A Chattering-free Sliding Mode Controller for Autonomous Vehicle under Gaussian Uncertainties

Abstract

This work focuses on designing a novel controller with a view to addressing the contemporary control issues of autonomous vehicles under gaussian uncertainties. The main objectives are to attain the exponential stability, address the tracking inaccuracy as well as minimizing the chattering effect on the control actions. The bicycle model has been modified by adding input noises and model errors. Then sliding control technique has been employed to derive control laws. The controller has been modified to address the chattering effect using saturation functions. Simulation results show that, the autonomous vehicle could track the trajectory and ensures the stability. As future works, the work is subjected to update in terms of accuracy in tracking and integration of optimal control tool with sliding mode controller.

1. Introduction

With the extensive development of technologies, the terms Autonomous Vehicles, Autonomous Driving, Self-Driving Cars are obtaining huge popularity day by day. At the same time, due to the increasing traffic in the roads the unwanted road occurrences are also viably significant. According to World Health Organization road safety report, every year approximately 1.35 million people died on road-crash around the world [1]. The top causes of the road accidents are reported to be the fault of the drivers of different age groups [2]. Therefore, the autonomous driving capability is potential solution to ensure driving safety in different road scenario at the different levels. At present, a number of companies like Tesla and Google are working on autonomy of driving and still now have claimed to achieve level 3 and level 4 automation although none of them have attained the level of absolute autonomy so far.

Many researchers are continuously working on the control system of the autonomous vehicle. In the recent years, the research thrust is on the steering control. [3] presents the combination of Lyapunov and gain scheduling method incorporating the dynamics of steering system in controller design for the first time. [4] claims to ensure the steering stability and minimizing the error by integrating backstepping variable structure control (BVSC) and artificial neural network approximator. In the [5], the authors have combined adaptive control and neural network. [6] proposes such a method where computational complexity of virtual control input is addressed using command filtering adaptive control considering the disturbances. [7-10] presents the combination of backstepping control and sliding mode control strategy which claim to improve the steering actions. Besides, with the PSO technique, [7] has optimized the control parameters. The backstepping controller of [9] is more of estimation-based system. The control of [10] is super twisting sliding mode control based on backstepping approach. [11] has introduced cascade backstepping controller having outer and inner loop where in each loop there are backstepping controller and augmented observer. In the [12], the authors have proposed a barrier function-based sliding mode control specific to two timescale nonlinear system.

In the literature, both qualitatively and quantitatively large part of works are seen to be covered in the field of steering control of autonomous vehicle. Though many researchers have claimed to handle lateral stabilization as well as minimization of deviation, much works are still to be done. None have yet claimed to ensure stability, performance and noise reduction at the same time. Therefore, the contribution of this paper is to provide the longitudinal and steering control with stability, performance as well as enhanced noise-reduction capability using the sliding mode controller.

2. Problem Formulation

2.1 Kinematic Equation

In this section, we model the autonomous vehicle as car-like bicycle mechanism. That means, the vehicle kinematic equations are derived based on the consideration that two-wheel mechanism represents the entire vehicle. In fig. 1, {O}is a base frame or world frame which is fixed to the scenario. {B} is body frame which moves with the motion of the vehicle. L denotes the length of the wheel base which is actually the distance between the front and the rear wheel. The notation v is the linear velocity of the vehicle whereas ω denotes the angular velocity. The configuration of the vehicle is $p = (x, y, \theta)$ where (x, y) is the position and θ is the orientation or heading with respect to the frame {O}. The steering angle and the Instantaneous Center of Rotation are specified by φ and ICR respectively. The distance from center of frame {B} to ICR can be denoted by the notation r.

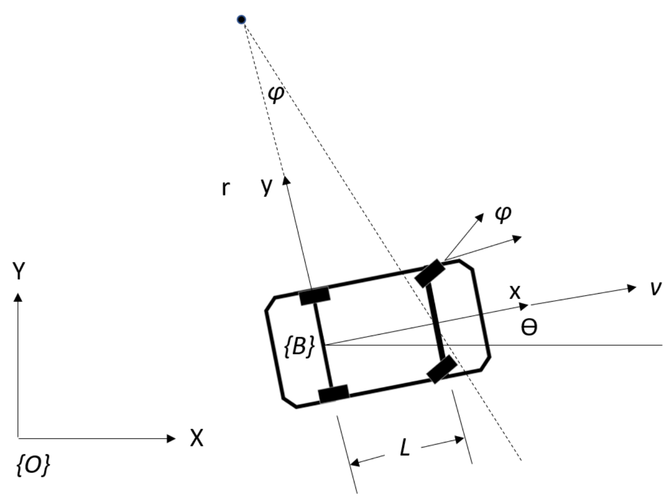

Fig. 1: Schematic Kinematic Diagram of Autonomous Vehicle

Thus, the kinematic equation can be derived as,

$$\dot{p} = \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\Theta} \end{bmatrix} = \begin{bmatrix} v\cos\theta \\ v\sin\theta \\ \omega \end{bmatrix} \tag{1}$$

Using the relation of linear and angular velocity in addition to basic trigonometry, the angular velocity can be given by,

$$\omega = \frac{v}{L} \tan \varphi \tag{2}$$

2.2 Uncertainties in Equation

The uncertainties in a dynamic model are caused by a number of factors. Many dynamic system inputs are susceptible to the external disturbances caused by environmental noise. Sensor noises are also common phenomenon while detecting the state variables. Finally, while mapping the real systems to system of differential equations, assumption in the process of modelling also cause errors which raises uncertainties in the systems. Considering all of the above conditions, introducing disturbances in the autonomous vehicle model is done in this work. Suppose, g_v and g_ω are denoted as input disturbances for the inputs v and ω respectively. The modelling errors can be specified as g_x, g_y and g_θ corresponding to the configuration of the vehicle. The input disturbances and errors can be modelled as normally distributed random variable as follows,

$$g_*(*) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}(\frac{*-\mu_*}{\sigma_*})^2} \tag{3}$$

In equation (3), the notation * is replaced by one of $(x, y, \theta, v, \omega)$ as per the usage in the equations. The notation σ and μ are standard deviation and mean of the distribution for the corresponding disturbances. Considering the uncertainties, the equation (1) now becomes,

$$\dot{p} = \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\Theta} \end{bmatrix} = \begin{bmatrix} (v + g_v) \ \cos(\theta + g_\theta) + g_x \\ (v + g_v) \sin(\theta + g_\theta) + g_y \\ (\omega + \ g_\omega) + \ g_\theta \end{bmatrix} \tag{4}$$

2.3 Error Kinematic Equation

The desired configuration of the vehicle can be given by the following equation. Where, v_d and ω_d are desired velocity and steering angle respectively.

$$\dot{p}_d = \begin{bmatrix} \dot{x}_d \\ \dot{y}_d \\ \dot{\Theta}_d \end{bmatrix} = \begin{bmatrix} v_d \cos \theta_d \\ v_d \sin \theta_d \\ \omega_d \end{bmatrix} \tag{5}$$

So, the error configuration in {O} frame is given by,

$$\begin{bmatrix} e_x \\ e_y \\ e_\theta \end{bmatrix} = \begin{bmatrix} x_d - x \\ y_d - y \\ \theta_d - \theta \end{bmatrix} \tag{6}$$

Transforming the error configuration in the body frame {B},

$$e_p = \begin{bmatrix} e_x \\ e_y \\ e_\theta \end{bmatrix} == \begin{bmatrix} (x_d - x)\cos\theta + (y_d - y)\sin\theta \\ -(x_d - x)\sin\theta + (y_d - y)\cos\theta \\ \theta_d - \theta \end{bmatrix} \tag{7}$$

Now, differentiating the equation (7) and using the equation (4) the kinematic error equations with uncertainties are obtained as,

$$\dot{e}_x = e_y\omega - v\cos g_\theta + v_d\cos e_\theta - g_v\cos g_\theta - g_x\cos\theta - g_y\sin\theta \tag{8}$$

$$\dot{e}_y = -e_x\omega + v_d\sin\theta_e + v\sin g_\theta + g_v\sin g_\theta + g_x\sin\theta - g_y\cos\theta \tag{9}$$

$$\dot{e}_\theta = \omega_d - \omega - g_\omega + g_\theta \tag{10}$$

If the uncertainties corresponding to $e_x, e_y, and\ e_\theta$ are $\eta_{ex}, \eta_{ey}\ and\ \eta_{e\theta}$, they can be written as,

$$\eta_{ex} = -g_v\cos g_\theta - g_x\cos\theta - g_y\sin\theta \tag{11}$$

$$\eta_{ey} = g_v\sin g_\theta + g_x\sin\theta - g_y\cos\theta \tag{12}$$

$$\eta_{e\theta} = -g_\omega + g_\theta \tag{13}$$

3 Controller Design

In this section, a stable sliding mode controller (SMC) has been designed that takes kinematic errors and (v_d, ω_d) as inputs and provides (v, ω) as output which is actually input to the plant to be controlled. Against the three state-error, there are two inputs. However, the inputs of system are coupled in the differential equation. Hence, the surfaces can be formulized in such a way that, one surface would incorporate two state-error. In this case, e_y and e_θ couples in the same sliding surface. Therefore, the sliding surfaces can be given by,

$$s_1 = \dot{e}_x + k_1 e_x \tag{15}$$

$$s_2 = \dot{e}_y + k_2 e_y + k_3 e_\theta \tag{16}$$

where k_1, k_2 and k_3 are positive values. The validity of the equations (15) and (16) are proved in two steps. First, if $s_1, s_2 \to 0$, the control objectives should be achieved. The control objective of the problem is to keep the errors to zero.

$$(e_x, e_y, e_\theta) \to (0,0,0)$$

So, when $s_1 \to 0$, $=>$ $\dot{e}_x + k_1 e_x = 0, => e_x \to 0$. When $s_2 \to 0, => \dot{e}_y + k_2 e_y + k_3 e_\theta = 0$. Before solving the linear differential equation, we suppose $e_y = \alpha, e_\theta = \beta$, to avoid contradiction between exponential function and error variables. So, the ODE now stands as, $\dot{\alpha} + k_2\alpha + k_3\beta = 0$. Solving the equation implies, $\alpha(t) = e^{-k_2 t}(\int e^{k_2 t}(-k_3\,\beta(t)dt + c),\ =>\ \alpha(t) = e^{-k_2 t}(-k_3\left(\frac{\beta(t)e^{k_2 t}}{k_2} - \int \dot{\beta}(t)\frac{e^{k_2 t}}{k_2}\right) + c,\ =>\ \alpha(t) = ce^{-k_2 t} - \frac{k_3}{k_2}\beta(t) + \frac{k_3}{k_2}\int \dot{\beta}(t)\,dt, =>$ $\alpha(t) = ce^{-k_2 t}, =>\ e_y \to 0$, putting the value of e_y in $\dot{e}_y + k_2 e_y + k_3 e_\theta = 0$ it can be also shown that, $e_\theta \to 0$. Therefore, the control objective is shown to be achieved. Second, in the derivation of the sliding surfaces, input variables are should be present, which can be understood by observation of the equation (15) and (16). Therefore, the sliding surfaces are valid for this problem. Now, the derivatives of sliding surfaces are given by,

$$\dot{s}_1 = \ddot{e}_x + k_1\dot{e}_x \tag{17}$$

$$\dot{s}_2 = \ddot{e}_y + k_2\dot{e}_y + k_3\dot{e}_\theta \tag{18}$$

To demonstrate asymptotic stability, a Lyapunov candidate function is defined.

$$\mathrm{V} = \frac{1}{2}s^T s \tag{19}$$

Differentiating the equation (19)

$$\dot{\mathrm{V}} = \mathrm{s}_1\dot{s}_1 + \mathrm{s}_2\dot{s}_2 \tag{20}$$

If $\dot{\mathrm{V}}$ is negative definite, the system will be asymptotically stable. In others words, $\dot{\mathrm{V}} = -Pssign(s)$ in order to bring the system asymptotically stable because the term $-Pssign(s)$ itself is negative definite as P>0. Using the equations (17) and (18) in the equation (20),

$$\begin{aligned}\dot{\mathrm{V}} \quad &= \mathrm{s}_1(\ddot{e}_x + k_1(e_y\omega - v\cos g_\theta + v_d\cos e_\theta + \eta_{ex})) + \mathrm{s}_2(\ddot{e}_y + k_2(-e_x\omega + v_d\sin\theta_e \\ &\qquad + v\sin g_\theta + \eta_{ey}) + k_3(\omega_d - \omega + \eta_{e\theta})) \\ &= \mathrm{s}_1(\ddot{e}_x + k_1e_y\omega - k_1v\cos g_\theta + k_1v_d\cos e_\theta + k_1\eta_{ex}) + \mathrm{s}_2(\ddot{e}_y - k_2e_x\omega \\ &\qquad + k_2v_d\sin e_\theta + k_2\, v\sin g_\theta + k_2\eta_{ey} + k_3\omega_d - k_3\omega + k_3\eta_{e\theta})\end{aligned}$$

Choosing the control laws in the following way we get,

$$v = \frac{\ddot{e}_x + k_1\eta_{ex} + k_1v_d\cos e_\theta + k_1e_y\omega + Psign(\mathrm{s}_1)}{k_1\cos g_\theta} \tag{21}$$

$$\omega = \frac{\ddot{e}_y + k_2v_d\sin e_\theta + k_2\, v\sin g_\theta + k_2\eta_{ey} + k_3\omega_d + k_3\eta_{e\theta} + Qsign(\mathrm{s}_2)}{k_2e_x + k_3} \tag{22}$$

Putting the values of equation (21-22) in $\dot{\mathrm{V}}$, we get,

$$\dot{\mathrm{V}} = -P\mathrm{s}_1 sign(\mathrm{s}_1) - Q\mathrm{s}_2 sign(\mathrm{s}_2) \tag{23}$$

In equation (23), the $\dot{\mathrm{V}}$ is Hence, the controller is asymptotically stable. The (v, φ) are given by,

$$v = \frac{\ddot{e}_x + k_1\eta_{ex} + k_1v_d\cos e_\theta + k_1e_y\omega + Psign(\mathrm{s}_1)}{k_1\cos g_\theta} \tag{24}$$

$$\varphi = \tan^{-1}\frac{L}{v}\left(\frac{\ddot{e}_y + k_2v_d\sin e_\theta + k_2\, v\sin g_\theta + k_2\eta_{ey} + k_3\omega_d + k_3\eta_{e\theta} + Qsign(\mathrm{s}_2)}{k_2e_x + k_3}\right) \tag{25}$$

One of the challenges of sliding mode control is the chattering effect which eventually can be addressed replacing the $sign(*)$ function by $sat(*)$ function in the control laws. Hence the control laws are given by,

$$v = \frac{\ddot{e}_x + k_1\eta_{ex} + k_1v_d\cos e_\theta + k_1e_y\omega + Psat(\mathrm{s}_1)}{k_1\cos g_\theta} \tag{26}$$

$$\varphi = \tan^{-1}\frac{L}{v}\left(\frac{\ddot{e}_y + k_2 v_d \sin e_\theta + k_2\, v\sin g_\theta + k_2\eta_{ey} + k_3\omega_d + k_3\eta_{e\theta} + Q sat(\mathrm{s}_2)}{k_2 e_x + k_3}\right) \tag{27}$$

Where,

$$\mathrm{sat(s)} = \begin{Bmatrix} sign(s), & |s| > Psi \\ \dfrac{s}{Psi}, & |s| \le Psi \end{Bmatrix} \tag{28}$$

4 Simulation Result

In this section we have shown the result of the simulations of trajectory tracking control based on the design undertaken in the previous section. The whole simulation procedure can be separated into three main steps: initialization, solving nonlinear equations and plotting the results. Initialization steps can further be separated into trajectory initialization, model parameters (model and controller) initialization and randomly initialize the initial conditions. The nonlinear equations of autonomous vehicles then solved using Euler method. Finally, the results are plotted against time and space. The following algorithms shows the procedure of simulation.

Algorithm. Trajectory Tracking

```
Initialization
1. Initialize the trajectories (vd,deltad)
2. Assign values to parameters L, k1,k2,k3,P,Q,h
3. Initialize x(1), y(1),theta(1),
   xd(1),yd(1),thetad(1), ex(1),ey(1),etheta(1),
   D2_ex(1), D2_ey(1),
   D2_etheta(1),s1(1),s2(1),gx(1),gy(1),gtheta(1),
   gv(1),gdelta(1)
4. N <- size of vd
5. M <- N
Euler Method
6.   for i = 1 to M do
7.       Update xd(i+1), yd(i+1) and thetad(i+1)
8.       Compute gx(i),gy(i),gtheta(i), gv(i),gdelta(i)
 9.      compute D2_ex(i), D2_ey(i)
10.      compute s_dot(i),s_dot(i)
11.      Compute and update v(i) and delta(i)
12.      Solve x(i+1),y(i+1) and theta(i+1)
13.      Update ex(i+1),ey(i+1) and etheta(i+1)
14.      update s1(i+1),s2(i+1)
     end for
  Plot
15.  Plot desired and real states (y vs x and yd vs xd)
16.  Plot error state values
17.  Plot the control inputs
```

Here, *D2_ex(i), D2_ey(i)* are the second derivative of e_x *and* e_y. The plots of the solutions are given below. Fig. 2 illustrates how the autonomous vehicle can track the desired path obtained from the desired states.

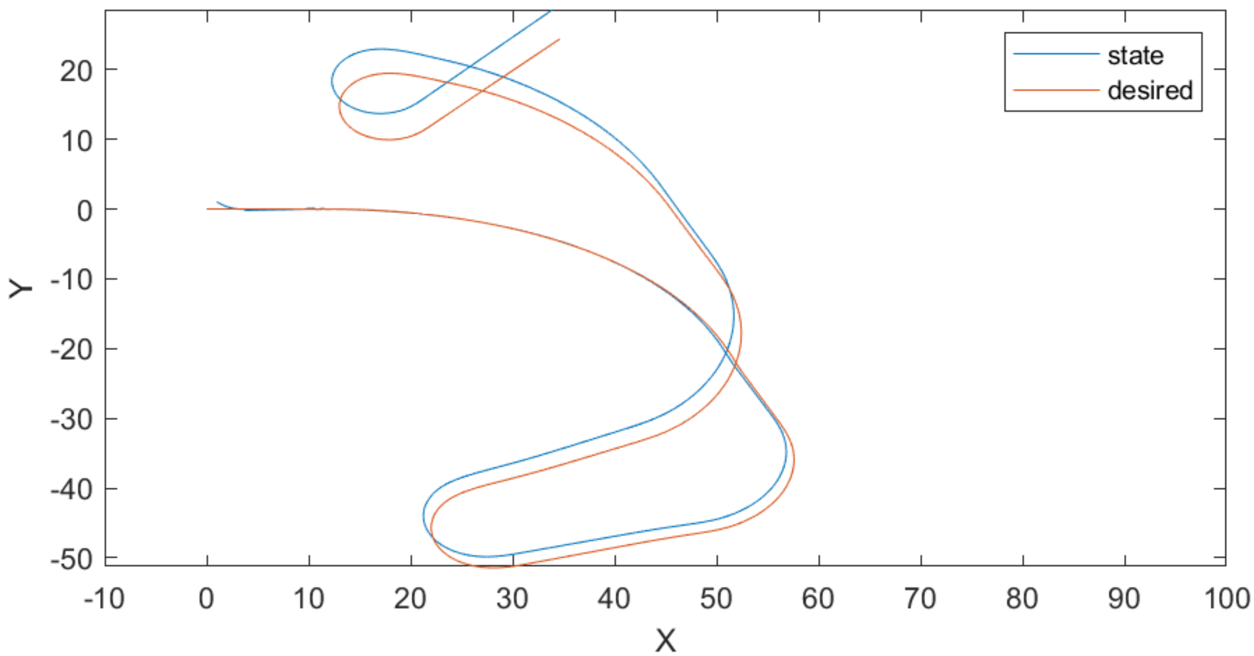

Fig. 2: X-Y plane of the Car-like Vehicle for desired and actual state

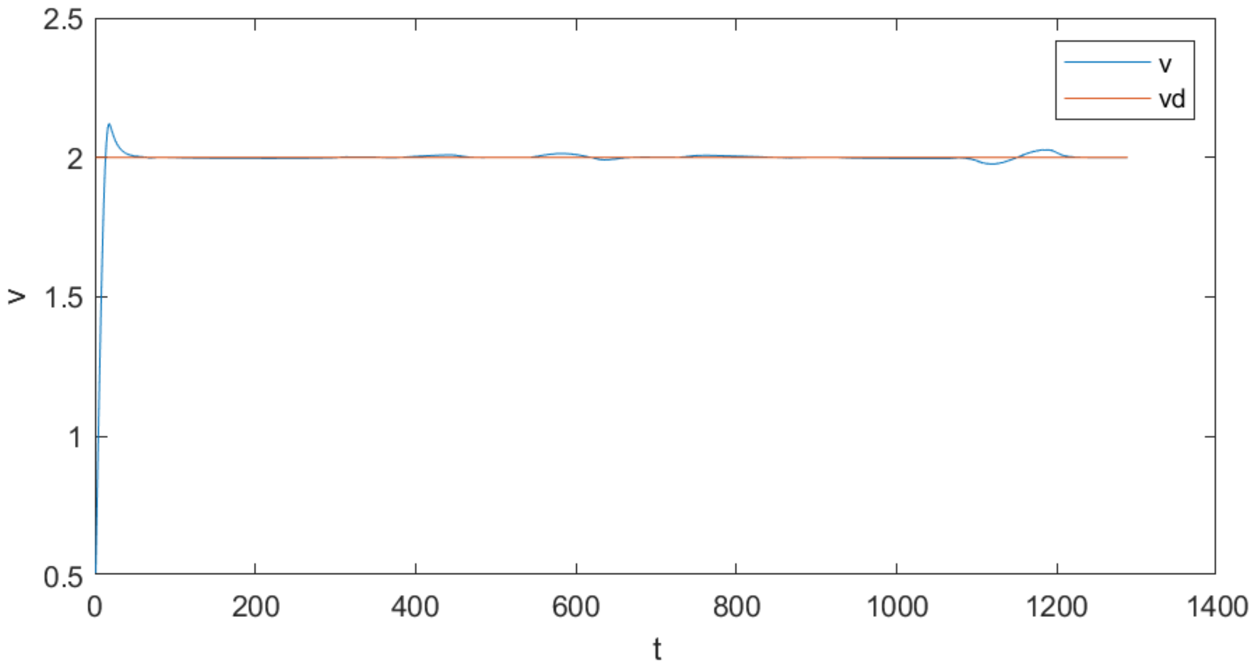

Fig. 3: Desired and Actual Linear velocity with time

The vehicle could follow the path. However, from the figure it is seen that, there exists some tracking

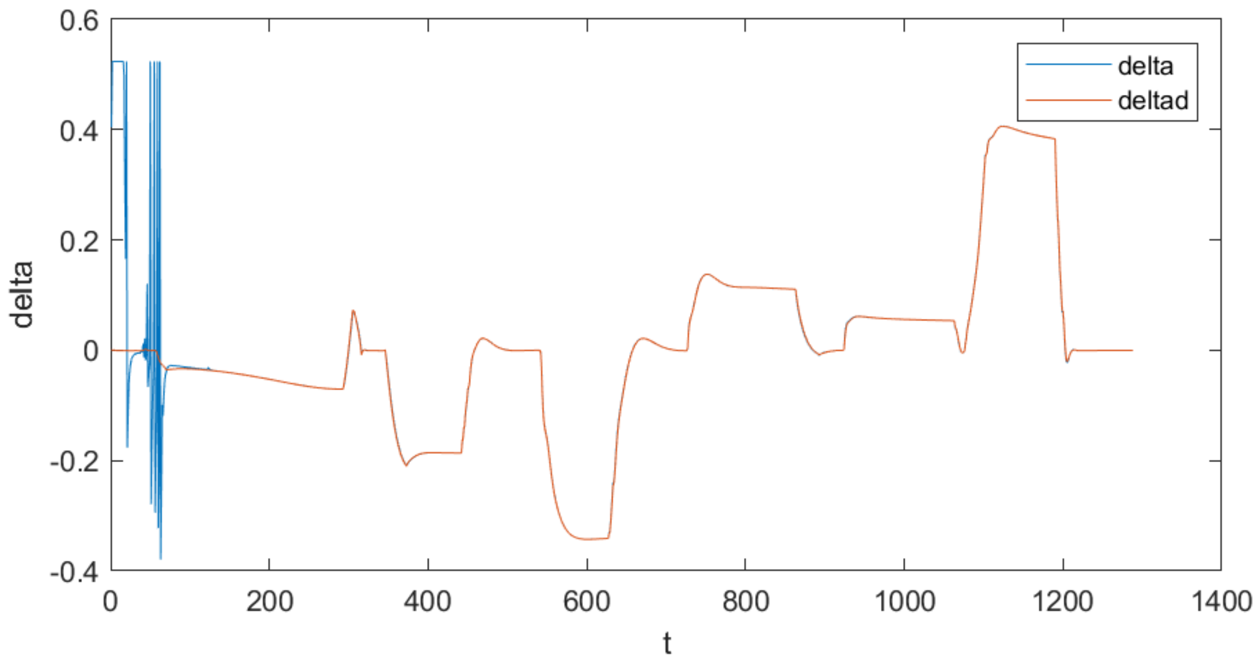

Fig. 4: Desired and Actual steering angle with time

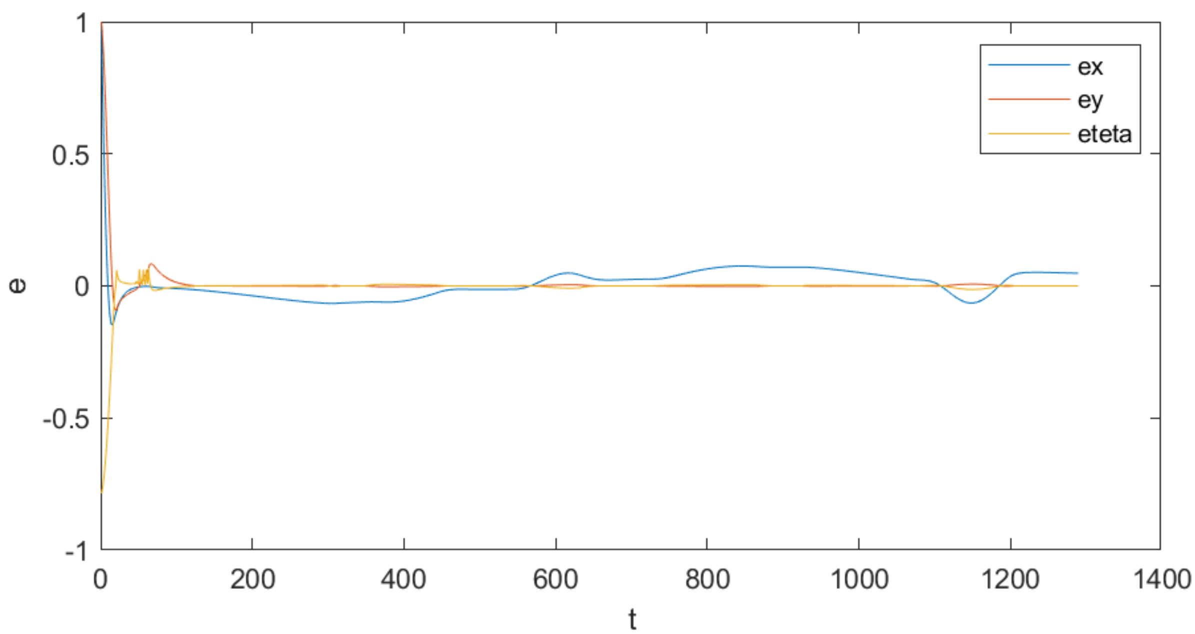

Fig. 5: Error values with time

deviations. Fig. 3 depicts how the actual velocity can follow the desired velocity. The curve of actual velocity is almost fall into the curve of desired velocity. However, at some points deviations are seen to be persisted. Fig. 4 demonstrates the actual and desired steering angle curves. At the beginning, the there are irregularities. With the convergence of the algorithm, the actual angles exactly follow the desired values. Fig. 5 shows the error values of the states of the system. According to the figure, all of the nonlinear ODE corresponding to autonomous vehicle could achieve asymptotical stability with the presence of some deviations in some points specially for the curve of e_y. Fig. 6-7 illustrates how the chattering effect of sliding mode controller could be addressed using the sat(s) function.

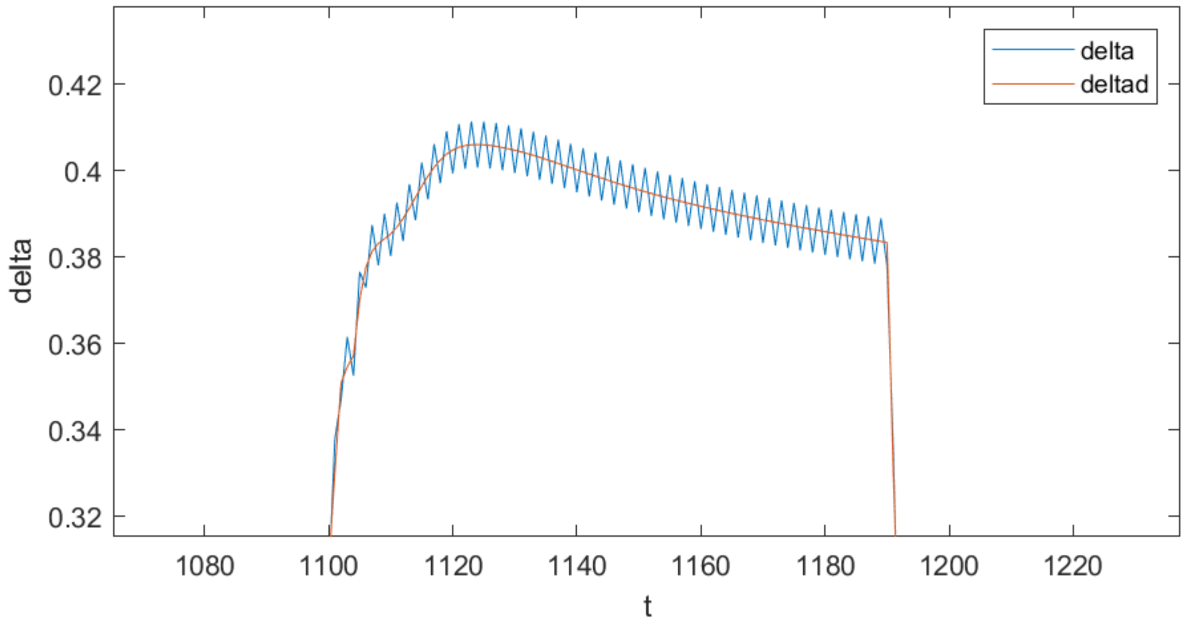

Fig. 6: Steering angle with chattering effect

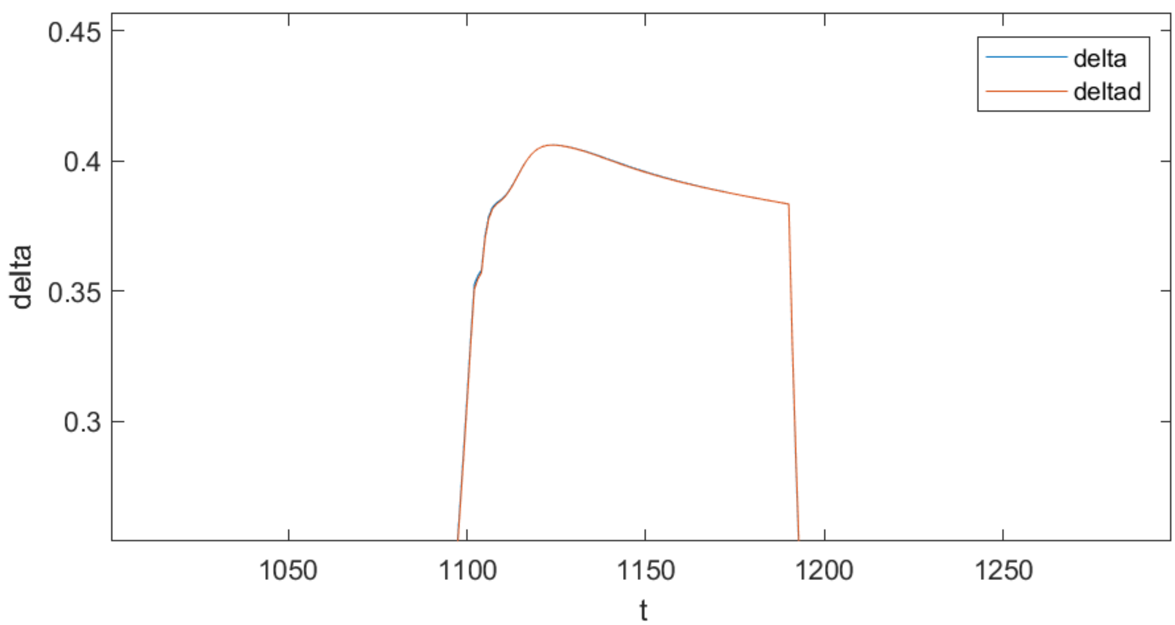

Fig. 7: Chattering Effect Addressed in Steering angle

The principal reason of the deviation from the desired values is due to the noise g_y which eventually limits the curves to be fully track the desired curves. The change of controller gain k1 and k2 could be a probable solution for this problem. However, this change can affect in other variables and parameters forming the control law which necessitates adding a capability of nonlinear optimal control facility with this controller.

5 Conclusion

The conclusion of this work can be summarized in the following points

1. A model of autonomous vehicle has been put forward which includes input disturbance as well as model errors in form of gaussian noise.
2. A sliding model controller has been designed which stabilize the system at the same time minimizes the sliding effect (chattering)
3. The controller can make the model follow the desired path at the same time minimizing the effect of disturbances and noises.

As to future works, the tracking performances of the system are subject to improvement. Adding the nonlinear optimal control to the facility is also under the process of improving the performance. Finally, combining the sliding mode controller with Koopman theory, an emerging field in the control techniques can also be undertaken.

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