

# Binomial Probability Distribution

- A fixed number of observations (trials),  $n$ 
  - e.g., 20 tosses of a coin
- Binary random variable
  - e.g., Head or tail in coin toss
  - Often called as success or failure
  - Prob of success is  $p$ , and prob of failure is  $1-p$
- Constant probability for each observation

# Binomial example

- Take the example of 5 coin tosses
- What's the probability that you flip exactly 3 heads in 5 coin tosses?

# Binomial distribution

- Solution:
- One way to get exactly 3 heads: HHH TT
- What's the probability of this exact arrangement?
  - $P(\text{heads}) \times P(\text{heads}) \times P(\text{heads}) \times P(\text{tails}) \times P(\text{tails})$   
 $= (1/2)^3 \times (1/2)^2$
- Another way to get exactly 3 heads: THH HT
- Probability of this exact outcome  $= (1/2) \times (1/2)^3 \times (1/2)$   
 $= (1/2)^3 \times (1/2)^2$

# Binomial distribution

- In fact,  $(1/2)^3 \times (1/2)^2$  is the probability of each unique outcome that has exactly 3 heads and 2 tails
- So, the overall probability of 3 heads and 2 tails is:  
 $(1/2)^3 \times (1/2)^2 + (1/2)^3 \times (1/2)^2 + (1/2)^3 \times (1/2)^2 +$   
..... for as many unique arrangements as there are
- But how many are there??

$$\binom{5}{3}$$

ways to  
arrange 3  
heads in  
5 trials

$${}_5C_3 = \frac{5!}{3!2!} = 10$$

Outcome	Probability
THHHT	$(1/2)^3 \times (1/2)^2$
HHHTT	$(1/2)^3 \times (1/2)^2$
TTHHH	$(1/2)^3 \times (1/2)^2$
HTTHH	$(1/2)^3 \times (1/2)^2$
HHTTH	$(1/2)^3 \times (1/2)^2$
THTHH	$(1/2)^3 \times (1/2)^2$
HTHTH	$(1/2)^3 \times (1/2)^2$
HHTHT	$(1/2)^3 \times (1/2)^2$
THHTH	$(1/2)^3 \times (1/2)^2$
HTHHT	$(1/2)^3 \times (1/2)^2$
10 arrangements $\times (1/2)^3 \times (1/2)^2$	

The probability  
of each unique  
outcome (note:  
they are all  
equal)

$$\begin{aligned}
 \therefore P(3 \text{ heads and } 2 \text{ tails}) &= \binom{5}{3} \times P(\text{heads})^3 \times P(\text{tails})^2 \\
 &= 10 \times \left(\frac{1}{2}\right)^5 = 31.25\%
 \end{aligned}$$

# Binomial distribution, generally

Note the general pattern emerging → if you have only two possible outcomes (call them 1/0 or yes/no or success/failure) in  $n$  independent trials, then the probability of exactly  $X$  “successes” =

The diagram shows the binomial distribution formula 
$$\binom{n}{X} p^X (1-p)^{n-X}$$
 enclosed in a purple box. Four arrows point from text labels to parts of the formula: 

- An arrow from " $n$  = number of trials" points to the top  $n$  of the binomial coefficient.
- An arrow from " $X$  = # successes out of  $n$  trials" points to the bottom  $X$  of the binomial coefficient.
- An arrow from " $p$  = probability of success" points to the  $p^X$  term.
- An arrow from " $1-p$  = probability of failure" points to the  $(1-p)^{n-X}$  term.