(24)

(e) $x[n] = 2\cos(\frac{\pi}{4} n) + \sin(\frac{\pi}{8} n) - 2\cos(\frac{\pi}{2} n + \pi/6)$

$\cos(\frac{\pi}{4} n)$ has period $N_{01} = \frac{2\pi}{\frac{2\pi}{8}} = 8$

$\sin(\frac{\pi}{8} n)$ " " $N_{02} = \frac{2\pi}{\frac{2\pi}{16}} = 16$

$\cos(\frac{\pi}{2} n + \frac{\pi}{6})$ " " $N_{03} = \frac{2\pi}{\frac{2\pi}{4}} = 4$

$\therefore$ $x[n]$ has a fundamental period is L.C.M of 8, 16, 4 = 16 Ans

1.27.

(a) $y(t) = x(t-2) + x(2-t)$

① Not memoryless at $t=0$ $y(0) = x(-2) + x(2)$

②

$x_1(t) \Rightarrow y_1(t)$

$x_2(t) = x_1(t-t_0) \Rightarrow y_2(t)$.

$y_2(t) = x_2(t-2) + x_2(2-t)$

$= x_1(t-2-t_0) + x_1(2-(t-t_0))$

$\neq y_1(t-t_0)$

$y_1(t-t_0) = x_1(t-t_0-2) + x_1(2-t+t_0)$

(5)

$x_1(t) \rightarrow y_1(t)$

$x_2(t) \rightarrow y_2(t)$

$x_3(t) = a\,x_1(t) + b\,x_2(t) \Rightarrow y_3(t)$

$\therefore y_3(t) = x_3(t-2) + x_3(2-t)$

$= a\,x_1(t-2) + b\,x_2(t-2) + a x_1(2-t) + b\,x_2(2-t)$

$= a\{x_1(t-2) + x_1(2-t)\} + b\{x_2(t-2) + x_2(2-t)\}$

$= a\,y_1(t) + b\,y_2(t)$.

So linear.

(4)

At $t=0$, $y(0) = x(-2) + x(2)$;

since at present o/p depends past & future o/p so the system is non causal.

(5) If has bounded input & bounded o/p so it is stable.

(b) $y(t) = [\cos(3t)]\, x(t)$

At $t = 0$, $y(0) = 1 \cdot x(0)$.

$y(\pi/2) = \cos\frac{3\pi}{2}\; x(\pi/2)$

$y(\pi) = \cos(\pi)\; x(\pi)$

(1) Present ~~input~~ o/p depends on present o/p so it is memory less.

(2)

$x_1(t) \rightarrow y_1(t)$

$x_2(t) = x_1(t-t_0) \rightarrow y_2(t)$.

$$y_2(t) = [\cos(3t)]\; x_2(t)$$
$$= [\cos(3t)]\; x_1(t-t_0)$$
$$\neq y_1(t-t_0) \quad \text{as } y_1(t-t_0) = \cos 3(t-t_0)\, x_1(t-t_0)$$

so it is not time invariant.

(3)

$x_1(t) \rightarrow y_1(t)$

$x_2(t) \rightarrow y_2(t)$

$a x_1(t) + b x_2(t) = x_3(t) \rightarrow y_3(t)$.

$$\therefore y_3(t) = \cos(3t)\, x_3(t) = \cos(3t)\{a x_1(t) + b x_2(t)\}$$
$$= a\cos(3t)\, x_1(t) + b\cos(3t)\, x_2(t)$$
$$= a\, y_1(t) + b\, y_2(t)$$

so it is linear.

④ Since present o/p depends on present I/P so it is causal.

⑤ It has bounded i/p has bounded o/p so it is stable.

(C) $y(t) = \int_{-\infty}^{2t} x(\tau)\,d\tau$

① Since I/P is integrated over $-\infty \to 2t$ so it present o/p ~~has~~ depends on previous I/P. Hence it is memory system.

②
$x_1(t) \rightarrow y_1(t)$

$x_2(t) \Rightarrow x_1(t-t_0) \rightarrow y_2(t)$

$$y_2(t) = \int_{-\infty}^{2t} x_2(\tau)\,d\tau$$

$$= \int_{-\infty}^{2t} x_1(\tau - t_0)\,d\tau$$

$$= y_1(t-t_0)$$

so it is time invariant.

③ $y_3(t) = \int_{-\infty}^{2t} x_3(\tau)\,d\tau.$

(4) It is casual, because present o/p depends on present & past value of I/P.

(5) Unstable → when $x(\tau) = e^{-t}$

(d) $y(t) = \begin{cases} 0, & t<0 \\ x(t) + x(t-2), & t \geq 0 \end{cases}$

(1) At, $t=0$, $y(0) = x(0) + x(-2)$

So, it is memory system.

(2)

$x_1(t) \rightarrow y_1(t)$

$x_2(t) \rightarrow y_2(t)$

$= x_1(t-t_0)$

$$y_2(t) = x_2(t) + x_2(t-2)$$

$$= x_1(t-t_0) + x_1(t-2-t_0)$$

$$= y_1(t-t_0)$$ so it is time invariant,

(3)

$x_1(t) \rightarrow y_1(t)$

$x_2(t) \rightarrow y_2(t)$

$x_3(t) = a\,x_1(t) + b\,x_2(t) \rightarrow y_3(t)$

$$y_3(t) = x_3(t) + x_3(t-2)$$

$$= a x_1(t) + b x_2(t) + a x_1(t-2) + b x_2(t-2)$$

$$= a[x_1(t) + x_1(t-2)] + b[x_2(t) + x_2(t-2)]$$

$= a y_1(t) + b y_2(t)$

so it is linear.

(4)

~~y(3)~~

$y(0) = x(0) + x(-2)$

$y(1) = x(1) + x(-1)$

$y(2) = x(2) + x(0)$

$y(3) = x(3) + x(1)$

Since the o/p depends on present I/p & past input so it is causal.

(5) Bounded I/p has bounded o/p so it is stable.

(e) $y(t) = \begin{cases} 0, & x(t) < 0 \\ x(t) + x(t-2), & x(t) \geq 0 \end{cases}$

~~(6) ...~~

~~y(t)~~

(f) $y(t) = x(t/3)$

① It is memory less system. • Present o/p depend present I/P.

② Time invariant :-

$x_1(t) \rightarrow y_1(t)$

$x_2(t) \rightarrow y_2(t)$

$= x_1(t-t_0)$

$$y_2(t) = x_2(t/3) = x_1\left(\frac{t-t_0}{3}\right)$$

$$= y_1(t-t_0)$$

so it is time invariant.

③

$x_1(t) \rightarrow y_1(t)$

$x_2(t) \rightarrow y_2(t)$

$a x_1(t) + b x_2(t) = x_3(t) \rightarrow y_3(t)$

$y_3(t) = x_3(t/3)$

$= a\,x_1(t/3) + b\,x_2(t/3)$

$= a\,y_1(t) + b\,y_2(t)$

so it is linear system.

(4) Causal :-

It is causal because present o/p depends only on present I/P.

(5) ~~linear~~ stable :-

Bounded ~~o~~ I/P has bounded o/p.

(9) $y(t) = \dfrac{d\,x(t)}{dt}$

(1) Memoryless :-

It .

(2) Time invariant :-

$x_1(t) \rightarrow y_1(t)$

$x_2(t) \rightarrow y_2(t)$ let $x_2(t) = x_1(t-t_0)$

$$y_2(t) = \frac{d\,x_2(t)}{dt}$$

$$= \frac{d}{dt}\{x_1(t-t_0)\}$$

$= y_1(t-t_0)$ so it is time invariant.

(3) Linear :-

$x_1(t) \rightarrow y_1(t)$

$x_2(t) \rightarrow y_2(t)$

$a\,x_1(t) + b\,x_2(t) = x_3(t) \rightarrow y_3(t)$

$$y_3(t) = \frac{d\,x_3(t)}{dt} = \frac{d}{dt}\{a\,x_1(t) + b\,x_2(t)\}$$

$= a\,y_1(t) + b\,y_2(t)$

so it is linear.

(4) Causal :-

(5) stable, if I/P is square wave we get a ~~pulse~~ spike so it is unstable system.

1.28

(a) $y[n] = x[-n]$

(1) Memory less :-

It is memory system. Present o/p depend past I/P.

(2) Time invariant :-

$x_1[n] \rightarrow y_1[n]$

$x_2[n] \rightarrow y_2[n]$ let $x_2[n] = x_1[n-n_0] \rightarrow y_2[n]$

$$y_2[n] = x_2[-n]$$
$$= x_1[-n+n_0]$$
$$= y_1[n-n_0]$$ so it is time invariant.

(3) Linear :-

$x_1[n] \rightarrow y_1[n]$ $\quad ax_1[n] + b x_2[n] = x_3[n] \rightarrow y_3[n]$

$x_2[n] \rightarrow y_2[n]$

$$y_3(n) = x_3(-n)$$
$$= a x_1(-n) + b x_2(-n)$$
$$= a y_1[n] + b y_2[n]$$

so it is linear.

(4) Causal :-

$y(0) = x(0)$

$y(1) = x(-1)$

$y(-1) = x(1)$

since present o/p depends on ~~past~~ future I/P so it is noncausal.

⑤ Stable :-

Bounded I/P has Bounded o/p so it is stable.

(6) $y(n) = x(n-2) - 2x(n-8)$

① Memory less :-

It is memory system,

$$y(0) = x(-2) - 2x(-8)$$

② Time invariant :-

$x_1(n) \rightarrow y_1(n)$

$x_2(n) = x_1(n-n_0) \Rightarrow y_2(n)$

$$y_2(n) = x_2(n-2) - 2x_2(n-8)$$

$$= x_1(n-2-n_0) - 2x_2(n-n_0-8)$$

$$= y_1(n-n_0)$$ so it is time invariant.

③ linear :-

$x_1(n) \rightarrow y_1(n)$

$x_2(n) \rightarrow y_2(n)$

$a x_1(n) + b x_2(n) = x_3(n) \rightarrow y_3(n)$

$$y_3(n) = x_3(n-2) - 2x_3(n-8)$$

$$= a x_1(n-2) + b x_2(n-3) - 2a x_1(n-8) - 2b x_2(n-8)$$

$$= a y_1(n) + b y_2(n)$$

so it is linear.

④ Causal :-

$y(n) = x(n-2) - 2x(n-8)$

$y(0) = x(-2) - 2x(-8)$

so it is causal.

$y(-10) = x(-12) - 2x(-20)$

$y(-1) = x(-1) - 2x(-$

$y(10) = x(8) - 2x(2)$

⑤ stable:

It is stable.

(c) $y(n) = n\,x(n)$

① It memory less system.

② $x_1(n) \to y_1(n)$ $\quad x_2(n) = x_1(n-n_0)$

$x_2(n) \to y_2(n)$

$$y_2(n) = n\,x_2(n)$$
$$= n\,x_1(n-n_0)$$
$$\neq y_1(n-n_0) = (n-n_0)\,x_1(n-n_0)$$

so it is time varying.

③ Linear:-

$x_1(n) \to y_1(n)$ $\quad a x_1(n) + b x_2(n) \Rightarrow x_3(n) \to y_3(n)$

$x_2(n) \to y_2(n)$

$$y_3(n) = n\,x_3(n)$$
$$= n\{a x_1(n) + b x_2(n)\}$$
$$= a n x_1(n) + b n x_2(n)$$
$$= a y_1(n) + b y_2(n)$$

so it is linear.

④ It is causal.

⑤ It is stable.

(d) $y(n) = Ev\{x[n-1]\}$

$$= \frac{x[n-1] + x[-(n-1)]}{2}$$

① It is memory system.

② <u>Time invariant :-</u>

$x_1(n) \to y_1(n)$

$x_2(n) = x_1(n-n_0) \to y_2(n)$

$y(n-n_0) = \frac{x(n-n_0-1)}{2} + \frac{x(-(n-n_0-1))}{2}$

$\frac{x(-n+n_0+1)}{2}$

$$y_2(n) = \frac{x_2(n-1) + x_2(-n+1)}{2}$$

$$= \frac{x_1(n-1-n_0) + x_2(-n+1-n_0)}{2}$$

so it is not time invariant.

③ <u>Linear :-</u>

$x_1(n) \to y_1(n)$ $\quad ax_1(n) + b x_2(n) = x_3(n) \to y_3(n)$

$x_2(n) \to y_2(n)$

$$y_3(n) = \frac{x_3[n-1] + x_3[-n+1]}{2}$$

$$= \frac{1}{2}\left[ax_1(n-1) + bx_2(n-1) + ax_1(-n+1) + bx_2(-n+1)\right]$$

$$= a y_1(n) + b y_2(n)$$

so it is linear.

④ $y(0) = \frac{1}{2}[x(-1) + x(1)]$ so it is non causal.

⑤ it is stable.