}^2 - 2x_2x_3$ to canonical form. b) Find the orthogonal transform which transforms the quadratic form $Q = 3{x_1}^2 + 5{x_2}^2 + 3{x_3}^2 - 2x_2x_3 + 2x_1x_3 - 2x_1x_2$ to canonical form.
19.	a) Find the canonical form which transforms the quadratic form $Q = {x_1}^2 + 3{x_2}^2 + 3{x_3}^2 - 2x_2x_3$. b) Find the canonical form which transforms the quadratic form $Q = 17{x_1}^2 + 17{x_2}^2 - 30x_1x_2$.
20.	a) Find the canonical form which transforms the quadratic form $Q = 5{x_1}^2 + 26{x_2}^2 + 10{x_3}^2 + 4x_2x_3 + 6x_1x_2 + 14x_1x_3$. b) Find the canonical form which transforms the quadratic form $Q = 2{x_1}^2 + 2{x_2}^2 + 2x_1x_2$.