University of Asia Pacific

Department of CSE

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Course Title: Numerical Methods

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Answer to the Q. No. 1 (a)

To know relative true error we need to know true error first. True error is the difference between ~~the~~ true value and approximate value.

Relative true error is the ratio between true error and true value.

$$\text{Relative true error } (\epsilon_t) = \frac{\text{True error}}{\text{True value}}.$$

Answer to the Q. No. 1(b)

Given,

$$f(x) = 3e^{2.5x} + 2$$

$$x = 2.12$$

$$\Delta x = 2$$

Now,

$$f'(x) = 7.5e^{2.5x}$$

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FDD equation,

$$f'(x) \cancel{(f')} = \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

$$f'(2.12) = \frac{f(2.12+2) - f(2.12)}{2}$$

$$= \frac{f(4.12) - f(2.12)}{2}$$

$$= \frac{\cancel{3e^{2.5\times4.2}+2) - 3e^{2.5\times 2.2}}}{\cancel{2}} = \frac{(3e^{2.5\times4.2}+2) - 3e^{2.\ldots}}{}$$

$$= \cancel{54943.12571}\ 54172.7488$$

True value,

$$f'(x) = 7.5e^{2.5x}$$

$$f'(2.12) = 7.5e^{2.5\times2.12}$$

$$= 1502.5261$$

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$$f'(2.12) = \frac{(3e^{2.5\times4.12}+2)-(3e^{2.5\times2.12}+2)}{2}$$

$$= 44298.42306$$

Actual value,

$$f'(x) = 7.5e^{2.5x}$$

$$f'(2.12) = 7.5e^{2.5\times2.12}$$

$$= 1502.5261$$

$$\therefore \text{Relative true error} = \frac{1502.5261 - 44298.42306}{1502.5261}$$

$$= -28.4826$$

$$\therefore |\epsilon_t| = |-28.4826| = 28.4826$$

Ans.

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Answer to the Q. NO. 2(a)

Error caused by approximating a ~~mathe~~ mathematical procedure that is change in the procedure is truncation error.

Ex:

$$f(x) = x^2$$

$$x = 3$$

$$\Delta x = 0.2$$

Procedure 1:

$$f'(3) = \frac{f(3+0.2) - f(3)}{0.2} \quad \left[f'(x) = \frac{f(x+\Delta x) - f(x)}{\Delta x} \right]$$

$$= \frac{3.2^2 - 3^2}{0.2}$$

$$= 6.2$$

Procedure 2:

$$f'(x) = 2x$$

$$f'(3) = 2 \times 3 = 6$$

$\therefore$ Truncation error $= |6 - 6.2| = |-0.2| = 0.2$

So we see due to change in procedure error occurs.

Answer to the Q.No. 2(b)

Given,

$f(x) = x^3 + 9x^2 + 7x + 5$ [function is changed by course teacher.]

$x_l = -4$

$x_u = -3$

$$f(x_l)f(x_u) = \{(-4)^3 + 9(-4)^2 + 7(-4) + 5\}\{(-3)^3 + 9(-3)^2 + 7(-3) + 5\}$$

$$= (57)(38)$$

$$= 2166$$

Here,

$f(x_l)f(x_u)$ isn't less than 0. In bisection method ~~we~~ there must ~~be a root~~ exist a root in the bracket if $f(x_l)f(x_u) < 0$. But is this case $f(x_l)f(x_u) > 0$.

So we can conclude that there doesn't exists any root in the bracket [-4, -3]. So we can't calculate any error and iteration here.

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Answer to the Q.No.3(a)

The equation of relative approximation error is ~~$|\in_a| = |$ Present approxim~~

$$|\in_a| = \left| \frac{\text{Present approximation} - \text{Past approximation}}{\text{Present approximation}} \right|$$

Here we devide the approximate error by present approximation to scale the magnitude of the approximate error into a general scale. So the error will be calculated and we'll get result for all magnitude in the same scale. No matter if the magnitude is value $\times 10^{-6}$ or value $\times 10^{6}$ or value $\times 10^{4}$ all the errors will be scaled ~~down~~ to the same scale using relative approximation error.

Answer to the Q. No. 3(b)

Given,

$$2-x^2=\sin(x)$$

$$\therefore \cancel{2-x^2}\ \sin(x)+x^2-2=0$$

$$\therefore f(x)=\sin(x)+x^2-2$$

$$\therefore f'(x)=\cos x+2x$$

initial guess, $x_0=-1.5$

from newton's raphson,

$$x_{i+1}=x_i-\frac{f(x_i)}{f'(x_i)}$$

Iteration 1:

$$x_1=x_0-\frac{f(x_0)}{f'(x_0)}$$

$$=-1.5-\frac{f(-1.5)}{f'(-1.5)}$$

$$=-1.5-\frac{\sin(-1.5)+(1.5)^2-2}{\cos(-1.5)+2(-1.5)}$$

$$=-1.5-(-0.11189)=-1.3881$$

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(6)

(8)

$$|\epsilon_a| = \left|\frac{x_1 - x_0}{x_1}\right| \times 100\%$$

$$= \left|\frac{-1.3881 + 1.5}{-1.5}\right| \times 100\%$$

$$= 0.0746 \times 100\%$$

$$= 7.46\%$$

As it isn't less than 5%. So there's no significant digit.

iteration 2:

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$$= -1.3881 - \frac{f(-1.3881)}{f'(-1.3881)}$$

$$= -1.3881 - \frac{\sin(-1.3881) + (-1.3881)^2 - 2}{\cos(-1.3881) + 2(-1.3881)}$$

$$= -1.3881 - 0.0552$$

$$= -1.4434$$

$$|\varepsilon_a| = \left|\frac{x_2 - x_1}{x_2}\right| \cancel{\phi} \times 100\%$$

$$= 0.038312 \times 100\%$$

$$= \cancel{3.830} \; 3.8312\%$$

~~itera~~
As it is less than 5% than so there is 1 significant digit.

itemation 3 :

$$\cancel{\phi}\ \cancel{x_3}\ x_3 = x_2 - \frac{f(x_2)}{f'(x_2}$$

$$= -1.4434 - \frac{f(-1.4434)}{f'(-1.4434)}$$

$$= -1.4434 - \frac{\sin(-1.4434) + (-1.4434)^2 - 2}{\cos(-1.4434) + 2(-1.4434)}$$

$$= -1.4434 + 0.0313$$

$$= -1.4121$$

$$|\epsilon_a| = \left|\frac{x_3 - x_2}{x_3}\right| \times 100\%$$

$$= 0.021655 \times 100\%$$

$$= 2.1655\%$$

As it is less than 5% then there is 1 significan digit

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