Riemann Surfaces

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Abstract

These notes will act as a an introductory text with a collection of theorems and definitions for differential equations.

Contents

1 Riemann Surfaces as Complex Analytic Manifolds

Definition 1.0.1: Complex Charts

Let M be a topological space. A complex chart on M is a homeomorphism $\phi : U \to \mathbb{C}^n$ of an open subset $U \subset M$ onto $\phi(U) \subset \mathbb{C}^n$. The coordinates on $\mathbb{C}^n$ determine complex valued functions $z_1, \dots, z_n : U \to \mathbb{C}$, called complex coordinates on U.

Definition 1.0.2: Complex Analytic Atlas

A complex atlas on a topological space M is a collection of complex charts

$$\Phi = \{\phi_i : U_i \to \mathbb{C}^{n_i} | i \in I\}$$

such that the collection $\{U_i | i \in I\}$ covers M. We say that the complex atlas is analytic if the transition maps

$$\phi_j \circ \phi_i^{-1} : \phi_i(U_i \cap U_j) \to \phi_j(U_i \cap U_j)$$

are holomorphic for all $i, j \in I$.

Definition 1.0.3: Complex Analytic Manifolds

A topological space M is said to be a complex analytic manifold if

- M is Hausdorff
- M is equipped with a complex analytic atlas Φ

Definition 1.0.4: Holomorphic Mappings of Complex Manifolds

A mapping $f : M \to N$ of complex manifolds is said to be holomorphic if the functions $f_i(z_1, \dots, z_n)$ for $i = 1, \dots, n$ given by coordinate functions on M, which is $z_1, \dots, z_n$, are holomorphic in their domain of definition.

Definition 1.0.5: Dimension

The dimension of a chart $\phi : U \to \mathbb{C}^n$ is the number n. For a connected complex manifold M, the number is independent of the choice of charts and is called the dimension of M.

Definition 1.0.6: Riemann Surfaces

A Riemann surface is a connected complex analytic manifold of dimension 1.