Human vs Zombie Population Model

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1 Abstract

Nowadays, zombies are popular in the movie and television industry. This paper will assume an outbreak of zombie virus and people will turn to zombies if infected by the virus. In the paper we will model the population of human and zombie in different time spans. We divide the time span into three periods: outbreak period, coexistence period, and after vaccine period. And our goal is to determine the lastest time of the invention of the vaccine in order to prevent the extinction of human being.

2 Introduction

Demonstrated by numerous movies and TV series, a zombie virus break-out is a very famous way to the end of the world. Usually after the initial break-out, zombies will soon take over the entire world, there really isnt much for the humans to do other than run, unless before the end of the world, an anti-virus was successfully developed so that infected humans can be cured. Our goal is to mathematically model the situation of a zombie attack, and to simulate this situation using Matlab in order to find the latest time at which the anti-virus can be developed in order to avoid human extinction.
There are roughly two kinds of zombie attacks. The first kind is like the movie Resident Evil, where there is someone who is immune to the virus and become inhuman because of it. The other kind is like The Walking Dead, in which human just fight zombies with human strength. Therefore, to simplify our model and to remove any heroic complex, we decided that all the humans in our model only fight zombie (no bad guys), and we model their fight using a modified Lanchesters combat model.

3 Assumption

- Every human is able to fight zombies, regardless of gender and age.
- One person can only fight one zombie.
- Birth rate is constant during each time period.
- During the outbreak period, the population of huamn decreases exponentially and population of zombie increases exponentially.
- Before the existence of vaccine, humans bitten or scratched by zombies will turn into zombies.
- After the invention of vaccine, humans are immune to zombie virus.
- The distribution of vaccine is instant.
- A proportion of bodies caused by natural death will turn into zombies.

4 Model

4.1 Variables

S	human population
Z	zombie population
N	net growth rate per unit of human/zombie population
SZ	human-zombie interaction
a	natural death rate
b	birth rate
c	rate of turning into zombie when die naturally
α	zombie death rate during the battle
β	human death rate during the battle
γ	human infection rate during battles(dead in phase 2, cured in phase 3); also zombie death rate during the battle

4.2 Mathematical Model

This model is inspired by Lanchester's combat model

$$\frac{dx}{dt} = -ax - by + P(t)$$

where

$$x(t) : \text{number of soldiers in force x}$$

$$y(t) : \text{number of soldiers in force y}$$

We sperate the whole time span into three periods, call them phase 1, phase 2 and phase 3.

4.2.1 Phase 1

We apply the logistic growth model here.

$$\frac{dS}{dt} = -NS$$

$$\frac{dZ}{dt} = NZ$$

The reason we apply the logistic growth model here is because during the outbreak period, people would panic and do not know what to do. Thus they will be attacked by zombies, and the human population will decrease exponentially and the zombie population will increase exponentially. In addition, we will restrict the time span of phase 1 to be small, in order to make it realistic.

4.2.2 Phase 2

In phase 2, we assume that birth rate is constant but very small, and during the battle, a proportion of zombies(α) are killed, a proportion of people(β) are killed and thus being turned into zombies, and a proportion of people(γ) are infected. In addtion, we assume that a proportion(c) of dead bodies will turn into zombies. Therefore, the model is given by

$$\frac{dS}{dt} = b - aS - \beta SZ - \gamma SZ$$

$$\frac{dZ}{dt} = \beta SZ - \alpha SZ + caS$$

4.2.3 Phase 3

In phase 3, we take out the γSZ term because people will not turn to zombie after they are wounded if they are treated with the vaccine, and we assume every person has access to the vaccine.

$$\frac{dS}{dt} = b - aS - \beta SZ$$

$$\frac{dZ}{dt} = \beta SZ - \alpha SZ - \gamma SZ + caS$$

4.3 Equilibrium Point and Stability

4.3.1 Phase 1

Set the two differention equations equal to 0, we have

$$f(S,Z) = \frac{dS}{dt} = -NS = 0$$

$$g(S,Z) = \frac{dZ}{dt} = NZ = 0$$

thus the equilibrium point is $(0,0)$. At (0,0),

$$A = \begin{bmatrix} f_S & f_Z \\ g_S & g_Z \end{bmatrix} = \begin{bmatrix} -N & 0 \\ 0 & N \end{bmatrix}$$

det(A) $= -N^2 < 0$, therefore it is unstable.

4.3.2 Phase 2

In phase 2, we assume that the birth rate is extremly small, i.e.,$b \approx 0$
Thus set

$$f(S,Z) = \frac{dS}{dt} = -aS - \beta SZ - \gamma SZ = 0$$

$$g(S, Z) = \frac{dZ}{dt} = \beta SZ - \alpha SZ + caS = 0$$

Therefore the equilibrium point is $(0, Z)$, where $Z \in \mathbb{R}_+$.
At $(0, Z)$,

$$A = \begin{bmatrix} f_S & f_Z \\ g_S & g_Z \end{bmatrix} = \begin{bmatrix} -a - (\beta + \gamma)Z & 0 \\ (\beta - \alpha)Z + ca & 0 \end{bmatrix}$$

det(A)=0, Tr(A) $= -a - (\beta + \gamma)Z < 0$.

When the determinant is zero and the trace is negative, the equilibrium point is nuetrally stable. Intuitively it makes sense since zombies will not die of natural cause; without the existence of human, the population of zombie will reamin unchanged.