COURSE : MATHEMATICS FOR COMPUTER ENGINEERS

COURSE CODE : 21MAT31A

MODULE – 4 : PROBABILITY DISTRIBUTIONS

Question Bank

Q.No	Questions
1.	a) The Random Variable X has the following probability mass function, find (i) k (ii) P(X<3) (iii) P(3<X≤ 5) (iv) variance <table><tr><td>X</td><td>0</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td></tr><tr><td>P(X)</td><td>K</td><td>3K</td><td>5K</td><td>7K</td><td>9K</td><td>11K</td></tr></table> b) A random variable X has the following probability distributions. Find (i) k (ii) p(x<6) (iii) p(x>6) (iv) Mean and also find the probability distribution <table><tr><td>x</td><td>0</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td></tr><tr><td>P(x)</td><td>0</td><td>K</td><td>2k</td><td>2k</td><td>3k</td><td>k^2</td><td>$2k^2$</td><td>$7k^2 + k$</td></tr></table>
2.	a) Find 'k' such that the following distribution represents a finite probability distribution. Hence find (i)mean (ii) p(x≤1) (iii) p(x>1) (iv) p(-1<x≤2) <table><tr><td>X</td><td>-3</td><td>-2</td><td>-1</td><td>0</td><td>1</td><td>2</td><td>3</td></tr><tr><td>P(x)</td><td>k</td><td>2k</td><td>3k</td><td>4k</td><td>3k</td><td>2k</td><td>k</td></tr></table> b) A random variable (X=x) has the following probability function for various values of x. Find (i)k (ii) p(x<1) (iii) p(x>-1) <table><tr><td>X</td><td>-2</td><td>-1</td><td>0</td><td>1</td><td>2</td><td>3</td></tr><tr><td>P(x)</td><td>0.1</td><td>k</td><td>0.2</td><td>2k</td><td>0.3</td><td>k</td></tr></table>
3.	a) Find the mean and variance of geometric distributions. b) 3% of the product produced by a machine is found to be defective. Find the probability that first defective occurs in the (i) 5th item inspected (ii) first five inspected (iii) mean (iv) variance.
4.	a) What is the probability that the marketing representative must select (i) more than 6 people (ii) six people, before he finds one who attended the last home? Given 1-p=0.8. b) If the probability that a target is destroyed on any one shot is 0.5. What is the probability that it would be destroyed on (i) 6th attempt (ii) more than 6 attempt (iii) Mean (iv) variance?
5.	a) A weighted coin when tossed has a 0.9 chance of turning head . what is the probability that the first time the coin lands on head is after the 3rd flip. b) A certain basket ball player has a 65% chance of making a free throw. Assume all the free throws are independent. What is the probability that

	(i) he makes his first free throw on the 3rd try (ii) it takes him 2 or 3 tries to make his first free throw.
6.	a) Derive mean and variance for the Poisson Distribution b) 2% of the fuses manufactured by a firm are found to be defective. Find the probability that a box containing 200 fuses which contains (i) No defective fuse (ii) 3 or more defective fuses (iii) atleast one defective fuse

7.	a) A certain screw making machines produces on an average two defective out of 100 and pack them in boxes of 500. Find the probability that the box contains (i)three defectives (ii) At least one defective.
	
	b) The number of industrial injuries per working week in a particular factory is known to follow a Poisson distribution with mean 0.5. Find the probability that
	
	(a) in a particular week there will be:
	   (i) less than 2 accidents,
	   (ii) more than 2 accidents;
	
	(b) in a three week period there will be no accidents.

8.	a) The number of misprints on a page of the Daily Mercury has a Poisson distribution with mean 1.2. Find the probability that the number of errors (i) on page four is 2 (ii) on page three is less than 3.
	
	b) A shop sells a particular make of video recorder. Assuming that the weekly demand for the video recorder is a Poisson variable with mean 3, find the probability that the shop sells
	   (i) at least 3 in a week,
	   (ii) at most 7 in a week.

9.	a) The number of runs scored by Ali in an innings of a cricket match is distributed according to a Poisson distribution with mean 4.5. Find the probability that he will score
	   (i) exactly 4 in his next innings
	   (ii) at least three in his next innings.
	
	b) The number of parasites on fish hatched in the same season and living in the same pond follows a Poisson distribution with mean 3.6. Find, giving your answers to 3 decimal places, the probability that a fish selected at random will have
	   (a) 4 or less parasites, (b) exactly 2 parasites.

10.	a) The number of bacteria in one millilitre of a liquid is known to follow a Poisson distribution with mean 3. Find the probability that a 1 ml sample will contain no bacteria. If 100 samples are taken, find the probability that at most ten will contain no bacteria.
	
	b) A van hire firm has twelve vehicles available and has found that demand follows a Poisson distribution with mean 9.5. In a month of 25 working days, on how many days would you expect:
	   (a) demand to exceed supply;
	   (b) all vehicles to be idle;

	(c) it to be possible to service 3 of the vans?
11.	a) Wireless sets are manufactured with 25 soldered joints each. On the average 1 joint in 500 is defective. How many sets can be expected to be free from defective joints in a consignment of 10,000 sets.
	b) A continuous random variable has the following density function P(x)=$\begin{cases} kx^2, -3 \le x \le 3 \\ 0 \;\; elsewhere \end{cases}$ Evaluate (i) k, (ii) P(1$\le x \le$ 2), (iii) P($x \le$ 2), (iv) P(x>1).
12.	a) A continuous random variable has the density function $f(x) = \frac{k}{1+x^2}$, $-\infty < x < \infty$. Determine (i) k, (ii) $P(x > 0)$, $(iii)P(0 < x < 1)$.
	b) The probability density function of continuous random variable is given by $(x) = ke^{-|x|}$, $-\infty < x < \infty$. Prove that $k = {1}/{2}$ and also find mean and variance.
13.	a) The probability density function f(x) of continuous random variable is given by f$(x) = \begin{cases} kx(1 - x), 0 \le x \le 1 \\ 0 \;\;\;\;\;\; otherwise \end{cases}$ Determine k, $P(0 < x < 1/3)$.
	b) Derive mean and S.D for the Exponential Distribution.
14.	a) In a certain town the duration of a shower is exponentially distributed with mean 5min. what is the probability that the shower will last for (i) 10min or more (ii) less than 10min (iii) between 10 to 12min.
	b) The life of a compressor manufactured by a company is known to be 200 months on an average following an exponential distribution. Find the probability that the life of a compressor of that company is (i) < 200 months, (ii) between 100 months and 25 years.
15.	a) Studies of a single-machine-tool system showed that the time the machine operates before breaking down is exponentially distributed with a mean 10 hours. 1. Find the probability that the machine operates for at least 12 hours before breaking down. 2. Find the probability that the machine operates between 8 to 12 hours?
	b) The sale per day in a shop is exponentially distributed with an average sale amounting to Rs.100 and net profit is 8%. Find the probability that the net profit exceeds Rs. 30 on a day.
16.	a) The length of telephone conversation in a booth has been exponentially distributed and found on an average to be 5min. find the probability that a random call made from this booth (i) ends less than 5min (ii) between 5 and 10 min.
	b) If x is an exponential variate with mean 3. Find (i) P(x)>1 (ii) P(x)<3.
17.	a) Derive mean and variance for normal distribution.
	b) In examination 7% of students score less than 35% marks and 89% of students score less than 60% marks, Find the mean and standard deviation, if the marks are normally distributed. It is given that if p (z) =$\frac{1}{\sqrt{2\pi}} \int_0^z e^{-z^2/2} \, dz$ then p(1.2263) = 0.39 p(1.4757) = 0.43.
18.	a) A sales tax officer has reported that the average sales of the 500 business that he has to deal with during a year is Rs.36,000 with a standard deviation of 10,000. Assuming that the sales in these business are normally distributed, find (i) the number of business as the sales of which exceed Rs.40,000 (ii) the percentage of business the sales of which are likely to range between Rs.30,000 and Rs. 40,000. [P(0.4)=0.1554, P(0.6)=0.2257]
	b) A manufacturer knows from experience that the resistance of resistors he produces is normal

	with mean 100 ohms and SD 2 ohms. What percentage of resistors will have resistance between 98 ohms and 102 ohms?[P(1)=0.3413].
19.	a) A sample of 100 dry battery cells tested to find the length of life produced by a company and following results are recorded: mean life is 12 hrs, SD is 3 hrs. Assuming data to be normally distributed, find the expected life of a dry cell. (i) have more than 15 hrs (ii) between 10 and 14 hrs.[P(0.667)=0.2486,P(1)=0.3413]. b) The mean weight of 1,000 students during medical examination was found to be 70kg and S.D weight 6kg. Assume that the weight are normally distributed, find the number of students having weight (i) less than 65kg (ii) more than 75kg (iii) between 65kg to 75kg.[P(0.83)=0.2967]
20.	a) Given that the mean height of students in a class is 158cms with SD of 20cms. Find how many students heights lie between 150cms and 170cms, if there are 100 students in the class.[P(0.4)=0.1554, P(0.6)=0.2257] b) Suppose 2% of the people on the average are left handed. Find (i) the probability of finding 3 or more left handed (ii) the probability of finding ≤1 left handed.