Dragon 3D Sketchpad

- A Solid Geometry Intelligent Educational Platform*

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***Abstract*—Solid geometry is the emphasis and the difficulty in high school mathematical teaching. This paper described the design and implementation, Dragon 3D Sketchpad, a solid geometry intelligent educational platform for assisting the teaching and learning of solid geometry. It was a dynamic solid geometry software and had the ability of automated geometry reasoning, it could generate short and readable proofs in traditional style. We gave the idea and explained the design and elementary functions of the platform and some cases to demonstrate this intelligent educational platform were shown.**

***Index Terms*—dynamic solid geometry, automated geometric reasoning, intelligent knowledge platform, intelligent educational platform**

I. Introduction

A computer system in some special field of knowledge which people use to refer knowledge, apply knowledge, diffuse knowledge, learn knowledge and develop knowledge is called an intelligent knowledge platform[2, 3]. If we control the scope of knowledge in high school or university, the intelligent knowledge platform can called intelligent educational platform[2].

Because of mathematics is a foundation and important course in high school, so educational software often firstly develops the mathematical software[1]. For example, The Geometer's Sketchpad[5], Cabri Geometry[6], Cinderella[7], The Super Sketchpad[4], etc. Their functions include some or most mathematical knowledge in high school. Among them The Super Sketchpad is designed specifically for the Chinese mathematical education and it is welcomed for its intelligent, interactive, dynamic and fun by teachers and students in high school. The students can use it understand abstract mathematical concepts and study the knowledge of geometry[4, 5].

However, because of the difficulties of development, most of the mathematical software just having plane geometry related features up to now. In fact, the geometry education is the focus of the high school education and the solid geometry education is a difficulty in geometry education. In solid geometry teaching, the teachers usually sketch on the blackboard and the diagram is more complicated than plane geometry so it is very difficult to imagine the geometric figure and interfere the comprehension of the solid geometry knowledge.

Therefore, under such background we have developed a solid geometry intelligent educational platform, Dragon 3D Sketchpad, for high school solid geometry education. It has not only the basic features of a dynamic geometry software, such as dynamic drawing, dragging, tracking, following, animation, measurement, transformation etc, but also have the abilities of automated geometry reasoning and can generate short and readable proofs. It can be used to as a solid geometry diagram editor and a prover to produce proofs in traditional style.

We hope that Dragon 3D Sketchpad can not only help students comprehend the concept of solid geometry, improve the learning efficiency, but also help teachers prepare lessons, reduce mechanical work and teach solid geometry knowledge.

II. The Design Principle

Without a doubt, the success of a educational software depends on if it is popular and accepted by the teachers and students. The design of Dragon 3D Sketchpad based on three principals, there were humanization, facing the specific curriculum and intelligence, which are the key factors determining the popularity of educational software and which are summarized after researching several educational software at home and abroad[8].

A. Humanization

No matter how complicated the software's design is, abundant its functions has, or powerful its intelligence has, if it is difficult to users, the software won't be popular and accepted by them. As an educational software for geometry teaching which is developed for teachers and students, its design must according to the teachers and students' usage patterns and requirements. For example, Similar to our software some good dynamic geometry software attributes drawing of diagram to that of geometric elements such as point, line, circle, and so on, which follows the mathematical tradition of axiom. But they all use different tools for drawing point, line and circle. It is ridiculous just thinking whether we change frequently our drawer or not in order to draw them in common situation. Why to use three pens if one a pen is enough? On the contrary, our software only use an intelligent pen to do these. After entering intelligent drawing state by clicking the icon, the mouse becomes an intelligent pen as a chalk by which we can without any shift between menus and tools draw almost all elementary geometric elements, such as free point, line segment, circle, polygon and polyhedron, etc.

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B. *Facing the specific curriculum*

The educational software which is facing the specific curriculum must have explicit elementary functions for teachers' teaching and students' learning of specific curriculum. Dragon 3D Sketchpad is designed for solid geometry teaching in high school, so we should first focus on dynamic interactive drawing function, furthermore we should implement intelligence in software, moreover we support dynamic measurement function, and in addition of above we provide plentiful multimedia resources and cases for teachers and students.

C. *Intelligence*

The intelligence of geometry educational software usually present in two aspects. One aspect is that the software can judge users' intention automatically by extracting information from users' operations and guide users to achieve their goals easily. The other is that the software has the ability of automated geometry reasoning which can help users to solve geometric problems automatically. Our software, Dragon 3D Sketchpad, completely implemented the two aspects which mentioned above. An elementary function called intelligent pen could give users some tips when they draw diagrams with mouse, and it could reduce the switching operation between different drawing tools. Meanwhile, Dragon 3D Sketchpad could automatically prove geometry theorems and generate short and readable proofs in tradition style.

III. THE MAIN ARCHITECTURE

Our platform is designed and implemented by a three-layer structure. Our system architecture showed in Figure 1 and it should be described in detail.

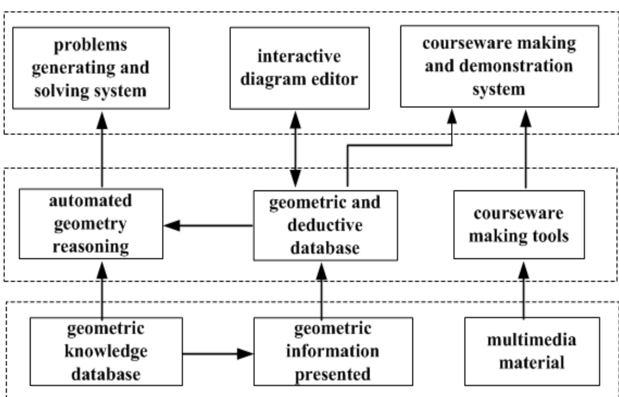

Figure 1: The Main architecture of the platform

A. *The Underlying Layer*

The underlying layer included geometric knowledge database, geometric information database and multimedia material database.

- The geometric knowledge database recorded the basic and elementary knowledge of solid geometry which we summed up from the textbooks, including concepts, facts, theorems, axioms, etc.
- The geometric information database included the geometric elements in process of diagram construction, because of the diagram constructions were built step by step, thus we record the information in the same way.
- The courseware material database usually included some pictures, texts, animations, etc, which could be used to help teacher preparing the lessons.

B. *The Middle Layer*

The middle layer was a bridge contacting the underlying layer and the application layer. It was composed by three modules which were dynamic geometry module, automated geometry reasoning module and courseware making module.

- The dynamic geometry module provided the features like usual dynamic geometry software such as The Geometer's Sketchpad[5], Cabri Geometry[6], Cinderella[7] and so on. And it was not only used as an interactive diagram editor for user but also used as an input tool for the automated geometry reasoning module. It implemented the elementary functions such as dynamic geometry construction, dynamic measurement, tracking, following, geometric transformation, etc.
- The automated geometry reasoning module is an important module in our platform, we have implemented the deductive database method[9]. This method is based on deductive database. We used it to generate the fix point in reasoning for a given geometric configuration under a fixed set of geometric rules or axioms. With this method, we could find a large portion of the facts about a given configuration and could generate proofs in traditional style. This module's mechanism in Figure 2.

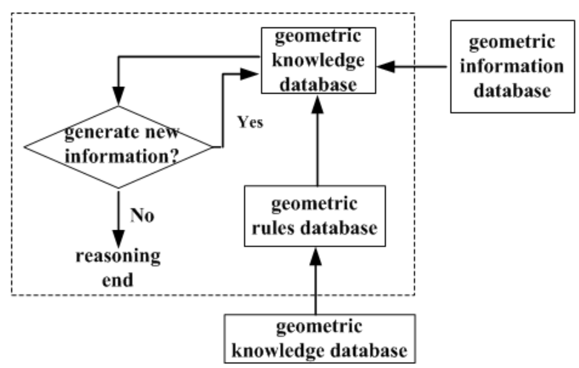

Figure 2: The mechanism of automated reasoning

- Courseware making tools module included buttons, audio materials, visual materials and page management feature set.

C. *The Application Layer*

The application layer provided a nice graphic user interface to deal with the user's operation and support the functions in middle layer to user. It included dynamic drawing module, question solving module and courseware making module.

- The user could used dynamic drawing module to generate diagrams interactively based on ruler and compass construction and drag it with geometric consistent.

- The question solving module is that, first the user could add new parameters or geometric information into the old geometric deductive database, then give out a question base on present diagram, finally use automated reasoning module to support the proofs in traditional style.
- The courseware making module is used to provide flexible, powerful and more open manipulation for teachers to ready lessons with our platform.

IV. FUNCTION MODULE AND DEMONSTRATION U

Through function decomposition, the platform is broken down into certain modules, including: Geometry drawing, relation drawing, geometric transformation, dynamic measurement, animation, tracking, following, algebra expression, courseware demo and automated reasoning modules. The method decomposition of the modules is applied to realize the communication and cooperation between these modules. And interfaces to external modules are reserved allowing for the extensibility of the platform. The main modules are introduced in detail below.

A. *Geometry Drawing*

With the drawing tools, the geometric elements commonly used in high school can be drawn freely. And the drawn element can be dynamic, namely when dragging its free points or half-free points, its geometric relations remain unchanged. As Figure 4 shows, the geometric figures currently supported by the platform are: point (free point, intersection point, line point, etc), line (line segment, straight line, ray, etc.), circle, quadratic curves (ellipse, hyperbolic curve and parabola), triangle, Equilateral polygon, arbitrary polygon, sphere, cone, cylinder, regular tetrahedron, regular polyhedron, convex polyhedron, etc. Almost the basic types of geometric elements used in the high school teaching are covered.

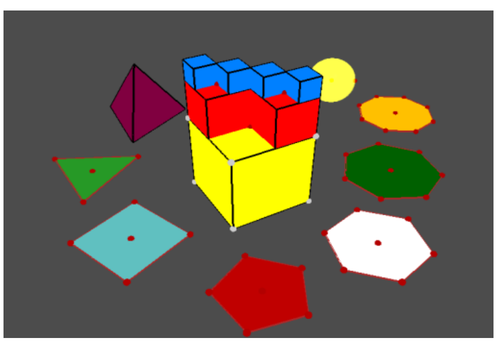

Figure 3: The function of geometry drawing

B. *Relation drawing*

Geometric properties, more often than not, is reflected in the relationships between different geometric elements. Besides drawing independent geometry elements, this platform enables users to draw group geometric elements which maintain a certain relation with one other. For example, a parallel plane can be drawn to an existing plane with the help of "parallel planes drawing". This function is called "relation drawing". The relation drawing provides these fixed relationships: midpoint, proportional division point, angular bisector, parallel line/plane, perpendicular line/plane, vertical cross line, and vertical-mid-plane, etc. Once group geometric elements were drawn, their relations were recorded. If one of them was being dragged or changed, the other would change accordingly, which belongs to the unique feature of dynamic geometry.

C. *Geometric transformation*

Geometric transformation is very important for its broad application. Commands provided are translation, rotation, scaling, axis symmetry and midpoint symmetry.

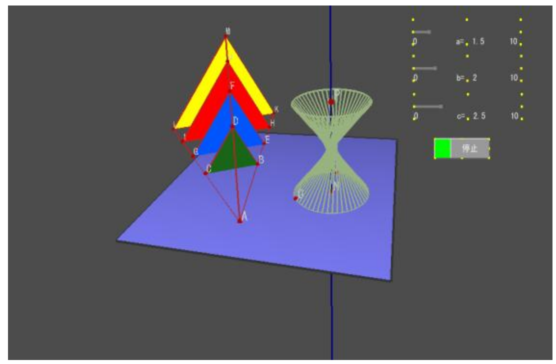

Figure 4: The display of scaling and rotation functions

D. *Dynamic measurement*

User can both measure the geometric quantity and algebraic expression. The result from geometric quantity and algebraic expression measurement will change with the variation of the geometric element and the parameter of expression. Through the dynamic measurement of geometric quantity and algebraic expression, user may find some geometric properties, and then put forward some conjecture. For example, the sum of the distance of one point which is in the regular tetrahedron to the four surfaces of the regular tetrahedron is equal to the high of the regular tetrahedron in Figure 5.

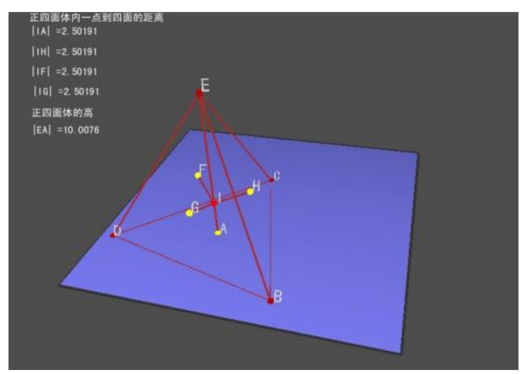

Figure 5: The function of dynamic measurement

E. *Tracking and following*

Free point is the primer object when researching tracking and following problem. Under specific conditions, the tracking will be produced when the free point after being restrained becomes half free point.

The motion which generate the tracking and following may be aroused by either the variety of parameter or the motion of half free point, while the variety of parameter can be represented as the motion of coordinate point, so the tracking and following may be considered as the result of the motion driven by one or more half free points which are called “active point”. And the tracking is the trajectory of the motion or the motion generated by the object which could move.

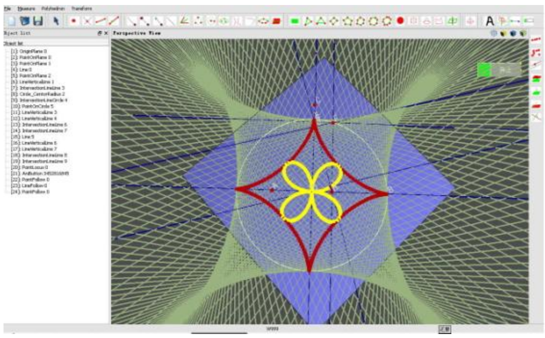

Figure 6: The demonstration of tracking and following

F. Automated geometric reasoning

The geometric information of the drawn objects is automatically stored in the initial information database. When the user clicks on the “reasoning” button, the automated reasoning module can use the information stored to reason deductively according to the mathematic rules. When no more new geometric information can be inferred, the reasoning process will stop and the result will be sorted and stored in different geometric database. So the user can choose from the library systematically and can also get all the deductive process step by step by clicking on the right mouse. If the user prefer to prove a certain conclusion on one’s own, they are two methods we will show in detail below. For example, a tetrahedron ABCD is drawn, then the midpoints of its four edges are pointed E, F, G, H and connected in Figure 7. There are two ways to prove that EFGH is a parallelogram. First way is to find it in the geometric database by clicking “reason” and choosing the parallel group. Second way is to click the “conclusion” button and choose “parallelogram” item. By inputting the four vertexes in order and clicking on the “reasoning” button, then the proof can also be generated.

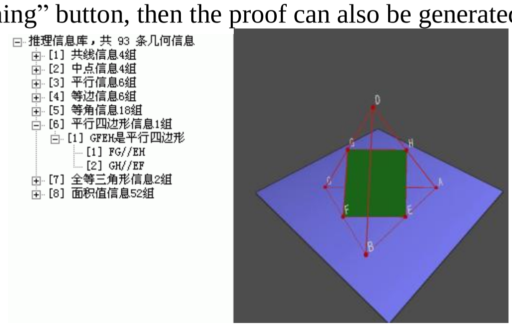

Figure 7: The tetrahedron and its geometric database

V. CONCLUSION AND FUTURE WORK

In this paper we introduced, Dragon 3D Sketchpad, an intelligent educational platform for solid geometry teaching in details. Besides the standard uses of dynamic geometry, the platform offers an interactive, direct, and graphical way for investigating and conjecturing in solid geometry. Up to now, the function of Dragon 3D Sketchpad is far from perfect. For example, there is not enough intelligence of the platform and the deductive database for automated geometry reasoning is also imperfect enough.

We will gradually improve the function of the platform and make it more practical and intelligent, make the module of automated geometry reasoning more perfect, add more rules into the deductive database and develop the interactive reasoning module for users to participate the reasoning process. Finally, we hopes, Dragon 3D Sketchpad, could have a significant impact on the way of teaching and learning solid geometry in high school.

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