Exercise Sheet for Week 9

Question 1

Let $f : X \to Y$ and $g : Y \to Z$ be functions. Prove the following statements.

- If f and g are injective then $g \circ f$ is injective
- If f and g are surjective then $g \circ f$ is surjective
- If f and g are bijective then $g \circ f$ is bijective.

Question 2

Let $f : X \to Y$ and $g : Y \to Z$ be functions. Prove or find a counter example to the following.

- If $g \circ f$ is injective, then g is injective
- If $g \circ f$ is injective, then f is injective
- If $g \circ f$ is surjective, then g is surjective
- If $g \circ f$ is surjective, then f is surjective

Question 3

Let $f : X \to Y$ and $g : Y \to Z$ be functions such that $g \circ f = \mathrm{id}_X$. Show that f is injective and g is surjective. Conclude that if $f \circ g = \mathrm{id}_Y$, then g is the inverse of f.

Question 4

Let $n \in \mathbb{N}$. Prove that $a \in \mathbb{N}$ is a primitive root of n if and only if $a + n\mathbb{Z}$ is a generator of the group $(\mathbb{Z}/n\mathbb{Z})^\times$.