Variations of the Turing Machine

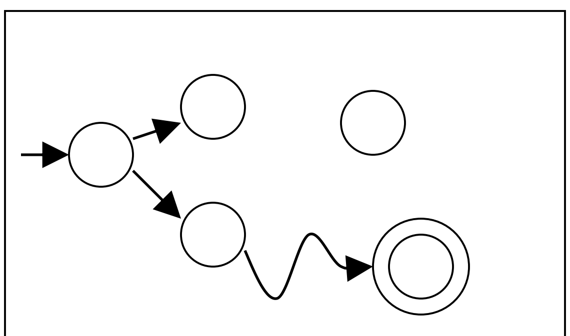
The Standard Model

Infinite Tape

◊	◊	a	a	b	a	b	b	c	a	c	a	◊	◊	◊

Read-Write Head (Left or Right)

Control Unit



Deterministic

Variations of the Standard Model

Turing machines with:

- Stay-Option
- Semi-Infinite Tape
- Off-Line
- Multitape
- Multidimensional
- Nondeterministic

The variations form different Turing Machine **Classes**

We want to prove:

Each **Class** has the same power with the Standard Model

Same Power of two classes means:

Both classes of Turing machines accept the same languages

Same Power of two classes means:

For any machine M_1 of first class

there is a machine M_2 of second class

such that: $L(M_1) = L(M_2)$

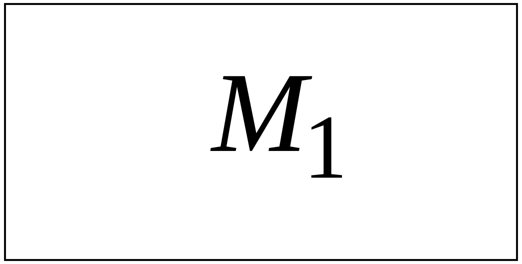
And vice-versa

Simulation: a technique to prove same power

Simulate the machine of one class with a machine of the other class

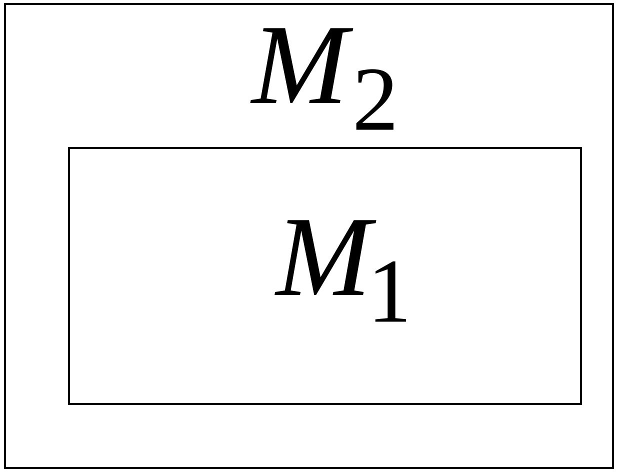
First Class

Original Machine



Second Class

Simulation Machine



Configurations in the Original Machine correspond to configurations in the Simulation Machine

$$\text{Original Machine:} \quad d_0 \succ d_1 \succ \cdots \succ d_n$$

$$\updownarrow \qquad \updownarrow \qquad \qquad \updownarrow$$

$$\text{Simulation Machine:} \quad d'_0 \overset{*}{\succ} d'_1 \overset{*}{\succ} \cdots \overset{*}{\succ} d'_n$$

Final Configuration

Original Machine: d_f

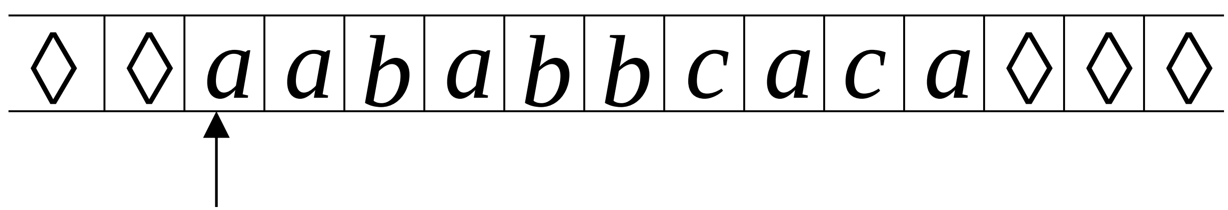
$\updownarrow$

Simulation Machine: d'_f

The Simulation Machine
and the Original Machine
accept the same language

Turing Machines with Stay-Option

The head can stay in the same position

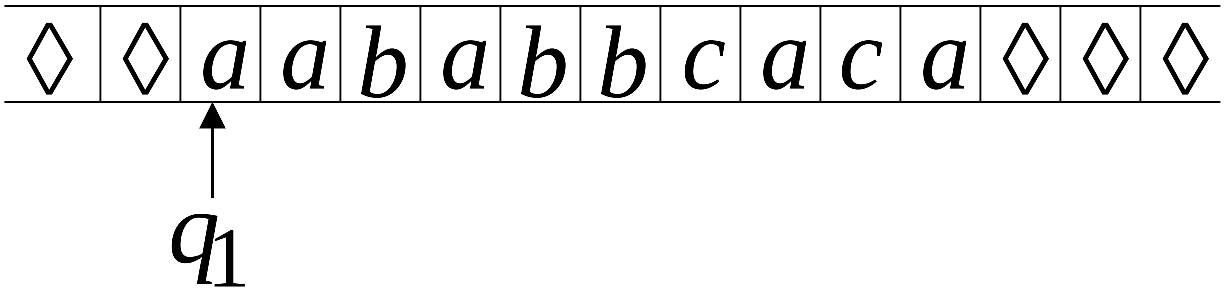


Left, Right, Stay

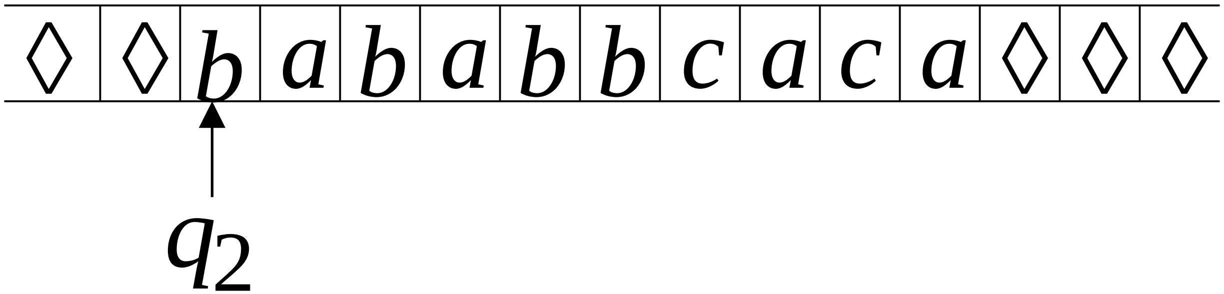
L,R,S: moves

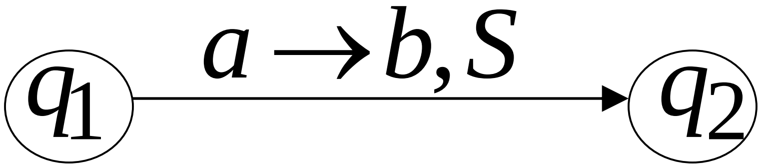
Example:

Time 1



Time 2





Theorem: Stay-Option Machines have the same power with Standard Turing machines

Proof:

Part 1: Stay-Option Machines
are at least as powerful as
Standard machines

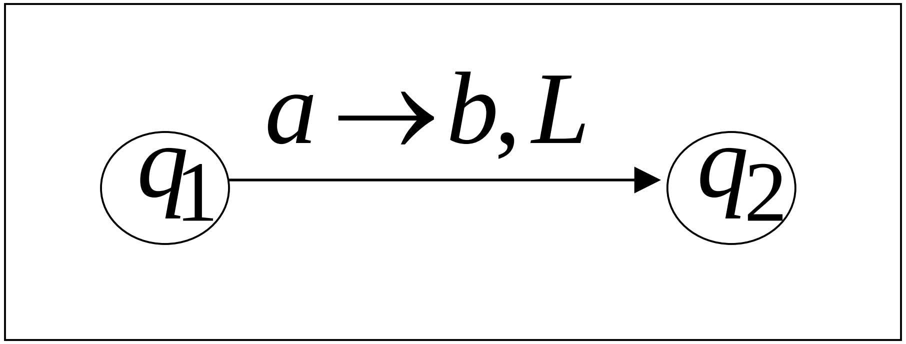
Proof: a Standard machine is also
a Stay-Option machine
(that never uses the S move)

Proof:

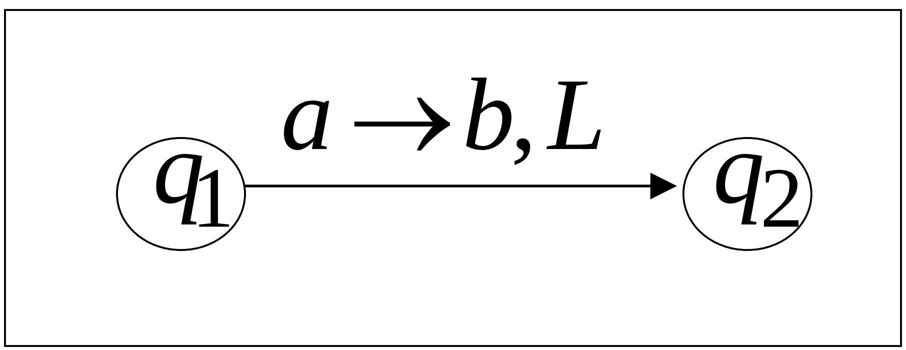
Part 2: Standard Machines are at least as powerful as Stay-Option machines

Proof: a standard machine can simulate a Stay-Option machine

Stay-Option Machine



Simulation in Standard Machine



Similar for Right moves

Stay-Option Machine

$$q_1 \xrightarrow{a \rightarrow b, S} q_2$$

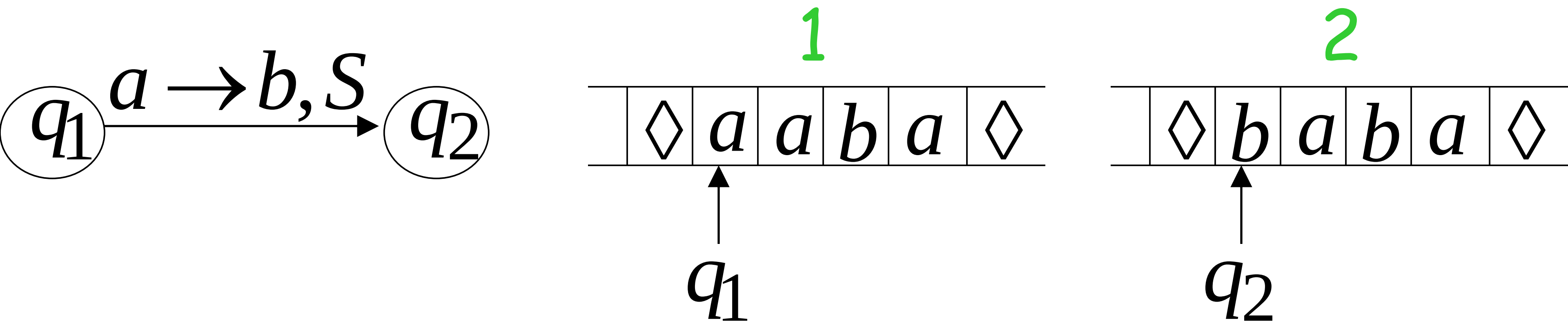
Simulation in Standard Machine

$$q_1 \xrightarrow{a \rightarrow b, L} q_2 \xrightarrow{x \rightarrow x, R} q_3$$

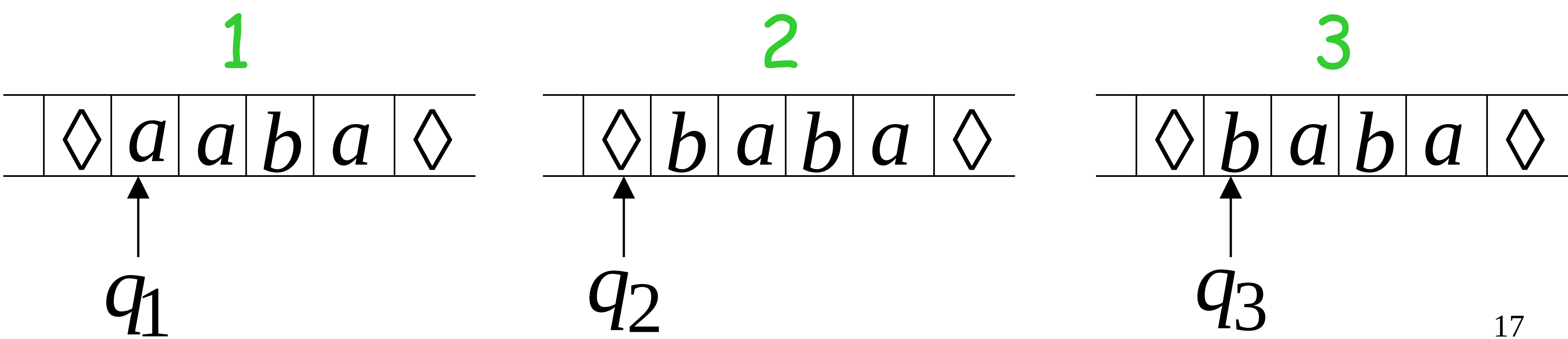
For every symbol x

Example

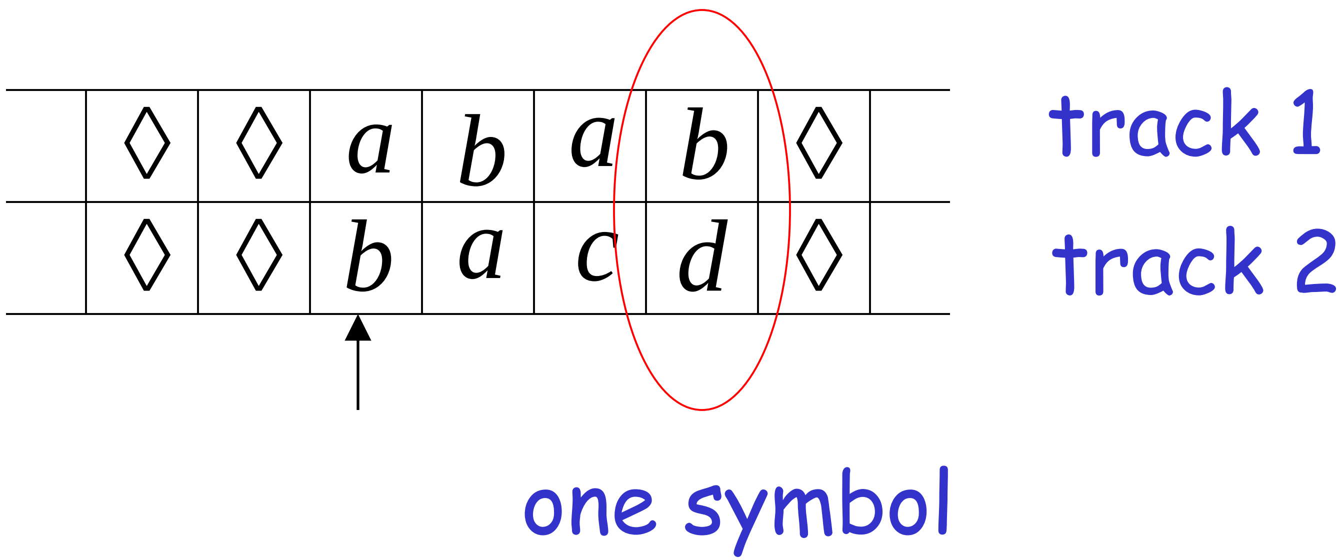
Stay-Option Machine:

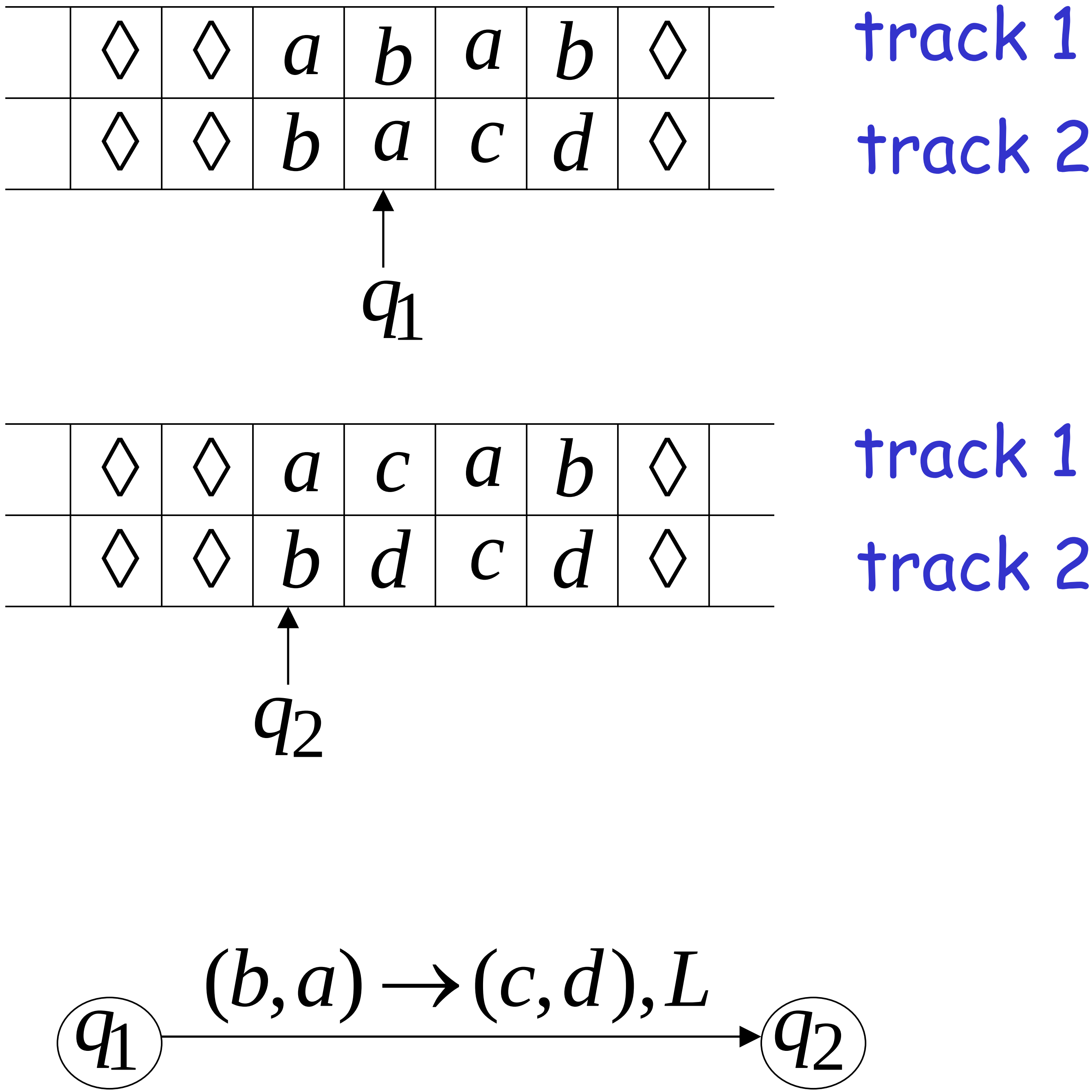


Simulation in Standard Machine:

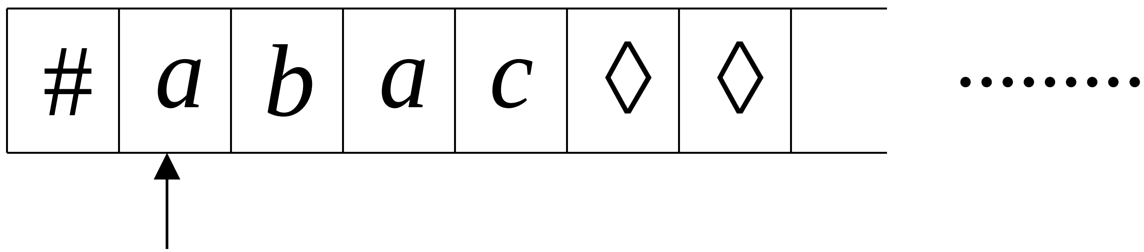


Standard Machine--Multiple Track Tape




◊ ◊ a b a b ◊
◊ ◊ b a c d ◊
track 1
track 2
q_1
◊ ◊ a c a b ◊
◊ ◊ b d c d ◊
track 1
track 2
q_2
$(b,a) \rightarrow (c,d), L$
q_1
q_2

Semi-Infinite Tape

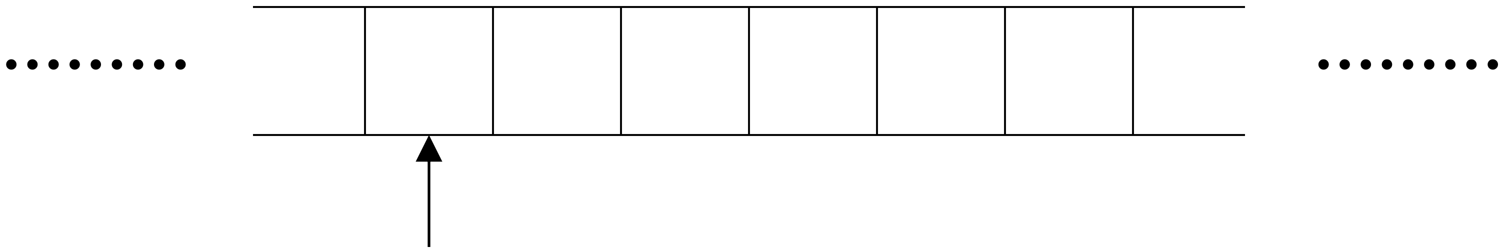


Standard Turing machines simulate Semi-infinite tape machines:

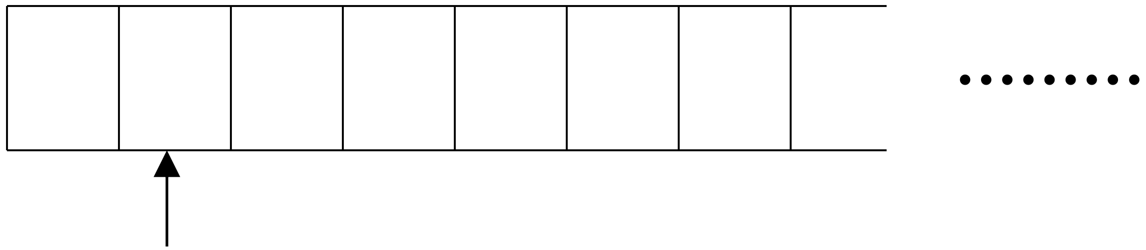
Trivial

Semi-infinite tape machines simulate Standard Turing machines:

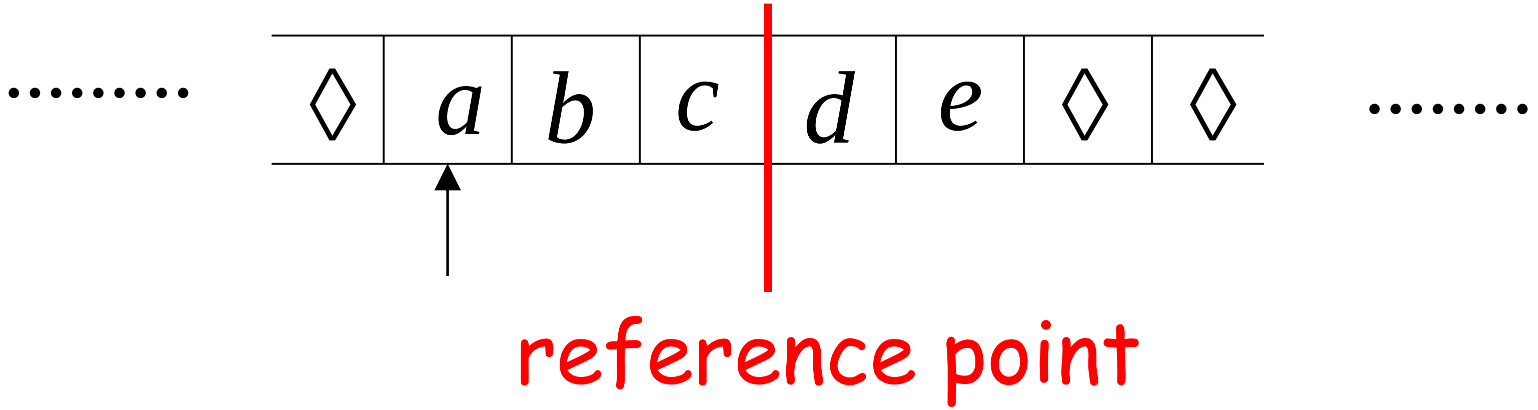
Standard machine



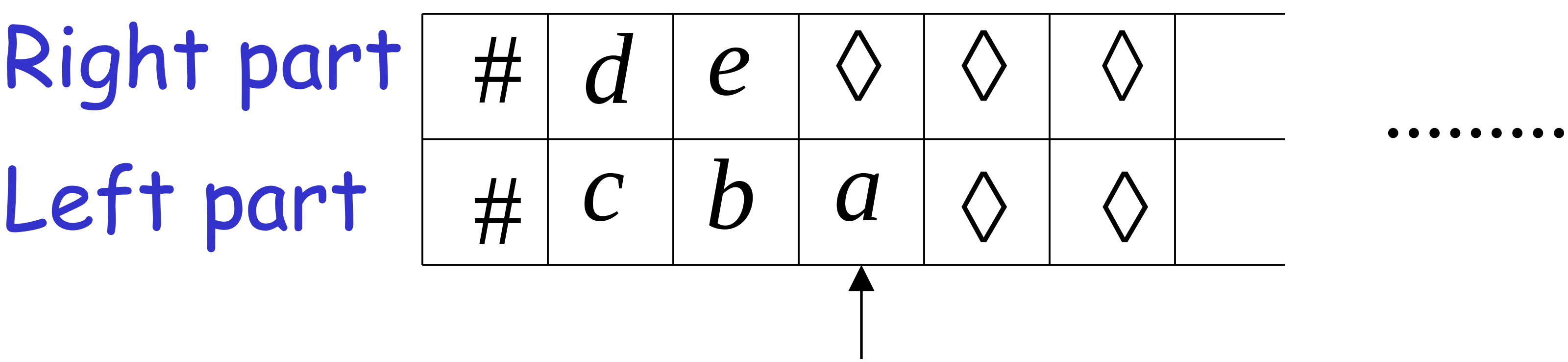
Semi-infinite tape machine



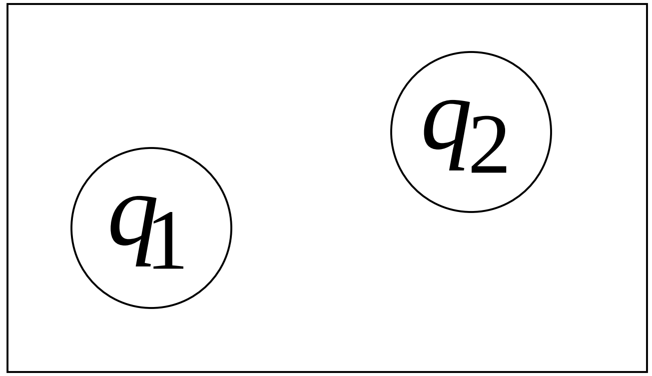
Standard machine



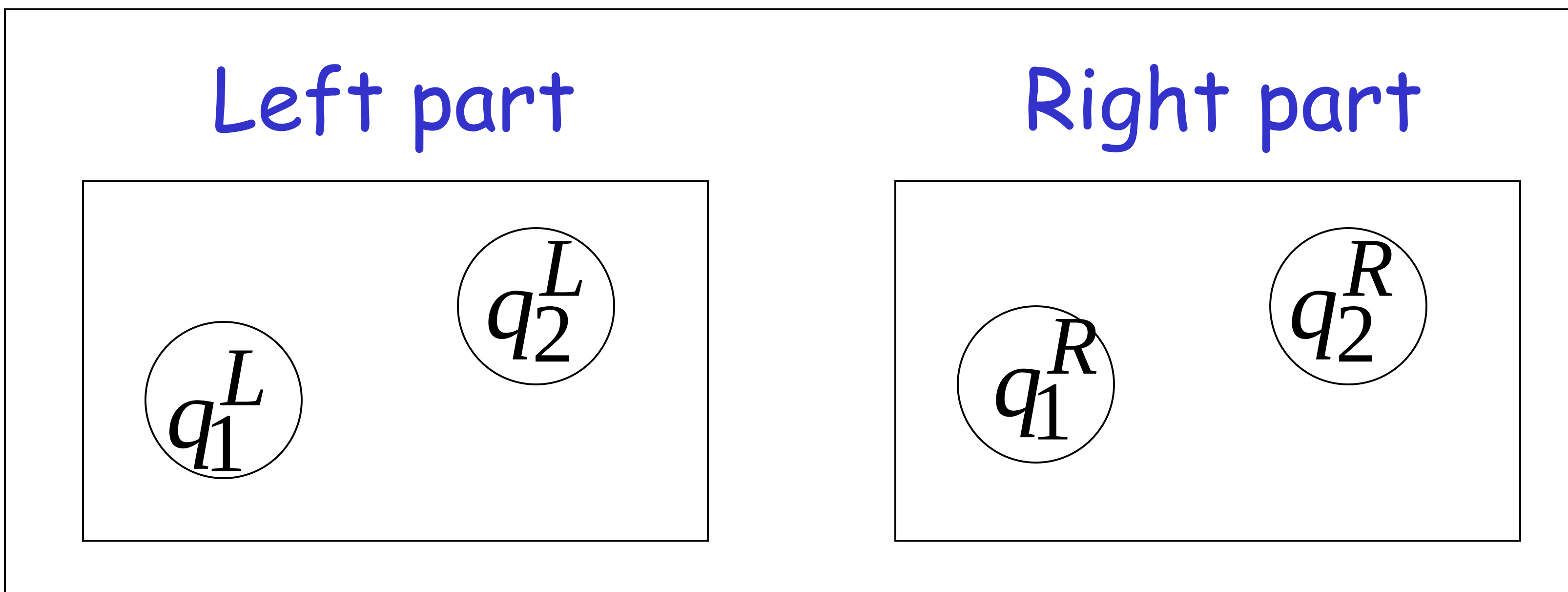
Semi-infinite tape machine with two tracks



Standard machine



Semi-infinite tape machine



Standard machine

$$q_1 \xrightarrow{a \rightarrow g, R} q_2$$

Semi-infinite tape machine

Right part

$$q_1^R \xrightarrow{(a,x) \rightarrow (g,x), R} q_2^R$$

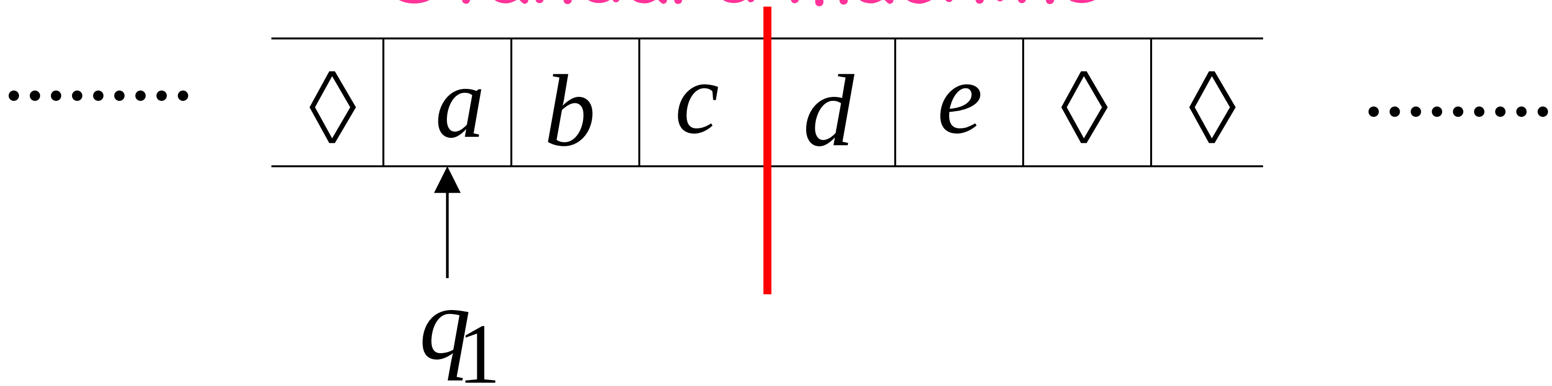
Left part

$$q_1^L \xrightarrow{(x,a) \rightarrow (x,g), L} q_2^L$$

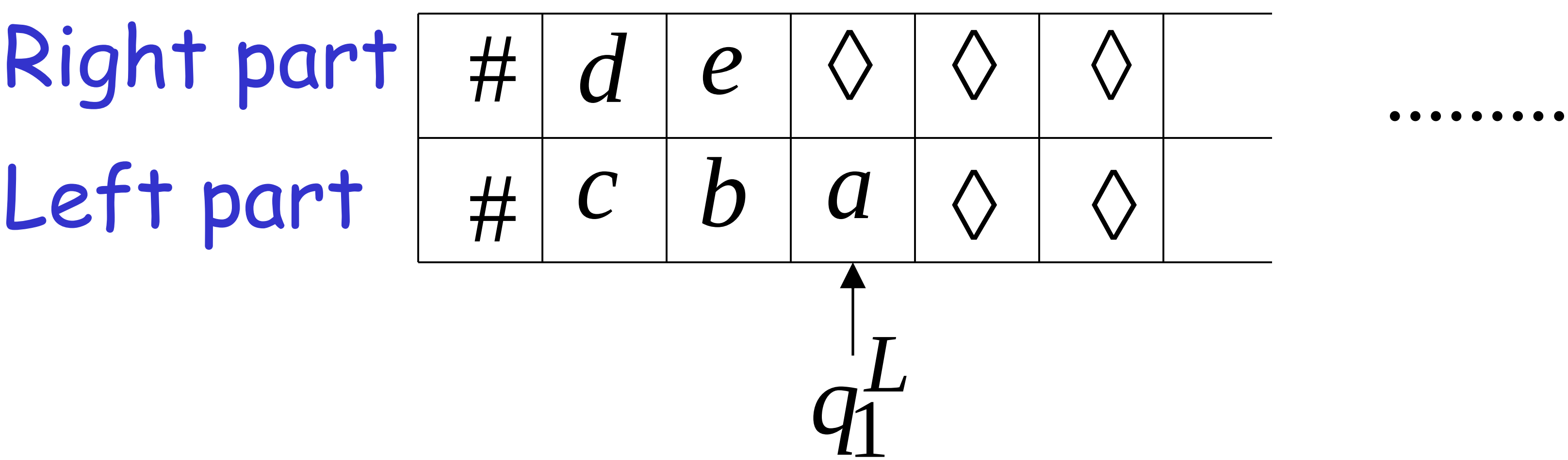
For all symbols x

Time 1

Standard machine



Semi-infinite tape machine



Time 2

Standard machine

....	◊	g	b	c	d	e	◊	◊	....
			↑ q_2						

Semi-infinite tape machine

Right part	#	d	e	◊	◊	◊	....
Left part	#	c	b	g	◊	◊	
			↑ q_2^L				

At the border:

Semi-infinite tape machine

Right part

$$q_1^R \xrightarrow{(\#,\#) \rightarrow (\#,\#), R} q_1^L$$

Left part

$$q_1^L \xrightarrow{(\#,\#) \rightarrow (\#,\#), R} q_1^R$$

Semi-infinite tape machine

Time 1

Right part	#	d	e	$\Diamond$	$\Diamond$	$\Diamond$	....
Left part	#	c	b	g	$\Diamond$	$\Diamond$	

q_1^L

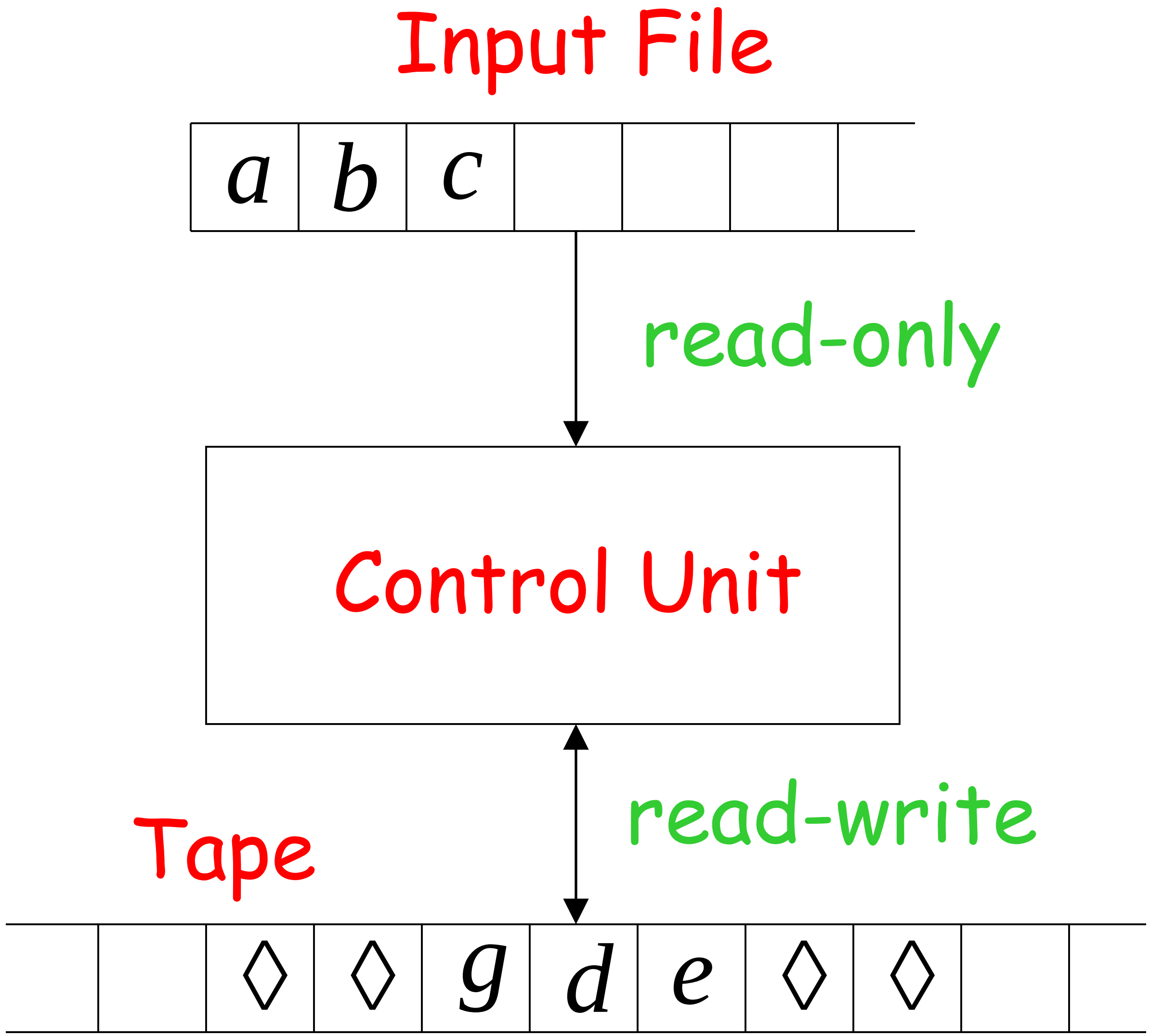
Time 2

Right part	#	d	e	$\Diamond$	$\Diamond$	$\Diamond$	....
Left part	#	c	b	g	$\Diamond$	$\Diamond$	

q_1^R

Theorem: Semi-infinite tape machines have the same power with Standard Turing machines

The Off-Line Machine



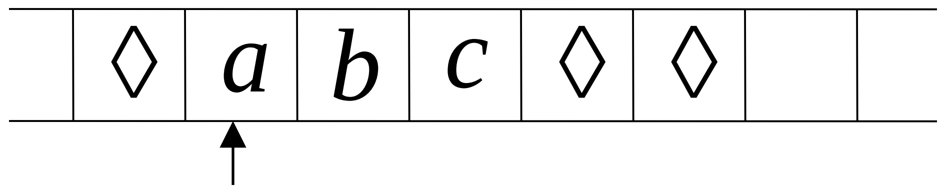
Off-line machines simulate Standard Turing Machines:

Off-line machine:

1. Copy input file to tape

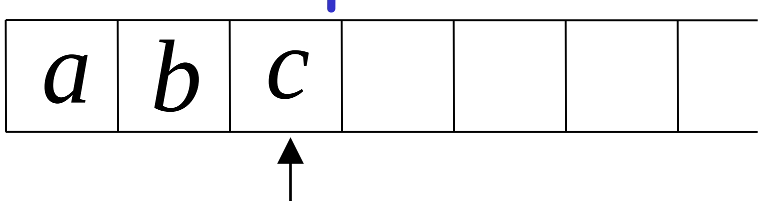
2. Continue computation as in Standard Turing machine

Standard machine

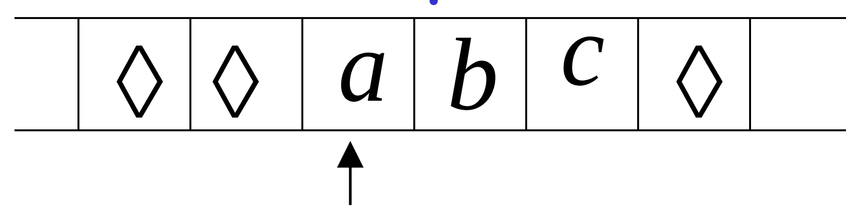


Off-line machine

Input File

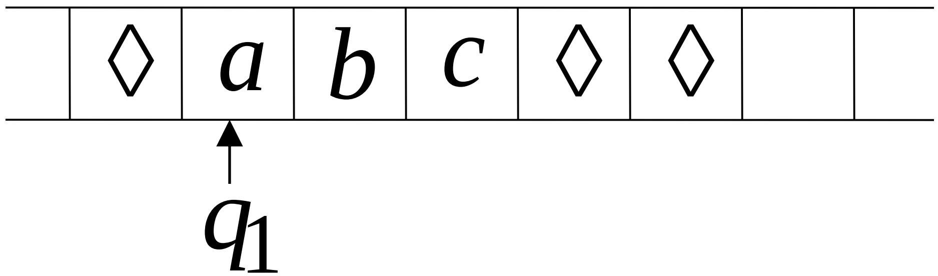


Tape



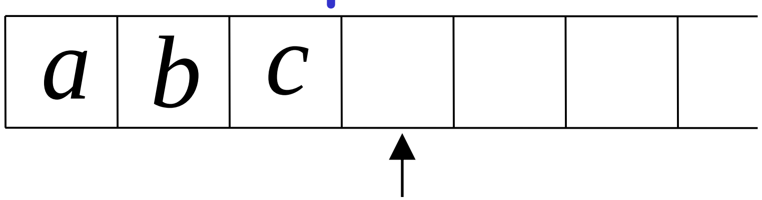
1. Copy input file to tape

Standard machine

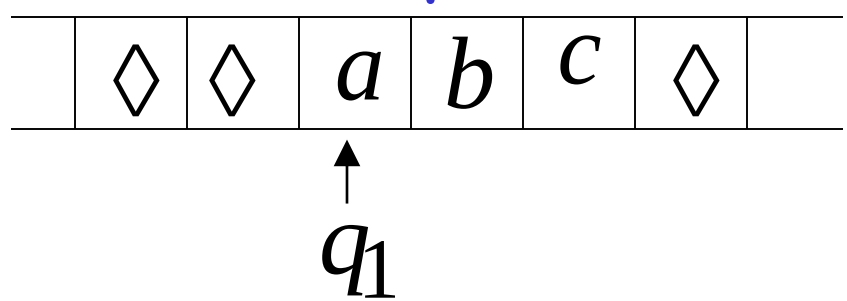


Off-line machine

Input File



Tape



2. Do computations as in Turing machine

Standard Turing machines simulate
Off-line machines:

Use a Standard machine with four track tape
to keep track of
the Off-line input file and tape contents

Off-line Machine

Input File

a	b	c	d			

Tape

	$\Diamond$	$\Diamond$	e	f	g	$\Diamond$

Four track tape -- Standard Machine

	#	a	b	c	d			Input File
	#	0	0	1	0			head position
		e	f	g				Tape
		0	1	0				head position

Reference point

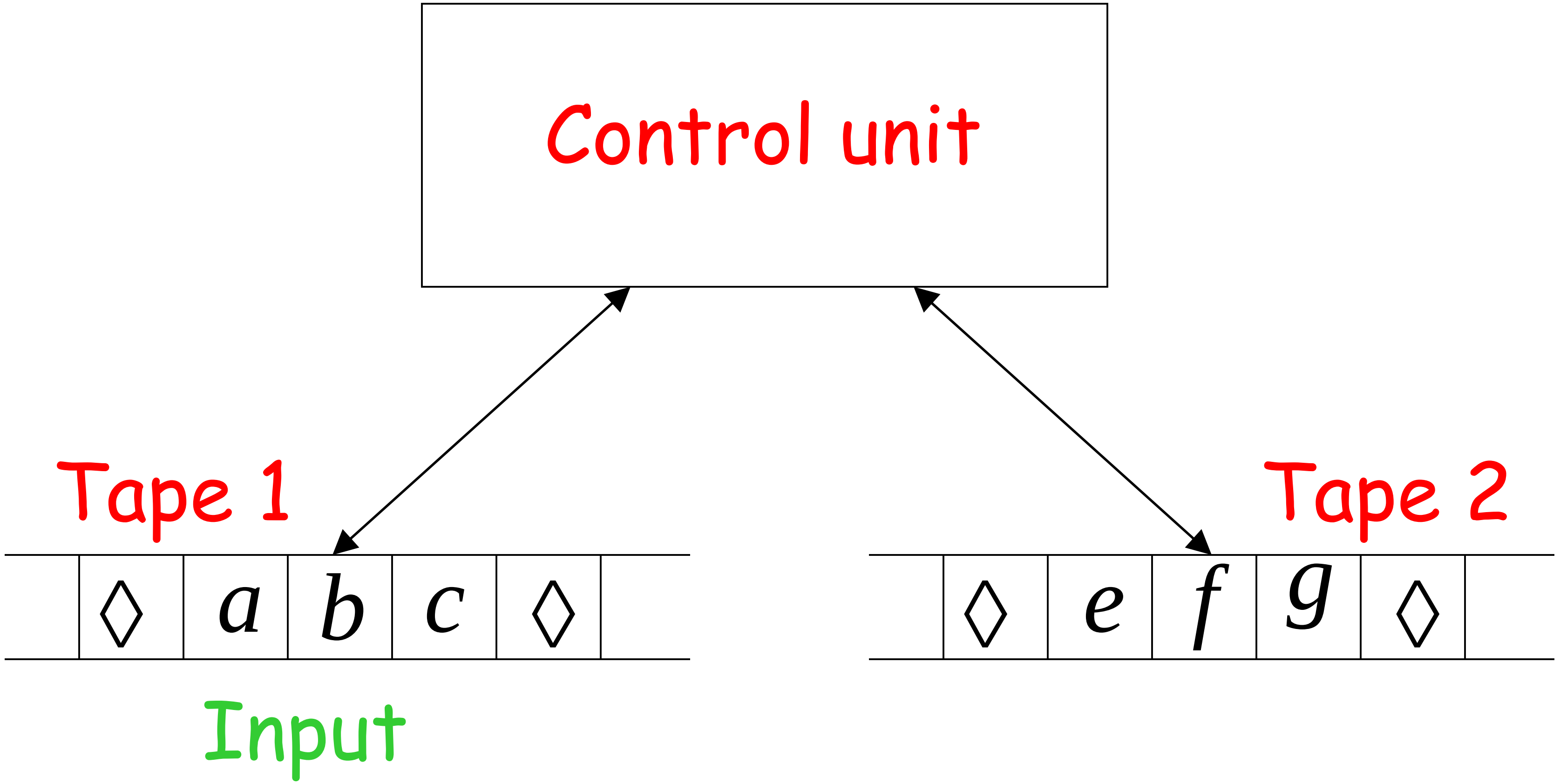
	#	a	b	c	d			Input File
	#	0	0	1	0			head position
		e	f	g				Tape
		0	1	0				head position

Repeat for each state transition:

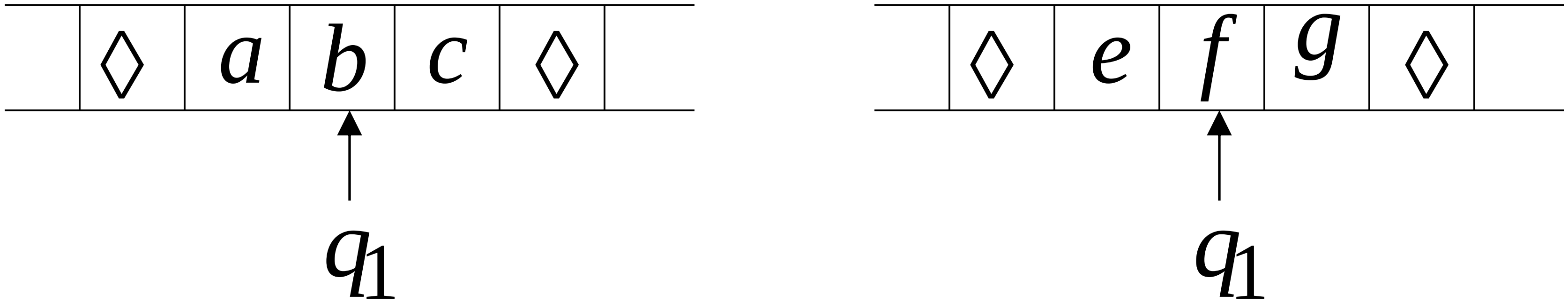
- Return to reference point
- Find current input file symbol
- Find current tape symbol
- Make transition

Theorem: Off-line machines have the same power with Stansard machines

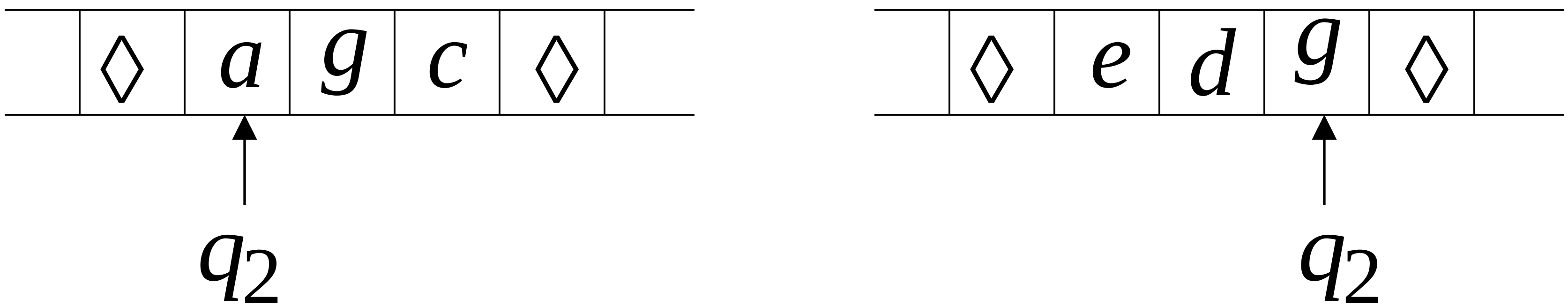
Multitape Turing Machines

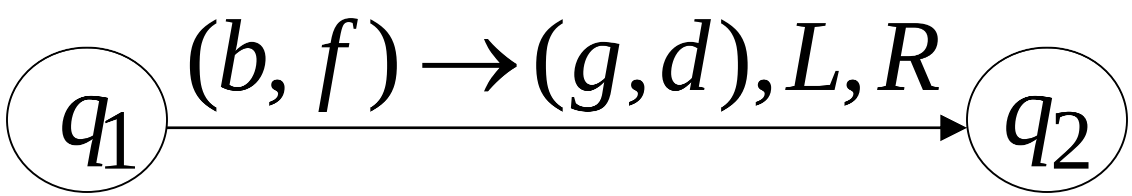


Tape 1 Time 1 Tape 2



Time 2





Multitape machines simulate Standard Machines:

Use just one tape

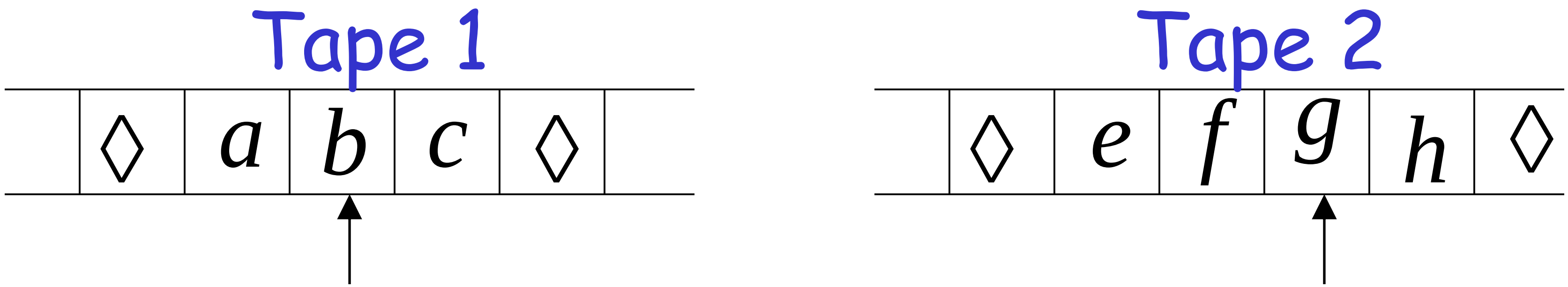
Standard machines simulate Multitape machines:

Standard machine:

- Use a multi-track tape

- A tape of the Multiple tape machine corresponds to a pair of tracks

Multitape Machine



Standard machine with four track tape

		a	b	c				Tape 1
		0	1	0				head position
		e	f	g	h			Tape 2
		0	0	1	0			head position

Reference point

	#	a	b	c			Tape 1
	#	0	1	0			head position
	#	e	f	g	h		Tape 2
	#	0	0	1	0		head position

Repeat for each state transition:

- Return to reference point
- Find current symbol in Tape 1
- Find current symbol in Tape 2
- Make transition

Theorem: Multi-tape machines have the same power with Standard Turing Machines

Same power doesn't imply same speed:

Language $L = \{a^n b^n\}$

	Acceptance Time
Standard machine	n^2
Two-tape machine	n

$$L = \{a^n b^n\}$$

Standard machine:

Go back and forth n^2 times

Two-tape machine:

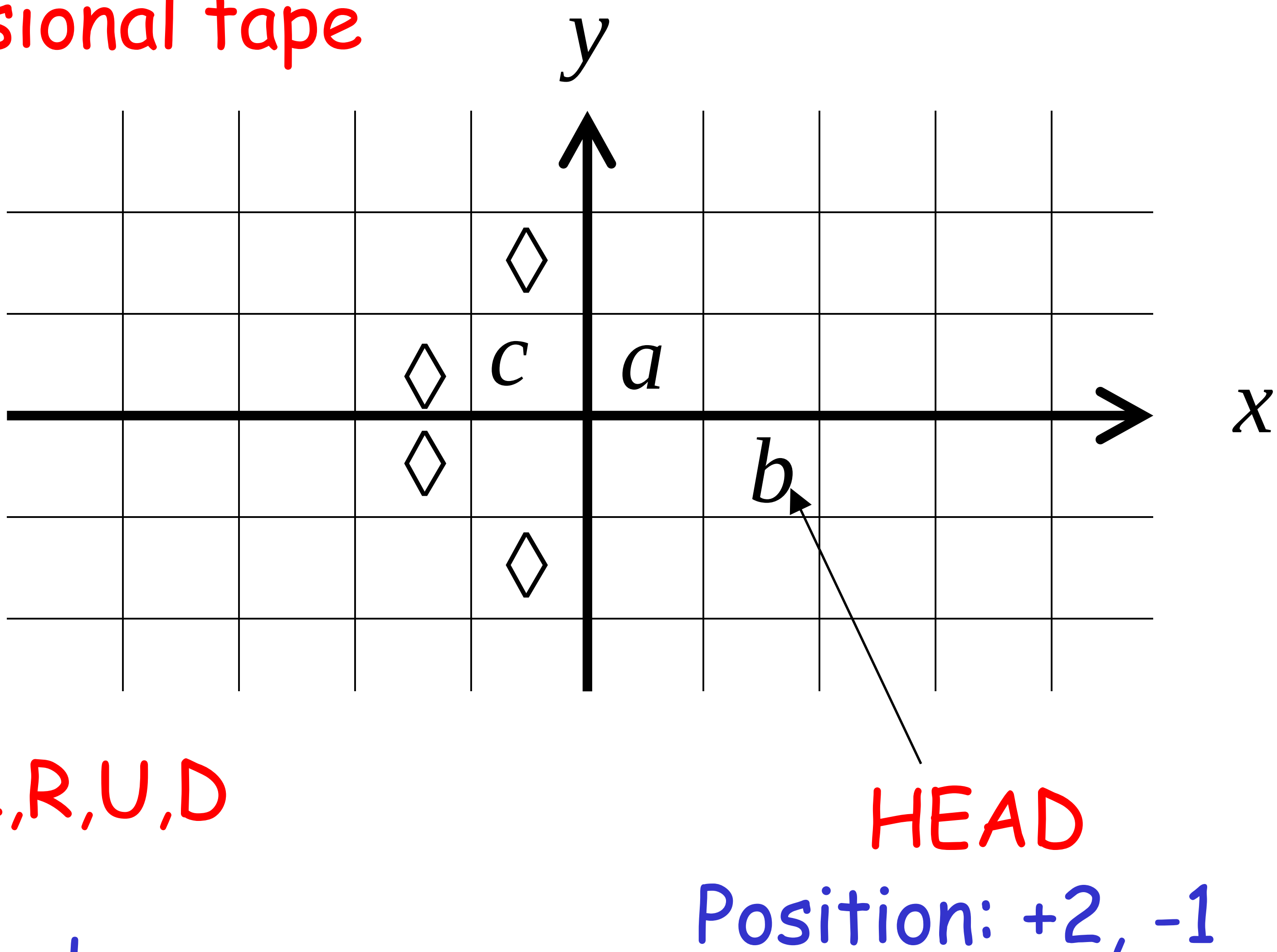
Copy b^n to tape 2 (n steps)

Leave a^n on tape 1 (n steps)

Compare tape 1 and tape 2 (n steps)

MultiDimensional Turing Machines

Two-dimensional tape



MOVES: L,R,U,D

U: up   D: down

Multidimensional machines simulate Standard machines:

Use one dimension

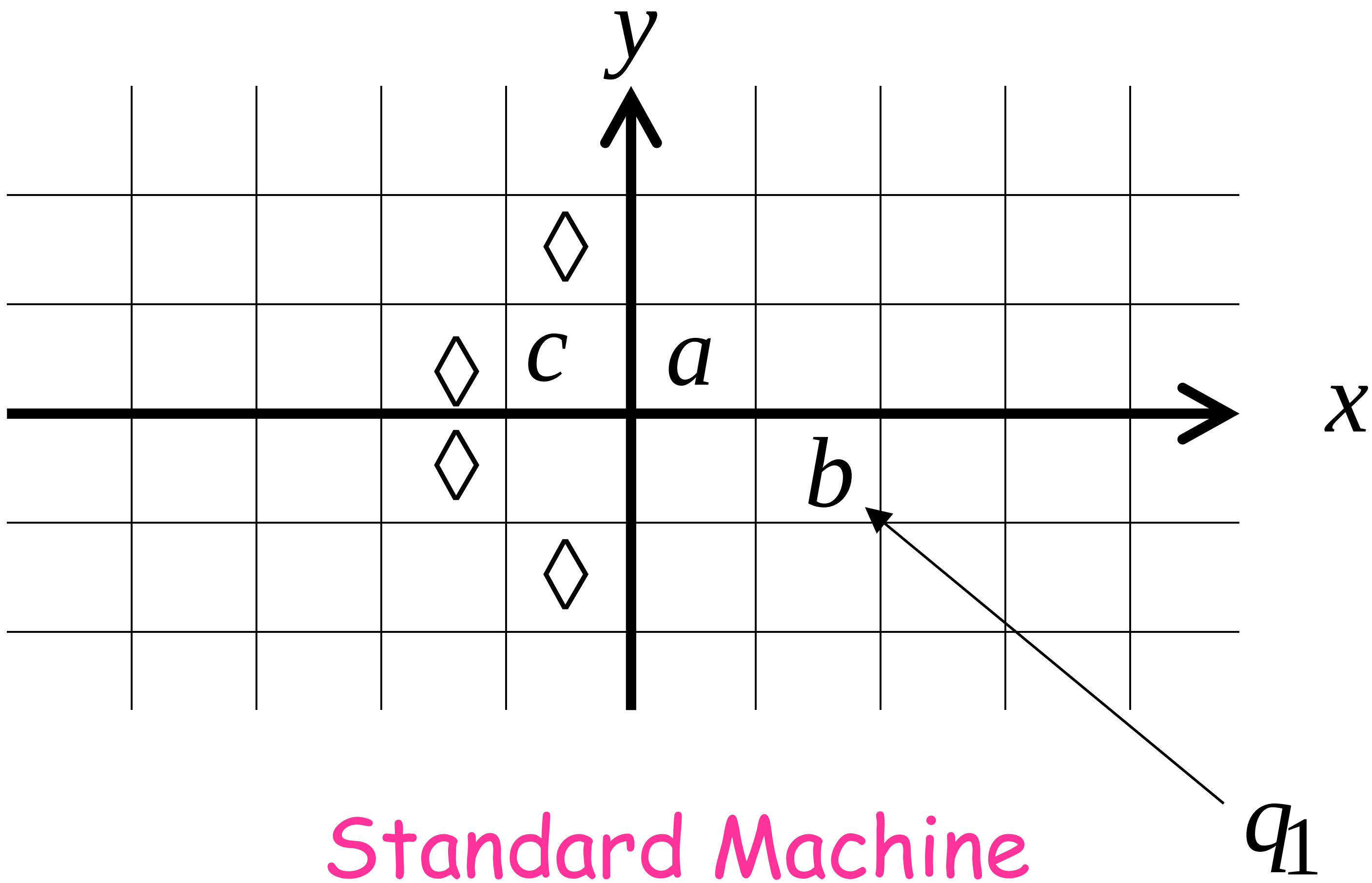
Standard machines simulate Multidimensional machines:

Standard machine:

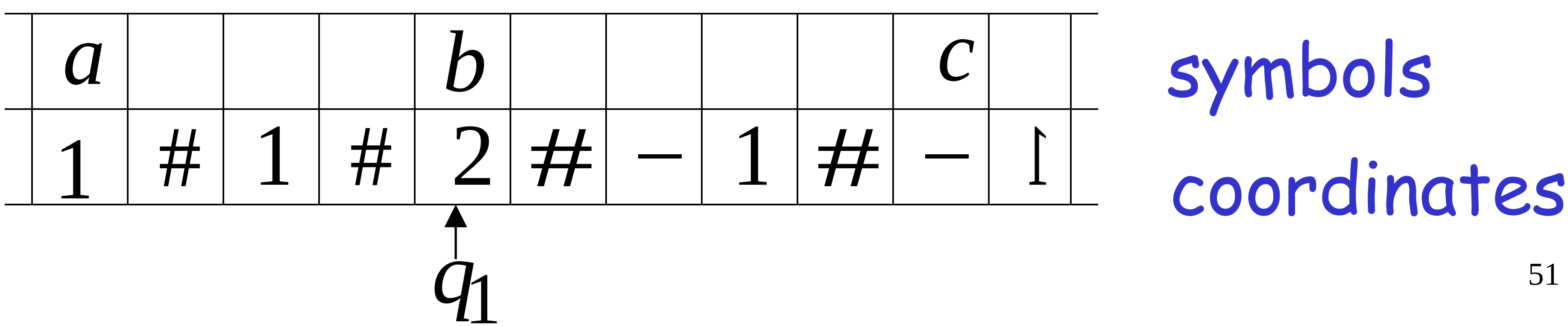
- Use a two track tape

- Store symbols in track 1
- Store coordinates in track 2

Two-dimensional machine



Standard Machine



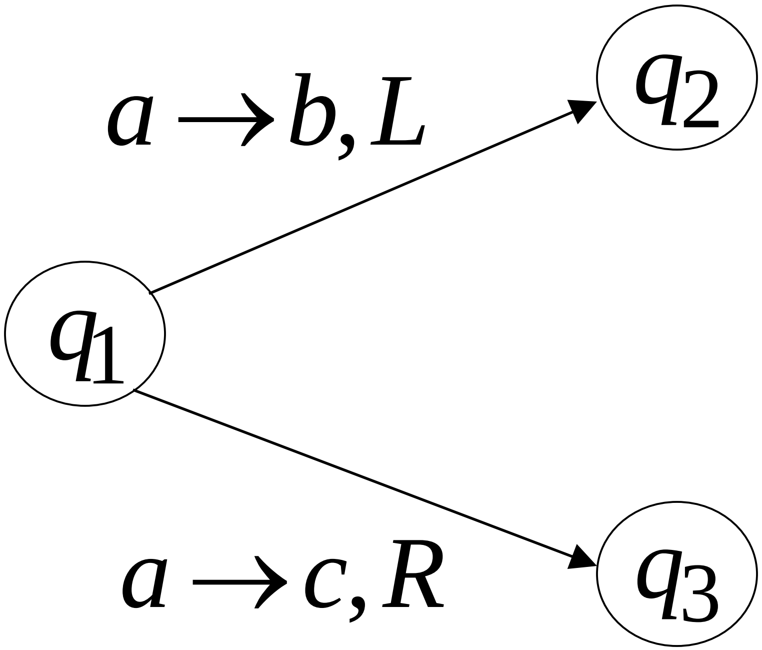
Standard machine:

Repeat for each transition

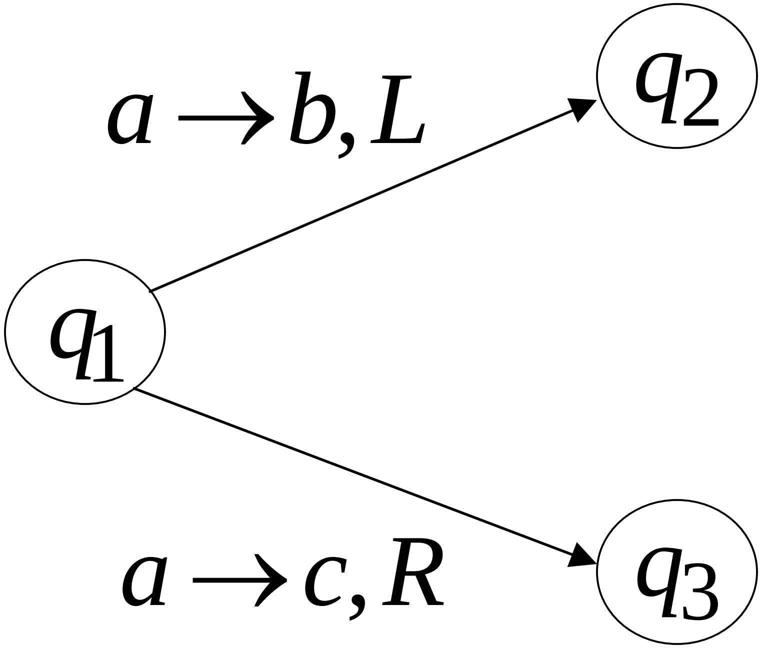
- Update current symbol
- Compute coordinates of next position
- Go to new position

Theorem: MultiDimensional Machines have the same power with Standard Turing Machines

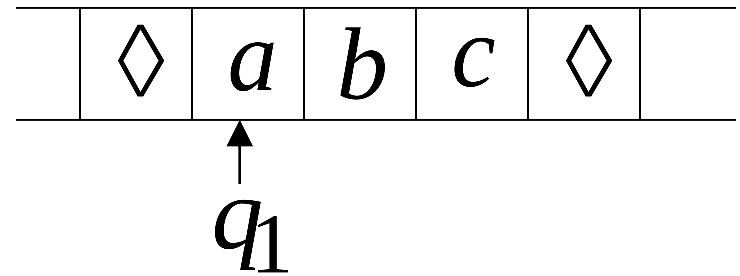
NonDeterministic Turing Machines



Non Deterministic Choice

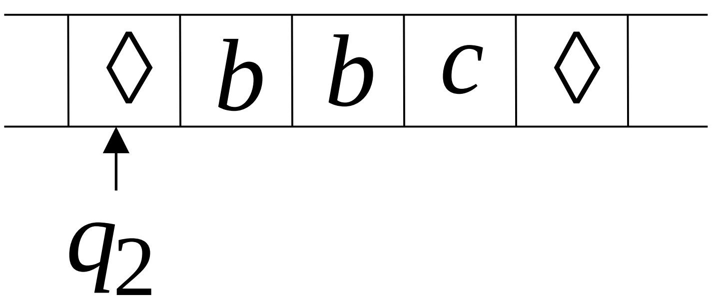


Time 0

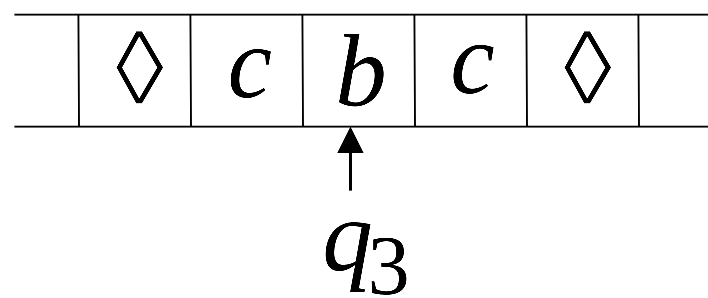


Time 1

Choice 1



Choice 2



Input string w is accepted if this a possible computation

$$q_0 w \overset{*}{\succ} x \, q_f \, y$$

Initial configuration

Final Configuration

Final state

NonDeterministic Machines simulate Standard (deterministic) Machines:

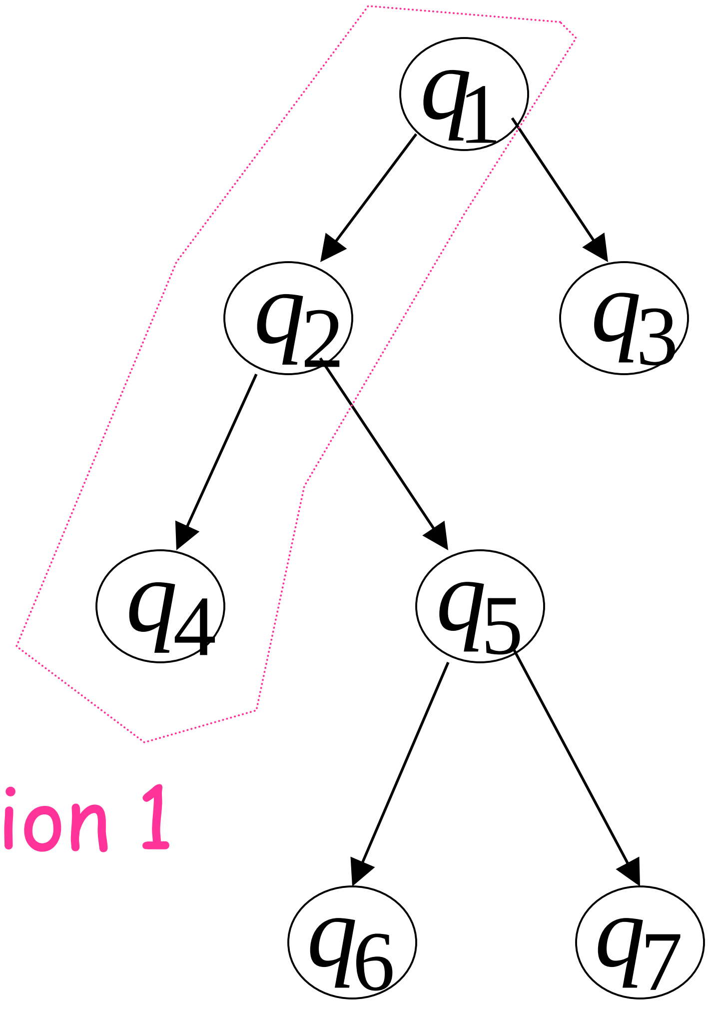
Every deterministic machine
is also a nondeterministic machine

Deterministic machines simulate NonDeterministic machines:

Deterministic machine:

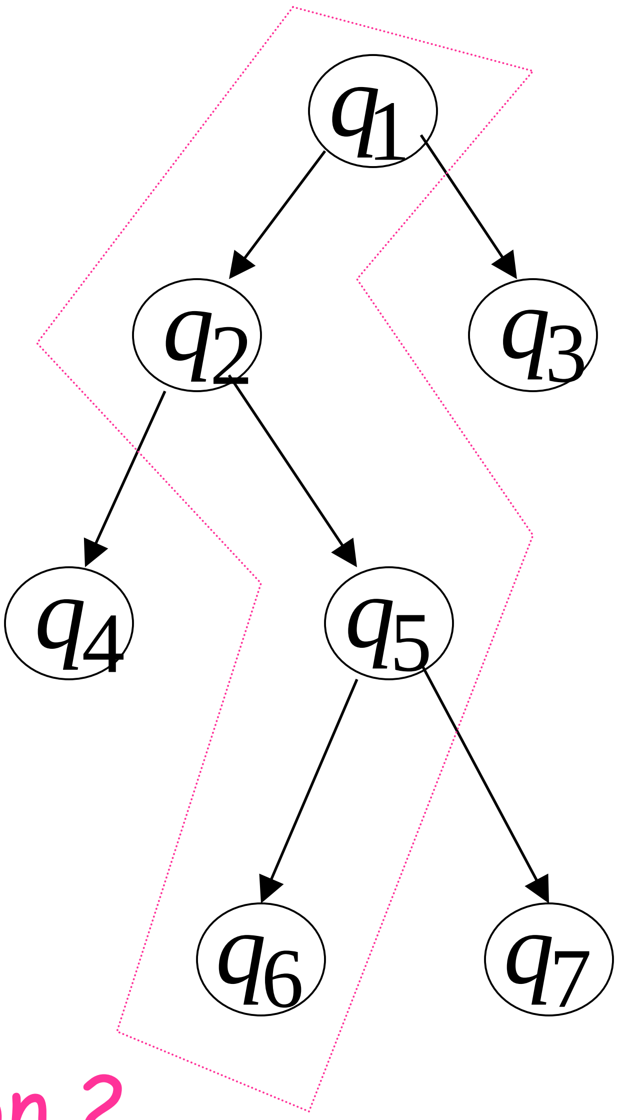
Keeps track of all possible computations

Non-Deterministic Choices



Computation 1

Non-Deterministic Choices



Computation 2

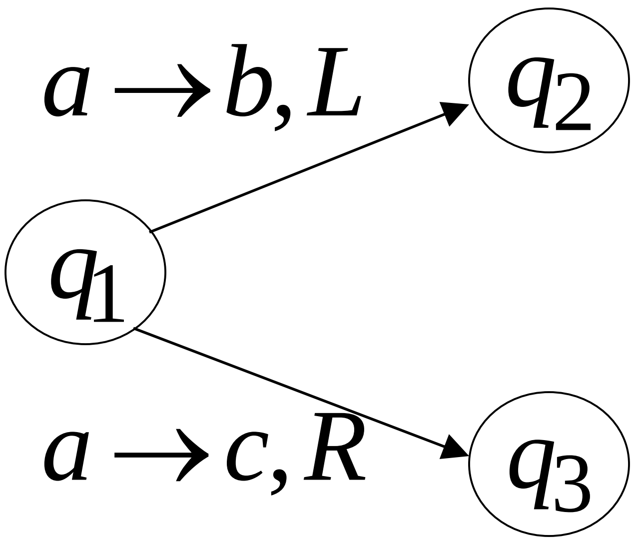
Simulation

Deterministic machine:

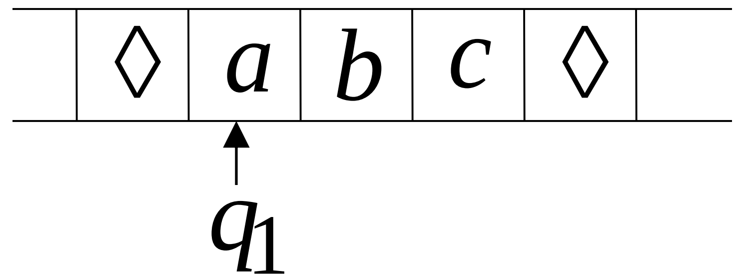
- Keeps track of all possible computations

- Stores computations in a two-dimensional tape

NonDeterministic machine



Time 0

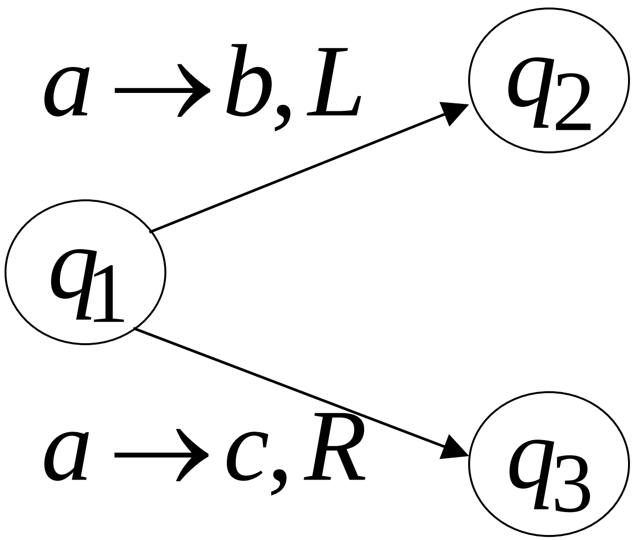


Deterministic machine

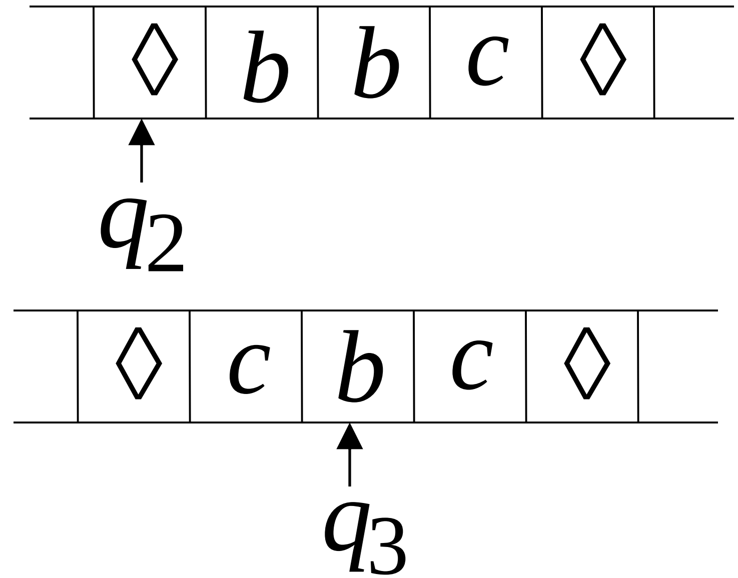
	#	#	#	#	#	#	
	#	a	b	c	#		
	#	q_1			#		
	#	#	#	#	#		

Computation 1

NonDeterministic machine



Time 1



Choice 1

Choice 2

Deterministic machine

	#	#	#	#	#	#	
#		b	b	c	#		
#	q_2				#		
#		c	b	c	#		
#			q_3		#		

Computation 1

Computation 2

Repeat

- Execute a step in each computation:

- If there are two or more choices in current computation:
    1. Replicate configuration
    2. Change the state in the replica

Theorem: NonDeterministic Machines have the same power with Deterministic machines

Remark:

The simulation in the Deterministic machine takes time exponential time compared to the NonDeterministic machine

Polynomial Time in NonDeterministic Machine:

NP-Time

Polynomial Time in Deterministic Machine:

P-Time

Fundamental Problem: P = NP ?