

Abelian Varieties

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Abstract

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1 Properties of Abelian Varieties

1.1 Basic Definitions

Let us start by recalling the definition of an abelian variety in Algebraic Geometry 3.

Definition 1.1.1: Abelian Varieties

An abelian variety over a field k is a group variety that is complete and connected.

Theorem 1.1.2: Rigidity Theorem

Corollary 1.1.3

The group law on any abelian variety is commutative, hence every abelian variety has a the structure of an abelian group.

1.2 Rational Maps into Abelian Varieties

Theorem 1.2.1

Let A be an irreducible abelian variety over k . Then for any non-singular irreducible variety V and rational map $\varphi : V \rightarrow A$, φ extends to a morphism $V \rightarrow A$.

1.3 Abelian Varieties are Projective

Theorem 1.3.1: Abelian Varieties are Projective

Every abelian variety over an algebraically closed field k is projective.

Theorem 1.3.2

Every abelian variety over \mathbb{C} is a compact complex submanifold of $\mathbb{P}^n(\mathbb{C})$.