Riemann Surfaces

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Abstract

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1 Riemann Surfaces

1.1 Basic Notations

Definition 1.1.1 (Complex Chart)**.** Let M be a topological space. We say that $\phi : U \subset M \to \mathbb{C}^n$ is a complex chart on M where U is open and $\phi(U)$ is open if ϕ is a topological homeomorphism.

Since the homeomorphism is bijective, we can assign to each point $p \in U$ a coordinate on $\mathbb{C}^n$, given by the coordinate functions $z_1, \dots, z_n$. The function z_i return the ith coordinate of p. Thus the coordinates in U has representation $\phi(p) = (z_1(p), \dots, z_n(p))$.

Definition 1.1.2 (Complex Atlas)**.** Let M be a topological space. A complex atlas on M is a countable colletion of complex charts $A = \{\phi_i : U_i \subset M \to \mathbb{C}^n | i \in I\}$ and $M \subset \bigcup_{i\in I} U_i$

Definition 1.1.3 (Analytic Atlas)**.** A compelx atlas A is an analytic atlas if the transition maps

$$\phi_j \circ \phi_i^{-1} : \phi(U_i \cap U_j) \to \phi_j(U_i \cap U_j)$$

are holomorphic for all $i, j \in I$.

Definition 1.1.4 (Complex Manifolds)**.** A Hausdorff Space M equipped with a complex analytic atlas A is called a complex manifold.

Definition 1.1.5 (Complex Analytic Chart)**.** A complex chart on a complex manifold M is said to be analytic if it can be added to A without destorying the analyticity of the atlas.

Definition 1.1.6 (Holomorphic Mappings)**.** Let M, N be complex manifolds. A mapping $f : M \to N$ of complex manifolds is said to be holomorphic if in local coordinates it is given by holomorphic functions.

Definition 1.1.7 (Riemann Surface)**.** A Riemann Surface is a connected complex analytic manifold of dimension 1.

1.2 Mappings of Riemann Surfaces