

Lecture 1:

* Reasons to use numerical method.

Lecture 2:

* True error = True value - Approximate error

* Relative true error = $\frac{\text{True error}}{\text{True value}}$

Lecture 3:

* Approximate error = Present approximation - Previous approximation

* Relative approximation error = $\frac{\text{Approximate error}}{\text{Present approximation}}$

* $|E_a| < \epsilon_s$; $|E_a| < 0.5 \times 10^{2-m} \%$
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 error Tolerance
 in %

* error% < 5% means it has 1 significant bit

$\% < 0.5\%$ $n = 2$ n

$\% < 0.05\%$ $n = 3$ n

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Lecture 4:

*) Round off error

*) Truncation error

Lecture 5:

*) Bin \rightarrow Dec

*) Dec \rightarrow Bin

*) Taylor series:

$$\rightarrow \cos(x) = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \dots$$

$$\rightarrow \sin(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \dots$$

$$\rightarrow e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots \quad [5 \text{ terms shown}]$$

*) General Taylor series:

$$\rightarrow f(x+h) = f(x) + f'(x)h + \frac{f''(x)h^2}{2!} + \frac{f'''(x)h^3}{3!} + \dots$$

*) Maclaurin series:

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots$$

*) FDD:

$$f'(x) = \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

Lecture 6:

* BDD:

$$\rightarrow f'(x) = \frac{f(x) - f(x-\Delta x)}{\Delta x}$$

* CDD:

$$\rightarrow f'(x) = \frac{f(x+\Delta x) - f(x-\Delta x)}{2\Delta x}$$

* Derive FDD, BDD from Taylor series.

Lecture 7:

* Higher order/second order derivative of FDD & BDD.

$$\rightarrow \text{FDD: } f''(x) = \frac{f(x_{i+2}) - 2f(x_{i+1}) + f(x_i)}{(\Delta x)^2}$$

$$\rightarrow \text{BDD: } f''(x) = \frac{f(x_{i-2}) - 2f(x_{i-1}) + f(x_i)}{(\Delta x)^2}$$

~~* Discrete functions~~

$$\rightarrow \text{CDD: } f''(x) = \frac{f(x_{i+1}) - 2f(x_i) + f(x_{i-1}))}{(\Delta x)^2}$$

* Discrete functions.

$$\rightarrow \text{Polynomial: } v(t) = a_0 + a_1 t + a_2 t^2 + a_3 t^3 \text{ [3rd order polynomial]}$$

Lecture 8:

*) Lagrang polynomial:

$$\rightarrow f_2'(x) = \frac{\{2x - (x_1 + x_2)\} f(x_0)}{(x_0 - x_1)(x_0 - x_2)} + \frac{\{2x - (x_0 + x_2)\} f(x_1)}{(x_1 - x_0)(x_1 - x_2)} + \frac{\{2x - (x_0 + x_1)\} f(x_2)}{(x_2 - x_0)(x_2 - x_1)}$$

$$\rightarrow f''(x) = \frac{2f(x_0)}{(x_0 - x_1)(x_0 - x_2)} + \frac{2f(x_1)}{(x_1 - x_0)(x_1 - x_2)} + \frac{2f(x_2)}{(x_2 - x_0)(x_2 - x_1)}$$

Lecture 9:

*) Bisection method

*) Drawbacks

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Lecture 10:

*) Newton's Raphson

$$\rightarrow x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}$$

$$|E_a| = \left| \frac{x_{i+1} - x_i}{x_{i+1}} \right| \times 100\%$$

*) Drawbacks

Lecture 11:

*) Secant method: $x_{i+1} = x_i - \frac{f(x_i)(x_i - x_{i-1})}{f(x_i) - f(x_{i-1})}$

$$\rightarrow |E_a| = \left| \frac{x_{i+1} - x_i}{x_{i+1}} \right| \times 100\%$$

→ Drawbacks