Positive Properties of Context-Free languages

Union

Context-free languages
are closed under: **Union**

L_1 is context free

L_2 is context free

$\Rightarrow$ $L_1 \cup L_2$ is context-free

Example

Language | **Grammar**

$$L_1 = \{a^n b^n\}$$

$$S_1 \rightarrow aS_1b \mid \lambda$$

$$L_2 = \{ww^R\}$$

$$S_2 \rightarrow aS_2a \mid bS_2b \mid \lambda$$

Union

$$L = \{a^n b^n\} \cup \{ww^R\}$$

$$S \rightarrow S_1 \mid S_2$$

In general:

For context-free languages $L_1, \; L_2$

with context-free grammars $G_1, \; G_2$

and start variables $S_1, \; S_2$

The grammar of the **union** $L_1 \cup L_2$

has new start variable S

and additional production $S \to S_1 \mid S_2$

Concatenation

Context-free languages are closed under: **Concatenation**

L_1 is context free

L_2 is context free

$\Longrightarrow$ $L_1 L_2$ is context-free

Example

Language	Grammar
$L_1 = \{a^n b^n\}$	$S_1 \to aS_1b \mid \lambda$
$L_2 = \{ww^R\}$	$S_2 \to aS_2a \mid bS_2b \mid \lambda$

Concatenation

$L = \{a^n b^n\}\{ww^R\}$	$S \to S_1 S_2$

In general:

For context-free languages $L_1, \ L_2$

with context-free grammars $G_1, \ G_2$

and start variables $S_1, \ S_2$

The grammar of the **concatenation** L_1L_2

has new start variable S

and additional production $S \rightarrow S_1S_2$

Star Operation

Context-free languages are closed under: **Star-operation**

L is context free $\Rightarrow$ L^* is context-free

Example

Language | **Grammar**

$$L = \{a^n b^n\}$$

$$S \rightarrow aSb \mid \lambda$$

Star Operation

$$L = \{a^n b^n\}^*$$

$$S_1 \rightarrow SS_1 \mid \lambda$$

In general:

For context-free language L

with context-free grammar G

and start variable S

The grammar of the **star operation** L^*

has new start variable S_1

and additional production $S_1 \rightarrow SS_1 \mid \lambda$

Negative Properties of Context-Free Languages

Intersection

Context-free languages
are **not** closed under: **intersection**

L_1 is context free

L_2 is context free

$\Rightarrow$ $L_1 \cap L_2$ **not** necessarily context-free

Example

$$L_1 = \{a^n b^n c^m\}$$

Context-free:

$$S \to AC$$

$$A \to aAb \mid \lambda$$

$$C \to cC \mid \lambda$$

$$L_2 = \{a^n b^m c^m\}$$

Context-free:

$$S \to AB$$

$$A \to aA \mid \lambda$$

$$B \to bBc \mid \lambda$$

Intersection

$L_1 \cap L_2 = \{a^n b^n c^n\}$ **NOT** context-free

Complement

Context-free languages
are **<u>not</u>** closed under: **complement**

L is context free $\Rightarrow$ $\overline{L}$ **<u>not</u>** necessarily context-free

Example

$$L_1 = \{a^n b^n c^m\}$$

Context-free:

$$S \to AC$$

$$A \to aAb \mid \lambda$$

$$C \to cC \mid \lambda$$

$$L_2 = \{a^n b^m c^m\}$$

Context-free:

$$S \to AB$$

$$A \to aA \mid \lambda$$

$$B \to bBc \mid \lambda$$

Complement

$$\overline{\overline{L_1} \cup \overline{L_2}} = L_1 \cap L_2 = \{a^n b^n c^n\}$$

NOT context-free

Intersection
of
Context-free languages
and
Regular Languages

The intersection of

a context-free language and

a regular language

is a context-free language

L_1 context free

L_2 regular

$$\left.\begin{array}{l} L_1 \text{ context free} \\ \\ L_2 \text{ regular} \end{array}\right\} \Rightarrow L_1 \cap L_2 \text{ context-free}$$

Machine M_1

NPDA for L_1

context-free

Machine M_2

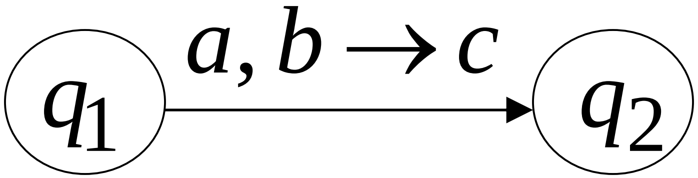
DFA for L_2

regular

Construct a new NPDA machine M that accepts $L_1 \cap L_2$

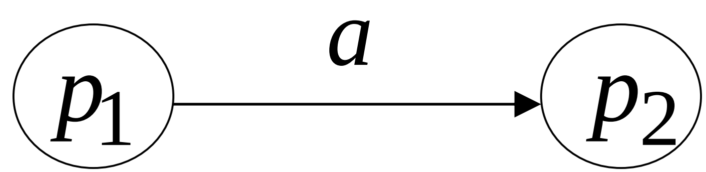
M simulates in parallel M_1 and M_2

NPDA M_1



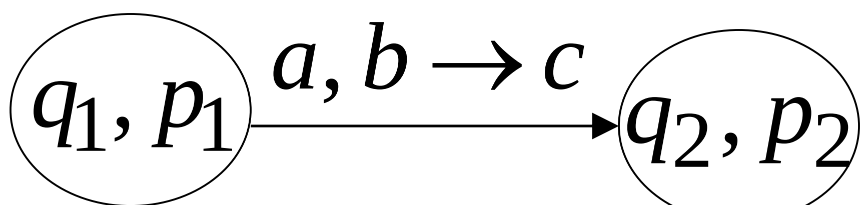
transition

DFA M_2

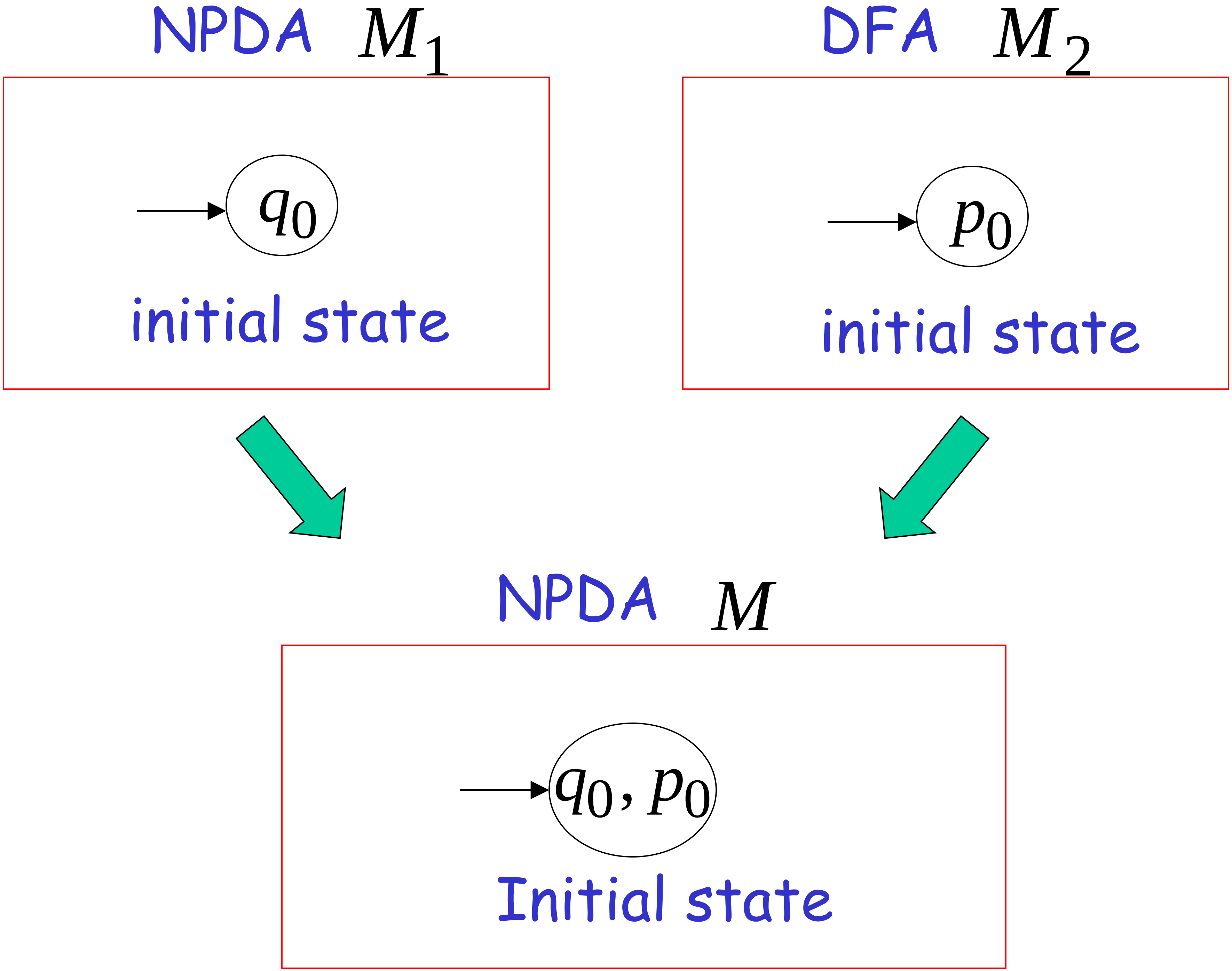


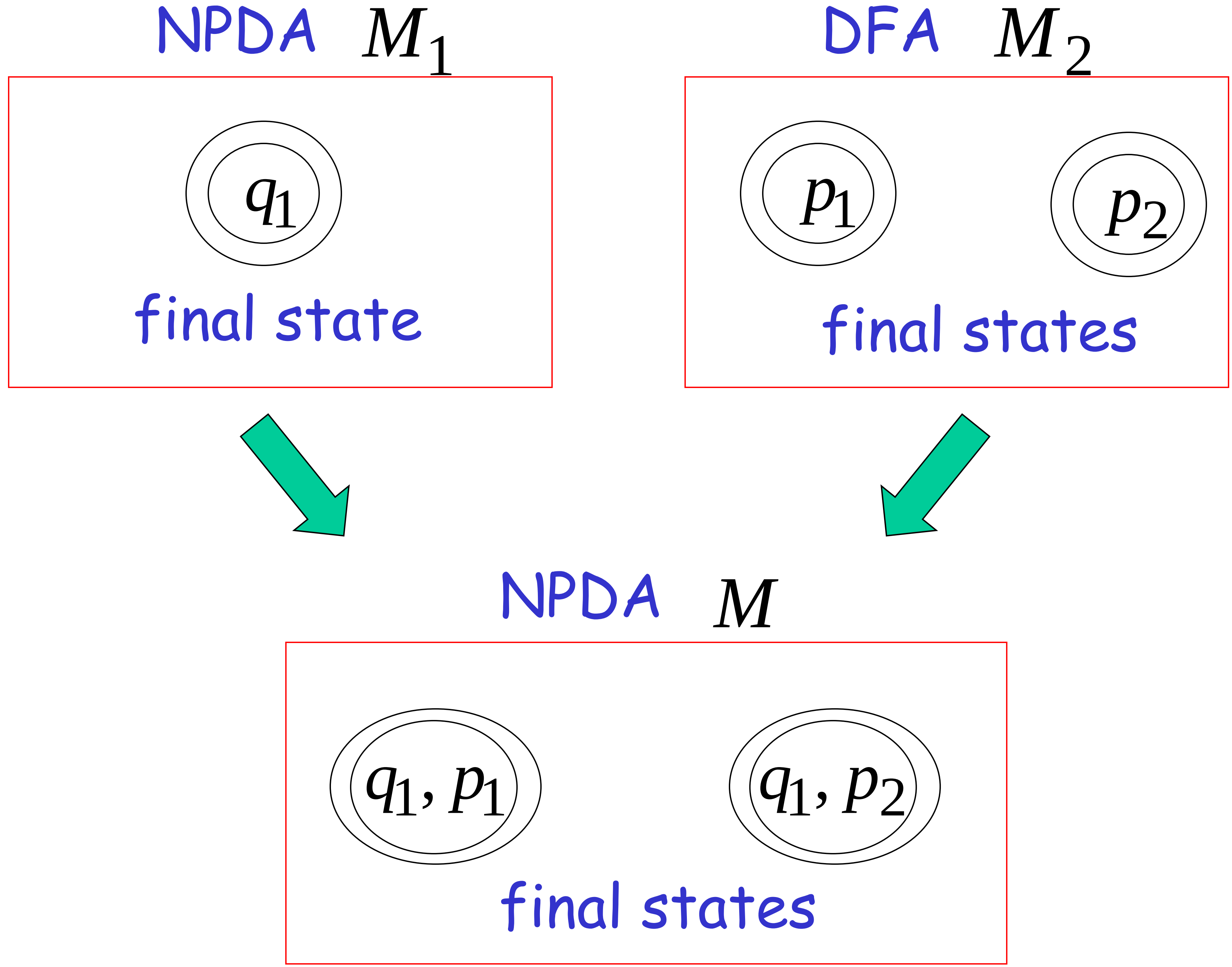
transition

NPDA M



transition


NPDA M_1
q_0
initial state
DFA M_2
p_0
initial state
NPDA M
q_0, p_0
Initial state


NPDA M_1
q_1
final state
DFA M_2
p_1
p_2
final states
NPDA M
q_1, p_1
q_1, p_2
final states

M simulates in parallel M_1 and M_2

M accepts string w if and only if

M_1 accepts string w and
M_2 accepts string w

$$L(M) = L(M_1) \cap L(M_2)$$

Therefore: $L(M_1) \cap L(M_2)$ is context-free

(since M is NPDA)

$$\Downarrow$$

$L_1 \cap L_2$ is context-free