

The Note of Graph Theory

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摘要

I hate definition!

1 definition

simple graph:

- a non-empty finite set $V(G)$ of elements **vertices**
- a finite set $E(G)$ of distinct unordered pairs of distinct elements **edges**
- at most **one edge** joining a given pair of vertices

general graph:

- multiple edges
- loops

graph

- a non-empty finite set $V(G)$ of elements **vertices**
- a finite family $E(G)$ of distinct unordered pairs of not necessarily distinct elements **edges**

- the use of **family** permits the existence of multiple edge
- vertex-set edge-family

$\{v, w\}$ is said to join the vertices v , w and is again abbreviated to vw .

Example : vv

isomorphism

- one-one correspondence between the vertices of G_1 and those of G_2 such that the number of edges joining any two vertices of G_1 equals the number of edges joining the corresponding vertices of G_2
- the **unlabelled graphs** and **labelled graphs** is different

connected graphs

- A graph is connected if it cannot be expressed as a union of graphs and disconnected otherwise.

Adjacency and degrees

- the v, w is **adjacency** if there is an edge vw joining them. and v, w are incident with such edge
- degree of a vertex is the number of edges incident with itself.
- A vertex of degree 0 is an isolated vertex
- A vertex of degree 1 is an end vertex

The **degree sequence** of a graph consists of degrees written in **increasing order**

Handshaking lemma In any graph the sum of all the vertex-degrees is an even number.

Corollary In any graph the number of vertices of odd degree is even.

Subgraphs

- A graph is a **subgraph** of graph G if each of its vertices belongs to $V(G)$ and each of its edges belongs to $E(G)$
- we can obtain subgraphs by deleting edges and vertices.
- $G - F$