

Exercise Sheet for Week 8

Question 1

1(a):

Find all the integer solution pairs (x, y) such that

$$387x + 83y = \gcd(387, 83)$$

1(b):

Is there integer solution pairs (x, y) to the equation

$$387x + 83y = 6$$

1(c):

Find all the integer solution pairs (x, y) for the equation

$$387x + 83y = \lambda$$

where $\lambda \in \mathbb{N}$.

1(d):

Let $a, b, d \in \mathbb{N}$ be natural numbers. When does the equation

$$ax + by = d$$

admit integer solution pairs (x, y) ?

Question 2

Find the solution to the system of congruent equations

$$x \equiv 3 \pmod{4}$$

$$x \equiv 5 \pmod{9}$$

$$x \equiv 2 \pmod{7}$$

You should be able to solve for x up to modulo $4 \cdot 9 \cdot 7$

Question 3

Let A, B be sets. Let $f : A \rightarrow B$ be a function. Let $C, D \subseteq A$ be subsets of A . Let $E, F \subseteq B$ be subsets of B . Prove or find a counter example for the following statements:

- $f^{-1}(f(A)) = A$
- $f(f^{-1}(B)) = B$
- $f(C \cup D) = f(C) \cup f(D)$
- $f(C \cap D) = f(C) \cap f(D)$
- $f^{-1}(E \cup F) = f^{-1}(E) \cup f^{-1}(F)$
- $f^{-1}(E \cap F) = f^{-1}(E) \cap f^{-1}(F)$

Bonus question: for those above that are false, can you find a necessary and sufficient criterion so that the statements hold?