Conic sections

Transformation from general to standard cartesian form

If we have the conic equation in general cartesian form and transform it to standard form and determine which conic it represents.

$$9x^2-4y^2-18x+32y-91=0$$

Starting equation

$$9x^2-18x \qquad -4y^2+32y \qquad -91=0$$

Arrange terms the terms like so.

pull out a 9   pull out a -1

$$9\left(x^2-2x\right) \qquad -4\left(y^2-8y\right) \qquad -91=0$$

Make sure the squared terms have a 1 in front.

take half → square it   take half → square it

$$9\left(x^2-2x+1-1\right) \qquad -4\left(y^2-8y+16-16\right)-91=0$$

Take a half of the number in front of the x or y and square it. Add and subtract it.

take half   take half

$$9\left((x-1)^2-1\right) \qquad -4\left((y-4)^2-16\right) \qquad -91=0$$

Take the first three terms and turn them into an (x^2 +___)² expression.

$$9(x-1)^2-9\cdot 1 \qquad -4(y-4)^2-4\cdot(-16) \qquad -91=0$$

Distribute any term outside the parentheses(), and simplify.

simplify   simplify

$$9(x-1)^2-9 \qquad -4(y-4)^2+64 \qquad -91=0$$

$$9(x-1)^2-9-4(y-4)^2+64-91=0+9-64+91$$

+9   -64+91

Move the constants to the other side of the equation.

$$9(x-1)^2-4\cdot(y-4)^2==36$$

needs to be 1, divide both sides

Divide both sides to make the right side of the equation = 1.

$$\frac{\cancel{9}(x-1)^2}{\cancel{36}}-\frac{\cancel{4}\cdot(y-4)^2}{\cancel{36}}==\frac{\cancel{36}}{\cancel{36}}$$

Divide if necessary.

Finally, our equation looks like this.

$$\frac{(x-1)^2}{4}-\frac{(y-4)^2}{9}==1$$

From this equation, we can see that the curve is a <u>hyperbola</u>.