Algebraic Curves

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Abstract

Contents

1 Algebraic Curves in Classical Algebraic Geometry

Definition 1.0.1: Algebraic Curves

An algebraic curve is an irreducible variety of dimension 1.

Theorem 1.0.2

Any irreducible curve is birationally equivalent to a unique non-singular projective curve.

2 Algebraic Curves in the Context of Schemes

Definition 2.0.1: Algebraic Curves

Let k be an algebraically closed field. A curve over k is an integral separated scheme X of finite type over k that has dimension 1.

Proposition 2.0.2

Let X be an algebraic curve. Then the arithmetic and geometric genus coincide. In particular,

$$p_a(X) = p_g(X) = \dim_k H^1(X, \mathcal{O}_X)$$

We will simply call the genus of a curve g from now on since the arithmetic genus is the same as the geometric genus.

2.1 Riemann-Roch Theorem

Definition 2.1.1: Canonical Divisor

Let X be an algebraic curve. The canonical divisor K of X is a divisor in the linear equivalence class of $\Omega^1_{X/k} = \omega_X$.

Theorem 2.1.2: Riemann-Roch Theorem

Let X be an algebraic curve. Let D be a divisor on X and let K be the canonical divisor of X. Denote $l(D) = \dim_k(H^0(X, \mathcal{L}(D)))$ and let g be the genus of X. Then

$$l(D) - l(K - D) = \deg(D) + 1 - g$$

2.2 Classification of Curves in $\mathbb{P}^3$