Classical Mechanics

Labix

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Abstract

Contents

1 Newton's Laws of Motion **3**
1.1 Space and Time . . . 3
1.2 Newton's Laws of Motion . . . 3
1.3 Friction . . . 5
1.4 Drag Force . . . 5
1.5 Circular Motion . . . 5

2 Work and Energy **6**
2.1 Kinetic Energy . . . 6
2.2 Work . . . 6
2.3 Work done by a Force as a Variable . . . 7
2.4 Power . . . 7

3 Simple Harmonic Motion **8**
3.1 Mass on a Spring . . . 8

1 Newton's Laws of Motion

1.1 Space and Time

Definition 1.1.1: Position

The position of a particle is defined in relation to a coordinate system centered on an arbitrary fixed reference point in space.

Definition 1.1.2: Velocity

The velocity of a particle with position $\mathbf{r}$ is given by

$$\mathbf{v} = \frac{d\mathbf{r}}{dt}$$

Definition 1.1.3: Acceleration

The acceleration of a particle with position $\mathbf{r}$ is given by

$$\mathbf{a} = \frac{d\mathbf{v}}{dt} = \frac{d^2\mathbf{r}}{dt^2}$$

Definition 1.1.4: Frame of Reference

A frame of reference is an abstract coordinate system whose origin, orientation and scale are specified by a set of reference points.

Definition 1.1.5: Inertial Frame of Reference

An intertial frame of reference is a frame of reference not undergoing accleration.

1.2 Newton's Laws of Motion

Definition 1.2.1: Mass

Mass is the quantity of matter, measured in kg.

Definition 1.2.2: Momentum

Momentum is the quantity of motion, defined to be

$$\mathbf{p} = m\mathbf{v}$$

Definition 1.2.3: Force

A force is an influence that can change the motion of an object.

Axiom 1.2.4: Newton's First Law

In the absense of forces, a particle moves with constant velocity $\mathbf{v}$.

Axiom 1.2.5: Newton's Second Law

For any particle of mass m, the net force $\mathbf{F}$ on the particle is always equal to the mass m times the particle's accleration. In other words,

$$\mathbf{F} = m\mathbf{a} = \frac{d\mathbf{p}}{dt}$$

Axiom 1.2.6: Newton's Third Law

If object 1 exerts a force $\mathbf{F}_{21}$ on onject 2, then the object 2 always exerts a reaction force $\mathbf{F}_{12}$ on object 1 given by

$$\mathbf{F}_{12} = -\mathbf{F}_{21}$$

Theorem 1.2.7: Conservation of Momentum

If the net external force $\mathbf{F}^{\text{ext}}$ on an N-particle system if 0, then then system's total momentum $\mathbf{P}$ is constant.

Proof. Suppose we have n particles named $\alpha_1, \dots, \alpha_n$. For each $i \in \{1, \dots, n\}$,

$$\mathbf{F}_{\alpha_i} = \sum_{k=1, k\neq i}^{n} \mathbf{F}_{\alpha_i\alpha_k} + \mathbf{F}_{\alpha_i}^{\text{ext}}$$

Now the total momentum of the system is given by

$$\mathbf{P} = \sum_{k=1}^{n} \mathbf{p}_{\alpha_k}$$

Differentiating it gives

$$\frac{d\mathbf{P}}{dt} = \sum_{k=1}^{n} \frac{d\mathbf{p}_{\alpha_k}}{dt}$$

Substituting $\mathbf{F}_{\alpha_i} = \frac{d\mathbf{p}_{\alpha_i}}{dt}$, we have

$$\begin{aligned}
\frac{d\mathbf{P}}{dt} &= \sum_{k=1}^{n} \sum_{j=1, j\neq k}^{n} \mathbf{F}_{\alpha_k\alpha_j} + \sum_{k=1}^{n} \mathbf{F}_{\alpha_k}^{\text{ext}} \\
&= \sum_{k=1}^{n} \sum_{j=k+1}^{n} (\mathbf{F}_{\alpha_k\alpha_j} + \mathbf{F}_{\alpha_j\alpha_k}) + \sum_{k=1}^{n} \mathbf{F}_{\alpha_k}^{\text{ext}} \\
&= \sum_{k=1}^{n} \mathbf{F}_{\alpha_k}^{\text{ext}} \\
&= \mathbf{F}^{\text{ext}}
\end{aligned}$$

Thus if the net external force $\mathbf{F}^{ext} = 0$, $\mathbf{P}$ is a constant. □

Theorem 1.2.8: Law of Gravity

Suppose two mass m_1, m_2 are in play and they are r distance apart. Newton's law of gravity states that there is a force acting on m_2 by m_1 and vice versa given by

$$F = \frac{Gm_1m_2}{r^2}$$

1.3 Friction

Definition 1.3.1: Static Friction

Friction arises when one onject is in contact with another. Denote $f_{\max}$ the maximal of static friction before it changes into kinetic friction. If $\mathbf{F} < f_{\max}$, then the static friction $\mathbf{f} = -\mathbf{F}$. We also have $f_{\max} = \mu|\mathbf{N}|$, meaning the max friction is proportional to $\mathbf{N}$.

Definition 1.3.2: Coefficient of Friction

μ is called the coefficient of friction. We have $0 < \mu \leq 1$.

Definition 1.3.3: Kinetic Friction

When motion occurs, friction is still roughly proportional to the normal force, but the coefficient changes

$$f_k = \mu_k N$$

where μ_k is the coefficient of kinetic or dynamic friction and f_k is the kinetic friction.

1.4 Drag Force

Definition 1.4.1: Drag Force

Drag force is experienced when moaing fluid or gas, the force that hinders your movement.

$$D = \frac{1}{2}C\rho Av^2$$

where ρ is the air density, A is the effective cross-sectional area, C the drag coefficient.

Theorem 1.4.2: Falling

When free falling, the total force acting on the body is $D - F_g$. Drag force reaches maximum F_g when $a = 0$. The velocity required thus is given by

$$v_t = \sqrt{\frac{2F_g}{C\rho A}}$$

1.5 Circular Motion

Theorem 1.5.1

If a particle moves in a circle or a circular arc of radius R at constant speed v, the particle experiences centripetal acceleration $\mathbf{a}$ with

$$|\mathbf{a}| = \frac{v^2}{R}$$

Theorem 1.5.2

Thus the force experienced by the particle is given by

$$F = m\frac{v^2}{R}$$

2 Work and Energy

2.1 Kinetic Energy

Axiom 2.1.1: Principle of Energy Conservation

Energy in a system is conserved.

Definition 2.1.2: Kinetic Energy

Kinetic Energy K is energy associated with the state of motion of an object. For an object of mass m whose speed v is well below the speed of light,

$$K = \frac{1}{2}mv^2$$

2.2 Work

Definition 2.2.1: Work

Work W is energy transferred to or from an object by means of a force acting on the object.

Theorem 2.2.2

$$W = Fd\cos(\theta)$$

where θ is the angle between $\mathbf{F}$ and $\mathbf{d}$.

Definition 2.2.3

$$W = \mathbf{F} \cdot \mathbf{d}$$

where d is the displacement of the object.

Theorem 2.2.4: Work Kinetic Energy Theorem

The change in kinectic energy is equal to the network done.

$$W = T_2 - T_1$$

where T_1 is the initial kinetic energy and T_2 its final.

Theorem 2.2.5: Work Done by Gravitational Force

$$W_g = mgd\cos(\theta)$$

Theorem 2.2.6: Hooke's Law

$$\mathbf{F}_s = -k\mathbf{d}$$

where k is called the spring constant, $\mathbf{d}$ is the displacement.

Theorem 2.2.7

Work done by a spring force

$$W_s = \frac{1}{2}k(x_i^2 - x_f^2)$$

2.3 Work done by a Force as a Variable

Definition 2.3.1

The work done by F on the x component is given by

$$W = \int_{x_i}^{x_f} F(x)\,dx$$

Theorem 2.3.2

The work done

$$W = \int_{x_i}^{x_f} F_x\,dx + \int_{y_i}^{y_f} F_y\,dy + \int_{z_i}^{z_f} F_z\,dz$$

2.4 Power

Definition 2.4.1

power is defined as the rate at which work is done.

$$P = \frac{dW}{dt}$$

Definition 2.4.2

The instantaneous power at a point in time is given by $P = \mathbf{F} \cdot \mathbf{v}$

3 Simple Harmonic Motion

3.1 Mass on a Spring

Theorem 3.1.1

Springs produce a force F that is linear to the displacement of x.

$$F = -kx$$