

K Theory

Labix

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Abstract

- Notes on Algebraic Topology by Oscar Randal-Williams

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1.1

Theorem 1.1.1: Serre-Swan Theorem I

Let M be a smooth manifold. Let E be a smooth vector bundle over M . Then the space of smooth sections $\Gamma(E)$ of E is finitely generated and projective over $C^\infty(M)$.

If M is connected, then the space of smooth section is one-to-one with the finitely generated and projective modules over $C^\infty(M)$.

Theorem 1.1.2

Let M be a smooth and connected manifold. Then the category of smooth vector bundles $\text{SVect}(M)$ is equivalent to the category of finitely generated projective modules $\text{FinProj}_{C^\infty(M)}\text{Mod}$ via the global section functor

$$\Gamma : \text{SVect}(M) \rightarrow \text{FinProj}_{C^\infty(M)}\text{Mod}$$

defined by $E \mapsto \Gamma(E)$