Verification of current-state opacity in discrete event systems using an observer net

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Basics of Petri nets

1 Petri nets

A Petri net is a four-tuple $N = (P, T, Pre, Post)$, where P is a set of m places, graphically represented by circles, T is a set of n transitions, graphically represented by bars, $Pre : P \times T \to \mathbb{N}$ [1] and $Post : P \times T \to \mathbb{N}$ specify the arcs directed from places to transitions, and transitions to places, respectively. Pre and $Post$ can be represented by $m \times n$ matrices. The incidence matrix of the net N is accordingly defined by $C = Post - Pre$.

The input and output sets of a node $x \in P \cup T$, denoted by ${}^\bullet x$ and $x^\bullet$, respectively, are defined as ${}^\bullet x = \{y \in P \cup T \mid Pre(x, y) > 0\}$ and $x^\bullet = \{y \in P \cup T \mid Post(x, y) > 0\}$. A Petri net is said to be acyclic if there are no oriented cycles.

A marking is a mapping $M : P \to \mathbb{N}$ that assigns to each place a non-negative integer number of tokens, graphically represented by black dots. The marking of place p at a M is denoted by $M(p)$. A marking M can be also denoted as $M = \sum\limits_{p \in P} M(p) \cdot p$. A Petri net system $\langle N, M_0 \rangle$ is a net N with *initial marking* M_0.

A transition t is said to be *enabled* at a marking M if $M \geq Pre(\cdot, t)$ and its firing yields a marking $M' = M + C(\cdot, t)$. We write $M[\sigma\rangle$ to denote that a sequence of transitions $\sigma = t_{j1} \ldots t_{jk} \in T^*$, $k \in \mathbb{N}$, is enabled at M, and $M[\sigma\rangle M'$ to denote that firing the sequence σ yields M'. Given a sequence $\sigma \in T^*$, the function $\pi : T^* \to \mathbb{N}^n$ associates with σ the Parikh vector $y = \pi(\sigma) \in \mathbb{N}^n$, i.e., $y(t) = k$ if transition t appears k times in σ.

A marking M is said to be *reachable* in $\langle N, M_0 \rangle$ if there exists a sequence σ such that $M[\sigma\rangle M'$. The set of all markings reachable from M_0 defines the reachability set of $\langle N, M_0 \rangle$, denoted by $R(N, M_0)$, i.e., $R(N, M_0) = \{M \in \mathbb{N}^m \mid \exists \sigma \in T^* : \ M_0[\sigma\rangle M\}$. A Petri net is *bounded* if there exists a non-negative integer $k \in \mathbb{N}$ such that for any place $p \in P$ and any reachable marking $M \in R(N, M_0)$, $M(p) \leq k$ holds.

[1] In this work. we use $\mathbb{N}$, $\mathbb{Z}$ and $\mathbb{R}$ to denote the sets of non-negative integers, integers, and real number, respectively.

2 Labeled Petri nets

A labeled Petri net is a four-tuple $G = (N, M_0, E, \ell)$, where $\langle N, M_0 \rangle$ is a PN system, E is the alphabet (a set of labels) and $\ell : T \to E \cup \{\varepsilon\}$ is a labeling function that assigns to each transition $t \in T$ either a symbol from E or the empty word ε. The transition set T is partitioned into two disjoint sets $T = T_o \dot{\cup} T_{uo}$, where $T_o = \{t \in T | \ell(t) \in E\}$ is the set of observable transitions and $T_{uo} = T \setminus T_o = \{t \in T | \ell(t) = \varepsilon\}$ is the set of unobservable transitions. The labeling function can be extended to firing sequences $\ell : T^* \to E^*$, i.e., $\ell(\sigma t) = \ell(\sigma)\ell(t)$ with $\sigma \in T^*$ and $t \in T$.

Given a labeled net system $G = (N, M_0, E, \ell)$ and a marking $M \in R(N, M_0)$, we define the language generated from M as

$$\mathcal{L}(N, M) = \{w \in E^* | \exists \sigma \in T^* : M[\sigma\rangle \text{ and } \ell(\sigma) = w\}$$

Let $G = (N, M_0, E, \ell)$ be an LPN. A string belonging to $\mathcal{L}(N, M_0)$ is called an observation of G, and denoted by w. We define the set of markings consistent with w as

$$\mathcal{C}(w) = \{M \in \mathbb{N}^m | \exists \sigma \in T^* : M_0[\sigma\rangle M \text{ and } \ell(\sigma) = w\}$$

Definition. *Given an LPN $G = (N, M_0, E, \ell)$ and a marking $M \in R(N, M_0)$, the unobservable reach of M is defined as $\mathcal{U}(M) = \{M' \in \mathbb{N}^m | \exists \sigma_u \in T_{uo}^* : M[\sigma_u\rangle M'\}$.*

In simple words, the unobservable reach of a marking M is the set of markings reachable from M by firing only unobservable transitions.

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