Challenge problems

Physics Discord

24-04-2018

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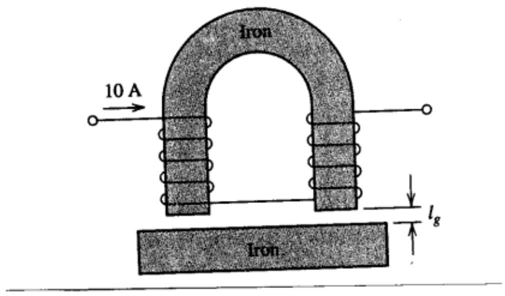
1) In the Rutherford gold foil experiment, an alpha particle with initial kinetic energy k MeV is shot directly at an $^{197}_{79}$Au nucleus. Find the distance of closest approach, assuming classical mechanics apply.

2) Consider the a wire carrying current of 10 A that wraps around an iron core as shown, with both sides having an equal number of turns around the core. There is a second iron bar underneath the core, a distance l_g m away from the iron core. The effective area of the air gap is $A\,\mathrm{m}^2$. Determine the number of turns needed to produce a flux of F T in the gaps.



3) A raindrop falls through a cloud made of suspended water droplets which are small, suspended in air, and approximately evenly distributed throughout the cloud. As the raindrop falls through the droplets, the droplets' masses are added to the raindop. What is the acceleration of the raindrop? (The raindrop being spherical may be assumed)

Solutions

1) The original particle has the kinetic energy k. The distance of closest approach will be when the potential energy of the particle is equal to k. Potential energy is given by

$$\begin{aligned} E_p &= \frac{k_e q_1 q_2}{r} = k \\ r &= \frac{k_e q_1 q_2}{k} \\ &= \frac{k_e \cdot 2e \cdot 79e}{k} \end{aligned}$$

Where e is the amount of charge in a proton and k_e is Coulomb's constant

2) The system can be viewed as a magnetic circuit where the loops are sources of magnetomotive force $\mathcal{F}$, the core is a magnetic current carrying wire and the gaps are magnetic resistances over which the magnetic potentail drops.

$$\begin{aligned} \mathcal{F} &= NI \\ \mathcal{F} &= \mathcal{R}\phi \\ \mathcal{R} &= \frac{l}{\mu_0 A} \\ \phi &= BA \\ \therefore N &= \frac{\mathcal{F}}{I} \\ &= \frac{\mathcal{R}\phi}{I} \\ &= \frac{BAl}{\mu_0 AI} \\ &= \frac{Bl}{\mu_0 I} \\ &= \frac{2Fl_g}{10\mu_0} \\ &= \frac{Fl_g}{5\mu_0} \end{aligned}$$

3) Let ρ be the density of the raindrop and λ the average mass density of the droplets in space. Writing down the expression for change in mass

$$\begin{aligned} \dot{M} &= 4\pi r^2 \dot{r} \rho \\ &= 3M \frac{\dot{r}}{r} \end{aligned}$$

Since the change in mass is due to the gain in mass from the droplets

$$\dot{M} = \pi r^2 \nu \lambda$$

Since $F = \frac{dp}{dt}$ and $p = mv$,

$$Mg = \dot{M}\nu + M\dot{\nu}$$

Additionally,

$$\begin{aligned}
v &= \frac{\dot{M}}{\pi r^2 \nu \lambda} \\
&= \frac{4\rho \dot{r}}{\lambda} \\
\dot{v} &= \frac{4\rho \ddot{r}}{\lambda}
\end{aligned}$$

Using previous equations,

$$\begin{aligned}
Mg &= \left(3M\frac{\dot{r}}{r}\right) + \left(\frac{4\rho\dot{r}}{\lambda}\right) + M\left(\frac{4\rho\ddot{r}}{\lambda}\right) \\
\therefore \frac{g\lambda}{\rho} &= 12\dot{r}^2 + 4r\ddot{r}
\end{aligned}$$

Let us define h as $\frac{g\lambda}{\rho}$. By dimensional analysis, r must take the form

$$r = Aht^2$$

Where A is a constant. $r(t)$ can only depend on h and t. Plugging this expression for r gives us

$$\begin{aligned}
h(Aht^2) &= 12(2Aht)^2 + 4(Aht^2)(Ah) \\
\implies A &= 48A^2 + 8A^2 \\
\therefore A &= \frac{1}{56}
\end{aligned}$$

So $\ddot{r} = 2Ah = \frac{g\lambda}{28\rho}$. Since $\dot{v} = \frac{4\rho}{\lambda}\ddot{r}$,

$$\dot{v} = \frac{g}{7}$$