University of Asia Pacific

Department of CSE

Semester Final Examination, Spring 2020

Name: Rashik Rahman

Reg ID: 17201012

Year: 3rd

Semester: 2nd

Course Code: CSE 313

Course Title: Numerical Methods

Date: 28.10.2020

"During Examination and upload time I will not take any help from anyone. I will give my exam all by myself."

University of Asia Pacific

Admit Card

Final-Term Examination of Spring, 2020

Financial Clearance	PAID

Registration No : 17201012

Student Name : Rashik Rahman

Program : Bachelor of Science in Computer Science and Engineering

SI.NO.	COURSE CODE	COURSE TITLE	CR.HR.	EXAM. SCHEDULE
1	CSE 313	Numerical Methods	3.00	
2	CSE 314	Numerical Methods Lab	0.75	
3	CSE 315	Peripheral & Interfacing	3.00	
4	CSE 316	Peripheral & Interfacing Lab	1.50	
5	CSE 317	Computer Architecture	3.00	
6	CSE 319	Computer Networks	3.00	
7	CSE 320	Computer Networks Lab	1.50	
8	CSE 321	Software Engineering	3.00	
9	CSE 322	Software Engineering Lab	0.75	

Total Credit: 19.50

1. Examinees are not allowed to enter the examination hall after 30 minutes of commencement of examination for mid semester examinations and 60 minutes for semester final examinations.

2. No examinees shall be allowed to submit their answer scripts before 50% of the allocated time of examination has elapsed.

3. No examinees would be allowed to go to washroom within the first 60 minutes of final examinations.

4. No student will be allowed to carry any books, bags, extra paper or cellular phone or objectionable items/incriminating paper in the examination hall.
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Answer to the Q.No.1 (a)

unkwon $= 12 \times 0.1 = 1.2$;

Given,

$$\begin{bmatrix} 2 & 8 & -11 \\ 1 & 6 & 4 \\ 16 & 1.2 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 7 \\ -6 \\ 10 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 16 & 1.2 & 3 \\ 1 & 6 & 4 \\ 2 & 8 & -11 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 7 \\ -6 \\ 10 \end{bmatrix}$$

N.B: Pivoted so that can converge.

From the matrix we get,

$$16x_1 + 1.2x_2 + 3x_3 = \cancel{7}\ 10 \quad \text{--- (i)}$$

$$x_1 + 6x_2 + 4x_3 = -6 \quad \text{--- (ii)}$$

$$2x_1 + 8x_2 - 11x_3 = \cancel{10}\ 7 \quad \text{--- (iii)}$$

From (i), (ii), (iii) we get,

$$x_1 = \frac{10 - 1.2x_2 - 3x_3}{16} \quad \text{--- (iv)}$$

$$x_2 = \frac{-6 - 4x_3 - x_1}{6} \quad \text{(v)}$$

$$x_3 = \frac{2x_1 + 8x_2 - 7}{11} \quad \text{(vi)}$$

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iteration 1:

initial guess,

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \\ 5 \end{bmatrix}$$

putting this in (iv), (v), (vi) we get,

$$x_1 = \frac{10 - 1.2\times3 - 3\times5}{16} = -0.5375$$

$$x_2 = \frac{-6 - 4\times5 - 1}{6} = -4.5$$

$$x_3 = \frac{2\times1 + 8\times3 - 7}{11} = 1.72727$$

$\therefore$ Now,
$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -0.5375 \\ -4.5 \\ 1.72727 \end{bmatrix}$$

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iteration 2:

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -0.5375 \\ -4.5 \\ 1.72727 \end{bmatrix}$$

putting these values in (iv), (v), (vi) we get

$$x_1 = \frac{10 - 1.2(-4.5) - 3 \times 1.72727}{16} = 0.6386$$

$$x_2 = \frac{-6 - 4 \times 1.72727 - (-0.5375)}{6} = -2.06193$$

$$x_3 = \frac{2(-0.5375) + 8(-4.5) - 7}{11} = -4.0068$$

$$\therefore \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0.6386 \\ -2.06193 \\ -4.0068 \end{bmatrix}$$

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iteration 3:

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0.6386 \\ -2.06193 \\ -4.0068 \end{bmatrix}$$

putting these value in (iv), (v), (vi) we get

$$x_1 = \frac{10 - 1.2 \times 0.6386 - 3(-4.0068)}{16} = 1.32838$$

$$x_2 = \frac{-6 - 4(-4.0068) - 0.6386}{6} = 1.56477$$

$$x_3 = \frac{2 \times 0.6386 + 8(-2.006193) - 7}{11} = -1.9793$$

So after 3rd iteration x_1, x_2, x_3 is

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1.32838 \\ 1.56477 \\ -1.9793 \end{bmatrix}$$

Ans.

Answer to the Q. NO. 1(b)

Observation: Initiall the system given ~~won't~~ ~~conve~~ won't converge but with pivoting it will converges.

Proof:

$$\begin{bmatrix} 2 & 8 & -11 \\ 1 & 6 & 4 \\ 16 & 1.2 & 3 \end{bmatrix}$$

$|2| \geq |8| + |-11|$; This isn't true.

$|6| \geq |1| + |4|$; This is true

$|3| \geq |16| + |1.2|$; This isn't true.

So the matrix isn't diagonally dominant but with pivoting we can make it dominat.

After pivoting we get,

$$\begin{bmatrix} 16 & 1.2 & 3 \\ 1 & 6 & 4 \\ 2 & 8 & -11 \end{bmatrix}$$

Here,

$|16| \geq |1.2| + |3|$; This is true.

$|6| \geq |1| + |4|$; This is true.

$|-11| \geq |8| + |2|$; This is true.

As all the conditions are true, so the matrix is now diagonally dominant and now the system will converge.

Verdict: After pivoting the system will converge.

Answer to the Q. No. 1(c)

The gauss seidal method allows the user to control round off error. Elimination method like gauss elimination and LU decomposition are prone to round of error. If the physics of the problem is understood a close initial guess can be made, decreasing the number of iteration needed. Also using gauss-seidal ~~we~~ method we can know that if the system will converge or not.

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Answer to the Q. No. 2 (a)

$t = 12 + 10 = 22$ s.

For quadratic we know,

$$v(t) = b_0 + b_1(t - t_0) + b_2(t - t_0)(t - t_1) \quad \text{--- (1)}$$

Here,

$t_0 = 8$; $v(8) = 227.04$ ms^{-1}

$t_1 = 36$; $v(36) = 1004.597$ ms^{-1}

$t_2 = 65.75$; $v(65.75) = 1902.249$ ms^{-1}

Now to find b_0, b_1, b_2 we'll use tree method

$$\begin{array}{llll}
t_i; & v(t) & & \\
 & \rightarrow b_0 & \rightarrow b_1 & \rightarrow b_2 \\
t_0 = 8; & 227.04 & & \\
 & & 27.76989 & \\
t_1 = 36; & 1004.597 & & 0.41616 \\
 & & 30.1732 & \\
t_2 = 65.75; & 1902.249 & &
\end{array}$$

N.B.:

$$b_i = \frac{v(t_{i+1}) - v(t_i)}{t_{i+1} - t_i}$$

So,

$b_0 = 227.04$

$b_1 = 27.76989$

$b_3 = 0.41616$

from (i) we get,

$$v(t) = b_0 + b_1(t-t_0) + b_2(t-t_0)(t-t_1)$$

$$v(22) = 227.04 + 27.76989(22-8) + 0.41616(22-8) \times (22-36)$$

$$= 534.2511$$

So at $t = 22s$, velocity would be $534.2511 \ ms^{-1}$

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Answer to the Q. No. 2(b)

For linear interpolation

$$v(t) = b_0 + b_1(t - t_0)$$

here,

$t = 12 + 10 = 22\,s.$

$t_0 = 8\,s$

$t_1 = 36\,s.$

$b_0 = v(t_0) = 227.04$

$$b_1 = \frac{v(t_1) - v(t_0)}{t_1 - t_0} = \frac{1004.597 - 227.04}{\cancel{38}\,36 - 8}$$

$$= 27.76989$$

$$\therefore v(22) = 227.04 + 27.76989(22 - 8)$$

$$= 615.818\ ms^{-1}$$

$$|\epsilon_a| = \left|\frac{\text{quadratic approx} - \text{Linear approx}}{\text{quadratic appox}}\right| \times 100\%$$

$$= \left|\frac{534.2511 - 615.818}{534.2511}\right| \times 100\%$$

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$= 15.268\%$

$\therefore$ Relative approximate error is 15.268% and this is how I'll calculate it.

Answer to the Q. No. 3(a)

Given,

$$\int_a^b f(x)\,dx = \int_a^b e^x\,dx$$

Here,

$a = 2.4$

$b = a + 2 = 4.4$

$\therefore$ step size, $h = \dfrac{4.4 - 2.4}{4} = \dfrac{2}{4} = \dfrac{1}{2} = 0.5$

~~$f(x_0) = f(1) = e^1 = 2.718$~~

~~$f(x_1) = f(2)$~~

~~$f(x_0) = f(2.4) = e^{2.4} = 11.023$~~

~~$f(x_1) = f(2.4+1) = e^{3.4} = 29.96$~~

$f(x_2) = f$

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$f(x_0) = f(2.4) = e^{2.4} = 11.023$

$f(x_1) = f(2.4+0.5) = e^{2.9} = 18.174$

$f(x_2) = f(2.9+0.5) = e^{3.4} = 29.964$

$f(x_3) = f(3.4+0.5) = e^{3.9} = 49.402$

$f(x_4) = f(3.9+0.5) = e^{4.4} = 81.451$

from 4 segment simpson's 1/3 rule we know,

$$x = \frac{b-a}{3\times n}\left[f(x_0) + 4\sum_{i=1}^{n-1} f(x_i) + 2\sum_{i=2}^{n-2} f(x_i) + f(x_n)\right]$$

$$= \frac{4.4-2.4}{3\times 4}\left[f(2.4) + 4\{f(x_1)+f(x_3)\} + 2f(x_2) + f(x_4)\right]$$

$$= 0.167\left[11.023 + 4(18.174 + 49.402) + 2\times 29.964 + 81.451\right]$$

$$= 0.167(152.402 + 4(67.576))$$

$$= 0.167 \times 422.706$$

$$= \cancel{70.452}\ 70.4524$$

So the value is 70.4524.

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Answer to the Q. No. 3(b)

~~$E_t = E_x$~~

$$\text{Exact} = \int_a^b e^x dx = \int_{2.4}^{4.4} e^x dx$$

$$= \left[e^x\right]_{2.4}^{4.4}$$

$$= e^{4.4} - e^{2.4}$$

$$= 70.42769$$

True error,

$$E_t = \text{exact value} - \text{approx value}$$

$$= 70.42769 - 70.4524$$

$$= -0.002471$$

Relative true error,

$$E_a = \left|\frac{\text{Exact value} - \text{approx value}}{\text{Exact value}}\right| \times 100$$

$$= \left|\frac{70.42769 - 70.4524}{70.42769}\right| \times 100\%$$

$$= 0.0351\%$$

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Answer to the Q. No. 4(a) OR

$$A = 4\sin\theta(1+\cos\theta)$$

iteration 2:

here,

$x_l = 0$

$x_u = 2.4$

$h = 2.4 - 0 = 2.4$

$x_1 = x_l + 0.618 \times h = 0 + 0.618 \times 2.4$

$= 1.4832$

$x_2 = 2.4 - 0.618 \times 2.4 = 0.9168$

$f(x_2) = f(0.9168) = 4\sin(0.9168)(1+\cos(0.9168))$

$= 5.10597$

$f(x_1) = f(1.4832) = 4\sin(1.4832)(1+\cos(1.4832))$

$= 4.3332$

$f(x_2) > f(x_1)$ So. now,

$x_u = 1.4832$

$x_1 = \cancel{0.9168}\ \cancel{1.4832}\ 0.9168$

$x_l = 0;\ h = 1.4832 - 0 = \cancel{0.5564}\ 1.4832$

$\cancel{x_2 = 1.4832 - 0.618 \times (1.4832) 0.55}$

$x_2 = 1.4832 - 0.618 \times \cancel{0.5564}\ 1.4832 = \cancel{1.1393}\ 0.56658$

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iteration 2:

$x_u = 1.4832$

$x_1 = 0.9168$

$x_2 = \cancel{1.1333}\ 0.56658$

$x_l = 0$

$f(x_2) = f(\cancel{1.1333}\ 0.56658) = \cancel{5.15297}\ 3.9585$

$f(x_1) = f(0.91608) = 5.10597$

~~$f(x_2) > f(x_1)$ so now,~~

~~$x_u = 0.9168$~~

$f(x_1) > f(x_2)$ so now,

$x_l = 0.56658$

$x_2 = 0.9168$

$x_u = 1.4832;\ h = 1.4832 - 0.56658 =$

$x_1 = 0.56658 + 0.618 \times 0.91662 = 1.1331$

iteration 3:

$x_l = 0.56658$

$x_1 = 1.1331$

$x_2 = 0.9168$

$x_u = 1.4832$

$f(x_2) = f(0.9168) = 5.10597$

$f(x_1) = f(1.1331) = 5.1585$

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$f(x_1) > f(x_2)$

So,

$x_l = 0.9168$

$x_2 = 1.1381$

$x_u = 1.4832$

$x_1 = 0.9168 + 0.618(1.4832 - 0.9168)$

$= 1.2668.$

After 3rd iteration

$\therefore$ optimal point $= \dfrac{x_l + x_u}{2} = \dfrac{0.9168 + 1.2268}{2}$

$= 1.0718$

For optimal point A would be,

$A = 4 \sin(1.0718)(1 + \cos(1.0718))$

$= 5.193$

Answer to the Q. No. 4 (b) OR

For equal interval, with interval [a,b)

~~if f(~~

if $f\left(\frac{a+b}{2}+\frac{\epsilon}{2}\right) > f\left(\frac{a+2}{2}-\frac{\epsilon}{2}\right)$

then the interval in which the maximum occurs is $\left[\frac{a+2}{2}-\frac{\epsilon}{2}, b\right)$ else $\left[a, \frac{a+b}{2}+\frac{\epsilon}{2}\right]$

here ϵ is a gussed distance from mid-point. Now the problem is it's inefficient when ϵ is small it causes more iteration will be needed. Thus making it more time consuming. For the given seenario in 4 (OR) we have to guess ϵ, when to small it'd cause ~~error~~ more iteration when too large ~~will~~ will cause loss of optimal value. So ~~doing~~ using equal interval for the given seenario won't be appropreate ~~or even not possible in~~.