Assignment 1 (due in week 6): Preference, Risk Aversion, MPT, CAPM

Due: Friday, 2 April 2021, 9:00 pm Submit your solutions via the course's Moodle page.

3. Portfolio theory with matrix algebra:
   To solve this question you should first study closely the file and code on portfolio algebra/matrix algebra that I have uploaded in "Additional Materials" on Moodle.

   Go to Moodle and download the data for `PS1 Daily.xlsx`. This file contains two worksheets. HPR Daily contains the daily holding period returns for the Microsoft, Exxon Mobil, General Electric, JP Morgan Chase, Intel, and Citigroup, the S&P 500 Composite Index and the value-weighted market portfolio (including dividends) from CRSP. Prices Daily contains the prices for the six stocks and the S&P 500 Composite Index.

   - Calculate the mean, the variance and the pairwise covariances for the three stocks `MSFT`, `GE`, and `JPM` for the sample period between 2/1/1990 and 31/12/2002.
   - Define the following matrices that contain returns, expected returns, portfolio weights, and co-variances:

$$\mathbf{R} = \begin{pmatrix} r_{MSFT} \\ r_{GE} \\ r_{JPM} \end{pmatrix} \qquad \mu = \begin{pmatrix} \mu_{MSFT} \\ \mu_{GE} \\ \mu_{JPM} \end{pmatrix} \qquad \mathbf{x} = \begin{pmatrix} x_{MSFT} \\ x_{GE} \\ x_{JPM} \end{pmatrix}$$
$$\Sigma = \begin{pmatrix} \sigma^2_{MSFT} & \sigma_{MSFT,GE} & \sigma_{MSFT,JPM} \\ \sigma_{GE,MSFT} & \sigma^2_{GE} & \sigma_{GE,JPM} \\ \sigma_{JPM,MSFT} & \sigma_{JPM,GE} & \sigma^2_{PM} \end{pmatrix}$$

- Note that the expected portfolio return and variance equal:

$$\mu_p = \mathbf{x'}\mu$$
$$\sigma_p^2 = \mathbf{x'}\Sigma\mathbf{x}$$

  Further, the condition that the portfolio weights have to sum up to one can be expressed as $\mathbf{x'1} = 1$.

- Calculate the return and standard deviation of a portfolio where you equal-weight the three stocks - call the portfolio e. Additionally, consider a portfolio y with a weight vector $\mathbf{y'} = (0.8, 0.4, -0.2)$. Calculate the risk-return tradeoff of y as well as its covariance with portfolio e.

- In order to find the global minimum variance portfolio with weights $\mathbf{m'} = (m_{MSFT}, m_{GE}, m_{JPM})$, we have to solve the following problem:

$$\min_m \sigma_{p,m}^2 = \mathbf{m'}\Sigma\mathbf{m} \qquad \text{s.t.} \qquad \mathbf{m'1} = 1$$

  The corresponding first order conditions are (check this by hand!)

$$\begin{pmatrix} 2\Sigma & \mathbf{1} \\ \mathbf{1}' & 0 \end{pmatrix} \begin{pmatrix} \mathbf{m} \\ \lambda \end{pmatrix} = \begin{pmatrix} \mathbf{0} \\ 1 \end{pmatrix}$$

  Hence, the system is of the form

$$\mathbf{A}_m \mathbf{z}_m = \mathbf{b}$$

  and the solution for $\mathbf{z}_m$ is then

$$\mathbf{z}_m = \mathbf{A}_m^{-1}\mathbf{b}$$

  The first three elements of $\mathbf{z}_m$ are the portfolio weights $\mathbf{m'} = (m_{MSFT}, m_{GE}, m_{JPM})$ for the global minimum variance portfolio. Calculate the variance and the expected return of the minimum variance portfolio.

- Find another efficient portfolio. Namely, the efficient portfolio that gives a return equal to the expected return of `MSFT`. Note, that your minimization problem now becomes:

$$\min_x \sigma_{p,x}^2 = \mathbf{x'}\Sigma\mathbf{x} \qquad \text{s.t.} \qquad \mathbf{x'1} = 1 \qquad \text{and} \qquad \mu_p = \mathbf{x'}\mu = \mu_{MSFT}$$

  Derive the solution as above in terms of portfolio weights and calculate them in your code. In addition, calculate the expected return and the variance of efficient portfolio x as well as its covariance with the global minimum portfolio.

- Plot the entire efficient frontier for the three risky assets.

- Now, rerun your code with sample moments for the three stocks `MSFT`, `GE`, and `JPM` for the sample period between 2/1/2003 and 31/12/2014.

- Finally, compare your results for the three assets across the two sample periods. Comment on potential problems that might arise when you based investment decisions on your analysis. Also discuss potential solutions to the problems mentioned.