UNIVERSITÀ DEGLI STUDI DI PADOVA
DIPARTIMENTO DI INGEGNERIA DELL'INFORMAZIONE
IAS-Lab
Intelligent Autonomous Systems Laboratory

3D Data Processing

Lab 1: Semi-Global Matching Example

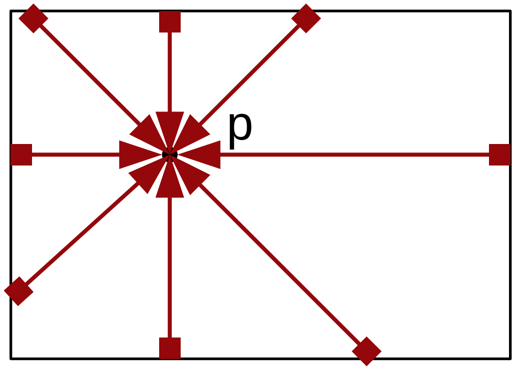
Semi-Global Matching

- Approximate global methods by aggregating costs for a number of directions (from 2D to multiple 1D areas of interest)
- Minimization along individual image rows can be performed efficiently in polynomial time using Dynamic Programming

H Hirschmuller "Stereo processing by semiglobal matching and mutual information", Pattern Analysis and Machine Intelligence, IEEE Transactions on 30 (2), 328-341

Semi-Global Matching

- For each direction, start from one end point and go toward **p**
- For each pixel along the direction, update the following dynamic programming equation (the result at step i depends on the result at step i-1):



$$E(p_i, d) = E_{data}(p_i, d) + E_{smooth}(p_i, p_{i-1}) - \min_{0 \le \Delta \le d_{\max}} E(p_{i-1}, \Delta)$$

Restrict the range of resulting values, without affecting the minimization procedure

where:

$$E_{smooth}(p,q) = \min \begin{cases} E(q, f_q) & \text{if } f_p = f_q \\ E(q, f_q) + c_1 & \text{if } |f_p - f_q| = 1 \\ \min_{0 \le \Delta \le d_{\max}} E(q, \Delta) + c_2(p,q) & \text{if } |f_p - f_q| > 1 \end{cases}$$

Semi-Global Matching

Consider the stereo matching problem for the following 7 x 1 left and right images:

2	3	1	2	3	3	1

1	2	3	1	4	0	2

1) Compute the right to left **cost volume**, considering positive disparities, d >= 0 with d_{max} = 3 and data cost defined by the sum of absolute differences (SAD) computed in 1 x 1 windows. Use value -1 to set "no disparity assigned"
2) Given the matching cost computed in 1), consider the Semi-Global Matching method with the following simplified dynamic programming equations:

$$E(p_i, d) = E_{data}(p_i, d) + E_{smooth}(p_i, p_{i-1})$$

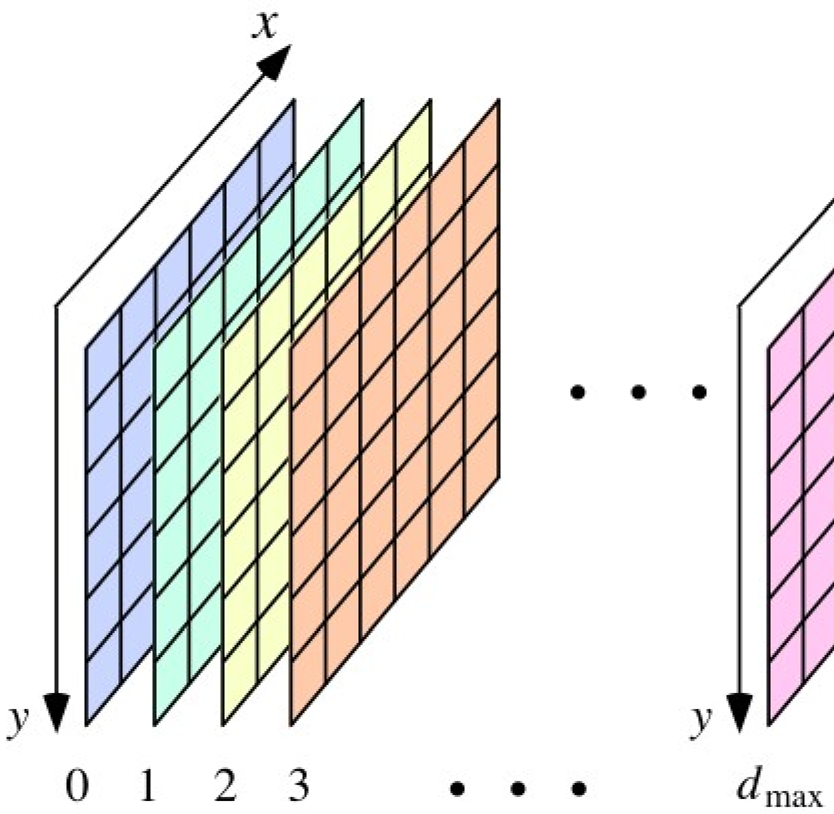
$$E_{smooth}(p, q) = \min \begin{cases} E(q, f_q) & \text{if } f_p = f_q \\ E(q, f_q) + c_1 & \text{if } |f_p - f_q| = 1 \\ \min_{0 \le \Delta \le d_{\max}} E(q, \Delta) + c_2 & \text{if } |f_p - f_q| > 1 \end{cases}$$

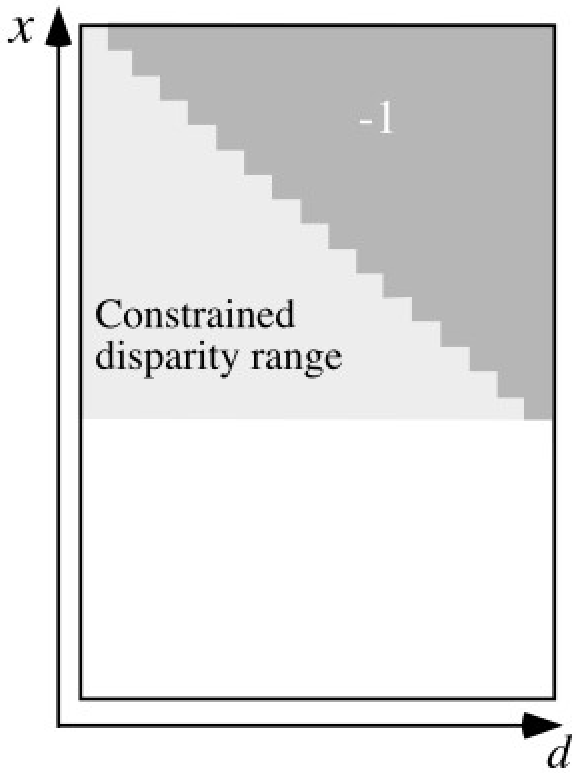
where c_1 = 1 and c_2 = 2. Compute the integration matrix for the highlighted pixel and scanline:

2	3	1	2	3	3	1

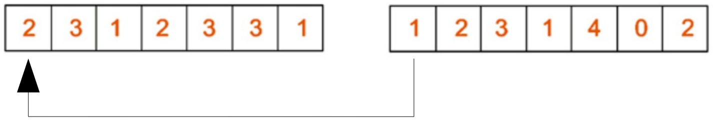
1	2	3	$\boxed{1}$	4	0	2

Cost Volume





Compute the Cost Volume



0	1	1	2	1	1	3	1
1	2	1	1	2	1	1	-1
2	0	0	0	2	3	-1	-1
3	1	1	0	0	-1	-1	-1

right to left

Compute Costs for One Path

2	3	1	2	3	3	1

1	2	3	1	4	0	2

$$E(p_i, d) = E_{data}(p_i, d) + E_{smooth}(p_i, p_{i-1})$$

$$E_{smooth}(p,q) = \min \begin{cases} \boxed{E(q, f_q)} & \text{if } f_p = f_q \\ \boxed{E(q, f_q) + c_1} & \text{if } |f_p - f_q| = 1 \\ \boxed{\min_{0 \le \Delta \le d_{\max}} E(q, \Delta) + c_2} & \text{if } |f_p - f_q| > 1 \end{cases}$$

$c_1 = 1$

$c_2 = 2$

0				
1				
2				
3				

Compute Costs for One Path

2	3	1	2	3	3	1

1	2	3	1	4	0	2

$$E(p_i, d) = E_{data}(p_i, d) + \cancel{E_{smooth}(p_i, p_{i-1})}$$

$$E_{smooth}(p,q) = \min \begin{cases} \boxed{E(q, f_q)} & \text{if } f_p = f_q \\ \boxed{E(q, f_q) + c_1} & \text{if } |f_p - f_q| = 1 \\ \boxed{\min_{0 \le \Delta \le d_{\max}} E(q, \Delta) + c_2} & \text{if } |f_p - f_q| > 1 \end{cases}$$

$c_1 = 1$

$c_2 = 2$

0	1			
1	2			
2	0			
3	1			

Compute Costs for One Path

2	3	1	2	3	3	1

1	2	3	1	4	0	2

$$E(p_i, d) = E_{data}(p_i, d) + E_{smooth}(p_i, p_{i-1})$$

$$E_{smooth}(p,q) = \min \begin{cases} \boxed{E(q, f_q)} & \text{if } f_p = f_q \\ \boxed{E(q, f_q) + c_1} & \text{if } |f_p - f_q| = 1 \\ \boxed{\min_{0 \le \Delta \le d_{\max}} E(q, \Delta) + c_2} & \text{if } |f_p - f_q| > 1 \end{cases}$$

c_1 = 1
c_2 = 2

0	1			
1	2			
2	0			
3	1			

1 + min(1, 2+1, 0+2)=2

1 + min(2, 1+1, 0+1, 0+2)=2

0 + min(0, 2+1, 1+1, 0+2)=0

1 + min(1, 0+1, 0+2)=2

Compute Costs for One Path

2	3	1	2	3	3	1

1	2	3	$\boxed{1}$	4	0	2

$$E(p_i, d) = E_{data}(p_i, d) + E_{smooth}(p_i, p_{i-1})$$

$$E_{smooth}(p,q) = \min \begin{cases} \boxed{E(q, f_q)} & \text{if } f_p = f_q \\ \boxed{E(q, f_q) + c_1} & \text{if } |f_p - f_q| = 1 \\ \boxed{\min_{0 \le \Delta \le d_{\max}} E(q, \Delta) + c_2} & \text{if } |f_p - f_q| > 1 \end{cases}$$

$\Rightarrow$ $c_1 = 1$, $c_2 = 2$

0	1	2		
1	2	2		
2	0	0		
3	1	2		

2 + min(2, 2+1, 0+2)=4

1 + min(2, 2+1, 0+1, 0+2)=2

0 + min(0, 2+1, 2+1, 0+2)=0

0 + min(2, 0+1, 0+2)=1

Compute Costs for One Path

2	3	1	2	3	3	1

1	2	3	$\boxed{1}$	4	0	2

$$E(p_i, d) = E_{data}(p_i, d) + E_{smooth}(p_i, p_{i-1})$$

$$E_{smooth}(p,q) = \min \begin{cases} \boxed{E(q, f_q)} & \text{if } f_p = f_q \\ \boxed{E(q, f_q) + c_1} & \text{if } |f_p - f_q| = 1 \\ \boxed{\min_{0 \le \Delta \le d_{\max}} E(q, \Delta) + c_2} & \text{if } |f_p - f_q| > 1 \end{cases}$$

c_1 = 1
c_2 = 2

0	1	2	4	
1	2	2	2	
2	0	0	0	
3	1	2	1	

1 + min(4, 2+1, 0+2)=3

2 + min(2, 4+1, 0+1, 0+2)=3

2 + min(0, 2+1, 1+1, 0+2)=2

0 + min(1, 0+1, 0+2)=1

Compute Costs for One Path

2	3	1	2	3	3	1

1	2	3	1	4	0	2

$$E(p_i, d) = E_{data}(p_i, d) + E_{smooth}(p_i, p_{i-1})$$

$$E_{smooth}(p,q) = \min \begin{cases} \boxed{E(q, f_q)} & \text{if } f_p = f_q \\ \boxed{E(q, f_q) + c_1} & \text{if } |f_p - f_q| = 1 \\ \boxed{\min_{0 \le \Delta \le d_{\max}} E(q, \Delta) + c_2} & \text{if } |f_p - f_q| > 1 \end{cases}$$

$c_1 = 1$

$c_2 = 2$

0	1	2	4	3
1	2	2	2	3
2	0	0	0	2
3	1	2	1	1