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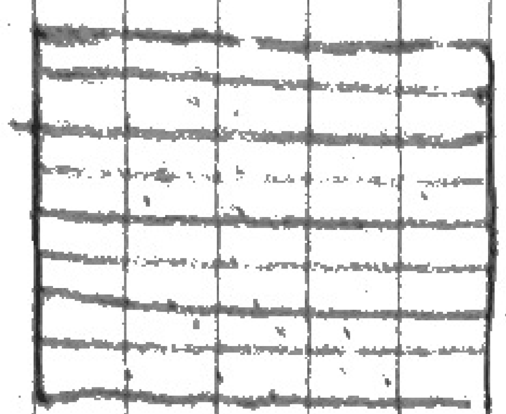
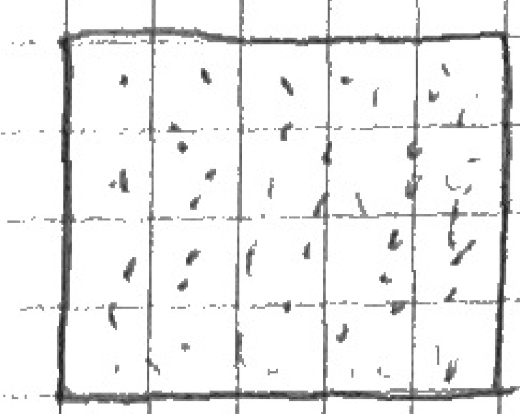
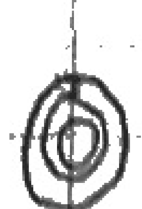
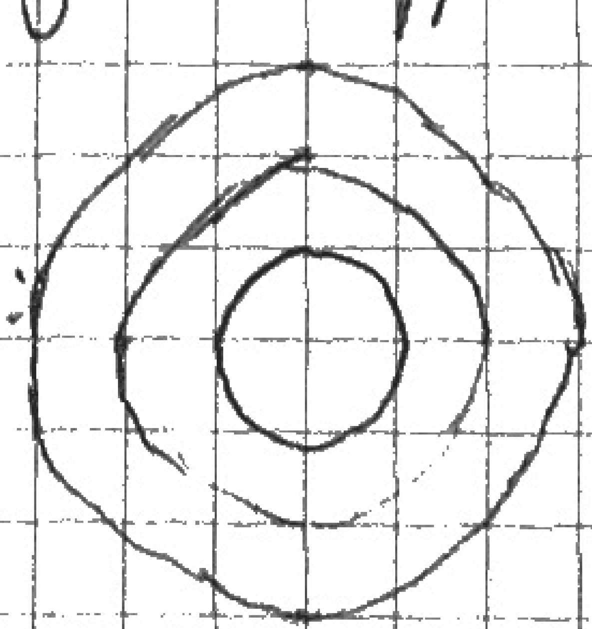
7. The two different type of images (1) & (2).

(1):  (2): 

a) The support of the Perona-Malik kernel is always symmetric since $\mu = \frac{1}{1+\frac{|\nabla f|^2}{\lambda^2}}$ is a scalar diffusion speed. Since there is a strong orientation in (1) the kernel for this image will have very small support, thus giving only little smoothing. In the case of (2) however no strong oriented structures exist thus the kernel will have large support giving much smoothing.

kernel_1:  kernel_2: 

Note that both kernels above are symmetric even though the sketches of them might not look that symmetric.

⇒ 2.

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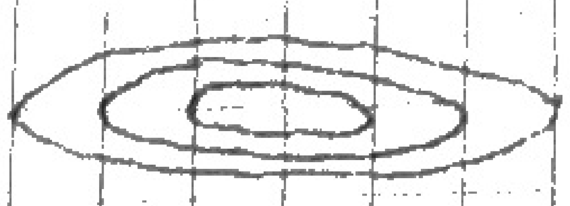
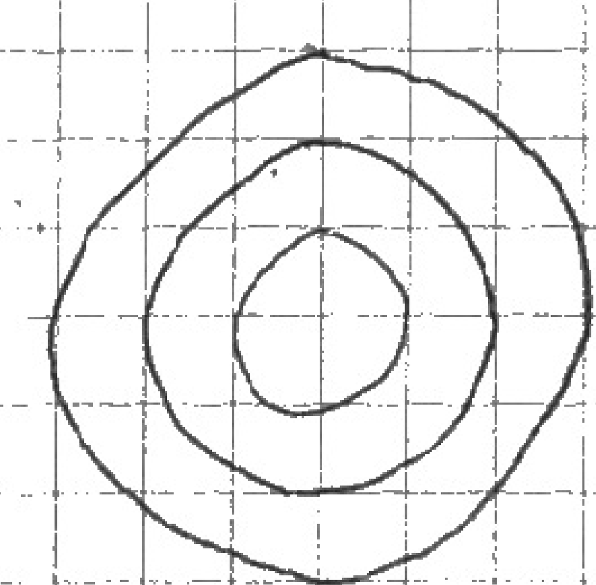
7. a) The resulting noise level will thus be practically the same for (1) but be greatly reduced for (2).

b) When using tensor driven diffusion the support of the kernel does not have to be symmetric. This will enable the filter to smooth along the hair in (1) but not across the hair. Thus the strong edges of the hair will be kept while still enabling noise to be removed. The kernel working on image (2) will be be about the same as in a) since there doesn't exist any structures to take into consideration.

kernel$_1$:  kernel$_2$: 

The filter will thus reduse the noise both in (1) & (2).

a bit less, though

good!

2p

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8. a) g_x can be formulated as the linear combination of an LP-filter and a number of HP-filters where the HP-filtering should be dependent on the local structure. The information about the local structure found in $c_k(\bar{x})$ is not specified but could for example contain the local structure tensor as $T(x) = \begin{pmatrix} c_1(x) & c_2(x) \\ c_2(x) & c_3(x) \end{pmatrix}$. We could the find

a filter g_x as below.

$g_x = g_{LP} + g_{HP}$ where $G_{HP}(u) = G_\rho(u)(\hat{e}^T\hat{u})^2$ (1)

where $G_\rho(u)$ is a radial function.

(1) can be expanded as

$$G_{HP}(u) = G_\rho(u) \langle \hat{e}\hat{e}^T | \hat{u}\hat{u}^T \rangle = G_\rho(u) \langle T(x) | \hat{u}\hat{u}^T \rangle =$$

$$= / \; T(x) = \sum_{i=1}^{3} \langle T(x) | \tilde{N}_i \rangle \tilde{N}_i \; / = \sum_{i=1}^{3} G_\rho(u) \langle T(x) | \tilde{N}_i \rangle \langle \tilde{N}_i | \hat{u}\hat{u}^T \rangle =$$

$$= \sum_{i=1}^{3} \langle T(x) | \tilde{N}_i \rangle \, G_\rho(u) \, (\hat{n}_i^T \hat{u})^2$$

$\Rightarrow$

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5a) The above gives that we can find g as

$$g = g_{LP} + \sum_{i=1}^{3} \langle T(x) | \hat{N}_i \rangle \, g_{HP,i}$$

The formulation above makes the computation simply being the case of convolving the LP- and HP-filters with the image and doing a linear combination of the HP filter using the pixel dependent coefficients of $\langle T(x) | \hat{N}_i \rangle$.

1

b) i) gives that

$$h = f * g_{LP}(x) + \sum_{i=1}^{K} \underbrace{\langle T(x) | \hat{N}_i \rangle}_{\text{pixel dependant scalar}} f * g_{HP,i}(x) \quad , K=3 \text{ for 2D-image}$$

1

$$T(x) = \begin{pmatrix} c_1(x) & c_2(x) \\ c_2(x) & c_3(x) \end{pmatrix}$$

2

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9

a) The L1-regularization term:

$L = |\nabla f| = \sqrt{f_x^2 + f_y^2}$ 1p

b) The Euler-Lagrange equation is given as:

$$L_f - \sum \partial x_w L_{f_{x_w}} = 0$$

No data term $\Rightarrow$ $L_f = 0$

$$L_{f_x} = \frac{\partial L}{\partial f_x} = \frac{1}{2\sqrt{f_x^2+f_y^2}} \cdot 2f_x = \frac{f_x}{\sqrt{f_x^2+f_y^2}} = \frac{f_x}{|\nabla f|}$$

$$L_{f_y} = \frac{\partial L}{\partial f_y} = \frac{1}{2\sqrt{f_x^2+f_y^2}} \cdot 2f_y = \frac{f_y}{\sqrt{f_x^2+f_y^2}} = \frac{f_y}{|\nabla f|}$$

$$\Rightarrow 0 - \left(\frac{\partial}{\partial x}\left(\frac{f_x}{|\nabla f|}\right) + \frac{\partial}{\partial y}\left(\frac{f_y}{|\nabla f|}\right)\right) = -\operatorname{div}\left(\frac{\nabla f}{|\nabla f|}\right) = 0$$ 1p

c) $\Rightarrow$

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9.c) The term in b) is problematic since (when $|\nabla f| = 0$) we risk division by zero. This can be handled by changing the resulting Euler-Lagrange equation to

$$-\operatorname{div}\left(\frac{\nabla f}{|\nabla f| + \varepsilon}\right) = 0$$ where ε is a very small constant.

1p

d) In the case of a 1D signal the equation from (b) will beome simply

$$-\left(\frac{\partial}{\partial x}\left(\frac{f_x}{|f_x|}\right)\right) = -\frac{\partial}{\partial x}\left(\operatorname{sign}(f_x)\right)$$

I.e. it will measure when the derivative changes. This will give a change in the image that tries to keep the sign of the derivative constant.

$$f^{(s+1)} = f^{(s)} + \alpha \frac{\partial}{\partial x}\left(\operatorname{sign}(f_x)\right)$$

yes, and this is a median filter

0.5p

3.5p

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10.

a) Probability outlier: p

Probability inlier: $(1-p)$

Probability that all 4 are inliers: $(1-p)^4$

1p

b) We first draw one point at random from A, which point we draw doesn't matter. We then draw one point from B. The probability that this point matches the already drawn point is $\frac{1}{100}$.

Answer: 0.01

1p

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11. a) The rotation invariance for the SIFT-descriptor is found by chosing the canonical frame from the strongest gradient/gradients at the feature. This is done by first calculating a histogram of gradients. The reference direction of the canonical frame is then chosen as the direction of the strongest gradient. In case of more than one very strong gradient it is also possible to create seperate descriptors aligned with the close-to-max gradients.

1p

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11. b) The affine illumination invariance is obtained by normalising the feature vector according to

$$\hat{h} = \frac{\bar{h}}{|\bar{h}|}.$$

(Further robustness can also be obtained by capping the elements of the feature vector at a certain threshold and then normalising again.)

The normalisation above handles when the illumination varies with a factor, i.e. $I_1 = k I_0$. However it doesn't take care of any offset. This has however already been taken care of since the feature vector is created from the gradients calculated at the feature (since taking the derivative removes the DC-level).

1p

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12 a) In the PnP-problem you have a set of known 3D-points and their projections in a camera (i.e. the projected positions in a camera). The unknown entities that we want to find is the rotation and translation of the camera in the same coordinate system as the 3D-points are given in.

Also to be noted is that the projected positions are given in normalised image coordinates. 1

b) The minimum n for the PnP problem is 3. 1

$\Rightarrow$

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12
c)

PnP can be used when a new view is added to the reconstruction. From the previous views we have already found 3D-points and their 3D-positions in some coordinate system. Hopefully some of these 3D points are projected into the new view. These correspondences are then found using for example SIFT. Thus we have 3D points and their projections in the camera (possibly we also have to transform the image coordinates to normalised image coordinates by multiplying them with K^{-1}, where K are the intrinsic camera parameters). PnP can now be used, using for example the previous camera as initial solution, to find the rotation and translation of the new camera.

The PnP problem differs from the bundle adjustment with regard to which parameters are optimised over. In the PnP case only the new camera, i.e. its rotation and translation, is optimised. In other words we fit the camera to the 3D points it can see. ⟹

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12c) In the bundle adjustment problem one tries to find the best fit of all cameras and 3D points (i.e. one allows both cameras and 3D points to change position) with regard to the given projections of the 3D points into the cameras.

2

4

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1a Let w_{kn} denote the partial membership for component k and sample n, $k=1,2$, $n=1,\dots,N$.

$$\tilde{w}_{kn} = p(I_n|T_k)\,p(T_k)$$

$$w_{kn} = \frac{\tilde{w}_{kn}}{\sum_{l=1}^{2} \tilde{w}_{ln}}$$

1p

b

$$p(T_k) = \frac{1}{N}\sum_n w_{kn}$$

$$\mu_k = \frac{1}{\sum_n w_{kn}} \sum_n w_{kn} I_n$$

$$\sigma_k = \frac{1}{\sum_n w_{kn}} \sum w_{kn} (I_n - \mu_k)^2$$

1p

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2 a and b denotes the <u>two images</u>, hence, ∇a is the gradient of image a. ∇a can be estimated using various <u>derivative filters</u> (1D or 2D), however, preferably a regularized derivative is estimated (using $\frac{-x}{\sigma^2} g$ and $\frac{-y}{\sigma^2} g$, where g is a Gaussian filter) but then $a * g$ and $b * g$ should be used instead.

w could be a box filter or a Gaussian filter that defines the region of interest and <u>how to weigh</u> the different points together.

2p

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3 $\varepsilon = \sum_x \| f_1(x) - f_2(x+d) \|^2$

A symmetric version is given by:

$$\varepsilon = \sum_x \| f_1(x - \tfrac{1}{2}d) - f_2(x + \tfrac{1}{2}d) \|^2$$ 1p

Using a first order Taylor expansion yields

$$\varepsilon \approx \sum_x \| f_1(x) - \frac{1}{2} d^T \nabla f_1(x) - f_2(x) - \frac{1}{2} d^T \nabla f_2(x) \|^2 =$$

$$= \sum_x \| f_1(x) - f_2(x) - \frac{1}{2} d^T (\nabla f_1(x) + \nabla f_2(x)) \|^2$$

Assume 1D

$\frac{d\varepsilon}{dd} = 0 \Rightarrow$

$$\sum_x (f_1'(x) + f_2'(x)) \cdot [f_1(x) - f_2(x) - \frac{1}{2} d (f_1'(x) + f_2'(x))] = 0$$

$$\Longleftrightarrow$$

$$\sum_x \frac{1}{2} d (f_1'(x) + f_2'(x))^2 = \sum_x (f_1'(x) + f_2'(x))(f_1(x) - f_2(x))$$

$$\Longleftrightarrow$$

$$d = 2 \cdot \frac{\sum_x (f_1'(x) + f_2'(x))(f_1(x) - f_2(x))}{\sum_x (f_1'(x) + f_2'(x))^2}$$ 3p

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4a This is done for computational reasons and for allowing larger motions to be found. A rough estimate is quickly found on a coarse level which is then refined on finer levels. 1p

b One should sample J in $\frac{x}{2} + 0.25$ and scale the result with 2 1p

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5 Either we let $\bar{v} = [v_1, v_2, 1]^T$ and try to minimize ε which gives $\underline{T_{2D}\bar{v} = \bar{e}}$ to solve or we let $\bar{v} = [r_1, r_2, r_3]$ with $\|\bar{v}\| = 1$ which gives

$T_{3D}\bar{v} = \lambda\bar{v}$ where the minimum of ε is given for $\bar{v}$ corresponding to the smallest eigenvalue of T_{3D}

2

The second approach is more computationally demanding since a 3D tensor is estimated instead of a 2D tensor and an eigenvalue decomposition of a 3x3 matrix instead of a 2x2 matrix inversion are performed.

However, the first approach can't handle a moving line/edge since T_{2D} will then be singular or close to. The second approach can in that case instead find the eigenvector corresponding to the largest eigenvalue and use this as the normal motion vector.

2

4

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6

6a $\varepsilon = \int \| f_1(x+d) - f_2(x) \|^2 dx + \beta \int [|\nabla d_1(x)|^2 + |\nabla d_2(x)|^2] dx$

The idea of the extra term is to ensure a smoothness constraint on d, i.e. assuming a smoothly varying d.

b This is not valid in the case of occlusion when there should be a sharp border in the motion field

1

1

2p