Combinatorial optimization: Number partitioning problem

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Introduction. Problem to be solved

- Given a set of numbers $\{d_1, \dots, d_n\}$ of even size

The objective is to divide the numbers into two groups of equal size, in such a way that the absolute value of the difference between the product of the numbers of each group is minimized.

Example: $\{1,2,3,4\}$ best solution $\{2,3\}$ and $\{1,4\}$ $6 - 4 = 2$

Mathematical description

Data: $d = \{\{d_1, ..., d_n\} \,|\, d_i \in \mathbb{R}\, \forall i = 1, ..., n\}$

Solution codification (uniqueness): $x = \{(x_1, ..., x_n) \,|\, x_i \in \{0,1\}\, \forall i = 1, ..., n\}$

Search space: $\Omega = \{(x_1, ..., x_n) \,|\, x_i \in \{0,1\}\, \forall i = 1, ..., n\; and\; \sum_{i=1}^{n} x_i = \frac{n}{2}\}$

Search space size: $\binom{n}{\frac{n}{2}}$

Objective function: $f(x) = \left|\prod_{i=1}^{n} d_i^{x_i} - \prod_{i=1}^{n} d_i^{1-x_i}\right|$

Local searches algorithm

Definition of two neighborhoods:

- 2-opt: $N = \{(x'_1, ..., x'_n) \mid x'_i \in \{0,1\}\, and\, \sum_{i=1}^{n} x_i \oplus x'_i = 2\, and\, \sum_{i=1}^{n} x'_i = \frac{n}{2}\}$ Size: $(\frac{n}{2})^2$
- 4-opt: $N = \{(x'_1, ..., x'_n) \mid x_i \in \{0,1\}\, and\, \sum_{i=1}^{n} x_i \oplus x'_i = 4\, and\, \sum_{i=1}^{n} x'_i = \frac{n}{2}\}$ Size: $\binom{\frac{n}{2}}{2}^2$

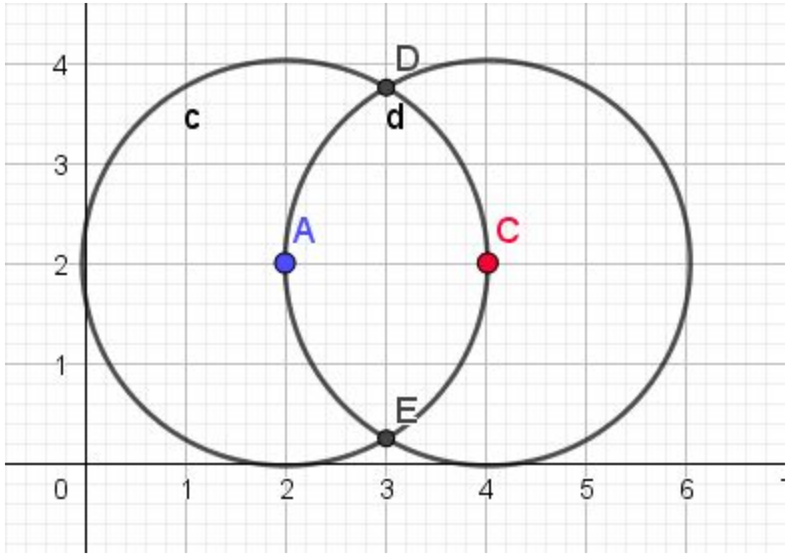
Two steps:

- Adaptively modify the neighborhood system
- Random multistart

Distence between local optima

- Distance: XOR of the vectors (properties of non-negativity, identity, symmetry and the triangle inequality)
- n= 50 ->20-30 units away from each other
- n= 100 are 40-50 units away.
- If we assume that this hypothesis is true it could be a useful idea to find new local optima or delimit the number of local optima

Example with 2-dim and restriction



N-dimensions-> N+1 solutions