PERFORMANCE ANALYSIS OF SPATIAL MODULATION WITH EUCLIDEAN DISTANCE BASED SELECTION COMBINING

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by

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to

DEPARTMENT OF ELECTRONICS & COMMUNICATION ENGINEERING

INDIAN INSTITUTE OF INFORMATION TECHNOLOGY KOTTAYAM

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April 2025

DECLARATION

I, **Adusumilli Laahiri** (**Roll No: 2021BEC0015**), hereby declare that, this report entitled **"Performance Analysis of Spatial Modulation with Euclidean Distance based Selection Combining"** submitted to Indian Institute of Information Technology Kottayam towards partial requirement of **BTech** in **Department of Electronics & Communication Engineering** is an original work carried out by me under the supervision of **Dr. Ananth A** and has not formed the basis for the award of any degree or diploma, in this or any other institution or university. I have sincerely tried to uphold the academic ethics and honesty. Whenever an external information or statement or result is used then, that have been duly acknowledged and cited.

Kottayam-686635 [**Adusumilli Laahiri - 2021BEC0015**]

April 2025

CERTIFICATE

This is to certify that the work contained in this project report entitled **"Performance Analysis of Spatial Modulation with Euclidean Distance based Selection Combining"** submitted by **Adusumilli Laahiri** (**Roll No: 2021BEC0015**) to the Indian Institute of Information Technology Kottayam towards partial requirement of **BTech** in **Department of Electronics & Communication Engineering** has been carried out by her under my supervision and that it has not been submitted elsewhere for the award of any degree.

Kottayam-686635 (Dr. Ananth A)

April 2025 Project Supervisor

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ABSTRACT

Considering the projected rapid growth in fifth generation subscribers, as estimated by Huawei to reach 2.8 billion by 2025, this project is significant in addressing the few of evolving demands and challenges of wireless communication systems, like spectral efficiency and energy efficiency.

This project focuses on Bit Error Rate (BER) comparision using simulation of few wireless communication systems like single-input single-output, single-input multiple-output, multiple-input multiple-output systems, with emphasis on the impact of various factors like modulation order, number of transmit antennas and number of receiver antennas on BER. The project also involved examining few modulation schemes to identify one that strikes a desirable balance between energy efficiency and spectral efficiency. Simulations are performed using Matlab.

The BER performance analysis was extended to include Spatial Modulation (SM), SM with Euclidean Distance based Selection Combining (ED-SC), and SM with Euclidean Distance based Generalized Selection Combining (ED-GSC). The BER performance of all these systems were studied using MATLAB. Further the theoretical analysis of SM ED-SC for N_t=2, N_r=2,4 was performed using Mathematica and these results were validated using simulation results.

Contents

Declaration **ii**

Certificate **iii**

Acknowledgements **iv**

Abstract **v**

List of Figures **ix**

List of Tables **x**

Abbreviations **xi**

1 Introduction **1**

2 Literature Survey **3**

3 System Model **6**

3.1 Parameters 6

3.2 Single-Input Single-Output System 7

3.3 Single-Input Multiple-Output System 8
3.4 Multiple-Input Multiple-Output 9
3.5 Spatial Modulation 12
3.5.1 Performance comparision of MIMO with SMX and SM 14
3.6 Spatial Modulation with ED based Selection Combining . . . 15
3.6.1 Idea behind ED-SC for SM 16
3.6.2 Metric for SC 19

4 Performance Analysis of SM with ED-SC 24
4.0.1 SM ED-SC with N_t=2, N_r=2, with BPSK 24
4.0.2 PDF of RVs 28
4.0.3 Conditional PDFs 31
4.0.4 SM ED-SC with N_t=2, N_r=4, with BPSK 37

5 Algorithm 41

6 Simulation Results 44

7 Conclusion 58
7.1 Conclusion 58

References 60

List of Figures

3.1 SISO Block Diagram 7
3.2 SIMO Block Diagram 8
3.3 MIMO Block Diagram 9
3.4 MIMO with SMX Block Diagram 11
3.5 SM Block Diagram 12
3.6 MIMO SMX Block Diagram 14
3.7 SM Block Diagram 14
3.8 SM with ED-SC Block Diagram 16
3.9 The probability that a Gaussian random variable $X \sim N(\mu, \sigma^2)$ exceeds x_0 19
3.10 Selection of receiver antenna 21

6.1 BER performance of SISO (PSK and QAM) 44
6.2 BER performance of SIMO with $N_r = 4$ and different M . . . 45
6.3 BER performance of SIMO with $M = 8$ and different N_r . . . 46
6.4 BER performance of MIMO (same message streams from all transmit antennas) with $N_t = 4$, $M = 8$, different N_r 47

6.5 BER performance of SIMO and MIMO for different N_r ($N_t =$ 4, $M = 8$) 48
6.6 BER performance of MIMO with SMX with ($N_t = 3$, $M = 2$) and different N_r 49
6.7 BER performance of SM with ($N_t = 4$, $M = 2$) different N_r . . 50
6.8 BER performance for 3 bpcu for SISO, SIMO, MIMO, MIMO with SMX, SM 51
6.9 BER performance comparison between SM and SM with ED-SC with N_t=2, N_r=2, M=2 52
6.10 BER performance comparison between SM and SM with ED-SC with N_t=2, N_r=4, M=2 53
6.11 BER performance comparison between SM with ED-SC with N_r=2 and N_r=4 54
6.12 BER performance of SM with ED-GSC system with fixed N_t=2, N_r=4 and varying N_{rf} 55
6.13 BER performance of SM, SM with ED-SC, BPSK SC, with single RF chain 56
6.14 BER performance of Sm with ED-SC N_t=2, N_r=4 simulation and theory, SM with ED-GSC for fixed N_t=2, N_r=4 and varying N_{rf} from 2 to 4 and SM with ML N_t=2, N_r=4 57

List of Tables

3.1 Spectral Efficiency (SE) 15

Abbreviations

5G	**5**th **G**eneration
AS	**A**ntenna **S**election
AWGN	**A**dditive **W**hite **G**aussian **N**oise
BER	**B**it **E**rror **R**ate
bpcu	**b**its **p**er **c**hannel **u**se
ED	**E**uclidean **D**istance
ED-GSC	**E**uclidean **D**istance based **G**eneralized **S**election **C**ombining
EE	**E**nergy **E**fficiency
IAS	**I**nter-**A**ntenna **S**ynchronization
ICI	**I**nter-**C**hannel **I**nterference
LOS	**L**ine **o**f **S**ight
M	**M**odulation order
ML	**M**aximum **L**ikelihood
MIMO	**M**ultiple-**I**nput **M**ultiple-**O**utput
N_r	**N**umber of **r**eceiver antennas
N_t	**N**umber of **t**ransmitter antennas
N_{rf}	**N**umber of **RF** chains
PSK	**P**hase **S**hift **K**eying
QAM	**Q**uadrature **A**mplitude **M**odulation
RF	**R**adio **F**requency

SC	**S**election **C**ombining
SE	**S**pectral **E**fficiency
SIMO	**S**ingle-**I**nput **M**ultiple-**O**utput
SISO	**S**ingle-**I**nput **S**ingle-**O**utput
SM	**S**patial **M**odulation
SMX	**S**patial **M**ultiplexing
SNR	**S**ignal-to-**N**oise **R**atio

Chapter 1

Introduction

In the world of wireless communication systems, along with the efficient transmission of information, there are increasing demands for both Spectral Efficiency (SE) and Energy Efficiency (EE).

This work delves into the performance comparison of wireless communication systems, centering on the Bit Error Rate (BER) as a key performance metric. BER is the number of bits received in error to the total number of bits transmitted over a certain period of time and is influenced by various factors including modulation schemes, channel characteristics, and detection techniques.

The work begins by observing the BER performance of a Single-Input Single-Output (SISO) system, comparing the BER of Quadrature Amplitude Modulation (QAM) and Phase Shift Keying (PSK) schemes. Subsequently, the investigation extends to Single-Input Multiple-Output (SIMO) configurations, focusing on BER evaluation for PSK modulation. The work then advances to Multiple-Input Multiple-Output (MIMO) systems, initially exploring strategies to enhance reliability by transmitting identical message streams from all transmit antennas. Following this, it shifts towards increasing data rates through Spatial Multiplexing (SMX), where distinct message streams are sent from each transmit antenna.

In the final phase, the focus moves to the performance evaluation of Spatial Modulation (SM)—a technique that offers a compelling trade-off between SE and EE. This phase includes a detailed BER analysis of SM, SM with Selection Combining (ED-SC), and SM with Generalized SC (ED-GSC). The BER simulations were carried out using MATLAB, while theoretical BER performance of the ED-SC scheme was calculated using Mathematica for N_t=2, N_r=2,4 and validated using the simulation results.

This analysis highlights the practical advantages of combining SM with receive diversity techniques to reduce energy consumption and receiver complexity (by reducing number of RF chains used) by trading off reliability.

Chapter 2

Literature Survey

The study aimed to evaluate and compare Bit Error Rate (BER) for modulation schemes of M-ary PSK (M-PSK) and M-ary QAM (M-QAM) in Additive White Gaussian Noise (AWGN), Rayleigh Fading and Rician fading channels. It was concluded that M-QAM outperforms M-PSK in the Rayleigh fading channel, and this performance gap widens as the modulation order M increases [1].

High data rate can be achieved without increasing bandwidth by using multiple antennas on both transmitter and receiver side. It was proposed that the use of multiple antennas on receiver side can reduce the effect of fading. Though increasing M in M-ary digital modulation schemes can increase data rate, it also increases BER [2].

This survey paper reviews various requirements, available technologies and challenges in Fifth Generation (5G) and beyond networks. It was observed that SE and increased battery life of devices are two of the key objectives of 5G networks. Several SE improvement techniques were mentioned, which included SM. The paper goes on reviewing how massive MIMO improves SE, but also has some critical drawbacks like EE, channel estimation etc. [3].

The survey paper on SM reviews basic principles and several variants of SM such as Space Shift Keying (SSK), generalized SM and states the research progress made. It describes how SM is a promising technique to achieve both SE and EE. SM exploits the OFF/ON status of transmit antennas to transmit extra information in addition to information transmitted by usual digital modulation techniques [4].

An optimal detection scheme based on Maximum Likelihood (ML) principle whose complexity is similar to sub optimal schemes is proposed for SM in which both the antenna index and symbol transmitted are detected together. The resultant BER obtained using optimal detection is compared with that of the sub optimal scheme where detection of antenna index and transmitted symbols are detected separately and a gain of 4 dB is observed in optimal detection at BER=10^{-5} [5].

The energy efficiency of SM has been studied extensively in [6], where the total power consumption at the transmitter is quantified, considering factors beyond just RF transmit power. Comparisons with multi-RF chain MIMO architectures, such as Space-Time Block Coding (STBC), demonstrate that SM achieves linear power savings relative to the number of RF chains employed in other architectures.

Optimal detection schemes for SM are proposed in [7], and their performance has been analysed comprehensively in [8]–[10]. A channel estimation method for correlated fading channels is detailed in [11], which utilizes pilot signals from a single antenna to estimate the entire channel by leveraging channel difference information.

The performance of SM under imperfect Channel State Information (CSI) for flat Rayleigh fading channels is addressed in [12], where upper bounds on

Average Bit Error Probability (ABEP) are derived for M-PSK and M-QAM modulations.

Various transmit antenna selection schemes have been developed to enhance SM performance. These include Euclidean Distance (ED)-based methods [13]–[15] and pairwise error probability-based selection [16].

Diversity combining is often employed to mitigate the effects of multipath fading in MIMO systems. SC, which uses the branch with the highest instantaneous Signal-to-Noise Ratio (SNR), reduces receiver complexity but discards lower-SNR data.

While not the most efficient approach, generalized SC addresses this limitation by considering N_{rf} additional branches with decreasing SNR, balancing EE and BER performance. Comparative study on SC and maximum ratio combining is presented in [17], and the performance analyses for GSC under Rayleigh and Nakagami m fading channels can be found in [18]–[20].

SSK, a subset of SM, transmits information purely through antenna states. The performance of SSK systems with ED-based SC and generalized ED-SC at the receiver has been investigated in [21].

Chapter 3

System Model

3.1 Parameters

In wireless transmission scenarios, signals traverse a complex environment marked by reflections, diffraction, and scattering. The result of these complex interactions is the presence of many signal components, or multipath signals at the receiver. Rayleigh fading model can capture the worst-case scenario of wireless communication, where there is no line of sight component and the signal is highly variable and unpredictable.

AWGN exhibits characteristics of randomness, with a Gaussian probability distribution, affecting all frequency components of the signal equally mirroring nature's stochastic processes. It is called additive be the noise gets added with the transmitted signal.

All the systems modeled in this project consider Rayleigh fading channel and AWGN such that they have independent and identically distributed (i.i.d.) complex normal Random Variable (RV) entries ($\mathcal{CN}(0, 1)$)

3.2 Single-Input Single-Output System

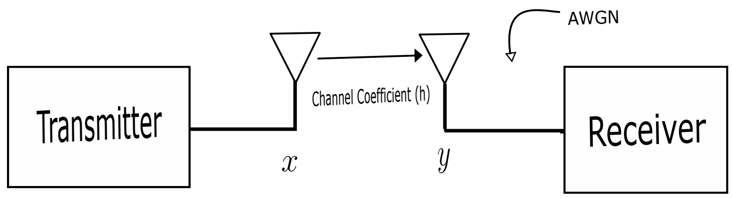

Figure 3.1: SISO Block Diagram

Fig. [3.1] shows the block diagram of SISO system with one transmit antenna and 1 receive antenna. The channel fading coefficient between these antennas is modeled on Rayleigh fading and the received signal is corrupted by AWGN.

Based on this the received baseband signal y can be represented as:

$$y = \sqrt{\rho} \cdot (hx) + n$$

where:

- y is the baseband received signal of dimensions 1×1.
- h is the channel fading coefficient
- x is the transmitted signal taken from M-PSK or M-QAM
- n is the AWGN
- ρ is the average Signal-to-Noise Ratio (SNR)

After the transmitted signal along with noise is received, the original signal can be detected using ML detection as follows :

$$\hat{x} = \arg\min_{x_i} |y - \sqrt{\rho} \cdot (hx_i)|^2$$

where:

- $\hat{x}$ is the estimated transmitted symbol
- $x_i \in \{x_1, x_2, \ldots, x_M\}$

- x_i is the i-th symbol in the constellation set
- $|\cdot|$ denotes the absolute operator

3.3 Single-Input Multiple-Output System

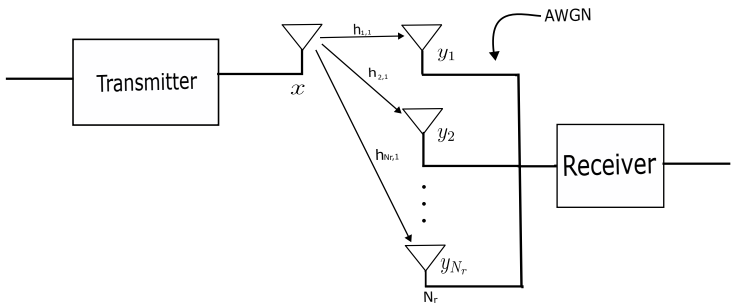

Figure 3.2: SIMO Block Diagram

Fig. [3.2] shows a block diagram of SIMO sytem consisting of a single transmit antenna and N_r number of receive antennas. The channel coefficients between these antennas is modeled on Rayleigh fading. The received signal is corruptedby AWGN.

Based on this the received baseband signal $\mathbf{y}$ can be represented as:

$$\mathbf{y} = \sqrt{\rho} \cdot (\mathbf{h}x) + \mathbf{n}$$

where:

- $\mathbf{y}$ is the received signal vector of size $N_r \times 1$
- $\mathbf{h}$ is the channel vector of size $N_r \times 1$
- x is the transmitted signal taken from M-PSK
- $\mathbf{n}$ is the AWGN vector of size $N_r \times 1$
- ρ is the SNR

After the transmitted signal along with noise is received, the original signal can be detected using ML detection as follows :

$$\hat{x} = \arg\min_{x_i} \|\mathbf{y} - \sqrt{\rho} \cdot (\mathbf{h}x_i)\|^2$$

where:

- $\hat{x}$ is the detected transmitted symbol
- $x_i \in \{x_1, x_2, \ldots, x_M\}$
- x_i is the i-th symbol in the constellation set
- $\| \cdot \|^2$ denotes the norm operator

3.4 Multiple-Input Multiple-Output

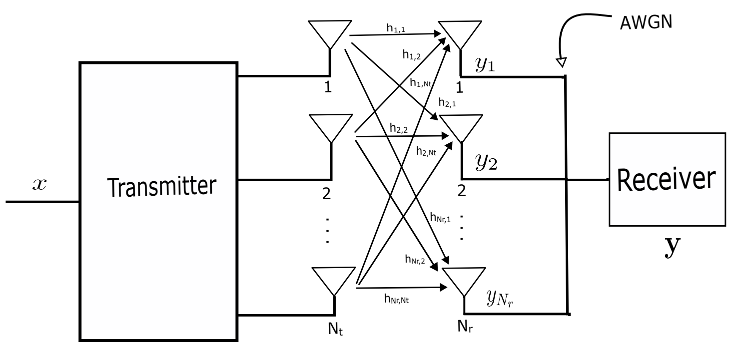

Figure 3.3: MIMO Block Diagram

Fig. [3.3] shows the block diagram of a MIMO system consisting of N_t number of transmit antenna and N_r number of receive antennas.The channel coefficients between these antennas is modeled on Rayleigh fading. The received signal is corrupted by AWGN. All the multiple transmit antennas transmit same message stream, which improves the reliability of the system.

Based on this the received baseband signal $\mathbf{y}$ can be represented as:

$$\mathbf{y} = \sqrt{\rho} \cdot (\mathbf{Hx}) + \mathbf{n}$$

where:

- $\mathbf{y}$ is the received signal vector of size $N_r \times 1$
- $\mathbf{H}$ is the channel matrix of size $N_r \times N_t$
- $\mathbf{x}$ is the transmitted signal vector of size $N_t \times 1$ with all elements being the same
- $\mathbf{n}$ is the AWGN vector of size $N_r \times 1$
- ρ is the average SNR

After the transmitted signal along with noise is received, the orginal signal can be detected using ML detection as follows :

$$\hat{\mathbf{x}} = \arg\min_{\mathbf{x_{ref}}} \|\mathbf{y} - \sqrt{\rho} \cdot (\mathbf{H}\mathbf{x_{ref}})\|^2$$

where:

- $\hat{\mathbf{x}}$ is the estimated transmitted symbol vector
- $\mathbf{x_{ref}}$ is the vector of reference constellation points
- $\| \cdot \|^2$ denotes the norm operator

Fig. [3.4] shows a block diagram of a MIMO system with SMX, consisting of N_t number of transmit antennas and N_r number of receive antennas. The channel coefficients between these antennas is modeled on Rayleigh fading. The received signal is corruptedby AWGN. All the transmit antenna transmit different message streams, which improves the data rate of the system.

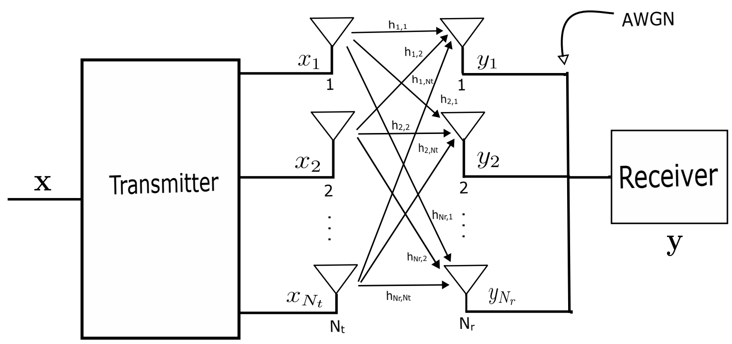

Figure 3.4: MIMO with SMX Block Diagram

Based on this the received baseband signal $\mathbf{y}$ can be represented as:

$$\mathbf{y} = \sqrt{\rho} \cdot (\mathbf{Hx}) + \mathbf{n}$$

where:

- $\mathbf{y}$ is the received signal vector of size $N_r \times 1$
- $\mathbf{H}$ is the channel matrix of size $N_r \times N_t$
- $\mathbf{x}$ is the transmitted signal vector of size $N_t \times 1$
  $\mathbf{x} = [x_1 \;\; x_2 \;\; \cdots \;\; x_{N_t}]^T$ where $x_1, x_2, \cdots, x_{N_t}$ are independent of each other.
- $\mathbf{n}$ is the AWGN vector of size $N_r \times 1$
- ρ is the received average SNR.

After the transmitted signal along with noise is received, the original signal can be detected using ML detection as follows :

$$\hat{\mathbf{x}} = \arg\min_{\mathbf{x_{ref}}} \|\mathbf{y} - \sqrt{\rho} \cdot (\mathbf{Hx_{ref}})\|^2$$

where:

- $\hat{\mathbf{x}}$ is the estimated transmitted symbol vector

- $\mathbf{x_{ref}}$ is the vector of reference constellation points
- $\|\cdot\|^2$ denotes the norm operator

3.5 Spatial Modulation

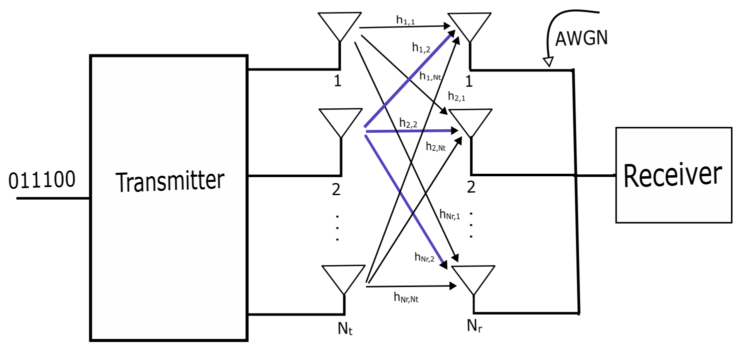

Figure 3.5: SM Block Diagram

Fig. [3.5] shows the block diagram of SM system with N_t number of transmit antennas and N_r number of receive antennas.The channel coefficients between these antennas is modeled on Rayleigh fading. The received signal is corruptedby AWGN.

$\log_2(N_t) + \log_2(M)$ bits can be transmitted for a single channel use, where M is modulation order of the system. The bits that are to be transmitted are split into $\log_2(N_t)$ and $\log_2(M)$ for transmission by antenna index and by modulation technique, respectively. Based on the bits for transmission through antenna index, a single antenna from array of transmit antennas is chosen to be active. Bits for transmission through modulation techniques are transmitted by M-PSK.

For example, let the stream of bits to be transmitted is 011100. Let's assume the case of N_t=4, M=2, N_r=4. Now a total of 3 bits can be sent in a single channel use. So 011 will be transmitted in 1^{st} instance. 1^{st} The bits 011 are split into 01, 1 where 01 is transmitted by activating 2^{nd} antenna and 1 is sent using BPSK.

Based on this the received baseband signal $\mathbf{y}$ can be represented as:

$$\mathbf{y} = \sqrt{\rho} \cdot (\mathbf{Hx}) + \mathbf{n}$$

where:

- $\mathbf{y}$ is the received signal vector of size $N_r \times 1$
- $\mathbf{H}$ is the channel matrix of size $N_r \times N_t$
- $\mathbf{x}$ is the transmitted signal vector of size $N_t \times 1$
  $\mathbf{x} = [x_1 \ \ x_2 \ \ \cdots \ \ x_{N_t}]^T$ among all the elements of the transmit signal vector $\mathbf{x}$, only one of the element will be non-zero indicating the active transmit antenna.
- $\mathbf{n}$ is the AWGN vector of size $N_r \times 1$
- ρ is the average SNR

After the transmitted signal along with noise is received, the original signal can be detected using ML detection as follows :

$$[\hat{j}_{\mathrm{ML}}, \hat{x}_{\mathrm{q}}] = \arg\min_{\mathbf{h}_j, x_{\mathrm{q}}} \|\mathbf{y} - \sqrt{\rho} \cdot (\mathbf{h}_j x_q)\|^2$$

where:

- $\hat{j}_{\mathrm{ML}}$ is the estimated antenna index
- $\hat{x}_{\mathrm{q}}$ is the estimated symbol index, where x_q is the reference constellation
- $\mathbf{h}_j$ is the channel vector when the jth antenna is active, $j \in \{1, 2, \ldots, N_t\}$

3.5.1 Performance comparision of MIMO with SMX and SM

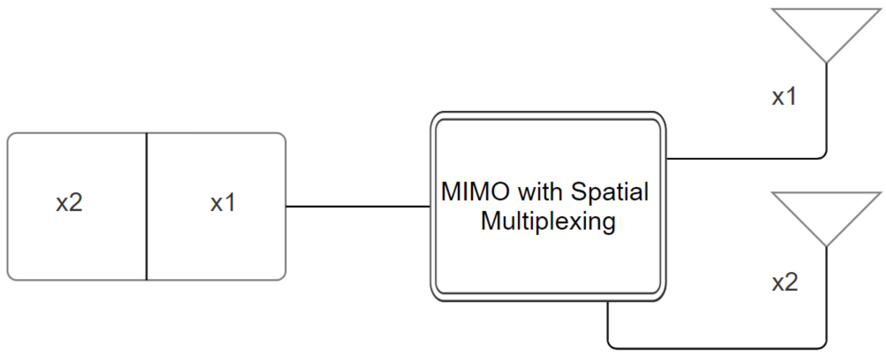

Figure 3.6: MIMO SMX Block Diagram

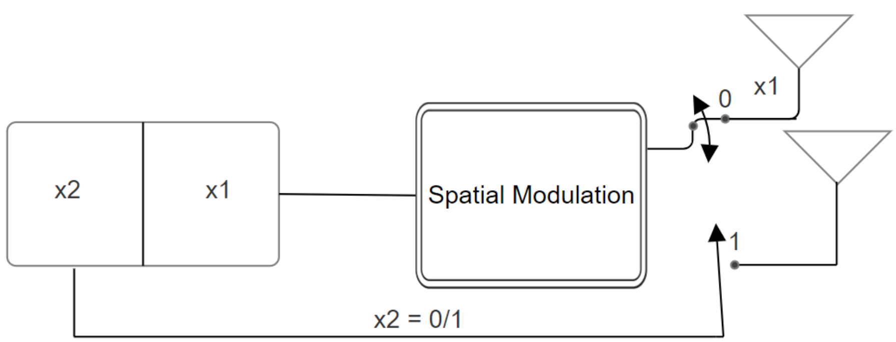

Figure 3.7: SM Block Diagram

Fig. [3.6] shows a block diagram depicting how a stream of information is transmitted by MIMO with SMX system. Fig. [3.7] shows a block diagram depicting how a stream of information is transmitted by SM system. The information bits are getting divided into two parts x_1 and x_2 and are transmitted through 2 antennas in MIMO with SMX. The information bits are getting divided into two parts x_1 and x_2 and x_1 is mapped to BPSK and x_2 is transmitted as the indexof active transmit antenna.

	Spectral Efficiency (bpcu)
SIMO	$\log_2(M)$
MIMO with SMX	$N_t * \log_2(M)$
SM	$\log_2(N_t) + \log_2(M)$

Table 3.1: Spectral Efficiency (SE)

From the table [3.1], SE is highest for MIMO with SMX followed by SM. Due to the activation of a single transmit antenna at any given time in SM, there are many advantages. (i. There is no requirement for inter antenna synchronization (IAS) or multiple RF chains, making the system less expensive to implement. ii. There is absence of inter channel interference (ICI). iii. The system becomes more energy efficient due to requirement of less RF chains at transmitter.)

3.6 Spatial Modulation with ED based Selection Combining

Fig. [3.8] shows block diagram of SM with ED-SC, consisting of N_t number of transmit antennas and N_r number of receive antennaa and 1 single RF chain on the receiver side.

In the SM with ED-SC scheme, instead of jointly processing all receive antennas, selection combining method is used at the receiver to choose a single receive antenna for detection. The best receive antenna is selected based on the minimum Euclidean distance criterion. This reduces receiver complexity while still leveraging the diversity gain provided by multiple receive antennas.

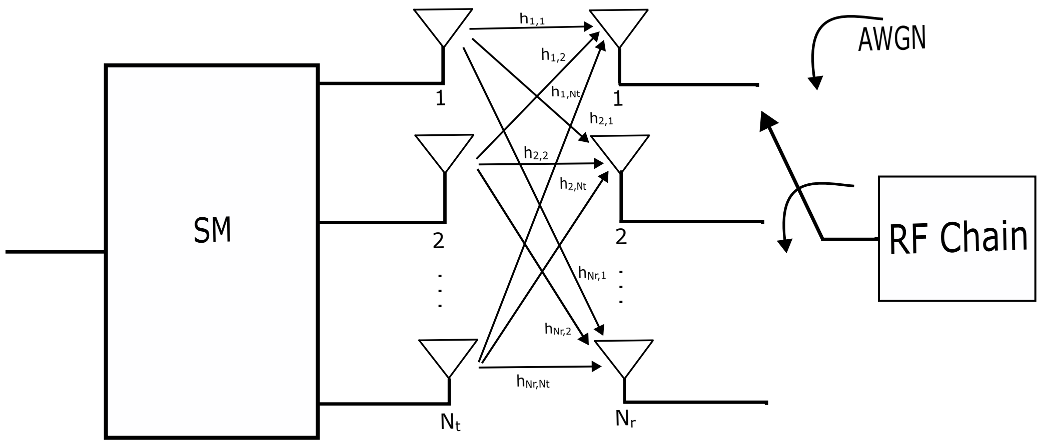

Figure 3.8: SM with ED-SC Block Diagram

3.6.1 Idea behind ED-SC for SM

The Instantaneous Bit Error Probability for SM is derived using the Pairwise Error Probability (PEP) and union bound techniques.

Let the PEP for SM be represented as $P(j, x_q \to i, x_p)$, where:

- j is the index of the active transmit antenna,
- x_q is the transmitted symbol,
- i is the detected antenna index at the receiver,
- x_p is the detected symbol.

The PEP $P(j, x_q \to i, x_p)$ denotes the probability of detecting i, x_p when j, x_q was actually transmitted. The baseband signal for SM is as follows:

$$\mathbf{y} = \sqrt{\rho} \cdot (\mathbf{Hx}) + \mathbf{n}$$

The ML detection rule for SM is:

$$[j_{ML}, \hat{x}_{\mathrm{q}}] = \arg\min_{\mathbf{j}, x_{\mathrm{q}}} \|\mathbf{y} - \sqrt{\rho} \cdot (\mathbf{h}_j x_{\mathrm{ref}})\|^2$$

The PEP $P(j, x_q \to i, x_p)$ can be written conditioned on the transmission of j, x_q as

$$P(j, x_q \to i, x_p) = P(j, x_q \to i, x_p \mid j, x_q)$$

Using the union bound, the bit error probability is:

$$P_e \leq \sum_{q=1}^{M} \sum_{p=1}^{M} \sum_{i=1}^{N_t} \sum_{j=1}^{N_t} \frac{N((j, x_q), (i, x_p)) P(j, x_q \to i, x_p)}{N_t M(\log_2 N_t M)}$$

We derive the PEP as:

$$P(j, x_q \to i, x_p) = P\left(\|\mathbf{y} - \sqrt{\rho} \cdot (\mathbf{h}_j x_q)\|^2 > \|\mathbf{y} - \sqrt{\rho} \cdot (\mathbf{h}_i x_p)\|^2\right) \tag{3.1}$$

$$= P\left(\|n\|^2 > \rho \|\mathbf{h}_j x_q - \mathbf{h}_i x_p\|^2 + \|n\|^2 + 2\,\mathrm{Re}\left(\sqrt{\rho}(\mathbf{h}_j x_q - \mathbf{h}_i x_p)^T n\right)\right) \tag{3.2}$$

$$= P\left(-\frac{\rho \|\mathbf{h}_j x_q - \mathbf{h}_i x_p\|^2}{2} > \mathrm{Re}\left\{\sqrt{\rho}(\mathbf{h}_j x_q - \mathbf{h}_i x_p)^T n\right\}\right) \tag{3.3}$$

Define:

$$\alpha = \sqrt{\rho}(\mathbf{h}_j x_q - \mathbf{h}_i x_p)^T, \qquad \alpha n = n_1$$

Then:

$$n_1 \sim \mathcal{N}\left(0, \sigma^2 \alpha^2\right)$$

Let $n_2 = \mathrm{Re}\{n_1\} \Rightarrow n_2 \sim \mathcal{N}\left(0, \frac{\sigma^2 \alpha^2}{2}\right)$

Then:

$$P\left(n_2 > \frac{\alpha^2}{2}\right) = \int_{\frac{\alpha^2}{2}}^{\infty} \frac{1}{\sqrt{\pi \alpha \sigma}} \exp\left(-\frac{n_2^2}{\alpha^2 \sigma^2}\right) dn_2$$

Using the definition of the Q-function:

$$Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^\infty \exp\left(-\frac{t^2}{2}\right) dt$$

We substitute:

$$t = \frac{n_2}{\sigma\alpha/\sqrt{2}} \Rightarrow dt = \frac{\sqrt{2}}{\sigma\alpha} dn_2$$

So:

$$P\left(n_2 > \frac{\alpha^2}{2}\right) = Q\left(\frac{\alpha}{\sqrt{2}\sigma}\right) = Q\left(\sqrt{\frac{\alpha^2}{2\sigma^2}}\right)$$

Therefore:

$$P(j, x_q \to i, x_p) = Q\left(\sqrt{\frac{\rho\|\mathbf{h}_j x_q - \mathbf{h}_i x_p\|^2}{2\sigma^2}}\right) \tag{3.4}$$

$$= Q\left(\sqrt{\frac{\rho}{2}\|\mathbf{h}_j x_q - \mathbf{h}_i x_p\|^2}\right) \quad \text{(for } \sigma = 1\text{)} \tag{3.5}$$

Finally, the bit error probability upper bound becomes:

$$P_e \leq \sum_{q=1}^{M}\sum_{p=1}^{M}\sum_{i=1}^{N_t}\sum_{j=1}^{N_t} \frac{N((j, x_q), (i, x_p))}{N_t M(\log_2 N_t M)} Q\left(\sqrt{\frac{\rho}{2}\|\mathbf{h}_j \mathbf{x}_q - \mathbf{h}_i \mathbf{x}_p\|^2}\right)$$

Fig. [3.9] shows a Gaussian random variable distribution with mean 0 & variance 1. The area under the bell curve for $x > x_0$ is denoted as $Q(x_0)$, & for $x > x_1$ as $Q(x_1)$. It can be observed that $x_1 > x_0$ & $Q(x_0) > Q(x_1)$. Hence, maximizing the argument inside the Q function minimizes its value. The idea of ED-SC for SM was conceived from ABEP term.

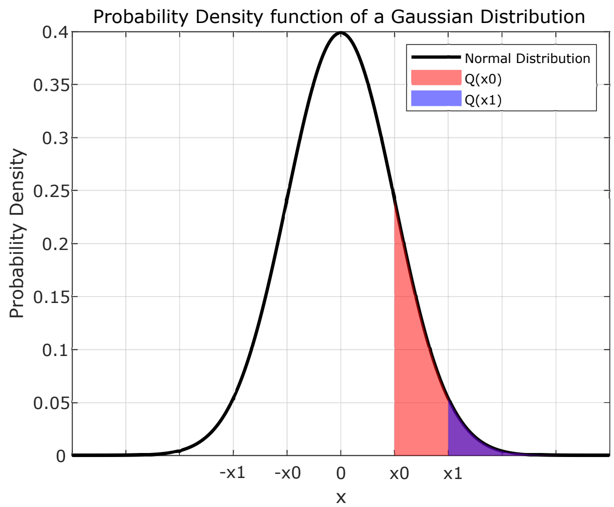

Figure 3.9: The probability that a Gaussian random variable $X \sim N(\mu, \sigma^2)$ exceeds x_0

3.6.2 Metric for SC

- The ABEP of SM can be reduced by maximizing the argument inside the Q-function, i.e., maximizing $\|\mathbf{h}_j\mathbf{x}_q - \mathbf{h}_i\mathbf{x}_p\|^2$ in

$$P_e \leq \sum_{q=1}^{M}\sum_{p=1}^{M}\sum_{i=1}^{N_t}\sum_{j=1}^{N_t} \frac{N((j,x_q),(i,x_p))}{N_t M(\log_2 N_t M)} Q\left(\sqrt{\frac{\rho}{2}\|\mathbf{h}_j\mathbf{x}_q - \mathbf{h}_i\mathbf{x}_p\|^2}\right)$$

- For a given SNR ρ is constant, maximizing the Euclidean distance between h_j, x_q and h_i, x_p can reduce the ABEP of the system.

- There are 3 cases of errors in SM

  1. When symbol is detected erroneously & antenna index is detected correctly

  2. When symbol is detected correctly & antenna index is detected erroneously

  3. When both symbol and antenna index are detected erroneously

$$v = \arg \max_{l \in \{1,2,\ldots,N_r\}} \left\{ \min_{\substack{i,\hat{i} \in \{1,\ldots,N_t\},\, i<\hat{i} \\ p,q \in \{1,\ldots,M\},\, p<q}} \left\{ \begin{array}{c} \left\| (\mathbf{h}_{l,i} - \mathbf{h}_{l,\hat{i}}) x_p \right\|^2, \\ \left\| \mathbf{h}_{l,i} x_p - \mathbf{h}_{l,\hat{i}} x_q \right\|^2, \\ \left\| x_p - x_q \right\|^2 \left\| \mathbf{h}_{l,\hat{i}} \right\|^2 \end{array} \right\} \right\} \tag{3.6}$$

For BPSK $M = 2$,
Possible symbols are 1,-1
Therefore the metric can be simplified to

$$v = \arg \max_{k \in \{1,2,\ldots,N_r\}} \left\{ \min_{\substack{i,\hat{i} \in \{1,2,\ldots,N_t\} \\ i<\hat{i}}} \left\{ |h_{ki}|^2, |h_{ki} - h_{k\hat{i}}|^2, |h_{ki} + h_{k\hat{i}}|^2 \right\} \right\}$$

After deciding the index of receiver antenna with maximum metric as v, the received signal at v^{th} antenna, that will be considered for signal detection will be as follows :

$$y_v = \sqrt{\rho} \cdot \mathbf{h}_v \mathbf{x} + n_r$$

where:

- y_r is the received scalar signal at the rth antenna
- $\mathbf{h}_r$ is the $1 \times N_t$ channel vector between all transmit antennas and the rth receive antenna
- $\mathbf{x}$ is the $N_t \times 1$ transmit signal vector with a single non-zero entry
- n_r is the AWGN at the rth antenna
- ρ is the SNR at the receiver

The ML detection will be as follows :

$$[j_{ML}, \hat{x_q}] = \arg \min_{x_{\text{ref}} \in \{1,2,\ldots,N_t\}} \| y_v - \sqrt{\rho} \cdot h_v x_{\text{ref}} \|$$

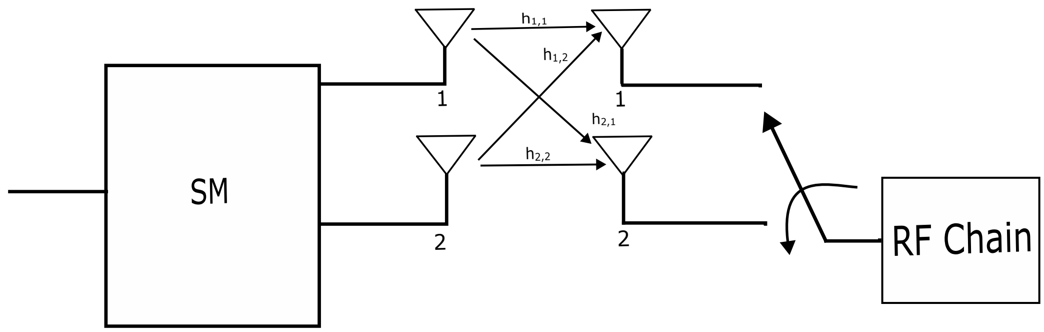

Figure 3.10: Selection of receiver antenna

where:

- y_v is the received signal at vth antenna
- $\hat{j}_{\mathrm{ML}}$ is the detected antenna index
- $\hat{x}_{\mathrm{q}}$ is the detected symbol index, where x_q is the reference constellation point
- h_v is the channel vector when the vth antenna is selected, $j \in \{1, 2, \ldots, N_t\}$

For an SM system with N_r=2, BPSK modulation scheme, ED-SC metric is as follows:

$$v = \arg \max_{k \in \{1,2\}} \left\{ \min_{\substack{i,\hat{i} \in \{1,2\} \\ i<\hat{i}}} \left\{ |h_{ki}|^2, |h_{ki} - h_{k\hat{i}}|^2, |h_{ki} + h_{k\hat{i}}|^2 \right\} \right\}$$

Instead of using single RF chain, if there are N_{rf} number of RF chains, GED-SC metric calculation will be as follows

$$v_1 = \arg \max_{k \in \{1,2,\ldots,N_r\}} \left\{ \min_{\substack{i,\hat{i} \in \{1,2,\ldots,N_t\} \\ i<\hat{i}}} \left\{ |h_{ki}|^2, |h_{ki} - h_{k\hat{i}}|^2, |h_{ki} + h_{k\hat{i}}|^2 \right\} \right\}$$

$$v_2 = \arg \max_{k \in \{1,2,\ldots,N_r\} \neq v_1} \left\{ \min_{\substack{i,\hat{i} \in \{1,2,\ldots,N_t\} \\ i<\hat{i}}} \left\{ |h_{ki}|^2, |h_{ki} - h_{k\hat{i}}|^2, |h_{ki} + h_{k\hat{i}}|^2 \right\} \right\}$$

$$v_{N_{rf}} = \arg \max_{\substack{k \in \{1,2,\ldots,N_r\} \\ k \neq v_1, v_2, \ldots, v_{N_{rf}-1}}} \left\{ \min_{\substack{i,\hat{i} \in \{1,2,\ldots,N_t\} \\ i<\hat{i}}} \left\{ |h_{ki}|^2, |h_{ki} - h_{k\hat{i}}|^2, |h_{ki} + h_{k\hat{i}}|^2 \right\} \right\}$$

After deciding the indices of the N_{rf} receiver antennas with the maximum metric as $\mathcal{S} = \{v_1, v_2, \ldots, v_{N_{rf}}\}$, the received signal at the selected antennas that will be considered for signal detection will be as follows:

$$\mathbf{y}_{\mathcal{S}} = \sqrt{\rho} \cdot \mathbf{h}_{\mathcal{S}} \mathbf{x} + \mathbf{n}_{\mathcal{S}}$$

where:

- $\mathbf{y}_{\mathcal{S}}$ is the received signal vector at the selected N_{rf} antennas
- $\mathbf{h}_{\mathcal{S}}$ is the $N_{rf} \times N_t$ submatrix of the channel matrix for selected antennas
- $\mathbf{x}$ is the $N_t \times 1$ transmit signal vector with a single non-zero entry
- $\mathbf{n}_{\mathcal{S}}$ is the AWGN vector at the selected antennas
- ρ is the SNR at the receiver

The ML detection will be as follows:

$$[j_{ML}, \hat{x_q}] = \arg \min_{\substack{j \in \{1,2,\ldots,N_t\} \\ x_{\text{ref}} \in \mathcal{A}}} \|\mathbf{y}_{\mathcal{S}} - \sqrt{\rho} \cdot \mathbf{h}_{\mathcal{S}} x_{\text{ref}}\|$$

where:

- $\mathbf{y}_{\mathcal{S}}$ is the received signal at the selected N_{rf} antennas

- $\hat{j}_{\mathrm{ML}}$ is the estimated antenna index
- $\hat{x}_{\mathrm{q}}$ is the detected symbol index, where x_q is the reference constellation point
- $\mathbf{h}_{\mathcal{S}}$ is the channel matrix of the selected antennas, $j \in \{1, 2, \ldots, N_t\}$

Chapter 4

Performance Analysis of SM with ED-SC

4.0.1 SM ED-SC with N_t=2, N_r=2, with BPSK

The ABEP of SM using PEP and union bound technique

$$P_e \leq \sum_{q=1}^{M}\sum_{p=1}^{M}\sum_{i=1}^{N_t}\sum_{j=1}^{N_t} \frac{N((j,x_q),(i,x_p))}{N_t M \log_2 N_t M} Q\left(\sqrt{\frac{\rho}{2}\|\mathbf{h}_j\mathbf{x}_q - \mathbf{h}_i\mathbf{x}_p\|^2}\right)$$

$$\|\mathbf{h}_j\mathbf{x}_q - \mathbf{h}_i\mathbf{x}_p\|^2 \text{ is the variable term}$$

By substituting $N_t = 2, N_r = 2, M = 2$

For $M = 2$, we consider Binary Phase Shift Keying (BPSK), where the modulation symbols are:

$$x_1 = +1, \quad x_2 = -1$$

For antenna selection, we consider:

- Transmit antenna pair: $(i, j) = (1, 2)$
- Modulation symbol pair: $(x_p, x_q) = (x_1, x_2) = (+1, -1)$
- Receive antennas: $l = 1, 2$

For each receive antenna l, we evaluate the following three expressions (substituting $x_1 = +1$, $x_2 = -1$):

- **For receive antenna $l = 1$:**
  1. $\|(h_{1,1} - h_{1,2}) \cdot (+1)\|^2 = \|h_{1,1} - h_{1,2}\|^2$
  2. $\|h_{1,1} \cdot (+1) - h_{1,2} \cdot (-1)\|^2 = \|h_{1,1} + h_{1,2}\|^2$
  3. $\|+1 - (-1)\|^2 \cdot \|h_{1,1}\|^2 = 4 \cdot \|h_{1,1}\|^2$
  4. $\|(h_{1,1} - h_{1,2}) \cdot (-1)\|^2 = \|h_{1,1} - h_{1,2}\|^2$
  5. $\|h_{1,1} \cdot (-1) - h_{1,2} \cdot (+1)\|^2 = \|h_{1,1} + h_{1,2}\|^2$
  6. $\|+1 - (-1)\|^2 \cdot \|h_{1,2}\|^2 = 4 \cdot \|h_{1,2}\|^2$
- **For receive antenna $l = 2$:**
  1. $\|(h_{2,1} - h_{2,2}) \cdot (+1)\|^2 = \|h_{2,1} - h_{2,2}\|^2$
  2. $\|h_{2,1} \cdot (+1) - h_{2,2} \cdot (-1)\|^2 = \|h_{2,1} + h_{2,1}\|^2$
  3. $\|+1 - (-1)\|^2 \cdot \|h_{2,2}\|^2 = 4 \cdot \|h_{2,2}\|^2$
  4. $\|(h_{2,1} - h_{2,2}) \cdot (-1)\|^2 = \|h_{2,1} - h_{2,2}\|^2$
  5. $\|h_{2,1} \cdot (-1) - h_{2,2} \cdot (+1)\|^2 = \|h_{2,1} + h_{2,2}\|^2$
  6. $\|+1 - (-1)\|^2 \cdot \|h_{2,2}\|^2 = 4 \cdot \|h_{2,2}\|^2$

It is evident that some expressions are repeated due to symmetry in BPSK (i.e., $x_p = +1, x_q = -1$ and vice versa yield the same metric). Therefore, we eliminate the duplicates and retain only the unique expressions. Each receive antenna ends up contributing 4 distinct random variables (RVs):

- $\|h_{l,1} - h_{l,2}\|^2$
- $\|h_{l,1} + h_{l,2}\|^2$
- $4 \cdot \|h_{l,1}\|^2$
- $4 \cdot \|h_{l,2}\|^2$

For receive antenna $l = 1$, we define the following random variables:

$$X_{11} = 4 \cdot \|h_{1,1}\|^2$$
$$X_{12} = 4 \cdot \|h_{1,2}\|^2$$
$$X_{1+2} = \|h_{1,1} + h_{1,2}\|^2$$
$$X_{1-2} = \|h_{1,1} - h_{1,2}\|^2$$

Let X be the minimum order statistic of the random variables of antenna 1, i.e.,

$$X = \min(X_{11}, X_{12}, X_{1+2}, X_{1-2})$$

For receive antenna $l = 2$, we define the corresponding random variables:

$$Y_{11} = 4 \cdot \|h_{2,1}\|^2$$
$$Y_{12} = 4 \cdot \|h_{2,2}\|^2$$
$$Y_{1+2} = \|h_{2,1} + h_{2,2}\|^2$$
$$Y_{1-2} = \|h_{2,1} - h_{2,2}\|^2$$

Let Y be the minimum order statistic of the random variables of antenna 1, i.e.,

$$Y = \min(Y_{11}, Y_{12}, Y_{1+2}, Y_{1-2})$$

We denote the event of choosing v^{th} receive antenna for SM symbol detection as A.

Using the selected antenna v, and considering $N_t = 2$, the ABEP can be expressed as:

$$P_e \leq \sum_{q=1}^{M}\sum_{p=1}^{M}\sum_{i=1}^{N_t}\sum_{j=1}^{N_t}\frac{N((j,x_q),(i,x_p))}{N_t M \log_2 N_t M} Q\left(\sqrt{\frac{\rho}{2}\|h_{v,j}\mathbf{x}_q - h_{v,i}\mathbf{x}_p\|^2}\right)$$

$$P_e \leq \frac{1}{N_t M \log_2 N_t M}\sum_{q=1}^{M}\sum_{p=1}^{M}\sum_{i=1}^{N_t}\sum_{j=1}^{N_t} N((j,x_q),(i,x_p))\mathbb{E}\left[Q\left(\sqrt{\frac{\rho}{2}\|h_{v,j}\mathbf{x}_q - h_{v,i}\mathbf{x}_p\|^2}\right)\middle| A\right]$$

$$P_e \leq \frac{1}{4*2}\sum_{p=1}^{M}\sum_{i=1}^{N_t}\sum_{j=1}^{N_t} N((j,x_q),(i,x_p))\mathbb{E}\left[Q\left(\sqrt{\frac{\rho}{2}\|h_{v,j}\mathbf{x}_q - h_{v,i}\mathbf{x}_p\|^2}\right)\middle| A\right]$$

Let's assume v is the 1st antenna of the 2 receive antennas.

$$\begin{aligned} P_e \leq\ & \frac{2}{4}\mathbb{E}\left[Q\left(\sqrt{\frac{\rho}{2}\|h_{11} - h_{12}\|^2}\right)\middle| A\right] + \frac{2}{4}\mathbb{E}\left[Q\left(\sqrt{\frac{\rho}{2}\|h_{11} + h_{12}\|^2}\right)\middle| A\right] \\ & + \frac{1}{4}\mathbb{E}\left[Q\left(\sqrt{\frac{\rho}{2}\|2h_{11}\|^2}\right)\middle| A\right] + \frac{1}{4}\mathbb{E}\left[Q\left(\sqrt{\frac{\rho}{2}\|2h_{12}\|^2}\right)\middle| A\right] \end{aligned}$$

$$\begin{aligned} P_e \leq\ & \frac{2}{4}\mathbb{E}\left[Q\left(\sqrt{\frac{\rho}{2}x_{1-2}}\right)\middle| A\right] + \frac{2}{4}\mathbb{E}\left[Q\left(\sqrt{\frac{\rho}{2}x_{1+2}}\right)\middle| A\right] \\ & + \frac{1}{4}\mathbb{E}\left[Q\left(\sqrt{\frac{\rho}{2}x_{11}}\right)\middle| A\right] + \frac{1}{4}\mathbb{E}\left[Q\left(\sqrt{\frac{\rho}{2}x_{12}}\right)\middle| A\right] \end{aligned} \tag{4.1}$$

where $\mathbb{E}[\cdot \mid \cdot]$ is the conditional expectation. Now, we derive the conditional probability density function (PDF) of $x_{11}, x_{12}, x_{1+2}, x_{1-2}$ given the event A.

If event A is selecting 1^{st} antenna,

$$Y < \min(X_{11}, X_{12}, X_{1+2}, X_{1-2})$$

$(X_{11}, X_{12}, X_{1+2}, X_{1-2})$ can be ordered in 4! ways, with 6 patterns which are unique and are repeat

$$Y < X_{11} < X_{12} < X_{1+2} < X_{1-2}$$
$$Y < X_{11} < X_{1+2} < X_{12} < X_{1-2}$$
$$Y < X_{11} < X_{1+2} < X_{1-2} < X_{12}$$
$$Y < X_{1+2} < X_{11} < X_{12} < X_{1-2}$$
$$Y < X_{1+2} < X_{11} < X_{1-2} < X_{12}$$
$$Y < X_{1+2} < X_{1-2} < X_{11} < X_{12}$$

4.0.2 PDF of RVs

$$X_{11} = 4|h_{11}|^2, \quad X_{12} = 4|h_{12}|^2$$

Since X_{11} and X_{12} are transformations of the random variables $|h_{11}|^2$ and $|h_{12}|^2$, and we know that the distribution of $|h_{11}|^2$ and $|h_{12}|^2$ is exponential with PDF e^{-x}, we have:

$$f_{X_{11}}(x) = f_{X_{12}}(x) = \frac{1}{4}e^{-x/4}, \quad x \geq 0$$

We know that $h_{11}, h_{12} \sim \mathcal{CN}(0, 1)$, and they are independent. Then:

$$h_+ = h_{11} + h_{12} \sim \mathcal{CN}(0, 2), \quad h_- = h_{11} - h_{12} \sim \mathcal{CN}(0, 2)$$

This follows from the fact that the sum or difference of two independent complex Gaussian variables $\mathcal{CN}(0, \sigma^2)$ also follows a complex Gaussian distribution:

$$\mathrm{Var}(h_{11} \pm h_{12}) = \mathrm{Var}(h_{11}) + \mathrm{Var}(h_{12}) = 1 + 1 = 2$$

Therefore:

$$h_{11} \pm h_{12} \sim \mathcal{CN}(0, 2)$$

If $h \sim \mathcal{CN}(0, \sigma^2)$, then the squared magnitude $|h|^2$ follows an exponential distribution:

$$|h|^2 \sim \text{Exponential}\left(\frac{1}{\sigma^2}\right)$$
$$|h_{11} \pm h_{12}|^2 \sim \text{Exp}\left(\frac{1}{2}\right)$$

$$f_{X_{1+2}}(x) = f_{X_{1-2}}(x) = \frac{1}{2}e^{-x/2}, \quad x \geq 0$$

Derivation of PDF of $Y = \min(Y_{11}, Y_{12}, Y_{1-2}, Y_{1+2})$

Assume the following:

$$Y_{11}, Y_{12} \sim \text{Exp}\left(\frac{1}{4}\right), \quad Y_{1-2}, Y_{1+2} \sim \text{Exp}\left(\frac{1}{2}\right)$$

Compute the distribution of:

$$W_1 = \min(Y_{11}, Y_{12})$$

Since both follow $\text{Exp}(\frac{1}{4})$, their individual CDFs are:

$$F_{Y_{11}}(y) = F_{Y_{12}}(y) = 1 - e^{-y/4}$$

Using the following formula from [22]

$$F_{W_1}(y) = F_{Y_{11}}(y) + F_{Y_{12}}(y) - F_{Y_{11}}(y)F_{Y_{12}}(y)$$

$$F_{W_1}(y) = 2(1 - e^{-y/4}) - (1 - e^{-y/4})^2 = 1 - e^{-y/2}$$

Thus,

$$W_1 \sim \text{Exp}\left(\frac{1}{2}\right)$$

$$W_2 = \min(Y_{1-2}, Y_{1+2})$$

Each follows $\text{Exp}(\frac{1}{2})$, so:

$$F_{W_2}(y) = 2(1 - e^{-y/2}) - (1 - e^{-y/2})^2 = 1 - e^{-y}$$

So,

$$W_2 \sim \text{Exp}(1)$$

Finally, compute:

$$Y = \min(W_1, W_2)$$

With $W_1 \sim \text{Exp}(\frac{1}{2})$, $W_2 \sim \text{Exp}(1)$, we get:

$$F_Y(y) = F_{W_1}(y) + F_{W_2}(y) - F_{W_1}(y)F_{W_2}(y)$$

$$F_Y(y) = (1 - e^{-y/2}) + (1 - e^{-y}) - (1 - e^{-y/2})(1 - e^{-y}) = 1 - e^{-3y/2}$$

Hence, the PDF is:

$$f_Y(y) = \frac{d}{dy} F_Y(y) = \frac{3}{2} e^{-3y/2}, \quad y \geq 0$$

$$X_{11}, X_{12} \sim \text{Exp}\left(\frac{1}{4}\right), \quad X_{1-2}, X_{1+2} \sim \text{Exp}\left(\frac{1}{2}\right), \quad Y \sim \text{Exp}\left(\frac{3}{2}\right)$$

4.0.3 Conditional PDFs

Lets consider the first condition and elaborate the process of finding conditional probability of each RV for it. In the end, the conditional probability of each RV is 4 times the sum of values obtained for all the conditions stated above.

$$Y < X_{11} < X_{12} < X_{1+2} < X_{1-2}$$

$f_{1_{X_{11},A}}(x_{11})$ denotes the joint probability density function (PDF) corresponding to the event that X_{11} is at particular position as in given unique order and event A

$f_{1_{X_{11}|A}}(x_{11})$ denotes the conditional PDF of X_{11} given event A, defined as:

$$f_{1_{X_{11}|A}}(x_{11}) = \frac{f_{1_{X_{11},A}}(x_{11})}{P(A)}$$

where $P(A)$ is the probability of event A.

$$\begin{aligned} f1_{X_{11},A}(x_{11}) = & \int_0^{x_{11}} \int_{x_{11}}^{\infty} \int_{x_{12}}^{\infty} \int_{x_{1+2}}^{\infty} f_{X_{11}}(x_{11}) f_{X_{1-2}}(x_{1-2}) \\ & \times f_{X_{12}}(x_{12}) f_{X_{1+2}}(x_{1+2}) f_Y(y)\, dx_{1-2}\, dx_{1+2}\, dx_{12}\, dy \end{aligned}$$

$$\begin{aligned} f1_{X_{12},A}(x_{12}) = & \int_0^{x_{12}} \int_0^{x_{11}} \int_{x_{12}}^{\infty} \int_{x_{1+2}}^{\infty} f_{X_{11}}(x_{11}) f_{X_{1-2}}(x_{1-2}) \\ & \times f_{X_{12}}(x_{12}) f_{X_{1+2}}(x_{1+2}) f_Y(y)\, dx_{1-2}\, dx_{1+2}\, dy\, dx_{11} \end{aligned}$$

$$f1_{X_{1+2},A}(x_{1+2}) = \int_0^{x_{1+2}} \int_0^{x_{12}} \int_0^{x_{11}} \int_{x_{12}}^{\infty} f_{X_{11}}(x_{11}) f_{X_{1-2}}(x_{1-2}) \times f_{X_{12}}(x_{12}) f_{X_{1+2}}(x_{1+2}) f_Y(y)\, dx_{1-2}\, dy\, dx_{11}\, dx_{12}$$

$$f1_{X_{1-2},A}(x_{1-2}) = \int_0^{x_{1-2}} \int_0^{x_{1+2}} \int_0^{x_{12}} \int_0^{x_{11}} f_{X_{11}}(x_{11}) f_{X_{1-2}}(x_{1-2}) \times f_{X_{12}}(x_{12}) f_{X_{1+2}}(x_{1+2}) f_Y(y)\, dy\, dx_{11}\, dx_{12}\, dx_{1+2}$$

Substituting marginal PDFs in above integrals and integrating, we obtain

$$\begin{aligned}
f1_{X_{11},A}(x_{11}) &= \frac{1}{20} e^{-\frac{9x_{11}}{4}} \sinh\left(\frac{3x_{11}}{4}\right) \\
f1_{X_{12},A}(x_{12}) &= \frac{1}{56} e^{-3x_{12}} \left(1 - 7e^{\frac{3x_{12}}{2}} + 6e^{\frac{7x_{12}}{4}}\right) \\
f1_{X_{1+2},A}(x_{1+2}) &= \frac{1}{112} e^{-3x_{1+2}} \left(-1 + 28e^{\frac{3x_{1+2}}{2}} - 48e^{\frac{7x_{1+2}}{4}} + 21e^{2x_{1+2}}\right) \\
f1_{X_{1-2},A}(x_{1-2}) &= \frac{1}{560} e^{-3x_{1-2}} \left(1 - 70e^{\frac{3x_{1-2}}{2}} + 160e^{\frac{7x_{1-2}}{4}} - 105e^{2x_{1-2}} + 14e^{\frac{5x_{1-2}}{2}}\right)
\end{aligned}$$

Final conditional PDFs after evaluating all combinations are as follows

$$
\begin{aligned}
f_{X_{11},A}(x_{11}) = {} & \frac{1}{40}e^{-3x_{11}}(-1+e^{x_{11}/2})^3(1+3e^{x_{11}/2}+6e^{x_{11}}) \\
& + \frac{1}{5}e^{-3x_{11}}(-1+e^{3x_{11}/2}) \quad + \frac{1}{28}e^{-3x_{11}}(1-7e^{3x_{11}/2}+6e^{7x_{11}/4}) \\
& + \frac{1}{8}e^{-3x_{11}}(1-4e^{3x_{11}/2}+3e^{2x_{11}}) \\
& + \frac{1}{63}e^{-3x_{11}}(-1+21e^{3x_{11}/2}-27e^{7x_{11}/4}+7e^{9x_{11}/4}) \\
& + \frac{1}{72}e^{-3x_{11}}(-1+12e^{3x_{11}/2}-27e^{2x_{11}}+16e^{9x_{11}/4}) \\
& + \frac{1}{440}e^{-3x_{11}}(1-22e^{3x_{11}/2}+55e^{2x_{11}}-66e^{5x_{11}/2}+32e^{11x_{11}/4}) \\
& + \frac{1}{1980}e^{-3x_{11}}(5-132e^{3x_{11}/2}+495e^{2x_{11}}-440e^{9x_{11}/4}+72e^{11x_{11}/4}) \\
& + \frac{1}{6930}e^{-3x_{11}}(20-924e^{3x_{11}/2}+1485e^{7x_{11}/4}-770e^{9x_{11}/4}+189e^{11x_{11}/4}) \\
& + \frac{1}{10}e^{-9x_{11}/4}\sinh\left(\frac{3x_{11}}{4}\right)
\end{aligned}
$$

$$
\begin{aligned}
f_{X_{12},A}(x_{12}) &= \frac{1}{40}e^{-3x_{12}}(-1+e^{x_{12}/2})^3(1+3e^{x_{12}/2}+6e^{x_{12}}) \\
&+ \frac{1}{5}e^{-3x_{12}}(-1+e^{3x_{12}/2}) + \frac{1}{28}e^{-3x_{12}}(1-7e^{3x_{12}/2}+6e^{7x_{12}/4}) \\
&+ \frac{1}{8}e^{-3x_{12}}(1-4e^{3x_{12}/2}+3e^{2x_{12}}) \\
&+ \frac{1}{63}e^{-3x_{12}}(-1+21e^{3x_{12}/2}-27e^{7x_{12}/4}+7e^{9x_{12}/4}) \\
&+ \frac{1}{72}e^{-3x_{12}}(-1+12e^{3x_{12}/2}-27e^{2x_{12}}+16e^{9x_{12}/4}) \\
&+ \frac{1}{440}e^{-3x_{12}}(1-22e^{3x_{12}/2}+55e^{2x_{12}}-66e^{5x_{12}/2}+32e^{11x_{12}/4}) \\
&+ \frac{1}{1980}e^{-3x_{12}}(5-132e^{3x_{12}/2}+495e^{2x_{12}}-440e^{9x_{12}/4}+72e^{11x_{12}/4}) \\
&+ \frac{1}{6930}e^{-3x_{12}}(20-924e^{3x_{12}/2}+1485e^{7x_{12}/4}-770e^{9x_{12}/4}+189e^{11x_{12}/4}) \\
&+ \frac{1}{10}e^{-9x_{12}/4}\sinh\left(\frac{3x_{12}}{4}\right)
\end{aligned}
$$

$$
\begin{aligned}
f_{X_{1-2},A}(x_{1-2}) &= \frac{1}{2}e^{-3x_{1-2}}(-1+e^{3x_{1-2}/2}) \\
&+ \frac{1}{7}e^{-3x_{1-2}}(1-7e^{3x_{1-2}/2}+6e^{7x_{1-2}/4}) \\
&+ \frac{1}{8}e^{-3x_{1-2}}(1-4e^{3x_{1-2}/2}+3e^{2x_{1-2}}) \\
&+ \frac{1}{56}e^{-3x_{1-2}}(-1+28e^{3x_{1-2}/2}-48e^{7x_{1-2}/4}+21e^{2x_{1-2}}) \\
&+ \frac{2}{63}e^{-3x_{1-2}}(-1+21e^{3x_{1-2}/2}-27e^{7x_{1-2}/4}+7e^{9x_{1-2}/4}) \\
&+ \frac{1}{36}e^{-3x_{1-2}}(-1+12e^{3x_{1-2}/2}-27e^{2x_{1-2}}+16e^{9x_{1-2}/4}) \\
&+ \frac{1}{280}e^{-3x_{1-2}}(1-70e^{3x_{1-2}/2}+160e^{7x_{1-2}/4}-105e^{2x_{1-2}}+14e^{5x_{1-2}/2}) \\
&+ \frac{1}{360}e^{-3x_{1-2}}(1-30e^{3x_{1-2}/2}+135e^{2x_{1-2}}-160e^{9x_{1-2}/4}+54e^{5x_{1-2}/2}) \\
&+ \frac{1}{630}e^{-3x_{1-2}}\left(2+e^{3x_{1-2}/2}(-105+180e^{x_{1-2}/4}-140e^{3x_{1-2}/4}+63e^{x_{1-2}})\right)
\end{aligned}
$$

$$
\begin{aligned}
f_{X_{1+2},A}(x_{1+2}) = &\frac{1}{2}e^{-3x_{1+2}}(-1+e^{3x_{1+2}/2}) \\
&+\frac{1}{7}e^{-3x_{1+2}}(1-7e^{3x_{1+2}/2}+6e^{7x_{1+2}/4}) \\
&+\frac{1}{8}e^{-3x_{1+2}}(1-4e^{3x_{1+2}/2}+3e^{2x_{1+2}}) \\
&+\frac{1}{56}e^{-3x_{1+2}}(-1+28e^{3x_{1+2}/2}-48e^{7x_{1+2}/4}+21e^{2x_{1+2}}) \\
&+\frac{2}{63}e^{-3x_{1+2}}(-1+21e^{3x_{1+2}/2}-27e^{7x_{1+2}/4}+7e^{9x_{1+2}/4}) \\
&+\frac{1}{36}e^{-3x_{1+2}}(-1+12e^{3x_{1+2}/2}-27e^{2x_{1+2}}+16e^{9x_{1+2}/4}) \\
&+\frac{1}{280}e^{-3x_{1+2}}(1-70e^{3x_{1+2}/2}+160e^{7x_{1+2}/4}-105e^{2x_{1+2}}+14e^{5x_{1+2}/2}) \\
&+\frac{1}{360}e^{-3x_{1+2}}(1-30e^{3x_{1+2}/2}+135e^{2x_{1+2}}-160e^{9x_{1+2}/4}+54e^{5x_{1+2}/2}) \\
&+\frac{1}{630}e^{-3x_{1+2}}\left(2+e^{3x_{1+2}/2}(-105+180e^{x_{1+2}/4}-140e^{3x_{1+2}/4}+63e^{x_{1+2}})\right)
\end{aligned}
$$

A PDF test was performed for all four derived probability density functions. This involved integrating each PDF from 0 to ∞, and verifying that the result equals 1. This confirms that all the PDFs are valid probability density functions. The probability of event A happening is the probability of selecting an antenna from N_r antennas :

$$P(A) = \frac{1}{N_r}$$

$$
\begin{aligned}
f_{X_{11}/A}(x_{11}) &= \frac{f_{X_{11},A}(x_{11})}{P(A)} = N_r f_{X_{11},A}(x_{11}) = 2 f_{X_{11},A}(x_{11}) \\
f_{X_{12}/A}(x_{12}) &= \frac{f_{X_{12},A}(x_{12})}{P(A)} = N_r f_{X_{12},A}(x_{12}) = 2 f_{X_{12},A}(x_{12}) \\
f_{X_{1-2}/A}(x_{1-2}) &= \frac{f_{X_{1-2},A}(x_{1-2})}{P(A)} = N_r f_{X_{1-2},A}(x_{1-2}) = 2 f_{X_{1-2},A}(x_{1-2}) \\
f_{X_{1+2}/A}(x_{1+2}) &= \frac{f_{X_{1+2},A}(x_{1+2})}{P(A)} = N_r f_{X_{1+2},A}(x_{1+2}) = 2 f_{X_{1+2},A}(x_{1+2})
\end{aligned}
$$

From eq(4.1)

$$
\begin{aligned}
P_e \leq \frac{2}{4}\mathbb{E}\left[Q\left(\sqrt{\frac{\rho}{2}x_{1-2}}\right)\middle|A\right] + \frac{2}{4}\mathbb{E}\left[Q\left(\sqrt{\frac{\rho}{2}x_{1+2}}\right)\middle|A\right] \\
+ \frac{1}{4}\mathbb{E}\left[Q\left(\sqrt{\frac{\rho}{2}x_{11}}\right)\middle|A\right] + \frac{1}{4}\mathbb{E}\left[Q\left(\sqrt{\frac{\rho}{2}x_{12}}\right)\middle|A\right]
\end{aligned}
$$

If the bit error probability is given by:

$$
P_e = \mathbb{E}\left[Q\left(\sqrt{\frac{\rho}{2}X}\right)|A\right] \tag{4.2}
$$

where the expectation in (4.2) is taken with respect to the variable X.

By utilizing the alternate integral expression for the Q-function [22], equation (4.2) can be rewritten as:

$$
P_b = \int_0^{\pi/2}\int_0^{\infty} \frac{1}{\pi}\exp\left(-\frac{\rho x}{4\sin^2\theta}\right) f_{X/A}(x)\,dx\,d\theta \tag{4.3}
$$

Using (4.3) in eq(4.1) and substituting various values for ρ in linear scale, we obtain our theoretical bit error probability.

4.0.4 SM ED-SC with N_t=2, N_r=4, with BPSK

In the SM ED-SC system with $N_t = 2$, $N_r = 4$, and BPSK modulation ($M = 2$), the antenna selection criterion remains the same as in the previous case, but now out of the 4 receive antennas, one is selected for SM detection based on the maximum metric value.

Let the event A denote that the first receive antenna is selected. Since all antennas are statistically identical under Rayleigh fading and selection is based on the maximum of i.i.d. metrics, the probability of any one antenna being selected is

$$P(A) = \frac{1}{N_r} = \frac{1}{4}$$

Random Variables: For the first receive antenna $l = 1$, the same four random variables as in the previous case are defined:

$$X_{11} = 4 \cdot \|h_{1,1}\|^2, \quad X_{12} = 4 \cdot \|h_{1,2}\|^2,$$
$$X_{1+2} = \|h_{1,1} + h_{1,2}\|^2, \quad X_{1-2} = \|h_{1,1} - h_{1,2}\|^2$$

For each of the other receive antennas $l = 2, 3, 4$, we define one corresponding minimum order metric random variable:

$$Y = \text{minimum order statistic for antenna 2},$$
$$W = \text{minimum order statistic for antenna 3},$$
$$Z = \text{minimum order statistic for antenna 4}$$

Assuming the first antenna is selected, this implies:

$$Z < W < Y < X_{11} < X_{12} < X_{1+2} < X_{1-2}$$

This is one of the possible orderings satisfying the event A.

For the 4 random variables from antenna 1, there are 4! = 24 total permutations and for 3 random variables z, w, y there are 3! = 6 permutations, only 6 of these permutations are distinct. So they can be derived and then multiplied with 4*6 to compensate not evaluating other permutations

$$
\begin{aligned}
&Z < W < Y < X_{11} < X_{12} < X_{1+2} < X_{1-2}\\
&Z < W < Y < X_{11} < X_{1+2} < X_{12} < X_{1-2}\\
&Z < W < Y < X_{11} < X_{1+2} < X_{1-2} < X_{12}\\
&Z < W < Y < X_{1+2} < X_{11} < X_{12} < X_{1-2}\\
&Z < W < Y < X_{1+2} < X_{11} < X_{1-2} < X_{12}\\
&Z < W < Y < X_{1+2} < X_{1-2} < X_{11} < X_{12}
\end{aligned}
$$

Considering $Z < W < Y < X_{11} < X_{12} < X_{1+2} < X_{1-2}$

$$
\begin{aligned}
f1_{X_{11},A}(x_{11}) = &\int_0^{x_{11}}\int_0^{y}\int_0^{w}\int_{x_{11}}^{\infty}\int_{x_{12}}^{\infty}\int_{x_{1+2}}^{\infty} f_{X_{11}}(x_{11})f_{X_{1-2}}(x_{1-2})f_{X_{12}}(x_{12})\\
&\times f_{X_{1+2}}(x_{1+2})f_Y(y)f_Z(z)f_W(w)\,dx_{1-2}\,dx_{1+2}\,dx_{12}\,dz\,dw\,dy
\end{aligned}
$$

$$
\begin{aligned}
f1_{X_{12},A}(x_{12}) = &\int_0^{x_{12}}\int_0^{x_{11}}\int_0^{y}\int_0^{w}\int_{x_{12}}^{\infty}\int_{x_{1+2}}^{\infty} f_{X_{11}}(x_{11})f_{X_{1-2}}(x_{1-2})f_{X_{12}}(x_{12})\\
&\times f_{X_{1+2}}(x_{1+2})f_Y(y)f_Z(z)f_W(w)\,dx_{1-2}\,dx_{1+2}\,dz\,dw\,dy\,dx_{11}
\end{aligned}
$$

$$f1_{X_{1+2},A}(x_{1+2}) = \int_0^{x_{1+2}} \int_0^{x_{12}} \int_0^{y} \int_0^{w} \int_0^{x_{11}} \int_{x_{12}}^{\infty} f_{X_{11}}(x_{11}) f_{X_{1-2}}(x_{1-2}) f_{X_{12}}(x_{12}) \times f_{X_{1+2}}(x_{1+2}) f_Y(y) f_Z(z) f_W(w)\, dx_{1-2}\, dz\, dw\, dy\, dx_{11}\, dx_{12}$$

$$f1_{X_{1-2},A}(x_{1-2}) = \int_0^{x_{1-2}} \int_0^{x_{1+2}} \int_0^{x_{12}} \int_0^{x_{11}} \int_0^{y} \int_0^{w} f_{X_{11}}(x_{11}) f_{X_{1-2}}(x_{1-2}) f_{X_{12}}(x_{12}) \times f_{X_{1+2}}(x_{1+2}) f_Y(y) f_Z(z) f_W(w)\, dz\, dw\, dy\, dx_{1+2}\, dx_{12}\, dx_{11}$$

By substituting marginal PDFs same as 2x2 and Z, W have same distribution as Y, we get $f1_{X_{11},A}(x_{11})$, $f1_{X_{12},A}(x_{12})$, $f1_{X_{1+2},A}(x_{1+2})$, $f1_{X_{1-2},A}(x_{1-2})$.

Then final conditional PDFs are sum of PDFs in all possible combinations.

The probability of selecting an antenna from N_r antennas is the probability of event A happening:

$$P(A) = \frac{1}{N_r}$$

$$f_{X_{11}/A}(x_{11}) = \frac{f_{X_{11},A}(x_{11})}{P(A)} = N_r f_{X_{11},A}(x_{11}) = 4 f_{X_{11},A}(x_{11})$$

$$f_{X_{12}/A}(x_{12}) = \frac{f_{X_{12},A}(x_{12})}{P(A)} = N_r f_{X_{12},A}(x_{12}) = 4 f_{X_{12},A}(x_{12})$$

$$f_{X_{1-2}/A}(x_{1-2}) = \frac{f_{X_{1-2},A}(x_{1-2})}{P(A)} = N_r f_{X_{1-2},A}(x_{1-2}) = 4 f_{X_{1-2},A}(x_{1-2})$$

$$f_{X_{1+2}/A}(x_{1+2}) = \frac{f_{X_{1+2},A}(x_{1+2})}{P(A)} = N_r f_{X_{1+2},A}(x_{1+2}) = 4 f_{X_{1+2},A}(x_{1+2})$$

From eq(4.1)

$$P_e \leq \frac{2}{4}\mathbb{E}\left[Q\left(\sqrt{\frac{\rho}{2}x_{1-2}}\right)\middle|A\right] + \frac{2}{4}\mathbb{E}\left[Q\left(\sqrt{\frac{\rho}{2}x_{1+2}}\right)\middle|A\right]$$
$$+\frac{1}{4}\mathbb{E}\left[Q\left(\sqrt{\frac{\rho}{2}x_{11}}\right)\middle|A\right] + \frac{1}{4}\mathbb{E}\left[Q\left(\sqrt{\frac{\rho}{2}x_{12}}\right)\middle|A\right]$$

If the bit error probability is given by:

$$P_e = \mathbb{E}\left[Q\left(\sqrt{\frac{\rho}{2}X}\right)|A\right] \tag{4.4}$$

where the expectation in (4.4) is taken with respect to the variable X.

By utilizing the alternate integral expression for the Q-function [22], equation (4.4) can be rewritten as:

$$P_b = \int_0^{\pi/2}\int_0^{\infty} \frac{1}{\pi}\exp\left(-\frac{\rho x}{4\sin^2\theta}\right) f_{X/A}(x)\, dx\, d\theta \tag{4.5}$$

Using (4.5) in eq(4.1) and substituting various values for ρ in linear scale, we obtain our theoretical bit error probability.

Chapter 5

Algorithm

The following SM with ED-SC and SM with ED-GSC algorithms have a MIMO setup with N_t transmit antennas and N_r receive antennas, where only one transmit antenna is active at a time. The system transmits symbols from a constellation of size M.

ED-SC selects a single best receive antenna based on a minimum distance metric derived from pairwise Euclidean distances. Once the best receive antenna is selected, ML detection is performed on the received signal at that antenna to estimate the transmit antenna index and the transmitted symbol.

ED-GSC extends ED-SC to the case where N_{rf} receive antennas can be simultaneously selected. Instead of selecting just one antenna, ED-GSC identifies the top N_{rf} antennas with the highest metrics. ML detection is then independently performed on each of these selected antennas, improving robustness and diversity gain.

TABLE I

ED-SC Algorithm

Step 1	Initialize the channel matrix $\mathbf{H}$ of dimension $N_r \times N_t$ with all entries being i.i.d. complex Gaussian.

Step 2	Compute the Euclidean distances for each receive antenna l:
	$d_{1,l}^{(i,\hat{i},p)} = \left\|(h_{l,i} - h_{l,\hat{i}})x_p\right\|^2, \quad d_{2,l}^{(i,\hat{i},p,q)} = \left\|h_{l,i}x_p - h_{l,\hat{i}}x_q\right\|^2,$
	$d_{3,l}^{(p,q,i)} = \|x_p - x_q\|^2 \cdot \|h_{l,i}\|^2$
	Initial $i = 1$, $\hat{i} < i$, $p, q \in \{1, 2, \ldots, M\}$, $p < q$
	while $i \leq N_t$ and $\hat{i} \leq N_t$ do
	Repeat d_1, d_2, d_3 calculations
	Increment i, update $\hat{i} = i + 1$
	end while

Step 3	Compute the minimum distance for each receive antenna l as $d_l = \min\{d_1, d_2, d_3\}$
Step 4	Repeat Steps 2 and 3 for all $l \in \{1, 2, \ldots, N_r\}$ to get $d_1, d_2, \ldots, d_{N_r}$
Step 5	Select the receiver antenna index as $v = \arg\max_{l \in \{1,2,\ldots,N_r\}} \{d_l\}$
Step 6	Perform ML detection of the SM symbol as $\hat{i} = \arg\min_{i \in \{1,2,\ldots,N_t\}} \left\|y_v - \sqrt{\rho}\, h_{v,i} x_{ref}\right\|^2$ where y_v is the received signal at the selected antenna v.

TABLE I

ED-GSC Algorithm

Step 1 Initialize the channel matrix $\mathbf{H}$ of dimension $N_r \times N_t$ with all entries being i.i.d. complex Gaussian.

Step 2 Compute the Euclidean distances for each receive antenna l:

$d_{1,l}^{(i,\hat{i},p)} = \left\|(h_{l,i} - h_{l,\hat{i}})x_p\right\|^2, \quad d_{2,l}^{(i,\hat{i},p,q)} = \left\|h_{l,i}x_p - h_{l,\hat{i}}x_q\right\|^2,$

$d_{3,l}^{(p,q,i)} = \|x_p - x_q\|^2 \cdot \|h_{l,i}\|^2$

Initial $i = 1$, $\hat{i} < i$, $p, q \in \{1, 2, \ldots, M\},\ p < q$

while $i \leq N_t$ and $\hat{i} \leq N_t$ do

Repeat d_1, d_2, d_3 calculations

Increment i, update $\hat{i} = i + 1$

end while

Step 3 Compute the minimum distance for each receive antenna l as

$d_l = \min\{d_1, d_2, d_3\}$

Step 4 Repeat Steps 2 and 3 for all $l \in \{1, 2, \ldots, N_r\}$ to get $d_1, d_2, \ldots, d_{N_r}$

Step 5 Initialize selected set $\mathcal{S} = \emptyset$

For $k = 1$ to N_{rf} do

$v_k = \arg\max_l\{d_l\}$

$\mathcal{S} = \mathcal{S} \cup \{v_k\}$

$d_{v_k} = -\infty$

end for

Step 6 Perform ML detection of the SM symbol as

For $k = 1$ to N_{rf} do

$\hat{i} = \arg\min_{i\in\{1,2,\ldots,N_t\}} \left\|y_{v_k} - \sqrt{\rho}\, h_{v_k,i}x_{ref}\right\|^2$

Chapter 6

Simulation Results

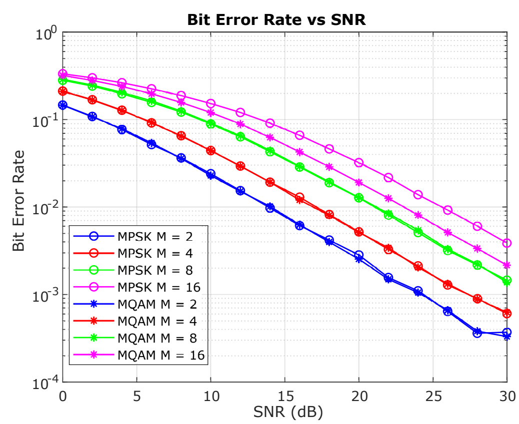

Figure 6.1: BER performance of SISO (PSK and QAM)

Fig. [6.1] shows BER performance of a SISO system with M-PSK and M-QAM modulation techniques for different M. As SNR increases, BER decreases. At lower modulation orders, i.e., $M < 8$, both PSK and QAM show similar BER for a range of SNR values. As M increases beyond 8, it can be observed that QAM has a better BER performance compared to PSK, and this gap increases as SNR increases.

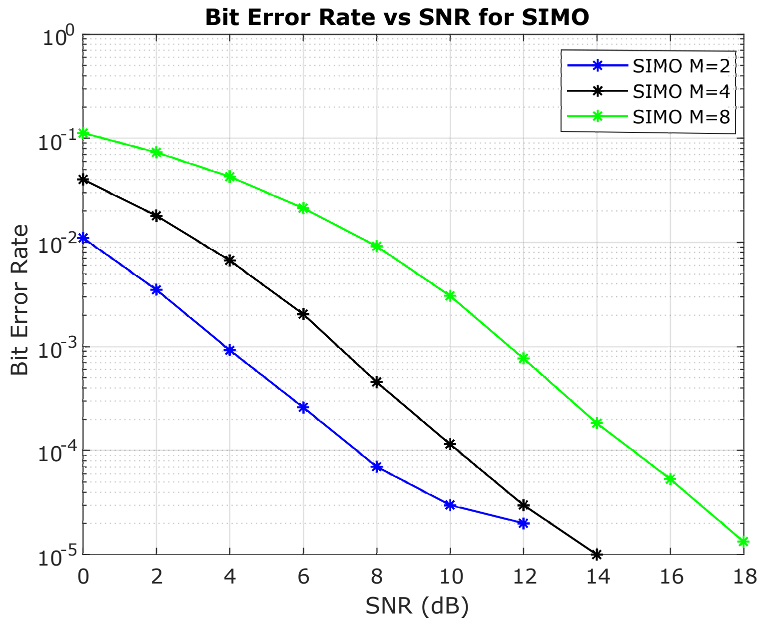

Figure 6.2: BER performance of SIMO with $N_r = 4$ and different M

Fig. [6.2] shows BER performance of a SIMO system for different modulation orders, when the number of receive antennas is kept at constant 4. As M increases, BER increases. This indicates that increasing modulation order to increase data rate leads to BER performance degradation.

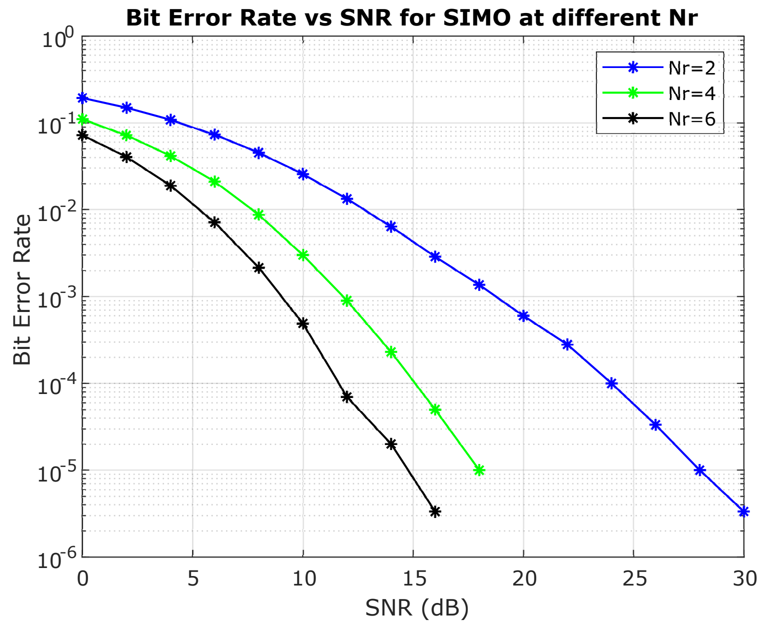

Figure 6.3: BER performance of SIMO with $M = 8$ and different N_r

Fig. [6.3] shows BER performance of a SIMO system for different number of receiver antennas, when modulation order is kept constant at 8. As N_r (number of receiver antennas) increases, BER decreases. It can also be observed that the slope of the curve keeps on increasing as N_r increases. This slope is the diversity order of the system. Increasing N_r increases diversity order and gives better BER performance.

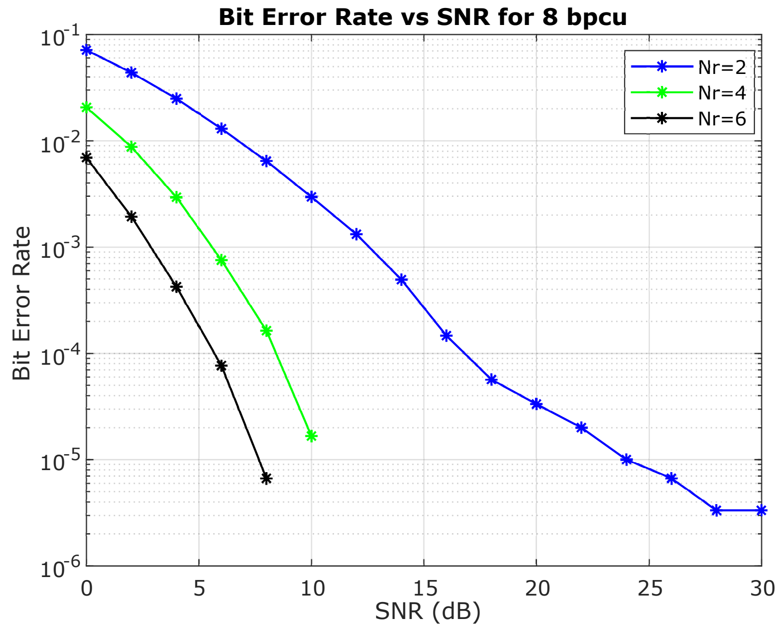

Figure 6.4: BER performance of MIMO (same message streams from all transmit antennas) with $N_t = 4$, $M = 8$, different N_r

Fig. [6.4] shows BER performance of a MIMO system for different number of receiver antennas, when modulation order is kept constant at 8 and number of transmit antennas are kept constant at 4. Similar to SIMO, it can be observed that increasing N_r decreases BER. The slope here increases more rapidly as compared to SIMO.

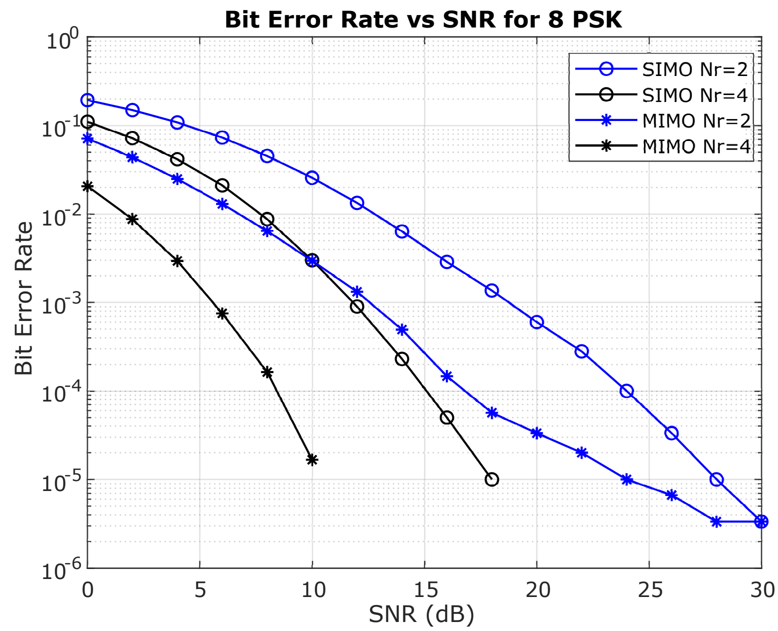

Figure 6.5: BER performance of SIMO and MIMO for different N_r ($N_t = 4$, $M = 8$)

Fig. [6.5] shows the comparison between SIMO and MIMO with all transmit antennas transmitting same message, when $N_t = 4$ and $M = 8$. The difference between the two systems is number of transmit antennas. It can be observed that MIMO gives better performance as compared to SIMO under same conditions due to transmit antenna diversity.

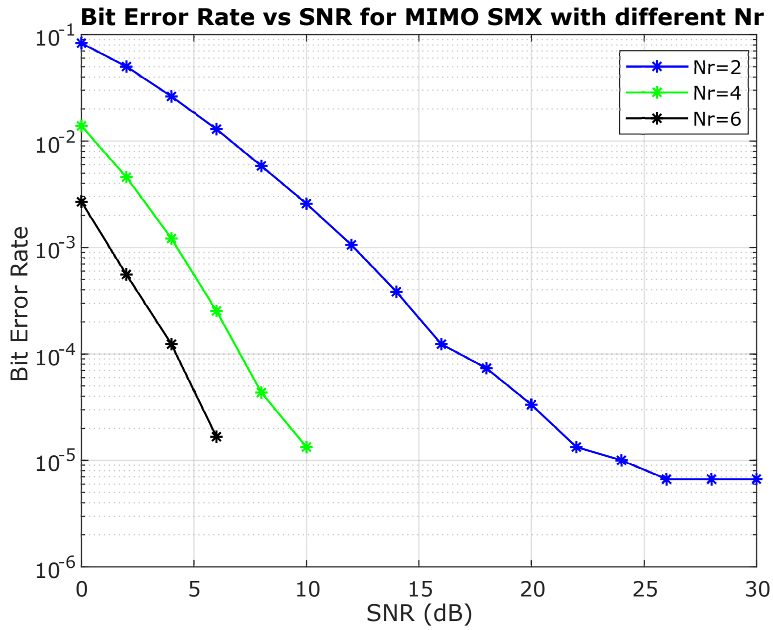

Figure 6.6: BER performance of MIMO with SMX with ($N_t = 3$, $M = 2$) and different N_r

Fig. [6.6] shows BER performance of a MIMO system with SMX for different number of receiver antennas, when $N_t = 3$ and $M = 2$. Increasing N_r for MIMO with SMX shows a similar trend of decreasing BER. The slope here increases even more rapidly as compared to the previous systems.

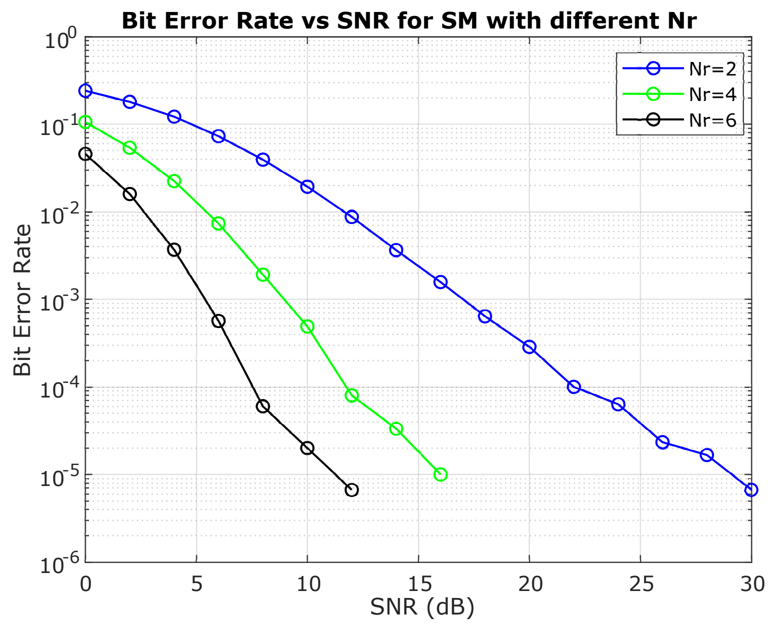

Figure 6.7: BER performance of SM with ($N_t = 4$, $M = 2$) different N_r

Fig. [6.7] shows BER performance of a SM system for different number of receiver antennas, when $N_t = 4$ and $M = 2$. There is a decrease in BER on increasing N_r. The slope does not increase that rapidly as compared to the MIMO with SMX, but it still shows some increase. Increasing N_r from 2 to 4 shows a greater improvement in BER performance than increasing it from 4 to 6.

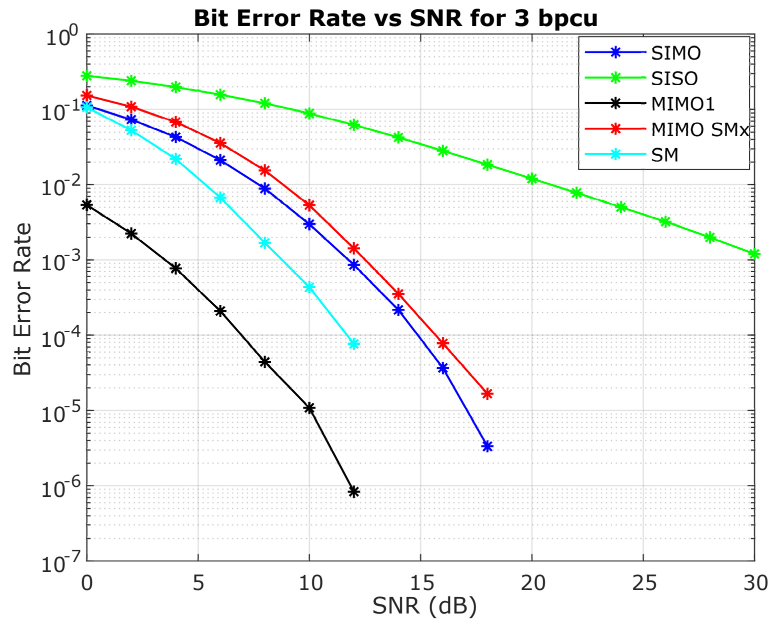

Figure 6.8: BER performance for 3 bpcu for SISO, SIMO, MIMO, MIMO with SMX, SM

Fig. [6.8] shows the comparison of BER performance for 3 bpcu for SISO, SIMO, MIMO, MIMO with SMX, SM. It can be seen that the SISO system has worst BER performance as compared to all other systems and MIMO with same message transmitted from all transmit antennas has the best BER performance. SM outperforms all systems except MIMO with same message transmitted from all transmit antennas.

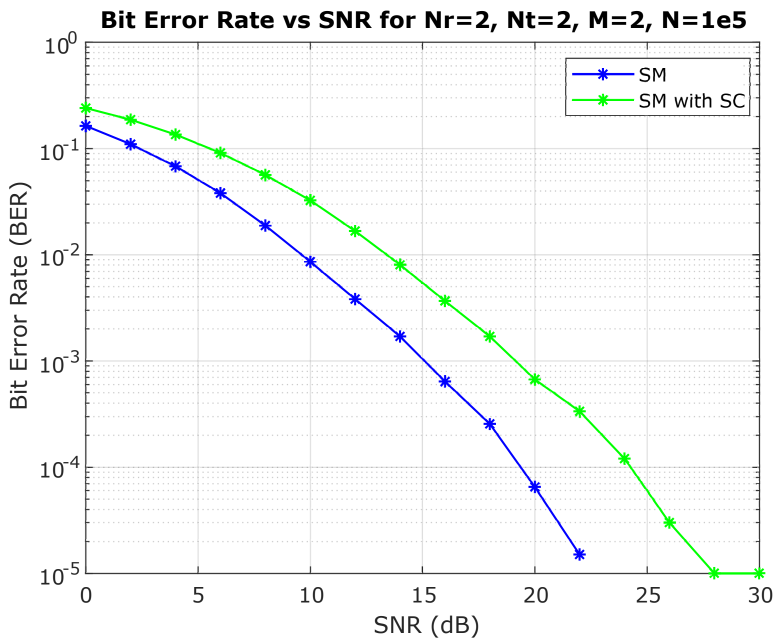

Figure 6.9: BER performance comparison between SM and SM with ED-SC with N_t=2, N_r=2, M=2

Fig. [6.9] shows the BER performance comparison between SM and SM with ED-SC with N_t=2, N_r=2 and BPSK modulation scheme.It can be seen that SM has a better BER performance compared to SM with ED-SC as we are trading of reliability for more EE and reduced receiver complexity. Coding gain can also be observed between SM and SM with ED-SC.

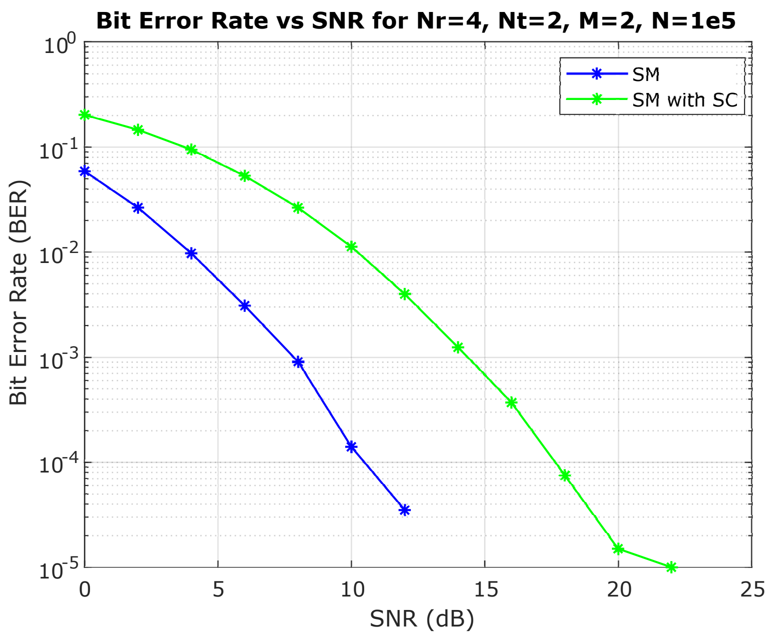

Figure 6.10: BER performance comparison between SM and SM with ED-SC with N_t=2, N_r=4, M=2

Fig. [6.10] shows the BER performance comparison between SM and SM with ED-SC with N_t=2, N_r=4 and BPSK modulation scheme.It can be seen that SM has a better BER performance compared to SM with ED-SC as we are trading of reliability for more EE and reduced receiver complexity. This improvement in BER performance is much more pronounced as compared to N_r=2. Coding gain can also be observed between SM and SM with ED-SC. Even the coding gain in this case is considerably larger than N_r=2.

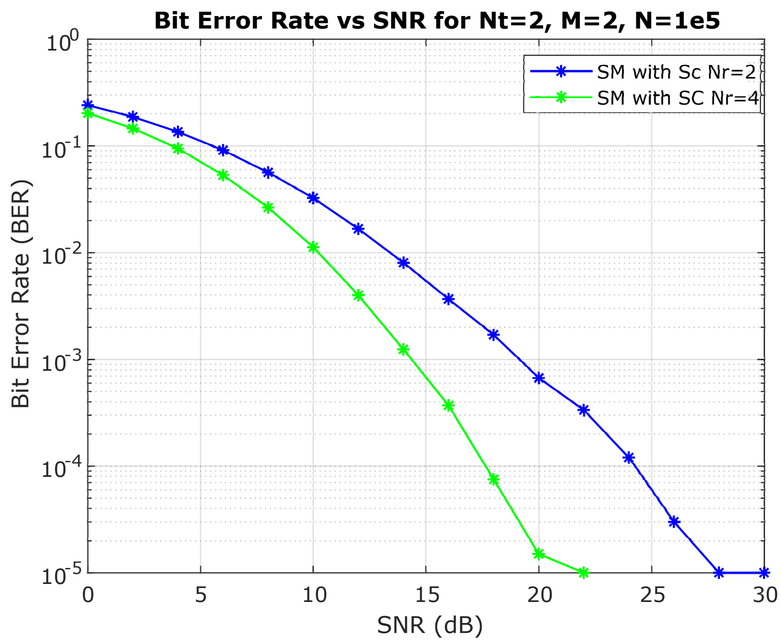

Figure 6.11: BER performance comparison between SM with ED-SC with N_r=2 and N_r=4

Fig. [6.11] shows BER performance comparison between SM with ED-SC with N_r=2 and N_r=4. It can be seen that as number of receiver antennas is increased in ED-SC, the BER performance improves i.e. increase in diversity order reduces error probability. Diversity order can be obtained as slope of the curves.

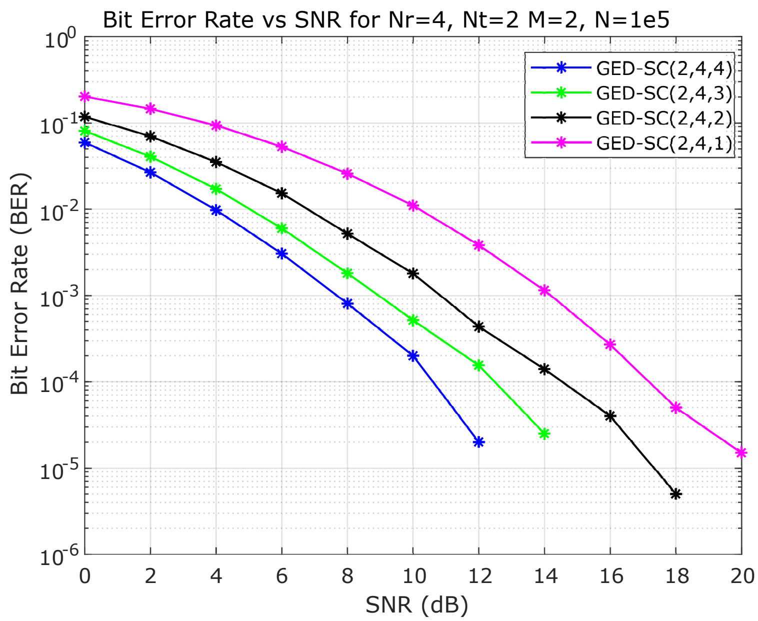

Figure 6.12: BER performance of SM with ED-GSC system with fixed N_t=2, N_r=4 and varying N_{rf}

Fig. [6.12] shows BER performance of SM with ED-GSC system with fixed N_t=2, N_r=4 and varying N_{rf} from 1 to 4. We can see a large improvement in BER performance when N_{rf} increases from 1 to 2. This improvement reduces while moving from 2 to 3 and 3 to 4. So it can be inferred that increasing N_{rf} improves system's BER performance while also increasing complexity and reducing EE. Coding gain can also be observed between the 4 cases.

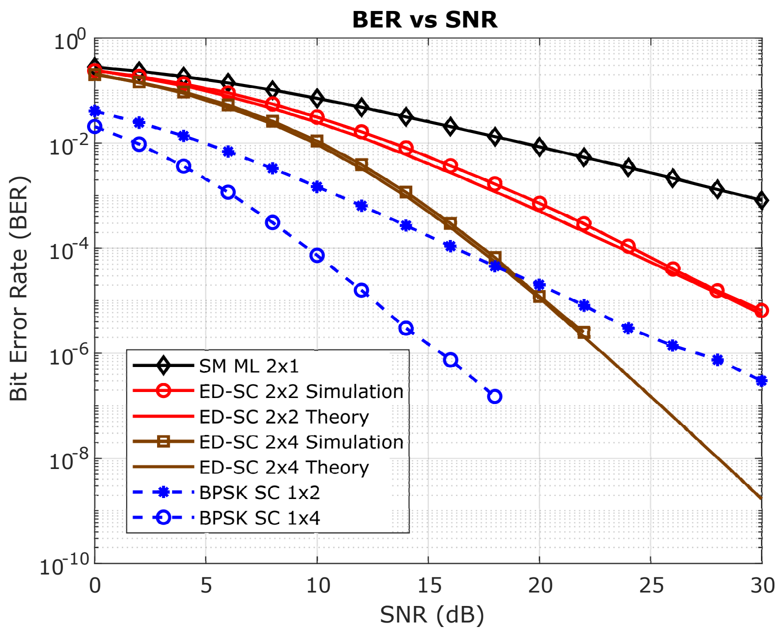

Figure 6.13: BER performance of SM, SM with ED-SC, BPSK SC, with single RF chain

Fig. [6.13] shows BER performance of systems like SM, SM with ED-SC, BPSK SC, with single RF chain. Also theoretical values for SM with ED-SC with N_t=2, N_r=2,4 are plotted to validate them against simulation. It can be seen that the theoretical calculation matches closely with simulation. The coding gain between ED-SC N_t=2, N_r=2 and BPSK SC 1x2, and between ED-SC N_t=2, N_r=4 and BPSK 1x4 can be seen. SM with ML detection and N_t=2, N_r1 has the worst BER performance, while BPSK SC N_t=1, N_r=4 has the best

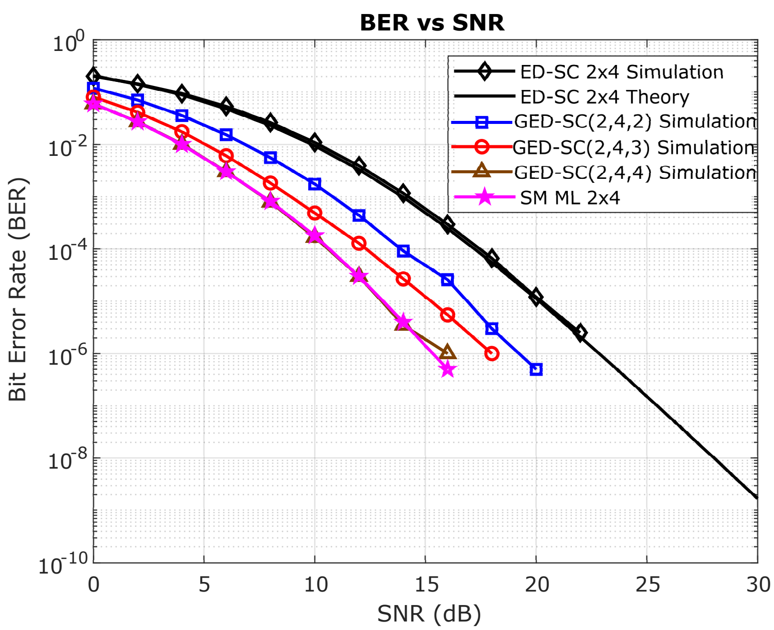

Figure 6.14: BER performance of Sm with ED-SC N_t=2, N_r=4 simulation and theory, SM with ED-GSC for fixed N_t=2, N_r=4 and varying N_{rf} from 2 to 4 and SM with ML N_t=2, N_r=4

Fig. [6.14] shows BER performance of Sm with ED-SC N_t=2, N_r=4 simulation and theory, SM with ED-GSC for fixed N_t=2, N_r=4 and varying N_{rf} from 2 to 4 and SM with ML N_t=2, N_r=4. It can be seen that the theoretical calculation matches closely with simulation.Performance of SM with ML N_t=2, N_r=4 matches with SM with ED-GSC for fixed N_t=2, N_r=4, N_{rf}=4. SM with ED-SC N_t=2, N_r=4 has the worst BER performance, while SM with ML N_t=2, N_r=4 and SM with ED-GSC for fixed N_t=2, N_r=4, N_{rf}=4 have the best. Coding gain can be observed between GED-SC with 2 transmit antennas, 4 receive antennas and 2,3,4 receive antenna selections.

Chapter 7

Conclusion

7.1 Conclusion

Two of the main objectives of 5G and beyond (B5G) networks are to improve spectral efficiency (SE) and energy efficiency (EE). To improve SE, the data rate of a system for a single channel use can be increased by increasing the modulation order M. However, this increase leads to degradation in BER performance. To improve BER performance, the system can be extended from SISO to SIMO. An increase in the number of receive antennas N_r improves the BER.

Moving on to MIMO systems, the first configuration considered involves all transmit antennas sending the same message stream to enhance reliability. This results in improved BER performance over SIMO. To increase the data rate, a MIMO system where each transmit antenna transmits a different message stream is introduced. While this significantly increases SE compared to previous systems, it also demands multiple active transmit and receive antennas, leading to requirements for inter-antenna synchronization

(IAS), multiple RF chains, and inter-channel interference (ICI). These factors compromise energy efficiency and make the system costly and complex to implement.

To overcome these limitations, Spatial Modulation (SM) offers a compelling alternative that provides a balance between SE and EE. SM outperforms all systems except the MIMO configuration with identical message streams from all transmit antennas, and achieves high SE. It is also energy efficient due to the absence of IAS and ICI, and the need for only a single RF chain.

Moreover, when comparing SM with ED-SC (2 transmit antennas, 2 receive antennas, and $M = 2$) and (2 transmit antennas, 4 receive antennas, and $M = 2$), a clear coding gain is observed in favor of ED-SC. As the number of receive antennas increases in ED-SC, the BER performance improves, confirming that higher diversity order reduces the error probability.

Theoretical analysis for ED-SC matches closely with simulations for 2 transmit antennas and both 2 and 4 receive antennas, where coding gains over BPSK SC (1×2 and 1×4) are also evident. Lastly, GED-SC systems with 2 transmit antennas and 4 receive antennas using 2, 3, and 4 receive antenna selections show improved BER performance, with the case of 4 selected antennas closely matching SM-ML with 2×4 configuration.

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