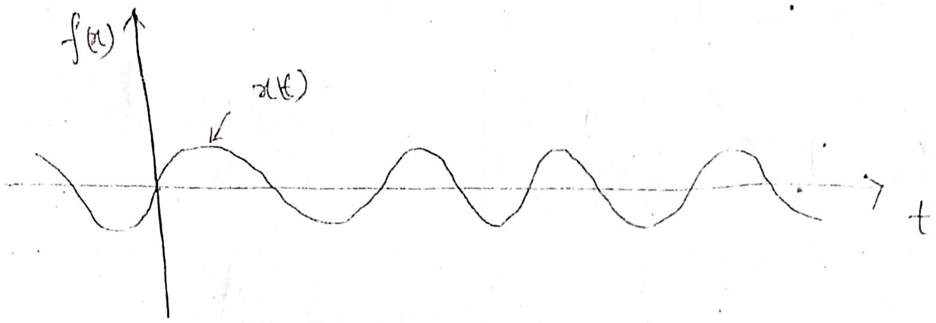
SIGNAL & SYSTEM



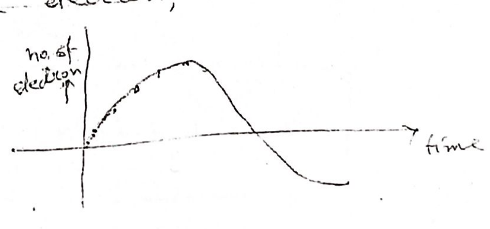
* Signal will move in what direction with respect to time?

Sol^n:- signal will move in the direction opposite to time direction.

assume → A man travel in a station (time) & a Goods train is signal. when the man ~~to~~ move stationary, Goods train has relative velocity in opposite dir of man.

* Why ~~signal~~ line current is sinusoidal & how electron will move in a wire.

Sol^n We get a current due to movement of free electron in a wire. current direction is opposite direction to electron. when switch is on ~~same~~ energied electrons will move in certain ~~direction~~ distance & transfers its energy (K.E only) to another electron loosing his ~~energy~~. life. so we get a continuous current flow. Because all the electrons are identical. Line current sinusoidal because generating power station supplies sinusoidal energy or excitation. ~~Such as of first one electron,~~ ~~Then 10 ele~~



LTI → 40
Random → 25
Fouriers → 20
Laplace & Z transform rest.

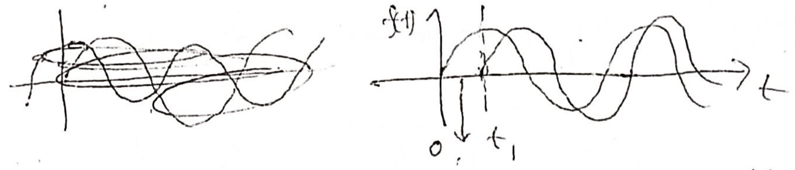
$x(t) = A\cos(\omega t + \theta)$

θ = phase angle

②

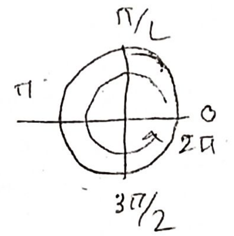
* What is the physical significance of phase angle (θ).

Soln Phase angle is nothing but a time difference bet two signal i.e. they start at different time;



t_1 time difference phase angle.

T = time period.



T ~~time~~ sec represents 2π

1 " " $\frac{2\pi}{T}$

t_1 " " $\frac{2\pi}{T} t_1$

$\therefore$ Phase angle $= \frac{2\pi}{T} t_1 = \theta$.

$$x(t) = A \sin\left(wt + 2\pi \frac{t_1}{T}\right)$$

$$= A \sin\left(wt + 2\pi \frac{t_1 v_p}{T v_p}\right)$$

$$= A \sin\left(wt + 2\pi \frac{x}{\lambda}\right)$$

v_p = velocity of signal;

λ = wave length.

$v = n\lambda$ n = freq.

$T = \frac{1}{n}$.

$v = \frac{1}{T}\lambda$

$vT = \lambda$

$$\boxed{\theta = \frac{2\pi t_1}{T} = 2\pi \frac{x}{\lambda}}$$

$$\boxed{e^{j\theta} = \cos\theta + j\sin\theta}$$

$$e^{jwt} = \cos wt + j \sin wt$$

Q 1. What is physical significance of j / write the difference bet i & j.

2. What is the graph of e^{jwt}

— us . .

In the case of mathematics there was a problem to find out the value of $\sqrt{-1}$, so to solve it mathematician assume its value is $i = \sqrt{-1}$.

But in case of ~~the physician~~, physics there was another problem as shown in bellow.

$$\boxed{\begin{aligned} \vec{A} &= +\vec{B} \\ \vec{A} &= \vec{B}\angle 0^\circ \end{aligned}}$$

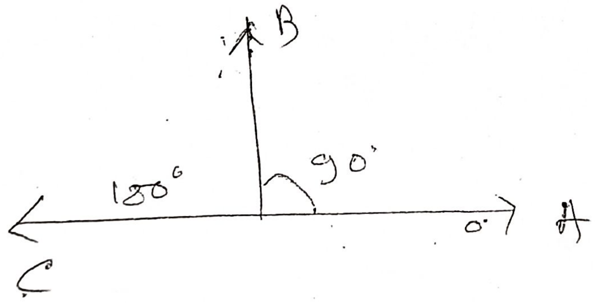
1895

$$\vec{A} = (-)\vec{B}$$ which represents as an angle as

$$\vec{A} = \vec{B}\angle 180^\circ$$

for denote the angle 90°

$$\boxed{\begin{aligned} \vec{A} &= j\vec{B} \\ \vec{A} &= \vec{B}\angle 90^\circ \end{aligned}}$$



$$\vec{A} = -\vec{C} \qquad \vec{B} = j\vec{A}$$

$$\begin{aligned} \vec{C} &= j\vec{B} \\ &= j(j\vec{A}) \\ &= j^2\vec{A} \end{aligned}$$

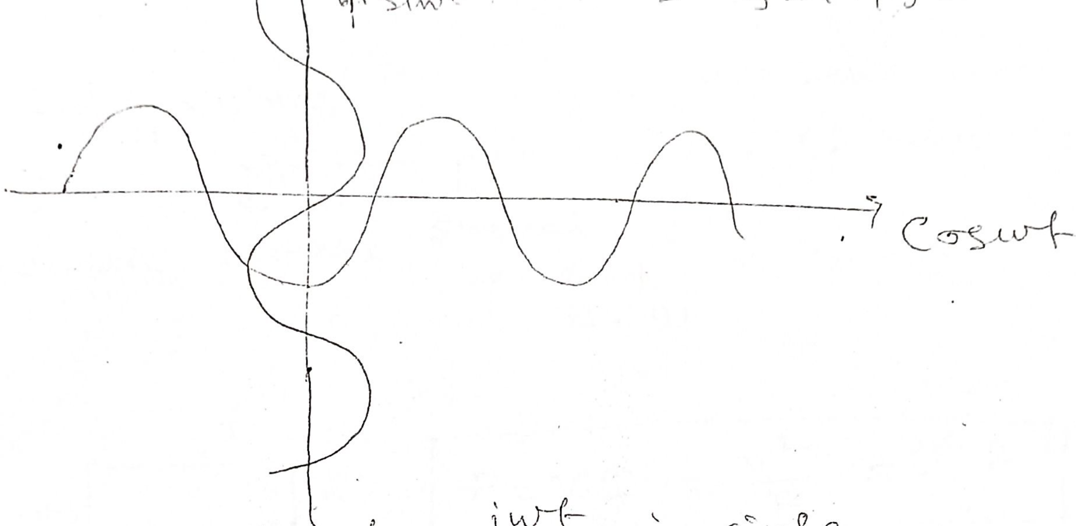
$$\vec{C} = -1\vec{A}$$

$$\Downarrow$$

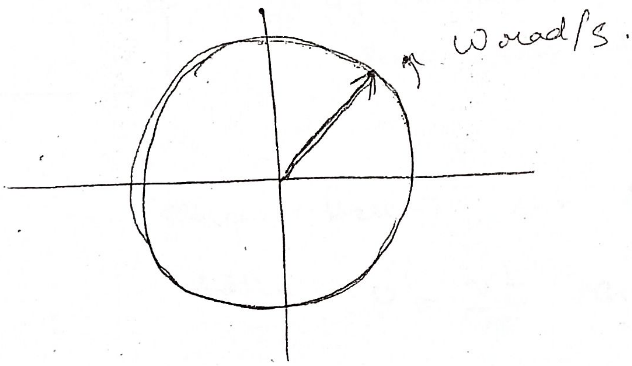
$$\therefore j^2 = -1.$$

$$\therefore j = \sqrt{-1}$$

$$e^{j\omega t} = \cos\omega t + j\sin\omega t$$

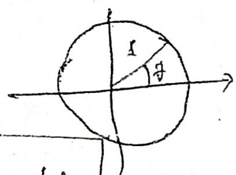


Graph of $e^{j\omega t}$ is circle



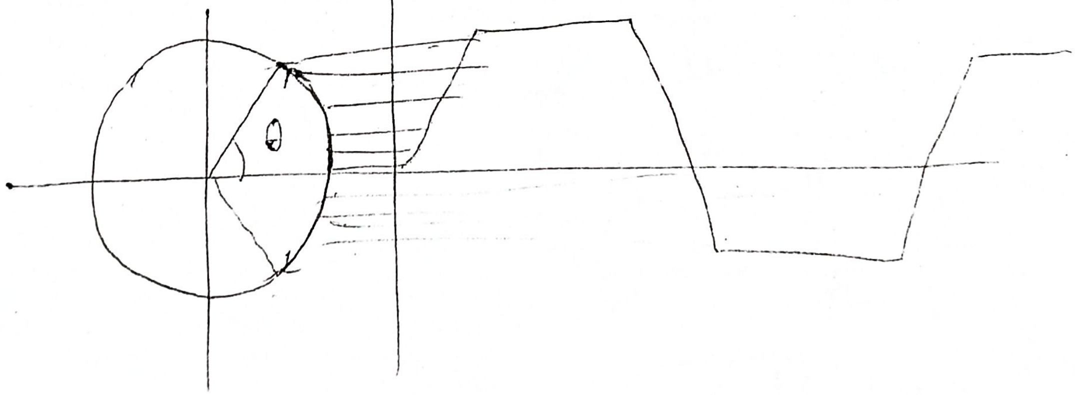
$$\frac{2\pi}{\omega} = T$$

$$|e^{j\theta}| = |\cos\theta + j\sin\theta| = 1$$



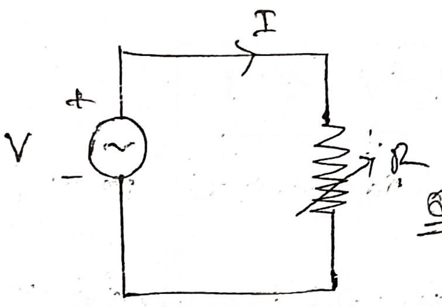
$$\boxed{e^{j\theta} = 1\angle\theta}$$

Square wave form from phasor diagram.



SIGNAL

- Deterministic
  - Periodic
  - Aperiodic
  - Special
    Ex. Ramp, $\delta(t)$, $u(t)$
- Non-Deterministic (Random Signal)



$$\boxed{P = VI = \frac{V^2}{R} = I^2 R}$$

Q. If we decrease the resistance power will decrease/increase?

Since here V is fixed we will take $P = \frac{V^2}{R}$ but not $P = I^2R$

In case of $P = I^2 R$,

There are two variable equal to P which deviate the law of variation.

So, we will take

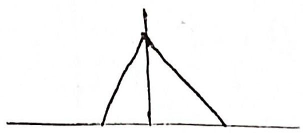
$$P = \frac{V^2}{R}.$$

$$\therefore P \propto \frac{1}{R}$$

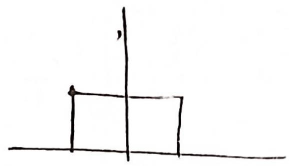
$$\boxed{R\downarrow \quad P\uparrow}$$

In full speed fan R is very small power is large where as low speed fan R is large, power is small.

(6) APERIODIC SIGNAL :-



or



Power $= P = VI = \frac{V^2}{R} = I^2 R$

If $R = 1\Omega$

$\boxed{P = V^2 = I^2}$ i.e. $\boxed{P \Rightarrow x^2(t)}$

$\Downarrow$

Normalized power / Energy

Normalized power / Energy is defined as E/P derived in 1Ω resistance.

General form of Power (P_α / P_{av} or E_α / E_T) :-

$$E_\alpha = \lim_{T\to\infty} \int_{-T/2}^{T/2} |x(t)|^2 \, dt \quad \text{[in continuous case]}$$

where T = time span. $= t_2 - t_1$

$$\boxed{E_\alpha = \lim_{N\to\infty} \sum_{n=-N}^{N} |x[n]|^2 = \sum_{n=-\infty}^{\infty} |x[n]|^2}$$

$\Downarrow$

Discrete case

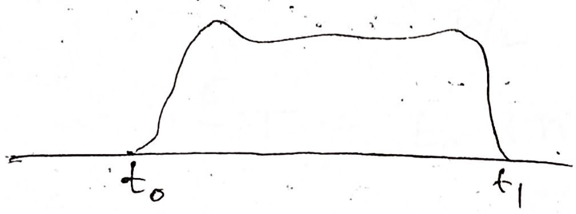
$$\boxed{P_{av} / P_{\alpha} = \underset{T \to \alpha}{lt} \frac{1}{T} \int_{-T/2}^{+T/2} |x(t)|^2 \, dt}$$

⇓ Continuous case

$$\boxed{P_{\alpha} = \underset{N \to \alpha}{lt} \frac{1}{2N+1} \sum_{n=-N}^{N} |x[n]|^2}$$

⇓ Discrete case.

Aperiodic signal :-



$$E_T = \underset{T \to \alpha}{lt} \int_{-T/2}^{T/2} |x(t)|^2 \, dt = \int_{t_0}^{t_1} |x(t)|^2 \, dt$$

$=$ finite value.

$$P_{av} / P_{\alpha} = \underset{T \to \alpha}{lt} \frac{E_T}{T} = 0$$

$\therefore$

$$\boxed{\begin{array}{c} E_T = \text{finite} \\ P_{av} = 0 \end{array}}$$

Energy signal = Aperiodic signal

PERIODIC SIGNAL :-

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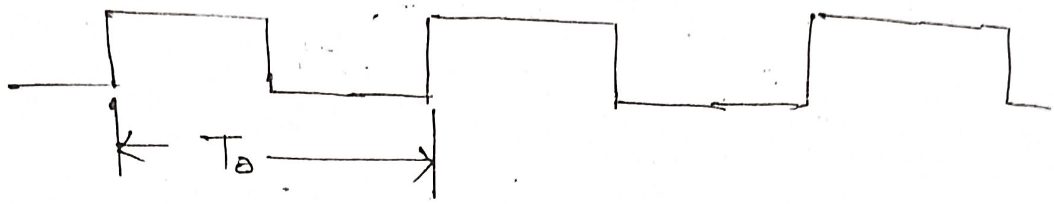

$$E_T = \lim_{T\to\infty} \int_{-T/2}^{T/2} |x(t)|^2 dt = \text{infinite} = \infty$$

We can prove it alternatively as we have one cycle energy is

$$E_c = \int_{-T_0/2}^{T_0/2} |x(t)|^2 dt = \text{finite}.$$

$$\therefore E_T = E_c \times n = \infty \quad \because n = \text{no. of infinite cycle}.$$

$$P_{av} = \lim_{T\to\infty} \frac{E_T}{T}$$

$$= \lim_{n\to\infty} \frac{E_c \times n}{T_0 \times n}$$

$$= \frac{E_c}{T_0}$$

$$= \frac{1}{T_0} \int_{-T_0/2}^{+T_0/2} |x(t)|^2 dt = \text{finite}$$

$$\boxed{\begin{aligned} E_T &= \text{infinite} = \infty \\ P_{av} &= \text{finite} \end{aligned}}$$

Periodic Signal = Power Signal.

$$= \frac{\sqrt{2}(1 - j\sqrt{3} + j + \sqrt{3})}{4}$$

$$= \frac{\sqrt{2}(1 + \sqrt{3} + j - j\sqrt{3})}{4} = \frac{1+\sqrt{3}}{2\sqrt{2}} - \frac{\sqrt{3}-1}{2\sqrt{2}} j$$

$$= \frac{1}{2\sqrt{2}}\left[\sqrt{(1+\sqrt{3})^2 + (\sqrt{3}-1)^2}\right] e^{\tan^{-1}\frac{1-\sqrt{3}}{1+\sqrt{3}}}$$

$$= \frac{1}{2\sqrt{2}}\left[\sqrt{1^2 + 2\sqrt{3} + 3 + 3 - 2\sqrt{3} + 1}\right] e^{\tan^{-1}(\sqrt{3}-2)}$$

$$= \frac{1}{2\sqrt{2}} \times 2\sqrt{2} \cdot e^{-j\,15} = e^{-j\pi/12}$$ Ans

$$\frac{1-\sqrt{3}}{1+\sqrt{3}} = \frac{1 - 2\sqrt{3} + 3}{1-3} = \frac{4 - 2\sqrt{3}}{-2} = \sqrt{3} - 2$$ Ans

1.3..

P_∞ and E_∞ :-

(a) $x(t) = e^{-2t} u(t)$

$$P_\infty = \lim_{T\to\infty} \frac{1}{2T} \int_{-T/2}^{T/2} |x_1(t)|^2 dt = \lim_{T\to\infty} \frac{1}{2T} \int_0^\infty e^{-4t} dt$$

$$= \lim_{T\to\infty} \frac{1}{2T} \times \frac{1}{-4}\left[e^{-4t}\right]_0^\infty$$

$$= \lim_{T\to\infty} \frac{1}{2T} \times \frac{1}{-4}[0 - 1]$$

(10) $E_\infty = \lim_{T\to\infty} \int_{-T/2}^{T/2} |x(t)|^2 \, dt$

$= \lim_{T\to\infty} \int_{0}^{T/2} (e^{-2t})^2 \, dt$

$= \int_0^\infty e^{-4t} \, dt = \left[\frac{e^{-4t}}{-4}\right]_0^\infty = -\frac{1}{4}[0-1]$

$= \frac{1}{4}$

(b) $x_2(t) = e^{j(2t + \pi/4)}$

$\boxed{|e^{j\theta}| = 1.}$

$E_\infty = \lim_{T\to\infty} \int_{-T/2}^{T/2} |x_2(t)|^2 \, dt$

$= \int_{-\infty}^{\infty} e^{j(4t+\pi/2)} \, dt$

$= e^{j\pi/2} \int_{-\infty}^{\infty} e^{4jt} \, dt$

$= \frac{e^{j\pi/2}}{4j} \left[e^{4jt}\right]_{-\infty}^{\infty} = \frac{e^{j\pi/2}}{4j}\left[e^{\infty} - e^{-\infty}\right]$

$= \frac{e^{j\pi/2}}{4j}[\infty - 0]$

$= \infty$ Ans

Alternative

$E_\infty = \lim_{T\to\infty} \int_{-T/2}^{T/2} |e^{j2t}|$

$= \int_{-\infty}^{\infty} 1 \, dt$

$= \infty.$

Given signal is periodic.

$P_\infty = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} \left|e^{j(2t + \pi/4)}\right|^2 dt$

Here, $\omega_0 t = 2$

$\therefore \omega_0 = 2$

$= \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} dt$

$= \frac{1}{T_0} [t]_{-T_0/2}^{T_0/2} = \frac{1}{T_0}\left[T_0/2 + T_0/2\right] = 1$ Ans

(c) $x_3(t) = \cos(t)$.

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$$E_\infty = \lim_{T\to\infty} \int_{-T/2}^{T/2} \cos^2(t)\, dt.$$

$\cos 2t = 2\cos^2 t - 1$

$\cos^2 t = (1+\cos 2t)/2$

$$= \int_{-\infty}^{\infty} \left(\frac{1+\cos 2t}{2}\right) dt.$$

$$= \int_{-\infty}^{\infty} \left(\frac{1}{2}\right) dt + \frac{1}{2}\int_{-\infty}^{\infty} \cos 2t\, dt = \infty$$

So

$$P_{av} / P_\infty = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} (\cos t)^2\, dt.$$

Since $\cos t$ is periodic signal so,

$$P_\infty = \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} \cos^2 t\, dt$$ where T_0 = period of $\cos t$.

$$= \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} \frac{1}{2}(1+\cos 2t)\, dt$$

$$= \frac{1}{T_0}\times\frac{1}{2} \int_{-T_0/2}^{T_0/2} dt + \frac{1}{T_0}\times\frac{1}{2} \int_{-T_0/2}^{T_0/2} \cos 2t\, dt$$

(over a period)

$$= \frac{1}{2T_0}\left[\frac{T_0}{2} + \frac{T_0}{2}\right] + 0$$

$$= \frac{T_0}{2T_0} = \frac{1}{2} \text{ Ans}$$

(d) $x_1[n] = \left(\frac{1}{2}\right)^n u[n]$ 12

$$\cancel{E_\infty = \lim_{T\to\infty} \int_{-T/2}^{T/2} |x_1[n]|^2}$$

$$E_\infty = \lim_{N\to\infty} \sum_{n=-N}^{N} |x_1[n]|^2$$

$$= \sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^{2n}$$

$$= \sum_{n=0}^{\infty} \left(\frac{1}{4}\right)^n$$

$$= \left(\frac{1}{4}\right)^0 + \sum_{n=1}^{\infty} \left(\frac{1}{4}\right)^n$$

$$\boxed{\sum_{n=1}^{\infty} a^n = \frac{a}{1-a}}$$

$$= 1 + \frac{\frac{1}{4}}{1-\frac{1}{4}}$$

$$= 1 + \frac{1}{4-1} = 1 + \frac{1}{3} = \frac{4}{3} \quad \underline{\underline{\text{Ans}}}$$

$$P_\infty = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} \left(\frac{1}{2}\right)^{2n} u(n)$$

$$= \lim_{N\to\infty} \frac{1}{2N+1} \times \sum_{n=0}^{\infty} \left(\frac{1}{4}\right)^n$$

$$= \lim_{N\to\infty} \frac{4/3}{2N+1} = 0 \quad \underline{\underline{\text{Ans}}}$$

(e) $x_2[n] = e^{j(\pi/2\, n + \pi/8)}$

$$E_\infty = \sum_{n=-\infty}^{\infty} \left| e^{j\left(\frac{\pi}{2} n + \frac{\pi}{8}\right)} \right|^2$$

$$= \sum_{n=-\infty}^{\infty} 1^n = \infty$$

$\omega_0 n = \frac{\pi}{2} n$

or, $\omega_0 = \pi/2$

or, $\frac{2\pi}{N_0} = \pi/2$

or, $N_0 = 4$

$$\therefore P_{av}/P_\infty = \frac{1}{2N_0+1} \sum_{n=-N_0}^{N_0} 1^n$$

$$= \frac{1}{2\times 4+1} \times 9$$

$$= \frac{1}{9} \times 9 = 1 \text{ Ans}$$

(f) $x_3[n] = \cos(\pi/4\, n)$

$$E_\infty = \sum_{n=-\infty}^{\infty} \cos^2 \frac{\pi}{4} n \qquad = \left(1 + \cos\frac{\pi}{2} n\right) 1/2$$

$$= \sum_{n=-\infty}^{\infty} \left(\frac{1}{2}\right) + \frac{1}{2} \sum_{n=-\infty}^{\infty} \cos\frac{\pi}{2} n$$

$$= \infty$$

$\omega_0 n = \frac{\pi}{4} n$

$\omega_0 = \frac{\pi}{4}$

$\frac{2\pi}{N_0} = \frac{\pi}{4}$

or, $N_0 = 8$

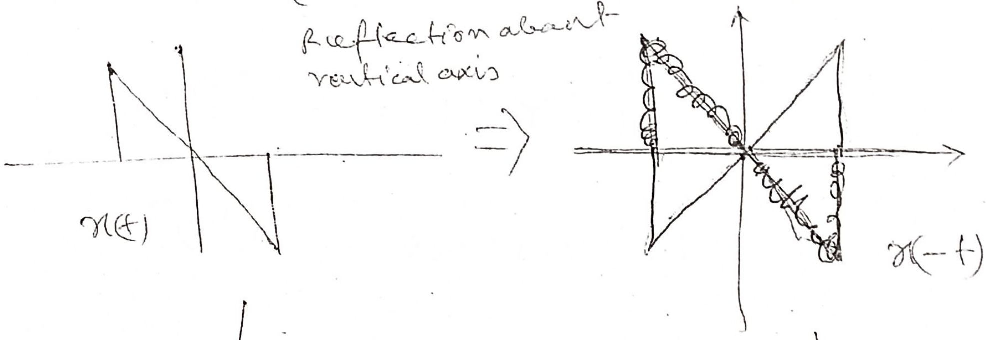
Periodic signal

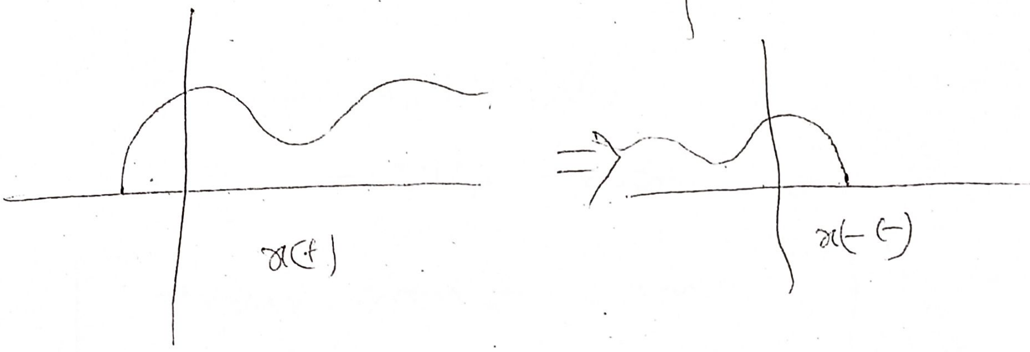
$$P_\infty = \frac{1}{2N_0+1} \sum_{n=-N_0}^{N_0} \left[\frac{1}{2} + \frac{1}{2}\cos\frac{\pi}{2} n\right]$$

$$= \frac{1}{2\times 8+1} \times \frac{1}{2} \times 17 + \frac{1}{2\times 8+1} \times \frac{1}{2} \times 0$$

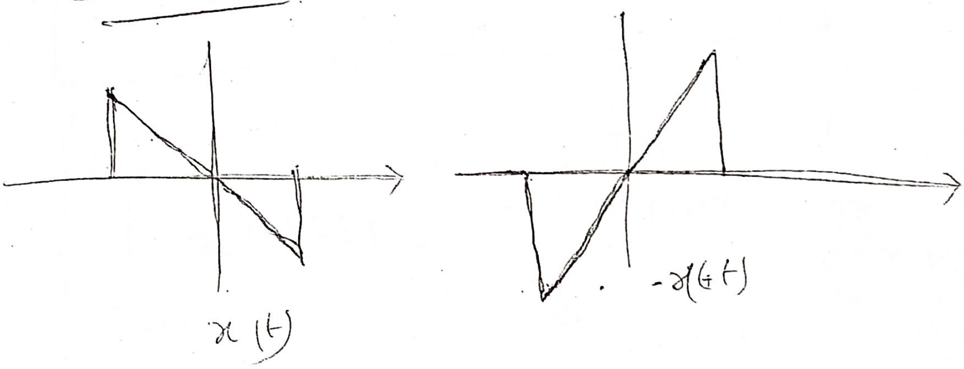
$$= \frac{1}{2} \text{ Ans}$$

$x(t) = x(-t)$ even fn





$x(t) = -x(-t)$ (odd)



$$\text{Ev } x(t) = \frac{1}{2}\left[x(t) + x(-t)\right] \rightarrow ①$$

$$\text{Od } x(t) = \frac{1}{2}\left[x(t) - x(-t)\right] \rightarrow ②$$

Prove eqn ①

$$x(t) = x_e(t) + x_o(t) \qquad x_e(t) = \frac{x(t)+x(-t)}{2}$$

$$x(-t) = x_e(-t) + x_o(-t)$$

$$= x_e(t) - x_o(t)$$

Similarly

$$x_o(t) = \frac{x(t) - x(-t)}{2}$$

$x_e(t)\, y_e(t)$ = odd or even?

$$z(t) = x_e(t)\, y_e(t)$$

$$z(-t) = x_e(-t)\, y_e(-t)$$

$$= x_e(t)\, y_e(t)$$

$$= z(t) = \text{even.}$$

$\boxed{\begin{array}{l} \text{even} \times \text{even} = \text{even.} \\ \text{Even} \times \text{odd} = \text{odd} \\ \text{odd} \times \text{odd} = \text{even} \end{array}}$

$$\int_{-T_0/2}^{T_0/2} x_e(t)\,dt = 2\int_{0}^{T_0/2} x_e(t)\,dt \quad \text{for } \underline{\text{even fn.}}$$

$$\int_{-T_0/2}^{T_0/2} x_o(t)\,dt = 0 \quad \underline{\text{for odd fn}}$$

= cos ...

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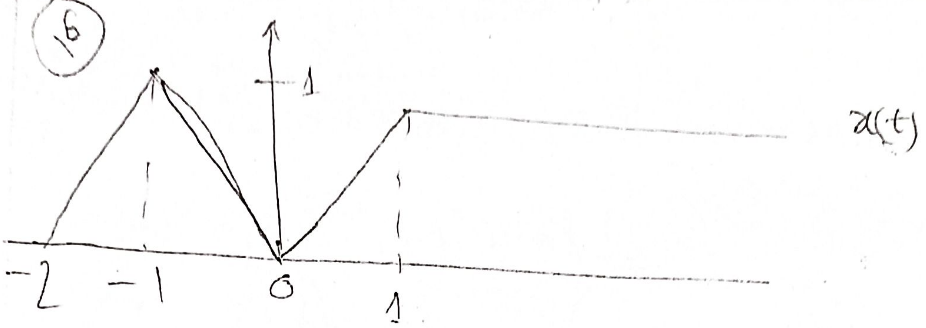
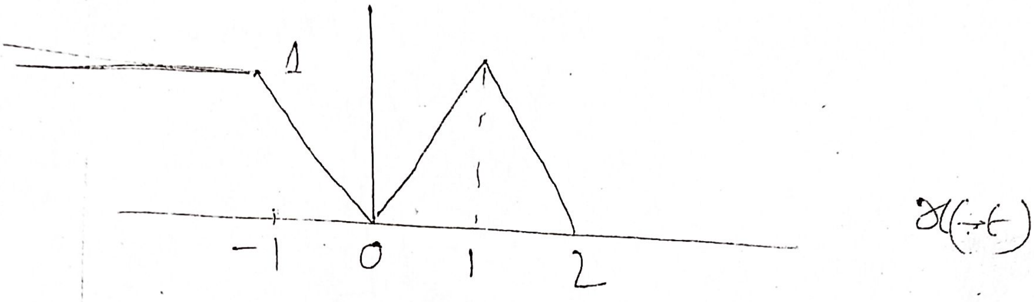
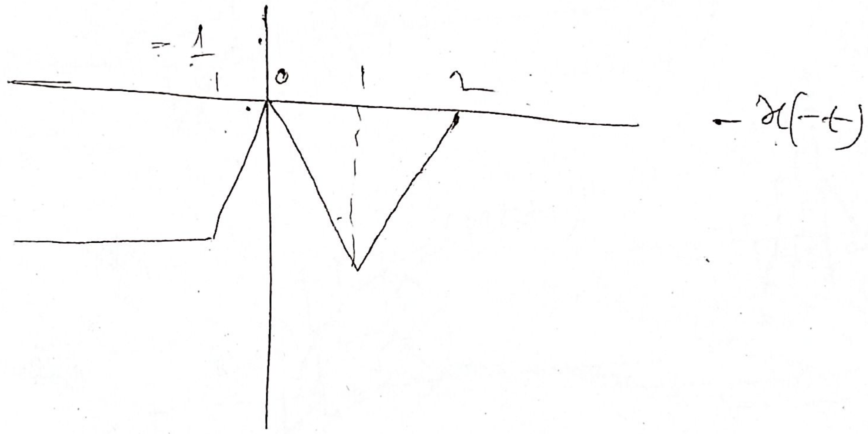
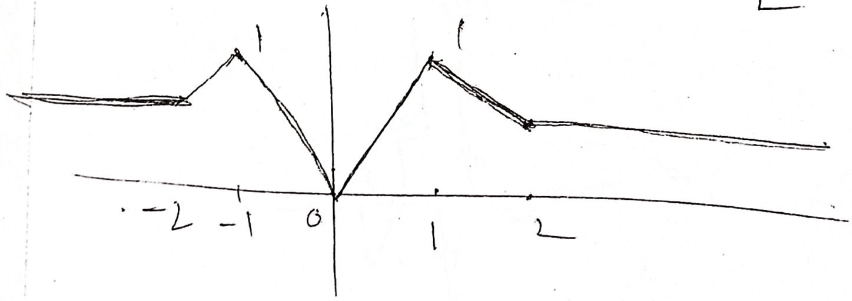


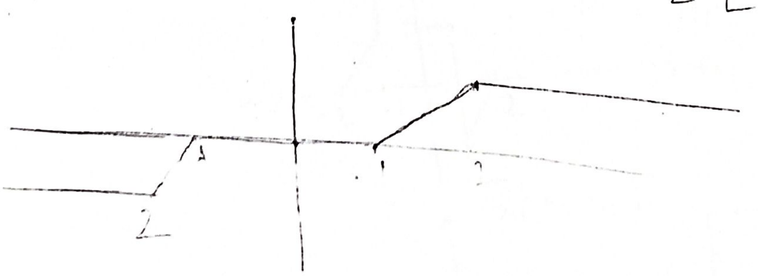




$X_e(t)$ = Even part of $x(t)$ $= \frac{1}{2}\left[x(t) + x(-t)\right]$



$X_o(t)$ = Odd part of $x(t)$ $= \frac{1}{2}\left[x(t) - x(-t)\right]$



(d) $x_1[n] = \left(\frac{1}{2}\right)^n u[n]$

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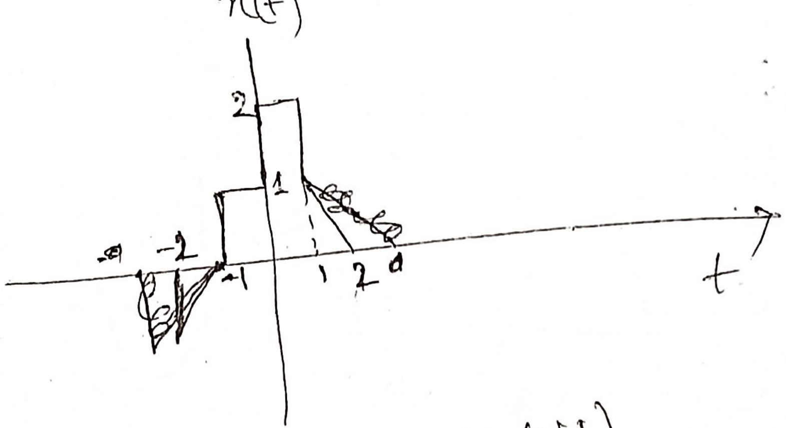
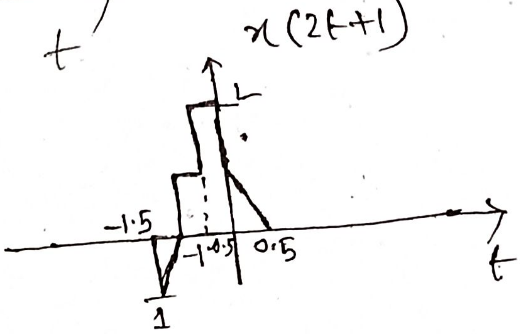
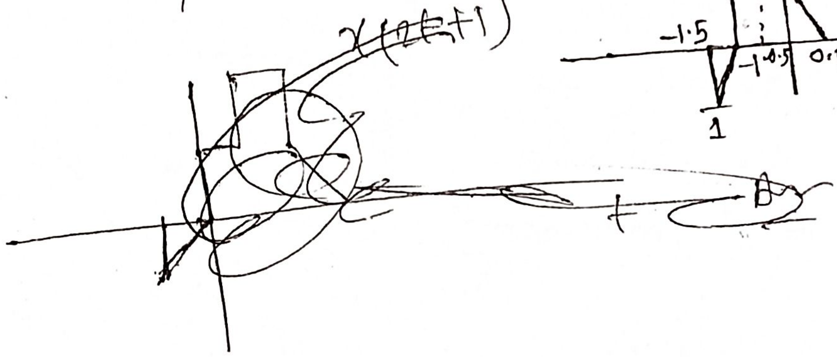
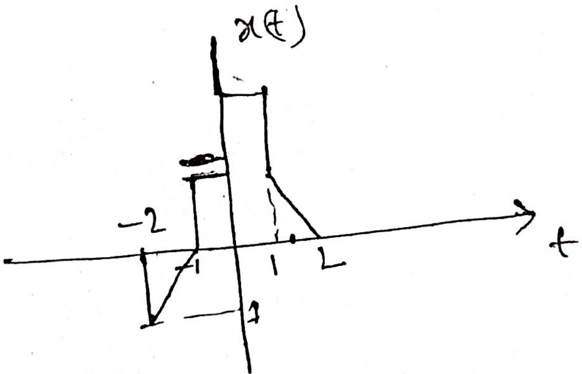
(e) $x(2t+1)$







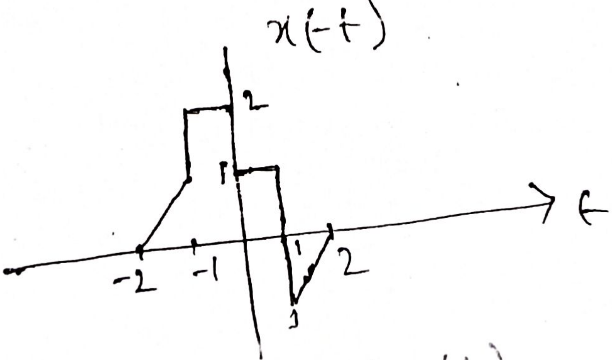
(d)

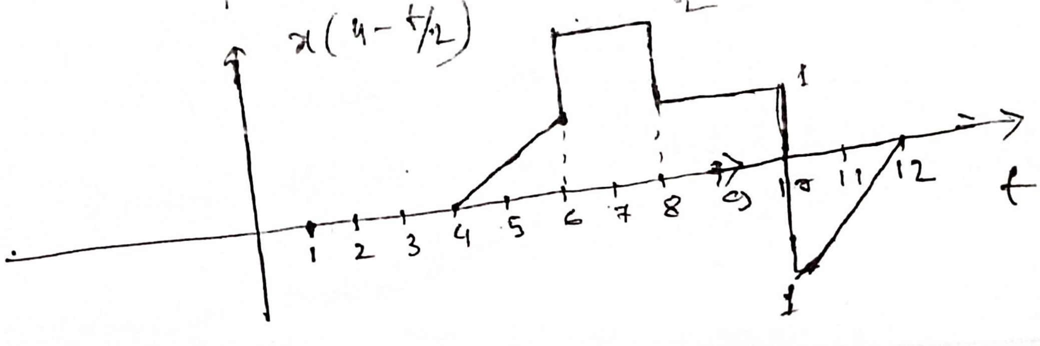


$x\left(4 - \frac{t}{2}\right)$

$= x\left\{\frac{1}{2}(8-t)\right\}$

$= x\{-[$





Periodic signal

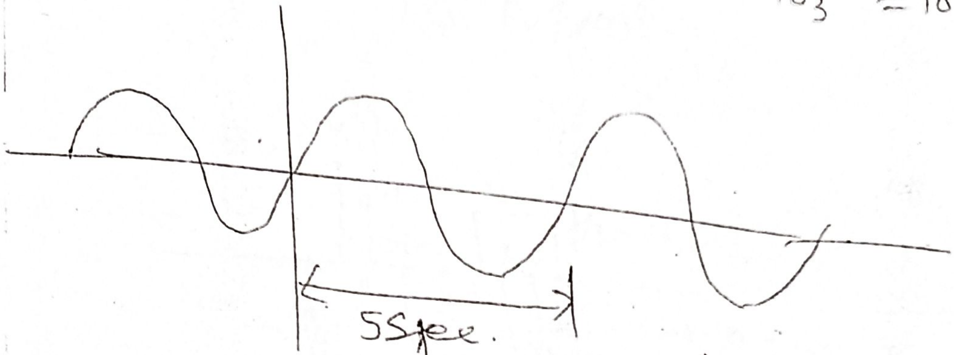
$$x(t) = x(t + T_x)$$

T_x = time period.

$T_{01} = 5$ sec.

$T_{02} = 7$ sec.

$T_{03} = 10$ sec.



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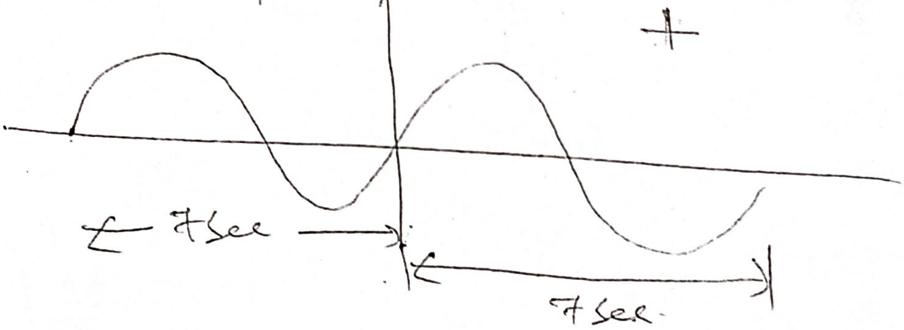
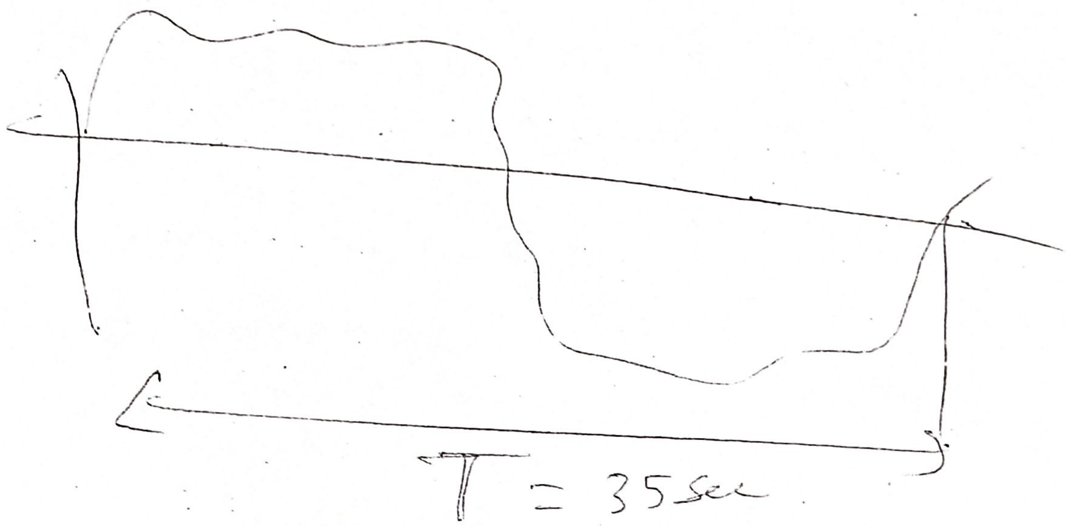




$T = 35$ sec

L.C.M of T_{01} & T_{02}

when $T_1 = \frac{22}{7}$ & $T_2 = 5$

Then their sum signal is aperiodic

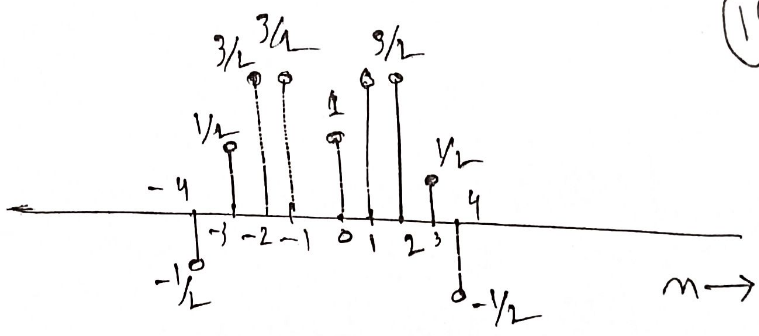


Fig: $Ev\{x(n)\}$

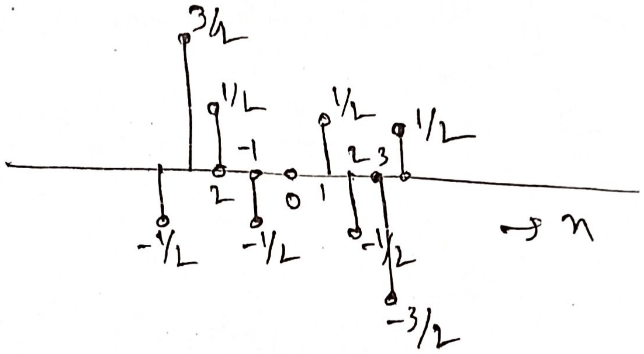


Fig: $Od\{x(n)\}$

1.25.

(a) $x(t) = 3\cos(4t + \pi/3)$

given signal is periodic with fundamental period $= \frac{\pi}{2}$ Ans

$\omega_0 = 4$

$\therefore T_0 = \frac{2\pi}{\omega_0}$

$= \frac{2\pi}{4} = \frac{\pi}{2}$

(b) $x(t) = e^{j(\pi t - 1)}$

It is periodic with fundamental

period $T_0 = \frac{2\pi}{\omega_0} = \frac{2\pi}{\pi} = 2$ Ans

(c) 2(a)

$$x(t) = \left[\cos\left(2t - \frac{\pi}{3}\right)\right]^2$$

$$= \cos^2\left(2t - \frac{\pi}{3}\right)$$

$$= \frac{1}{2}\left[1 + \cos\left(4t - \frac{2\pi}{3}\right)\right]$$

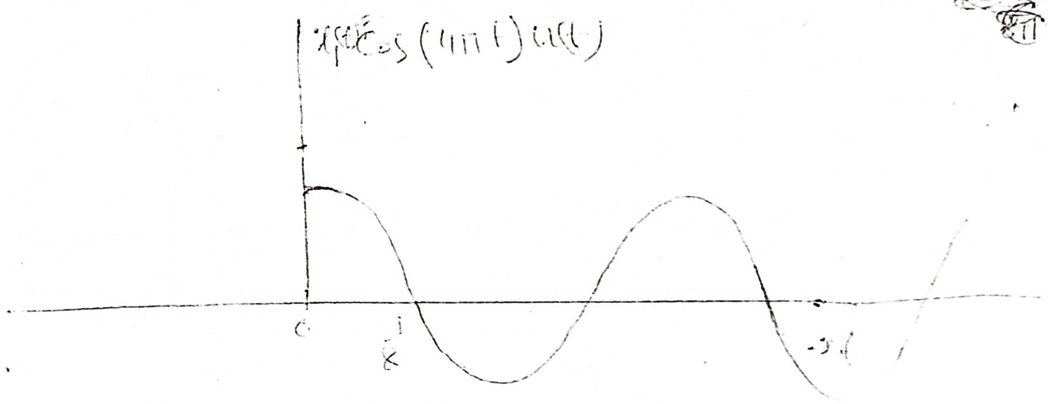
$$= \frac{1}{2} + \frac{1}{2}\cos\left(4t - \frac{2\pi}{3}\right)$$

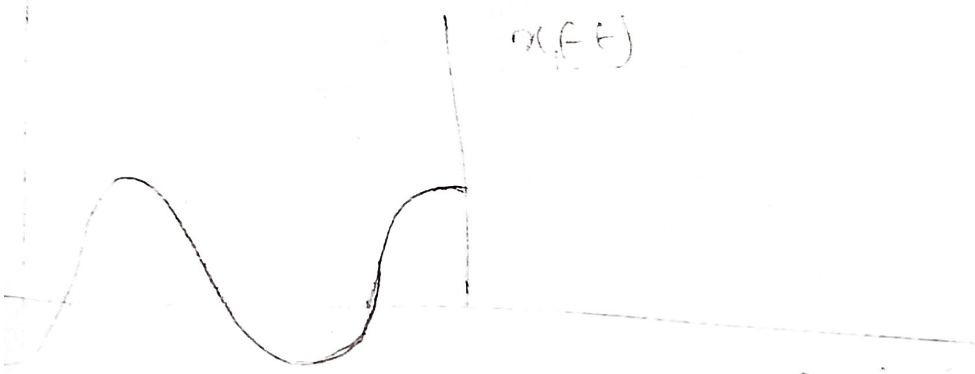
It is periodic with fundamental period

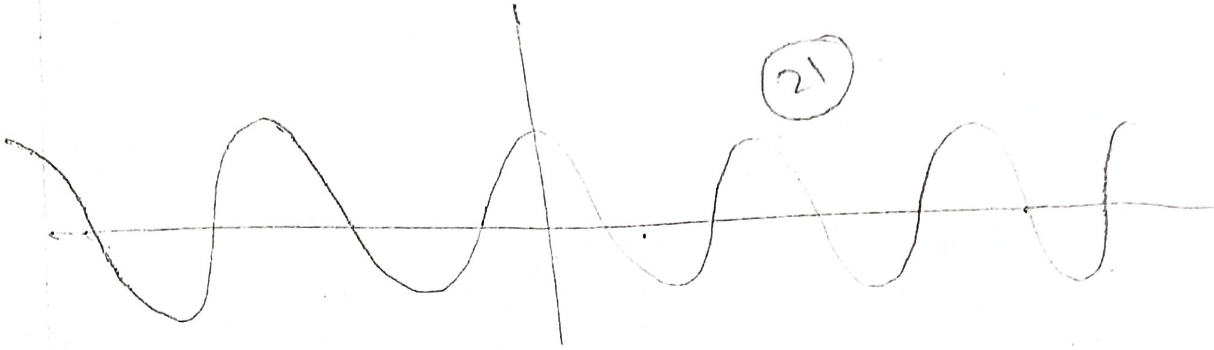
$$T_0 = \frac{2\pi}{\omega_0} = \frac{2\pi}{4} = \frac{\pi}{2} \quad \underline{\text{Ans}}$$

(d) $x(t) = Ev\{\cos(4\pi t)\, u(t)\}$

$\frac{1}{8}$







So $x(t)$ is periodic

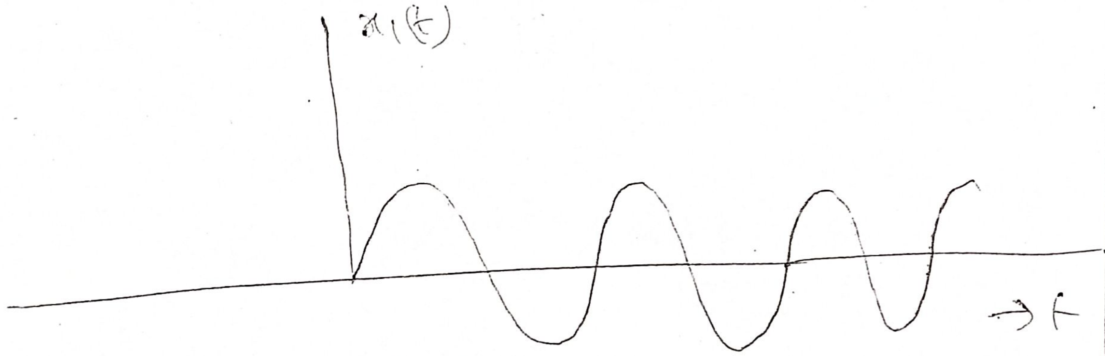
with fundamental period $= T_0 = \frac{2\pi}{\omega_0} = \frac{2\pi}{4\pi}$

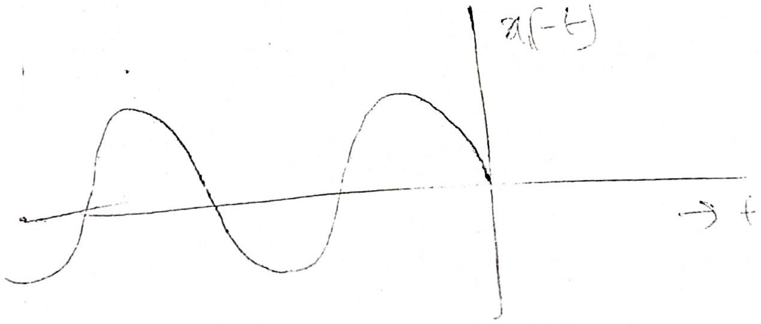
$= \frac{1}{2}$

Ans

(e) $x(t) = \mathcal{E}v\{\sin(4\pi t)\, u(t)\}$

$= \mathcal{E}v\{x_1(t)\}$





$\therefore \mathcal{E}v\{x_1(t)\} = \frac{x_1(t) + x_1(-t)}{2}$

there is discontinuity at $t=0$, so it is not periodic

(f) $x(t) = \sum_{n=-\infty}^{\infty} e^{-(2t-n)}$

It is periodic with period $= T_0 = \frac{2\pi}{\omega_0} =$

$= \pi$

1.26

(a) $x[n] = \sin\left(\frac{6\pi}{7}n+1\right)$

$= \sin\left[\left(\frac{2\pi}{7}\right)3n+1\right]$

$\therefore$ Here $\omega_0 = \frac{2\pi}{7}$ or, $N_0 = \frac{2\pi}{\omega_0} = \frac{2\pi}{\frac{2\pi}{7}} = 7$ Ans

(b) $x[n] = \cos\left(\frac{n}{8} - \pi\right)$

$= \cos\left[\left(\frac{2\pi}{8\times 2\pi}\right)n - \pi\right]$

Here $\omega_0 = \frac{2\pi}{8\times 2\pi}$ $\therefore N_0 = \frac{2\pi}{\omega_0} = \frac{2\pi}{\frac{2\pi}{8\times 2\pi}}$

$= 8\times 2\pi$

Since N_0 is fraction no it is not periodic.

(c) $x[n] = \cos\left(\frac{\pi}{8} n^2\right)$

It is not periodic

(d) $x[n] = \cos\left(\frac{\pi}{2} n\right) \cos\left(\frac{\pi}{4} n\right)$

$\cos\left(\frac{\pi}{2} n\right)$ is a periodic signal with fundamental period $= N_0 = \frac{2\pi}{\omega_0}$

$= \frac{2\pi}{\frac{2\pi}{4}} = 4$

$\cos\left(\frac{\pi}{4} n\right)$ is also periodic with fundamental period $N_0 = \frac{2\pi}{\omega_0} = \frac{2\pi}{\frac{2\pi}{8}} = 8$

24

(e) $x[n] = 2\cos\left(\frac{\pi}{4}n\right) + \sin\left(\frac{\pi}{8}n\right) - 2\cos\left(\frac{\pi}{2}n + \pi/6\right)$

$\cos\left(\frac{\pi}{4}n\right)$ has period $N_{01} = \frac{2\pi}{\frac{2\pi}{8}} = 8$

$\sin\left(\frac{\pi}{8}n\right)$ " " $N_{02} = \frac{2\pi}{\frac{2\pi}{16}} = 16$

$\cos\left(\frac{\pi}{2}n + \frac{\pi}{6}\right)$ " " $N_{03} = \frac{2\pi}{\frac{2\pi}{4}} = 4$

$\therefore$ $x[n]$ has a fundamental period is L.C.M of 8, 16, 4 = 16 Ans

1.27.

(a) $y(t) = x(t-2) + x(2-t)$

① Not memoryless at $t=0$ $y(0) = x(-2) + x($

②

$x_1(t) \Rightarrow y_1(t)$

$x_2(t) = x_1(t-t_0) \Rightarrow y_2(t)$.

$= x_1(\quad - t\quad) + x_1(\quad)$

$\neq y_1(t-t_0)$