Chapter 4

Dynamics: Newton's Laws of Motion

- Force
- Newton’s First Law of Motion
- Mass
- Newton’s Second Law of Motion
- Newton’s Third Law of Motion
- Weight – the Force of Gravity; and the Normal Force
- Applications Involving Friction, Inclines

Recalling Last Lecture

Force

Force is the interaction between two objects.

The force you apply on the cart changes its state of motion

Force

Force is a vector.

And we will represent it using the standard vector representation: $\vec{F}$
with the arrow representing the direction a force is applied on an object.

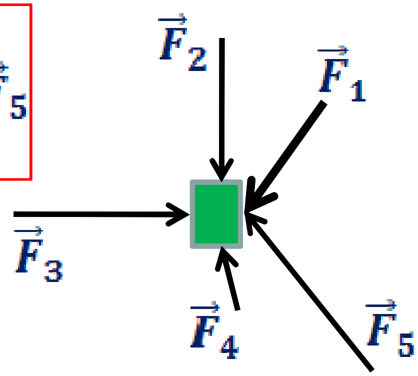
Force

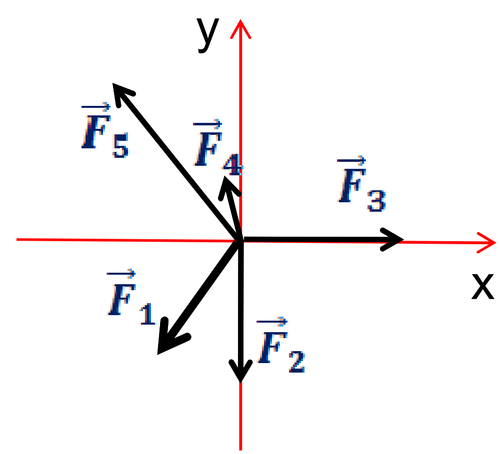
In general we can state that if a system of two or more forces are applied on a object, the net force on this object will be given by:

$$\vec{F} = \sum_{i=1}^{N} \vec{F}_i \qquad \textbf{(4.1)}$$

Example: Determine the net force $\vec{F}$ acting on an object subject five different forces: $\vec{F}_1$; $\vec{F}_2$; $\vec{F}_3$; $\vec{F}_4$; and $\vec{F}_5$

$$\vec{F} = \sum_{i=1}^{5} \vec{F}_i = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \vec{F}_4 + \vec{F}_5$$





Newton’s Laws of Motion

<u>**Newton’s first law of motion**</u> :

“ **Every Object continues in its state of rest, or uniform velocity in a straight line, as long as no net force acts on it** ”

Newton's Laws of Motion

Newton's first law **ONLY** holds in inertial frames.

Inertial frames are frames FIXED on a system which is either at rest or moving with constant velocity (no changes in direction and magnitude).

Or, in other words, **inertial frames are those where the Newton's first law is valid.**

Frames other than the inertial frames are called **non-inertial frames** (for obvious reasons).

Newton’s Laws of Motion

The Concept of Mass:

Mass measures that quantity of matter in a body (or object). It is given in units of ***Kg***.

→ **Mass is a measure of the inertia of an object.**

→ **Inertia is the tendency an object has to continue in its state of motion.**

→ **The larger is the mass of an object, the bigger is its inertia.**

Newton’s Laws of Motion

Newton’s Second Law of Motion:

“ The acceleration of an object is directly proportional to the net force acting on it, and is inversely proportional to its mass. The direction of the acceleration is in the direction of the net force acting on the object ”

This lecture continues in the next slides

Newton’s Laws of Motion

<u>Newton’s Second Law of Motion:</u>

Newton’s second law can be summarized with the following equation:

$$\vec{a} = \frac{\sum_{i=1}^{N} \vec{F}_i}{m}$$

This equation can be re-written to yield the well known relationship between force, mass and acceleration:

$$\boxed{\vec{F} = \sum_{i=1}^{N} \vec{F}_i = m\vec{a}}$$

Note that you can also write:

$$\boxed{\vec{F} = \sum_{i=1}^{N} \vec{F}_i = \sum_{i=1}^{N} (m\vec{a}_i) = m\vec{a} \quad \text{, where} \quad \vec{F}_i = m\vec{a}_i} \qquad \textbf{(4.2)}$$

Newton's Laws of Motion

<u>Newton's Second Law of Motion:</u>

<u>Note</u>: There are cases where the summation over all forces acting on an object are implicitly assumed to be present, and we can then write:

$$\vec{F} = \sum \vec{F}_i = m\vec{a}$$

in this case, the indices on the Greek symbol (sigma) for summation were dropped.

Newton's Laws of Motion

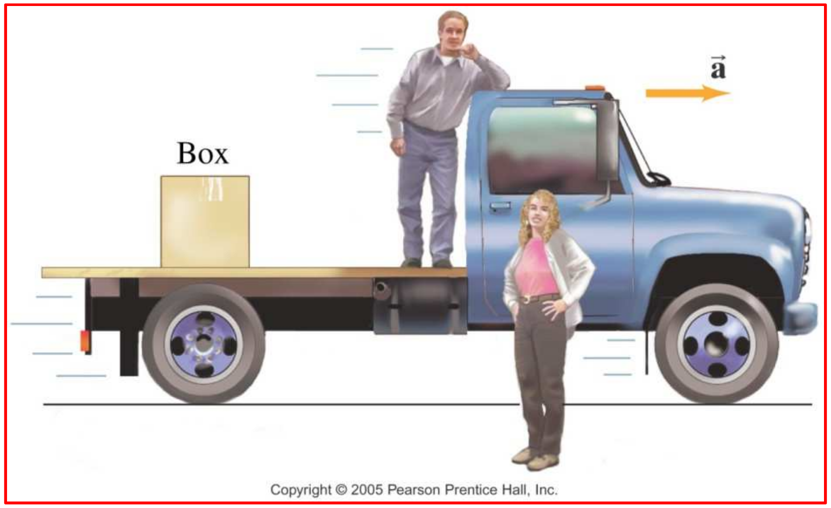
Newton's Second Law of Motion:

Note 1: There are cases where the summation over all forces acting on an object are implicitly assumed to be present, and we can then write:

$$\vec{F} = \sum \vec{F}_i = m\vec{a}$$

in this case, the indices on the Greek symbol (sigma) for summation were dropped.

Note 2: Equation 4.2 is only valid in inertial reference frames. In our previous example of non-inertial reference frame, the box will slide in the truck even if no force is being applied on it. So, if the frame is fixed on the accelerate car, it will look like the box is accelerating even if the net force on it is zero. So, the above equation does not apply.



Newton's Laws of Motion

Newton's Second Law of Motion:

Note 3: Note that since acceleration and force are vectors, we can then write:

$$\vec{F}_x = \sum_{i=1}^{N} \vec{F}_{ix} = \sum_{i=1}^{N} (m\vec{a}_{ix}) = m\vec{a}_x \quad ; \quad \vec{F}_y = \sum_{i=1}^{N} \vec{F}_{iy} = \sum_{i=1}^{N} (m\vec{a}_{iy}) = m\vec{a}_y$$

$$F_x = (\vec{F})_x = \sum_{i=1}^{N} (\vec{F}_i)_x = \sum_{i=1}^{N} F_{ix} = \sum_{i=1}^{N} m(\vec{a}_i)_x = \sum_{i=1}^{N} ma_{ix} = ma_x \quad ; \quad \boxed{F_y = ma_y}$$

$$F_y = (\vec{F})_y = \sum_{i=1}^{N} (\vec{F}_i)_y = \sum_{i=1}^{N} F_{iy} = \sum_{i=1}^{N} m(\vec{a}_i)_y = \sum_{i=1}^{N} ma_{iy} = ma_y \quad ; \quad \boxed{F_x = ma_x}$$

$$F = |\vec{F}| = \sqrt{F_x^2 + F_y^2} = m\sqrt{a_x^2 + a_y^2}$$

Newton's Laws of Motion

$$\boxed{1\ N = 1\ Kg\ .\ m/s^2}$$

$$\underset{F}{1\ N} = \underset{m}{1\ Kg}\ .\ \underset{a}{m/s^2}$$

TABLE 4–1
Units for Mass and Force

System	Mass	Force
SI	kilogram (kg)	newton (N) $(= kg \cdot m/s^2)$
cgs	gram (g)	dyne $(= g \cdot cm/s^2)$
British	slug	pound (lb)

Conversion factors: 1 dyne $= 10^{-5}$ N;
1 lb $\approx$ 4.45 N.

Newton's Laws of Motion

Newton's Second Law of Motion:

Problem 4.7 (textbook: What average force is needed to accelerate a *7.00-gram* pellet from rest to *125 m/s* over a distance of *0.800 m* along the barrel of a rifle (assumed to be pointing in the horizontal direction)?

Newton's Laws of Motion

Problem 4.7 (textbook):

The average force on the pellet is its mass times its average acceleration.

The initial conditions are:

$$v_0 = 0$$
$$v = 125\,\text{m/s}$$
$$x - x_0 = 0.800\ \text{m}$$

The average acceleration can be found using the following equation of motion:

$$v^2 = v_0^2 + 2a(x - x_0)$$

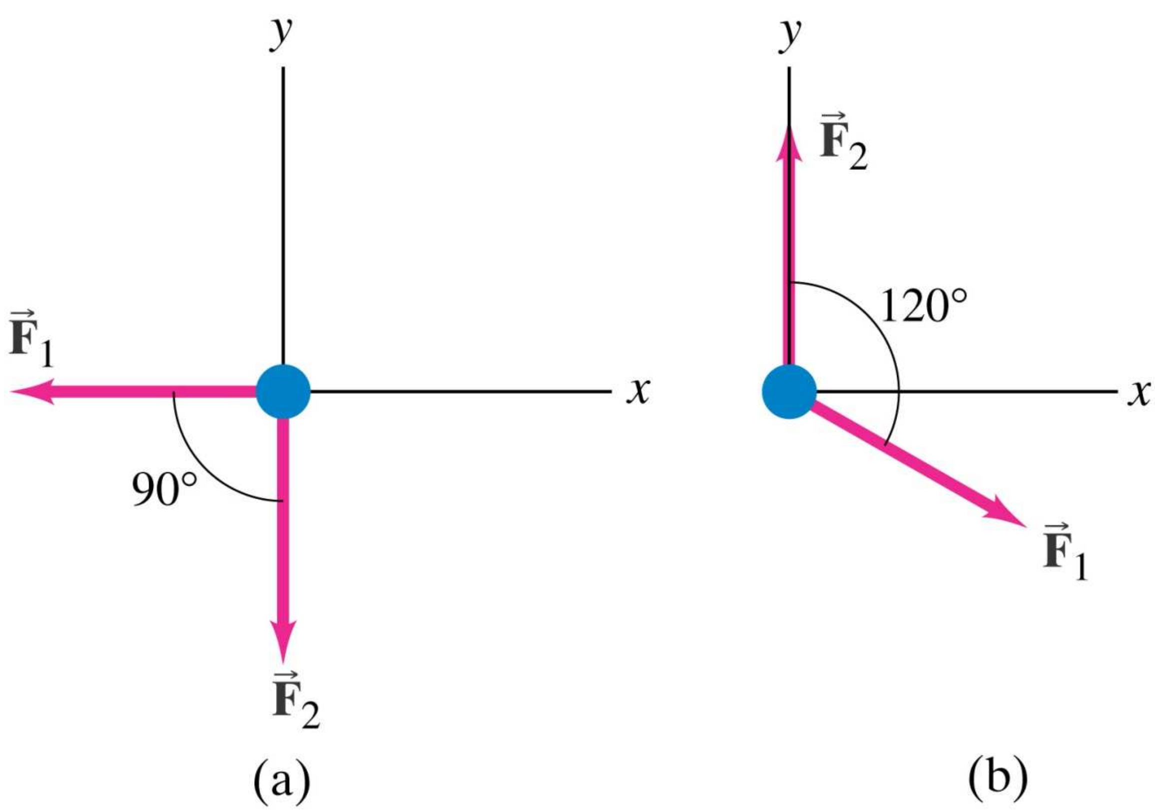
$$a_{avg} = \frac{v^2 - v_0^2}{2\left(x - x_0\right)} = \frac{\left(125\,\text{m/s}\right)^2 - 0}{2\left(0.800\ \text{m}\right)} = 9770\ \ \text{m/s}^2$$

$$F_{avg} = m a_{avg} = \left(7.00 \times 10^{-3}\ \text{kg}\right)\left(9770\ \ \text{m/s}^2\right) = \boxed{68.4\ \text{N}}$$

Newton's Laws of Motion

Newton's Second Law of Motion:

Problem 4.24 (textbook: The two forces $\vec{F}_1$ and $\vec{F}_2$ shown in Fig. 4–43a and b (looking down) act on a *27.0-kg* object on a frictionless tabletop. If $F_1 = 10.2\ N$ and $F_2 = 16.0\ N$, find the net force on the object and its acceleration for (*a*) and (*b*).



Problem 4.24 (textbook):

The net force in each case is found by vector addition with components.

a) $F_{\text{Net x}} = -F_1 = -10.2 \text{ N} \qquad F_{\text{Net y}} = -F_2 = -16.0 \text{ N}$

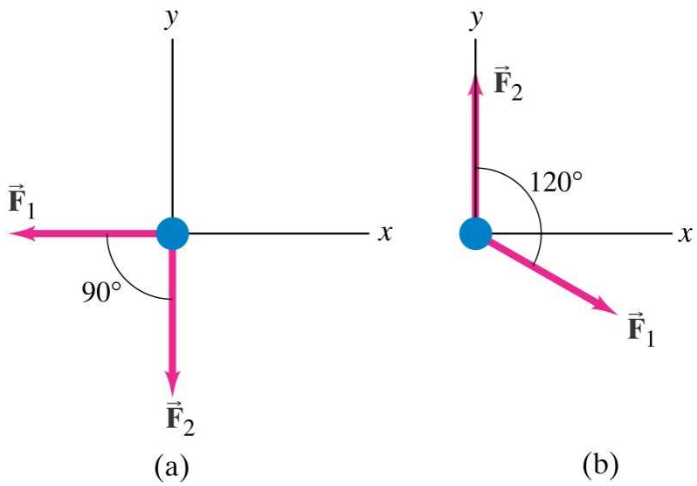
$$F_{\text{Net}} = \sqrt{(-10.2)^2 + (-16.0)^2} = \boxed{19.0 \text{ N}} \qquad \theta = \tan^{-1}\frac{-16.0}{-10.2} = 57.5^\circ$$

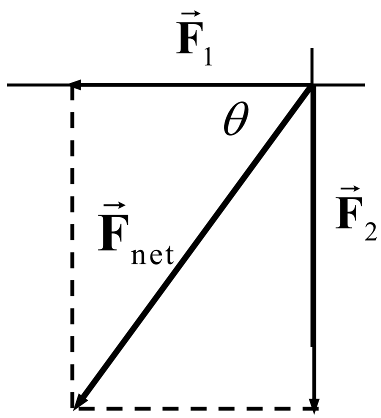
(4th quadrant)

It acceleration will be given by

$$a = \frac{F_{\text{Net}}}{m} = \frac{19.0 \text{ N}}{27.0 \text{ kg}} = \boxed{0.703 \text{ m/s}^2}$$

in the same direction as $\vec{F}_{Net}$





Problem 4.24 (textbook):

The net force in each case is found by vector addition with components.

b)

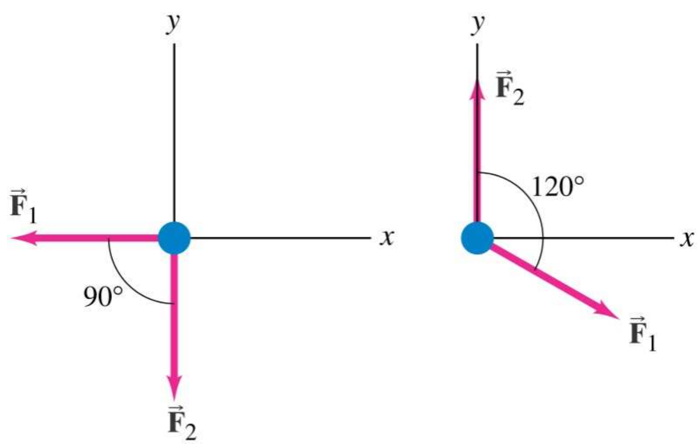
$$F_{\text{Net x}} = F_1 \cos 30^\circ = 8.83\ \text{N} \qquad F_{\text{Net y}} = F_2 - F_1 \sin 30^\circ = 10.9\ \text{N}$$

$$F_{\text{Net}} = \sqrt{(8.83\ \text{N})^2 + (10.9\ \text{N})^2} = \boxed{14.0\ \text{N}} \qquad \theta = \tan^{-1}\frac{10.9}{8.83} = \boxed{51.0^\circ}$$

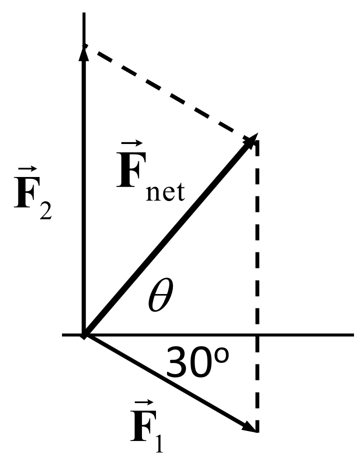
(1st quadrant)

$$a = \frac{F_{\text{Net}}}{m} = \frac{14.0\ \text{N}}{27.0\ \text{kg}} = \boxed{0.520\ \text{m/s}^2}$$

in the same direction as $\vec{F}_{Net}$



(a) (b)



Newton's Laws of Motion

Newton's Third Law of Motion:

We have seen that an applied **NET** force on an object changes its state of motion (velocity) by adding an acceleration to it.

But, from where does this force come from?

It is clear that when I push a table and it starts moving from rest, I am the source of the force.

But ………..

Newton’s Laws of Motion

Newton’s Third Law of Motion:

Very Important Note:

You **can not** change **your motion** by trying **to push yourself**.

To change your state of motion **an external net force** should be applied on you.

So, the **net force applied** on an object **by other objects** are the reason for giving acceleration to this object (changing its state of motion).

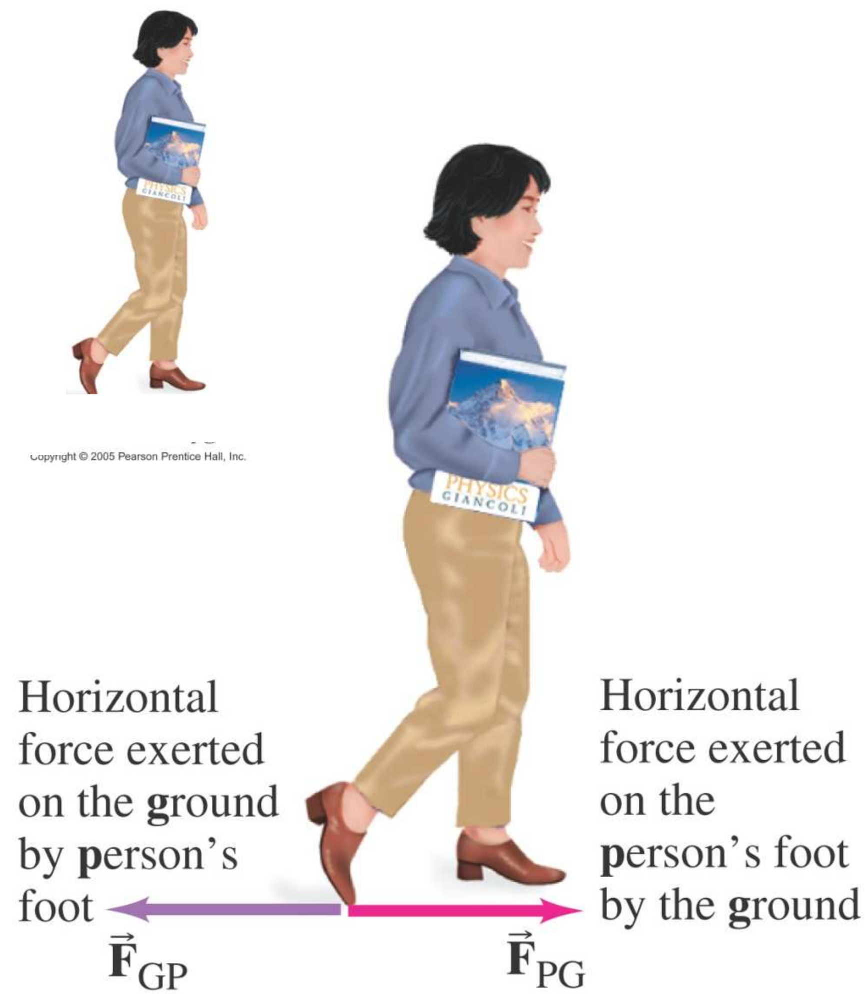
So, let me then ask you a couple of questions:

Newton's Laws of Motion

Newton's Third Law of Motion:

Example 1): When I start walking from rest, I push the ground backwards. Is the force **I am** exerting on the ground the **reason** for my **motion**?

Answer: NO, the fact is that the ground will react to my force with another one of same magnitude but opposite direction. This force is applied on me and is the reason for the change in my state of motion.



Newton’s Laws of Motion

Newton’s Third Law of Motion:

Example 2): An ice skater pushes against a wall and then starts moving in the opposite direction of the force she had applied on the wall? What is the source of her motion?

Answer: Again, it is the reaction of the wall to her force, with same magnitude and opposite direction.

Note that here she doesn’t need to keep pushing against the wall or ground to keep moving. This is because once she starts moving, and considering a frictionless ice surface, she will tend to keep her state of motion (Newton’s first law)) unless a external force is applied on her.

We will discuss friction in the next lecture.

Newton’s Laws of Motion

Newton’s Third Law of Motion:

The Newton’s third law says:

“ Whenever one object exerts a force on a second object, the second exerts an equal force in the opposite direction on the first ”

This law is best know in the following form:

“ To every action there is a corresponding reaction ”

or also as the law of **“ action and reaction ”.**

Newton's Laws of Motion

<u>Newton's Third Law of Motion:</u>

But why the second body reacts to the force applied by the first object?????

First, let me first say that every object has some elasticity no matter how hard it may look like.

- So, in principle, you can deform an object by applying a force on it.

- The intensity of the deformation is determined by the magnitude of the force applied.

Newton's Laws of Motion

Newton's Third Law of Motion:

So, when you push a table, the following happen:

1. The force you apply on the table tend to distort its shape;

2. In fact, what you do is to transmit some energy to the table. The atoms of the table are then excited and tend to move from their original position due to the addition of energy to the original energy stored in the table;

3. But nature likes to find its objects in their state of minimal energy. The table will then try to release the extra energy in order to return to its original and stable configuration;

4. In doing so, the table will exert a force on you with same magnitude and opposite direction;

5. You may even notice that the shape of your hands are also distorted. This is now due to the extra energy in the table being released.

The relation between force and displacement is called work. Work is energy. We shall discuss work in chapter 6.

Newton's Laws of Motion

Newton's Third Law of Motion:

The Newton's third law can be mathematically represented as:

$$\boxed{\vec{F}_{AB} = -\vec{F}_{BA}} \qquad \textbf{(4.3)}$$

Where

$\vec{F}_{BA}$ is the force applied by A on B

and

$\vec{F}_{AB}$ is the force resulting of the reaction of by B on A

Weight – the Force of Gravity and the Normal Force

We have seen that an object dropped near the surface of the earth will fall with constant acceleration, the gravitational acceleration $\vec{g}$.

But we have just seen that acceleration is related to force.

The force that produces the gravitational acceleration is called **gravitational force**, $\vec{F}_G$. We will discuss gravitational force in chapter 5.

$\vec{F}_G$ points **down toward to the center of the Earth**, and so does $\vec{g}$.

The relationship between $\vec{F}_G$ and $\vec{g}$ is given by Newton's second law:

$$\boxed{\vec{F}_G = m\vec{g}} \tag{4.4}$$

The **magnitude** of $\vec{F}_G$ on a object is called the **object's weight**.

For instance → a *1.00 Kg* object will weight

$$F_G = 1.00\ Kg \times 9.80\ m/s^2 = 9.80\ Kg \cdot m/s^2 = 9.80\ N$$

Weight – the Force of Gravity and the Normal Force

Normal Force:

When you weight your mass by climbing on a spring scale, you stay at rest while performing the measurement.

→ Since you are at rest, you are not changing your state of motion. So, is there a gravitational force applying on you?

→ Yes, there is.

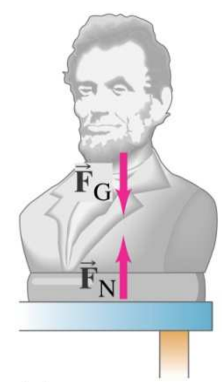
→ But, Newton's second law says that the net force applied on you should be zero to keep you in your original state of motion (rest in this case).

→ There should then be a force acting on the object that balances the gravitational force such that:

$$\vec{F}_{Net} = \vec{F}_G + \vec{F}_N = 0$$

$$\vec{F}_N = -\vec{F}_G$$

$\boxed{\vec{F}_N \text{ is called the } \textbf{normal force}.}$



Weight – the Force of Gravity and the Normal Force

Normal Force:

The normal force results from the deformation on the surface the object is located, and is always **perpendicular** to this surface.

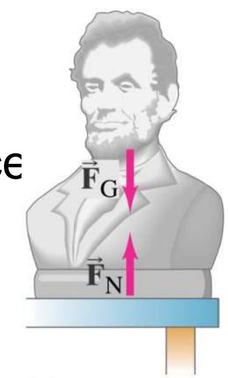
The normal force is **NOT** the resultant of Newton's third law:

→ Newton's third law applies to forces acting on different objects

→ Here **BOTH** the **normal** and **gravitational** forces acts on YOU

→ So, the normal force is not resultant of the scale reacting to you**, as you are not applying any force on it**.

The **normal force** is of different nature of the gravitational force
It is called a **contact force**.



(a)

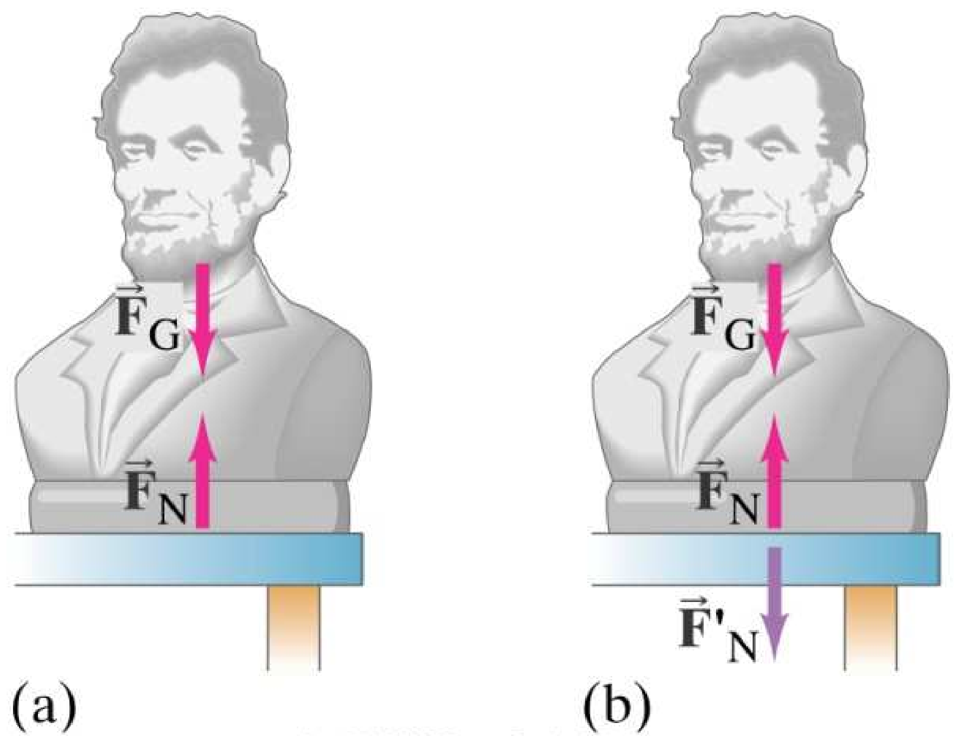
Contact forces are those that require two objects to be in contact.
Gravitational force does not require two objects to be in contact.

Weight – the Force of Gravity and the Normal Force

Normal Force:

The reaction to the normal force in the figure is the reaction of the statue, $\vec{F}'_N$, to the normal force applied by the table on it.

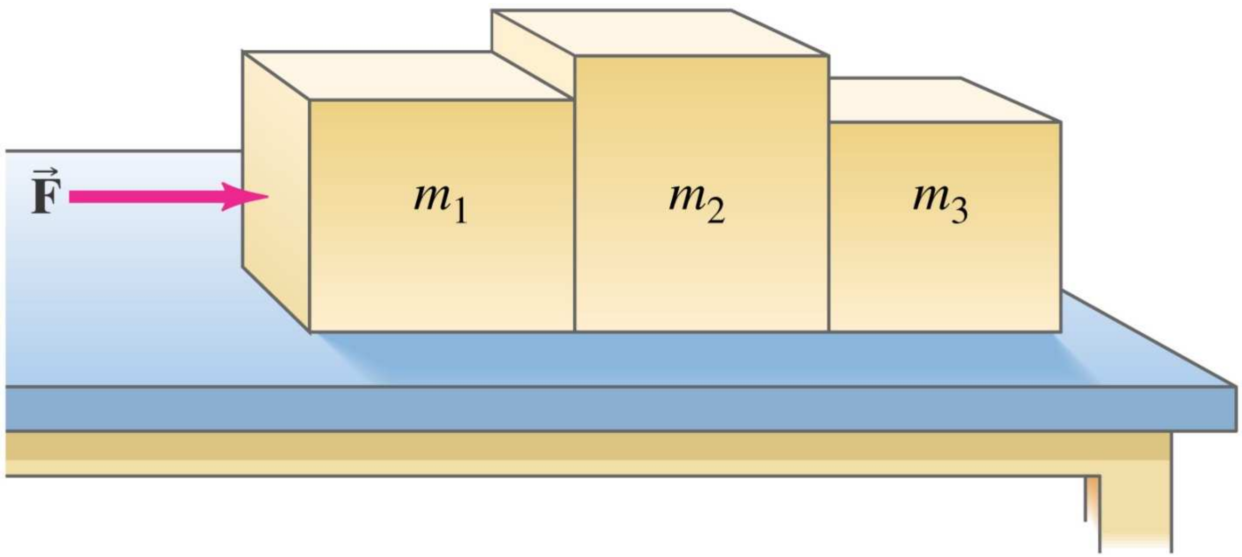
The reaction to the gravitational force acting on the statue by Earth is the gravitational force acting on Earth by the statue.
We shall discuss it in chapter 5.



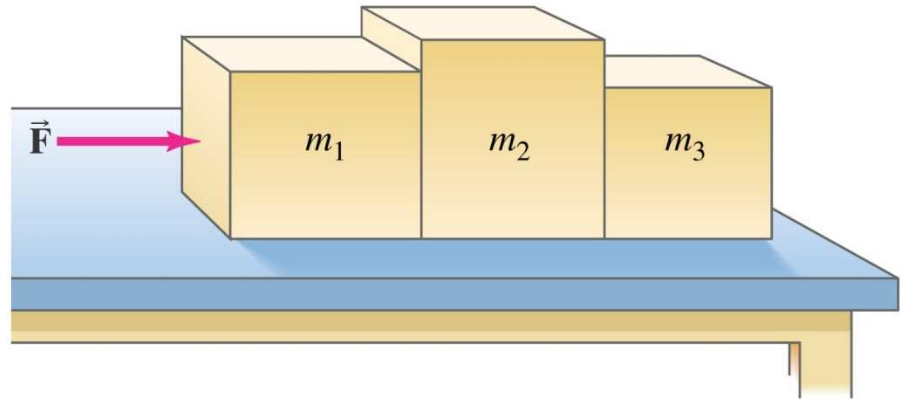
Newton's Laws of Motion

Newton's Second Law of Motion:

Problem 4.33 (textbook: Three blocks on a frictionless horizontal surface are in contact with each other, as shown in Fig. 4–51. A force $\vec{F}$ is applied to block ***1*** (mass m_1). (*a*) Draw a free-body diagram for each block. Determine (*b*) the acceleration of the system (in terms of m_1, m_2 and m_3), (*c*) the net force on each block, and (*d*) the force of contact that each block exerts on its neighbor. (*e*) If $m_1 = m_2 = m_3 = 12.0\ Kg$ and $F = 96.0\ N$, give numerical answers to (*b*), (*c*), and (*d*). Do your answers make sense intuitively?



Problem 4.33 (textbook):



a) In the free-body diagrams below,

$\vec{F}_{12}$ = force on block 1 exerted by block 2,

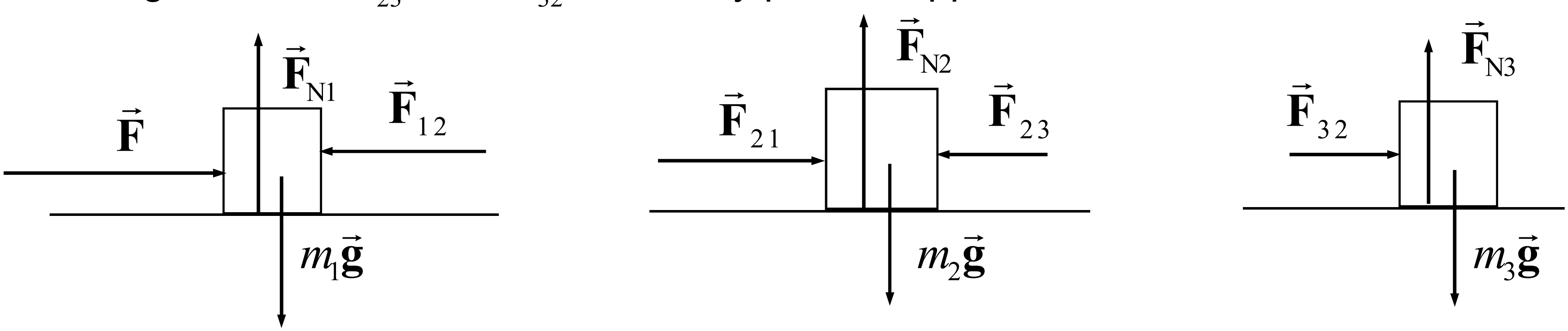
$\vec{F}_{21}$ = force on block 2 exerted by block 1,

$\vec{F}_{23}$ = force on block 2 exerted by block 3, and

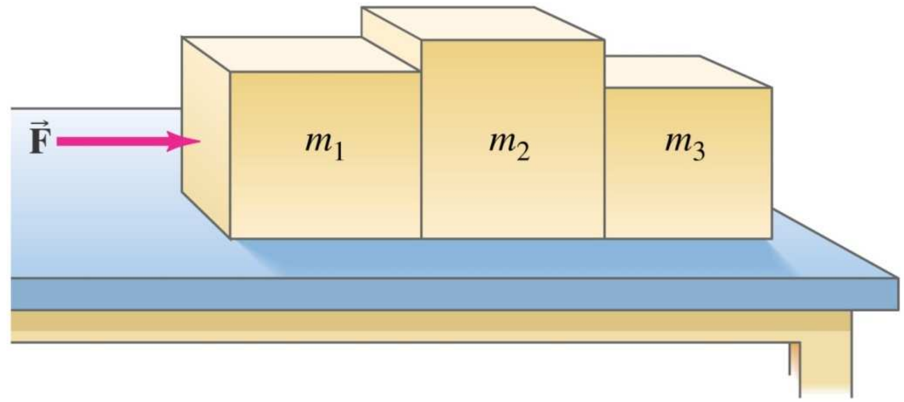
$\vec{F}_{32}$ = force on block 3 exerted by block 2.

By Newton's 3rd law:

The magnitudes of $\vec{F}_{21}$ and $\vec{F}_{12}$ are equal and they point I opposite directions , and the magnitudes of $\vec{F}_{23}$ and $\vec{F}_{32}$ and they point in opposite directions.



Problem 4.33 (textbook):

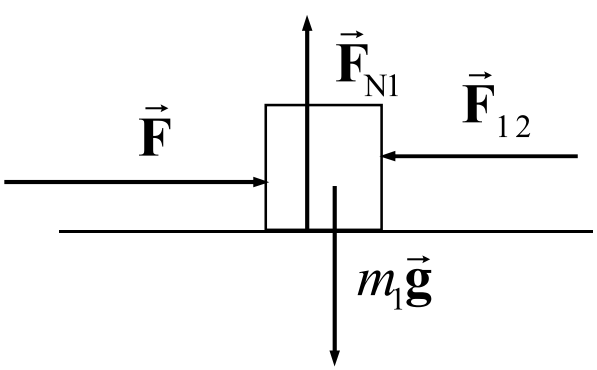


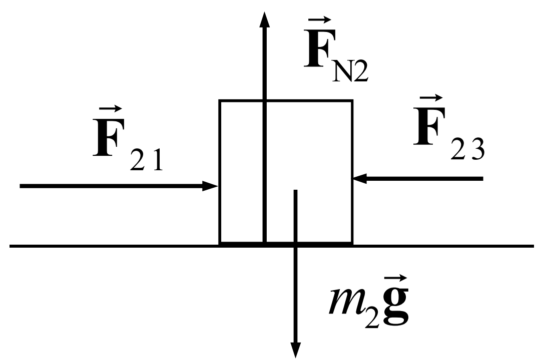
b) All of the vertical forces on each block add up to zero, since there is no acceleration in the vertical direction. Thus for each block,

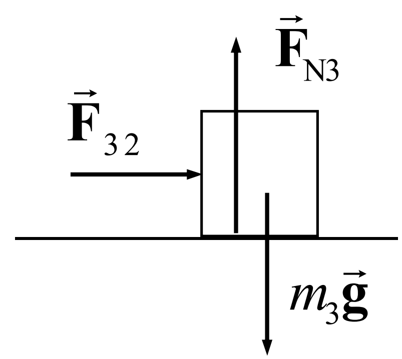
$$F_N = m g$$

For the horizontal direction, we have

$$\sum F = F - F_{12} + F_{21} - F_{23} + F_{32} = F = \left(m_1 + m_2 + m_3\right) a \quad \rightarrow \quad \boxed{a = \frac{F}{m_1 + m_2 + m_3}}$$







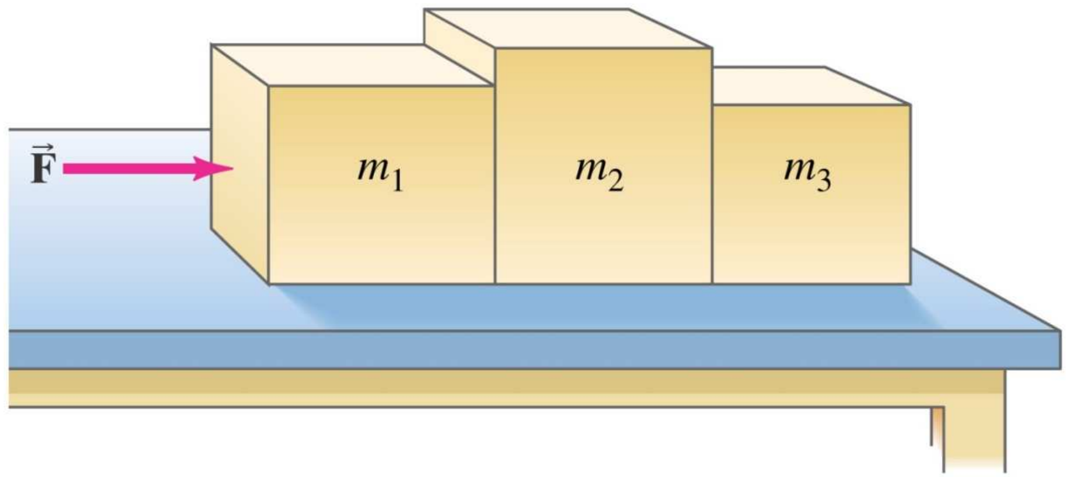
Problem 4.33 (textbook):

c) For each block, the net force must be *F* = *ma* by Newton's 2nd law. Each block has the same acceleration since they are in contact with each other.

$$\boxed{F_{1\,net} = \frac{m_1 F}{m_1 + m_2 + m_3}}$$

$$\boxed{F_{2\,net} = \frac{m_2 F}{m_1 + m_2 + m_3}}$$

$$\boxed{F_{3\,net} = \frac{m_3 F}{m_1 + m_2 + m_3}}$$



Problem 4.33 (textbook):

d) From the free-body diagram, we see that for m_3

$$F_{32} = F_{3\,net} = \boxed{\frac{m_3 F}{m_1 + m_2 + m_3}}$$

And by Newton's 3rd law

$$F_{32} = F_{23} = F_{3\,net} = \boxed{\frac{m_3 F}{m_1 + m_2 + m_3}}$$

Of course, $\vec{\mathbf{F}}_{23}$ and $\vec{\mathbf{F}}_{32}$ are in opposite directions.

Also from the free-body diagram, we see that for m_1:

$$F - F_{12} = F_{1\,net} = \frac{m_1 F}{m_1 + m_2 + m_3} \quad \rightarrow \quad F_{12} = F - \frac{m_1 F}{m_1 + m_2 + m_3} \quad \rightarrow \quad \boxed{F_{12} = \frac{(m_2 + m_3) F}{m_1 + m_2 + m_3}}$$

By Newton's 3rd law:

$$F_{12} = F_{21} = \boxed{\frac{(m_2 + m_3) F}{m_1 + m_2 + m_3}}$$

Problem 4.33 (textbook):

e) Using the given values:

$$a = \frac{F}{m_1 + m_2 + m_3} = \frac{96.0\ \text{N}}{36.0\ \text{kg}} = \boxed{2.67\ \text{m/s}^2}$$

Since all three masses are the same value, the net force on each mass is

$$F_{net} = ma = (12.0\ \text{kg})(2.67\ \text{m/s}^2) = 32.0\ \text{N}$$

This is also the value of F_{32} and F_{23}. The value of F_{12} and F_{21} is

$$F_{12} = F_{21} = (m_2 + m_3)a = (24\ \text{kg})(2.67\ \text{m/s}^2) = 64.0\ \text{N}$$

To summarize:

$$F_{\text{net }1} = F_{\text{net }2} = F_{\text{net }3} = \boxed{32.0\ \text{N}} \qquad F_{12} = F_{21} = \boxed{64.0\ \text{N}} \qquad F_{23} = F_{32} = \boxed{32.0\ \text{N}}$$

The values make sense in that in order of magnitude, we should have $F > F_{21} > F_{32}$, since F is the net force pushing the entire set of blocks, F_{12} is the net force pushing the right two blocks, and F_{23} is the net force pushing the right block only.

Assignment 4

Textbook (Giancoli, 6th edition), Chapter 4:

Due on Thursday, October 9, 2008

- **Problem 25 - page 99 of the textbook**
- **Problems 27 and 32 - page 100 of the textbook**
- **Problem 34 - page 101 of the textbook**