Critical points

Definition

We call x = c for the critical point of the function f(x) if f(c) exists and if either of the following are true.

$$f'(c) = 0 \qquad \text{or} \qquad f'(c) \text{ doesn't exist}$$

Example 1.

Determine all the critical points for the function.

$$f(x) = 6x^5 + 33x^4 - 30x^3 + 100$$

Solution

First of all we need the derivative of the function in order to find the critical points .

$$f := 6\ x^5 + 33\ x^4 - 30\ x^3 + 100$$

$$f' := \frac{d}{dx}\ f$$

$$f' = 30\ x^4 + 132\ x^3 - 90\ x^2$$

The first derivative is a polynomial and the only critical points will be those values of x which make the derivative zero. So we must solve equation

$$30x^4 + 132x^3 - 90x^2 = 0$$

$$\text{nonlinsolve}\left(f' == 0\ ,\ x\right) = \begin{bmatrix} -5 & 0 & 0.6 \end{bmatrix}$$

So the critical points are -5, 0 and 0.6.

Example 2.

Determine all the critical points for the function.

$$h(x) = x^2 \ln(3x) + 6$$

Solution

The function ln(x) is defined only when x > 0. The derivative is

$$h := x^2 \cdot \ln(3\,x) + 6$$

$$h' := \frac{d}{dx}\, h$$

$$h' = x + 2\,x\,\ln(3\,x)$$

The derivative and the function will not exist if $x \leq 0$. So the derivative will not be zero for $x = 0$ because of the natural logarithm. The equation that we have to solve is

$$\mathrm{nonlinsolve}(h' == 0\ ,\ x) = 0.202$$

This is the only critical point for this function.