1: Encuentra el espectro de potencias en 1d, 2d y 3d para la función de correlación

$$\xi(r) = \left(\frac{r}{r_0}\right)^{-\gamma}$$

Para 1 dim: $P(k) = \int_0^\infty \xi(r) \cos(kr) 2\, dr$

$$P(k) = \int_0^\infty \left(\frac{r}{r_0}\right)^{-\gamma} \cos(kr) 2\, dr = 2 r_0^{\gamma} \int_0^\infty r^{-\gamma} \cos(kr)\, dr$$

recordando que $\cos\alpha = \frac{e^{i\alpha} + e^{-i\alpha}}{2} \Rightarrow P(k) = r_0^{\gamma} \int_0^\infty r^{-\gamma} \left[e^{ikr} + e^{-ikr} \right] dr$

$$P(k) = r_0^{\gamma} \int_0^\infty r^{-\gamma} e^{ikr}\, dr + r_0^{\gamma} \int_0^\infty r^{-\gamma} e^{-ikr}\, dr ,$$

haciendo el cambio de variable $u = ikr \Rightarrow du = ik\, dr$

$$P(k) = \underbrace{\frac{r_0^{\gamma}}{ik} \int_0^\infty \left(\frac{u}{ik}\right)^{-\gamma} e^{u}\, du}_{\text{I}} + \underbrace{\frac{r_0^{\gamma}}{ik} \int_0^\infty \left(\frac{u}{ik}\right)^{-\gamma} e^{-u}\, du}_{\text{II}}$$

II $\frac{r_0^{\gamma}}{(ik)^{-\gamma+1}} \int_0^\infty u^{-\gamma} e^{-u}\, du$, recordando: $\Gamma(a) = \int_0^\infty z^{a-1} e^{-z}\, dz$

$$\Rightarrow \quad r_0^{\gamma} (ik)^{\gamma-1}\, \Gamma(-\gamma+1)$$

$$\textcircled{I}\ \frac{r_0^{\gamma}}{ik}\int_0^{\infty}\left(\frac{u}{ik}\right)^{-\gamma}e^{u}\,du, \qquad \text{si} \quad u=-v$$

$$-\frac{r_0^{\gamma}}{ik}\int_0^{\infty}\left(\frac{-v}{ik}\right)^{-\gamma}e^{-v}\,dv = r_0^{\gamma}(-ik)^{\gamma-1}\int_0^{\infty}v^{-\gamma}e^{-v}\,dv = r_0^{\gamma}(-ik)^{\gamma-1}\Gamma(-\gamma+1)$$

Así que el espectro de potencias es

$$P(k) = \Gamma(-\gamma+1)\left[(ik)^{\gamma-1} + (-ik)^{\gamma-1}\right]r_0^{\gamma}$$

Para 2 dim: $P(k) = \int_0^{\infty} \xi(r)\, J_0(kr)\, 2\pi r\, dr$

$$P(k) = \int_0^{\infty}\left(\frac{r}{r_0}\right)^{-\gamma} J_0(kr)\, 2\pi r\, dr,$$

recordando la función de Bessel J_0

$$J_0(x) = 1 - \frac{x^2}{2^2} + \frac{x^4}{2^2 4^2} - \frac{x^6}{2^2 4^2 6^2}\cdots$$

Tomando hasta el término de segundo orden

$$P(k) = \frac{2\pi}{r_0^{-\gamma}}\int_0^{\infty} r^{-\gamma+1}\left(1 - \frac{(kr)^2}{4}\right)dr$$

$$= 2\pi r_0^{\gamma}\int_0^{\infty} r^{-\gamma+1}\,dr - \frac{\pi k^2 r_0^{\gamma}}{2}\int_0^{\infty} r^{-\gamma+3}\,dr$$

$$= 2\pi r_0^{\gamma}\left.\frac{r^{-\gamma+2}}{-\gamma+2}\right|_0^{\infty} - \frac{\pi k^2 r_0^{\gamma}}{2}\left.\frac{r^{-\gamma+4}}{-\gamma+4}\right|_0^{\infty} = 2\pi r_0^{\gamma}\lim_{r\to\infty}\left[\frac{r^{-\gamma+2}}{-\gamma+2} - \frac{k^2}{4}\frac{r^{-\gamma+4}}{-\gamma+4}\right]$$

Para que no se indetermine $\gamma \neq 2, 4$

$$P(k) = 2\pi\, r_0^{\gamma} \lim_{r\to\infty} \left[\frac{r^{-\gamma+2}}{-\gamma+2} - \frac{k^2}{4} \frac{r^{-\gamma+4}}{-\gamma+4} \right], \quad \text{con} \quad \gamma \neq 2,4$$

Para 3 dim: $P(k) = \int_0^{\infty} \xi(r) r^2 \frac{\sin(kr)}{kr}\, dr$

$P(k) = \int_0^{\infty} r^2 \left(\frac{r}{r_0}\right)^{-\gamma} \frac{\sin(kr)}{kr}\, dr$, Sabemos que $\lim_{x\to 0} \frac{\sin x}{x} = 1$

entonces con $kr < 1 \Rightarrow \frac{\sin(kr)}{kr} \simeq 1$,

$\Rightarrow r < 1/k$

Mientras que para x grandes $\frac{\sin x}{x} \simeq 0$

$$\Rightarrow P(k) = r_0^{\gamma} \int_0^{1/k} r^{-\gamma+2}\, dr = r_0^{\gamma} \left.\frac{r^{-\gamma+3}}{-\gamma+3}\right|_0^{1/k} = \frac{r_0^{\gamma}}{-\gamma+3} k^{\gamma-3}, \quad \text{con} \quad \gamma \neq 3$$

$$P(k) = \frac{r_0^{\gamma}}{3-\gamma} k^{\gamma-3}$$