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Answer to the Q. No.1

a)

$$x_1 + 5x_2 + 3x_3 = 4$$
$$x_1 - x_2 + 6x_3 = 16$$
$$2x_1 + x_2 = 5$$

in $Ax=b$ format:

$$\begin{bmatrix} 1 & 5 & 3 \\ 1 & -1 & 6 \\ 2 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 4 \\ 16 \\ 5 \end{bmatrix}$$

b)

~~for~~ $x_1 + 5x_2 + 3x_2 = 4$

$$x_1 - x_2 + 6x_3 = 16$$
$$2x_1 + x_2 = 5$$

From this we get,

$$\begin{bmatrix} 1 & 5 & 3 \\ 1 & -1 & 6 \\ 2 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 4 \\ 16 \\ 5 \end{bmatrix}$$

~~Forward elimination,~~

~~Step 1~~

$$\Rightarrow \begin{array}{c} \begin{bmatrix} 1 & 5 & 3 & : & 4 \\ 1 & -1 & 6 & : & 16 \\ 2 & 1 & 0 & : & 5 \end{bmatrix} \end{array} \begin{array}{l} — \text{(i)} \\ — \text{(ii)} \\ — \text{(iii)} \end{array}$$

Forward elimination step 1,

After multiplying (i) with $1/1 = 1$ we get

$$\begin{bmatrix} 1 & 5 & 3 & : & 4 \end{bmatrix} \text{ — (iv)}$$

After subtracting (iv) from (ii) we get

$$\begin{array}{cccc} 1 & -1 & 6 & : 16 \\ 1 & 5 & 3 & : 4 \\ \hline 0 & -6 & 3 & : 12 \end{array}$$

$$\therefore \begin{bmatrix} 1 & 5 & 3 & : & 4 \\ 0 & -6 & 3 & : & 12 \\ 2 & 1 & 0 & : & 5 \end{bmatrix}$$

Again, after multiply (i) with $2/1 = 2$ we get

$$\begin{bmatrix} 2 & 10 & 6 & : & 8 \end{bmatrix}$$

$$\begin{array}{rrrcr} & 2 & 1 & 0 & : 5 \\ (-) & 2 & 10 & 6 & : 8 \\ \hline & 0 & -9 & -6 & : -3 \end{array}$$

∴ Matrix after step 1 is

$$\begin{array}{l} \left[\begin{array}{ccc:c} 1 & 5 & 3 & 4 \\ 0 & -6 & 3 & 12 \\ 0 & -9 & -6 & -3 \end{array}\right] \end{array} \begin{array}{l} - \text{(v)} \\ - \text{(vi)} \\ - \text{(vii)} \end{array}$$

Step 2,

~~(vii) ÷ (vi) we get~~

~~we m~~ multiply (vi) with $\frac{-9}{-6} = 1.5$, we get,

$$0 \quad -9 \quad 4.5 : 18 \; - \text{(viii)}$$

after subtract (viii) from (vii) we get,

$$\begin{array}{rrrrr} & 0 & -9 & -6 & : -3 \\ (-) & 0 & -9 & 4.5 & : 18 \\ \hline & 0 & 0 & -10.5 & : -21 \end{array}$$

So after step 2 ~~we get~~ the final matrix is

$$\left[\begin{array}{ccc:c} 1 & 5 & 3 & 4 \\ 0 & -6 & 3 & 12 \\ 0 & 0 & -10.5 & -21 \end{array}\right]$$

Back substitution:

$$\begin{bmatrix} 1 & 5 & 3 & : & 4 \\ 0 & -6 & 3 & : & 12 \\ 0 & 0 & -10.5 & : & -21 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & 5 & 3 \\ 0 & -6 & 3 \\ 0 & 0 & -10.5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 4 \\ 12 \\ -21 \end{bmatrix}$$

Thus we get

$$x_1 + 5x_2 + 3x_3 = 4 \quad \text{——— (ix)}$$

$$-6x_2 + 3x_3 = 12 \quad \text{——— (x)}$$

$$-10.5x_3 = -21 \quad \text{——— (xi)}$$

from (xi) we get

$$x_3 = \frac{-21}{-10.5} = 2$$

from (x) we get,

$$-6x_2 + 3 \times 2 = 12$$

$$-6x_2 = 12 - 6$$

$$-6x_2 = 6$$

$$x_2 = -1$$

from (ix) we get,

$$x_1 + 5\times -1 + 3\times 2 = 4$$

$$\Rightarrow x_1 - 5 + 6 = 4$$

$$\Rightarrow x_1 = 4 - 1$$

$$\Rightarrow x_1 = 3$$

$$\therefore \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 3 \\ -1 \\ 2 \end{bmatrix}$$

~~4~~

$$\begin{bmatrix} 1 & 5 & 3 \\ 1 & -1 & 6 \\ 2 & 1 & 0 \end{bmatrix} \begin{array}{l} \text{(i)} \\ \text{(ii)} \\ \text{(iii)} \end{array}$$

forward elimination

Step 1:

after multiplying (i) with $1/1 = 1$ and subtracting it from (ii) the replacing (ii) with the result we get,

$$\begin{bmatrix} 1 & 5 & 3 \\ 0 & -6 & 3 \\ 2 & 1 & 0 \end{bmatrix}$$

after ~~and~~ multiplying (i) with $2/1 = 2$ and subtracting it from (iii), then we replace (iii) with ~~as~~ the result we get,

$$\begin{bmatrix} 1 & 5 & 3 \\ 0 & -6 & 3 \\ 0 & -9 & -6 \end{bmatrix} \begin{matrix} \text{(iv)} \\ \text{(v)} \\ \text{(vi)} \end{matrix}$$

Step 2,

after multiplying (v) with $-9/-6 = 1.5$ then subtracting it from (vi) and then replacing (vi) ~~from~~ with result we get,

$$\begin{bmatrix} 1 & 5 & 3 \\ 0 & -6 & 3 \\ 0 & 0 & -10.5 \end{bmatrix}$$

∴ Determinant of A,

$$= U_{11} \times U_{22} \times U_{33}$$

$$= 1 \times -6 \times -10.5$$

$$= 63$$

Ans

d)

$$\begin{bmatrix} 1 & 5 & 3 \\ 1 & -1 & 6 \\ 2 & 1 & 0 \end{bmatrix}\begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} = \begin{bmatrix} 4 \\ 16 \\ 5 \end{bmatrix}$$; initial guess $\begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

$a_1 + 5a_2 + 3a_3 = 4$ — (i)

$a_1 - a_2 + 6a_3 = 16$ — (ii)

$2a_1 + 1a_2 \quad = 5$ — (iii)

∴ from (iii),
$a_3 = 0$

from (ii)

$-a_2 = 16 - a_1 - 6a_3$

$\Rightarrow -a_2 = 16 - 1 - 6 \times 1$

$\Rightarrow -a_2 = 16 - 7 = 9$

$\Rightarrow a_2 = -9$

from (i)

$a_1 + 5a_2 + 3a_3 = 4$

$\Rightarrow a_1 = 4 - 5a_2 - 3a_3$

$\Rightarrow a_1 = 4 - 5 - 3 = -4$

$\therefore$ After 1st iteration, we get,

$$\begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} = \begin{bmatrix} -4 \\ -9 \\ 0 \end{bmatrix}$$

$$|\epsilon_a|_1 = \left| \frac{a_1^{new} - a_1^{old}}{a_1^{new}} \right| \times 100 = \left| \frac{-4-1}{-4} \right| \times 100\cancel{0}$$

$$= 125\%$$

$$|\epsilon_a|_2 = \left| \frac{a_2^{new} - a_2^{old}}{a_2^{new}} \right| \times 100 = \left| \frac{-9-1}{-9} \right| \times 100$$

$$= 111.11\%$$

$$|\epsilon_a|_3 = \left| \frac{a_3^{new} - a_3^{old}}{a_3^{new}} \right| \times 100 = \text{Inconsistent.}$$

9

Answer to the Q.No.2

a)

Given,

$f(x) = 4x - 1 - \sin(x)$

initial,

$l = 0$
$u = 2$

~~$\therefore f(x_l) f(x_u) =$~~

$f(l) f(u) = (0 - 1 - \sin(0))(8 - 1 - \sin(2))$

$= (-1) (7 - 0.909)$

$= -6.091$

So root lies between 0 and 2.

iteration 1,

$m = \frac{l+u}{2} = \frac{0+2}{2} = 1$

$f(l) f(m) = (0 - 1 - \sin(0))(4 - 1 - \sin(1))$

$= (-1)(4 - 1 - 0.841)$

$= -2.159$

As $f(l) f(m) < 0$

$\therefore$ the root lies between l & m, So after iteration 1,

$$l = 0$$
$$u = 1$$

iteration 2,

$$m = \frac{0+1}{2} = 0.5$$

~~f(l) 0.1~~

$$f(l)f(m) = (0-1-\sin(0))(2-1-\sin(0.5))$$
$$= (-1)(0.521)$$
$$= -0.521$$

As $f(l)f(m) < 0$
∴ root lies between l and m, new l & u would be,

$$l = 0$$
$$u = 0.5$$

$$|\epsilon_m| = \left|\frac{m^{new} - m^{old}}{m^{new}}\right| \times 100$$

$$= \left|\frac{0.5 - 1}{0.5}\right| \times 100$$

$$= 100\%$$

iteration 3:

$$m = \frac{0+0.5}{2} = 0.25$$

$f(l)f(m) = (0-1-\sin(0))(1-1-\sin(0.25))$

$= (-1)(-0.247) = 0.247$

$\therefore$ As $f(l)f(m) > 0$ then root isn't here

$f(m)f(u) = (4\times0.25-1-\sin(0.25))(4\times0.5-1-\sin(0.5))$

$= (-0.247)(0.521) = -0.1286$

$\therefore$ $f(m)f(u) < 0$ thus root lies here,

$\therefore$ new,

$l = 0.25$

$u = 0.5$

$$|\epsilon_m| = \left|\frac{0.25-0.5}{0.25}\right| \times 100 = 100\%$$

b/

$f(x) = 4x - 1 - \sin x$

$\therefore f'(x) = 4 - \cos x$

iteration 1:

given,

$x_0 = 1$

$$\therefore x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$= 1 - \frac{4 - 1 - \sin(1)}{4 - \cos(1)}$$

$$= 1 - \frac{2.159}{3.4597}$$

$$= 0.376$$

iteration 2:

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$$= 0.376 - \frac{4 \times 0.376 - 1 - \sin(0.376)}{4 - \cos(0.376)}$$

$$= 0.376 - \frac{0.1371}{3.0699} = 0.3314$$

Ans.

5/

Absolute difference between the final estimates of the root in parts a & b is,

difference = $|0.25 - 0.3314|$

$= 0.0814$

Considering this value as the error of the bisection method we would need 4 more iteration to get accuracy of 10^{-4} that is 4 significant digits. Cause in iteration 13 we get error near to 0.0814, and in iteration 17 we get error 0.00461. This is the first to have four significant digit. Thus I can say in iteration 17th or doing four more itaration we can get accuracy of 10^{-4}.

Ar.