Machine learning algorithms and statistical models: An example of regularization and cross validation in logistic regression to predict high school dropout in Quebec

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Contents

Abstract: Code of analysis presented in part 2 of chapter 23 : regularized logistic regression

Keywords: Machine learning, regularization, cross-validation

##2.1 Description of sample, research goal and variables

###2.1.1 Sample

The data was initially collected from n = 6773 teenagers from 12 schools where the dropout rate is particularly high, around 36%, as to measure a set of risk factors associated to high school dropout. In total, ten out of twelve schools where located in economically disadvantaged neighbourhoods. A subsample was then invited to complete an interview in order to establish the stressors to which teenagers were exposed. The goal was to interview 45 teenagers by school (for a total of n = 545), following a plan with paired control cases. First, 15 students who had just dropped out of school were interviewed. Then, 15 paired students with a similar initial risk profile, but who were still in school, were also interviewed. Finally, 15 "normative" students, also still in school, with an average risk profile, were interviewed.

###2.1.2 Variables

The dependent variable is a dichotomic variable (coded 0 = no / 1 = yes) representing the fact that a student dropped out of school or not. A student is considered to have dropped out if he fulfills at least one of the following conditions: 1) having officially notified the school of a cessation of studies before obtaining a high school diploma, 2) having been transferred to the adult education system, 3) being absent from school for over a month without notifying the school direction of underlying motivations. For more details on variables and research methodology, see the original article by Dupéré & al. (2018). A specificity in the data structure if that the vast majority of variables are ordinals and were thus recoded in binary dummy variables. In general, the effects of regulation are more important: 1) with interval/ratio variables since they have a larger variance and 2) in presence of multicollinearity. Another characteristic of the data used here is that we are not in context of high dimensions since we have 25 variables for 1000 observations ($p < n$). Lastly, the original study was confirmatory in nature (hypothetical deductive) and not exploratory (inductive), which favours least flexible algorithms like the "classical" logistic regression.

Description of independent variables introduced in the regularized logistic regression model from the study by Dupéré & al. (2018)

Variable name - Variable type - Variable name in the data file

1. Sex - Dichotomous - MALE

2. Age - Interval - AGE
3. Immigrated parent - Dichotomous - PAR_IMM
4. Ethnicity - Dichotomous - MINORITY
5. Parents' education level - Interval - SCOLMAX
6. Employed mother - Dichotomous - TRAVAILM
7. Employed father - Dichotomous - TRAVAILP
8. Separated parents - Dichotomous - PAR_SEP
9. School adaptation - Interval - ADAPT
10. Dropout risk - Interval - SRDQ
11. Severe chronic difficulties - Interval - CHRONSEVACT
12. Severe stressors 0-3 months - Dichotomous - SEVER03DICO
13. Severe stressors 3-6 months - Dichotomous - SEVER36DICO
14. Severe stressors 6-9 months - Dichotomous - SEVER69DICO
15. Severe stressors 9-12 months - Dichotomous - SEVER912DICO
16. Moderate stressors 0-3 months - Dichotomous - MODER203DICO
17. Moderate stressors 3-6 months - Dichotomous - MODER236DICO
18. Moderate stressors 6-9 months - Dichotomous - MODER269DICO
19. Moderate stressors 9-12 months - Dichotomous - MODER2912DICO
20. Mild stressors 0-3 months - Dichotomous - LOW203DICO
21. Mild stressors 3-6 months - Dichotomous - LOW236DICO
22. Mild stressors 6-9 months - Dichotomous - LOW269DICO
23. Mild stressors 9-12 months - Dichotomous - LOW2912DICO
24. Severe distal stressors - Interval/ratio - EVDISTSEV
25. Moderate distal stressors - Interval/ratio - EVDISTMOD

2.2 Analysis' objectives

The main goal of this analysis is to select a model of logistic regression, in an exploratory/inductive manner, by using technics that are specific to machine learning in order to potentially predict high school dropout with the highest predictive accuracy possible from the 25 independent variables. We use data that was simulated from the original sample.

In summary, the classification task consists in finding both the optimal number of predictors of high school dropout and the algorithm of regularization that will give the best model adjustment to the data, considering the specificity of the variables introduced in this model.

##2.3 & 2.4 Regularized logistic regression: model and procedures + interpretations and result tables

The rest of this document presents the code used (originally in RStudio) for model selection according to the procedures described in section 2.3 and the results presented in section 2.4 of the chapter.

```
#Download packages for analysis (if installing packages for the first time, remove # at the beginning o

#install.packages("CUFF") #Package CUFF (Charles's Utility Function using Formula) for variable present
#install.packages ("dplyr") #Package dplyr for data manipulation
#install.packages("ggplot2")#Package ggplot2 for graphic creation
#install.packages("haven") #Package haven for data importation
#install.packages("knitr") #Package knitr for table production
#install.packages("xtable") #Package xtable for table production
#install.packages("pairwise") #Package xtable for table production

require(dplyr, quietly = TRUE, warn.conflicts = FALSE)
require(ggplot2, quietly = TRUE, warn.conflicts = FALSE)
require(CUFF, quietly = TRUE, warn.conflicts = FALSE)
require(haven, quietly = TRUE, warn.conflicts = FALSE)
```

```r
require(knitr, quietly = TRUE, warn.conflicts = FALSE)
require(xtable, quietly = TRUE, warn.conflicts = FALSE)
require(pairwise, quietly = TRUE, warn.conflicts = FALSE)

opts_chunk$set(echo = TRUE, prompt = TRUE, comment = "", cache = TRUE)
options(xtable.comment = FALSE)

#install.packages("glmnet", quietly = TRUE, warn.conflicts = FALSE, dependencies = TRUE)
#install.packages("latex2exp", quietly = TRUE, warn.conflicts = FALSE, dependencies = TRUE)

require(glmnet)
```

```
## Loading required package: glmnet

## Loading required package: Matrix

## Loaded glmnet 4.0-2
```

```r
require(latex2exp)
```

```
## Loading required package: latex2exp
```

```r
> #Download data from github
> SD.csv <- "https://github.com/Labo-Lacourse/Code_chap_23_logistic_regression_regularization.git"
>
> #Read data
> library(readr)
> SD.df <- read.csv("SD.csv")
> ls(SD.df)
```

```
 [1] "ADAPT"        "AGE"          "AIMES"        "AMB"
 [5] "CHRONSEVACT"  "DOUBLE"       "ECOLE"        "EVDISTMOD"
 [9] "EVDISTSEV"    "ID"           "IMP"          "LOW203DICO"
[13] "LOW236DICO"   "LOW269DICO"   "LOW2912DICO"  "MALE"
[17] "MINORITY"     "MODER203DICO" "MODER236DICO" "MODER269DICO"
[21] "MODER2912DICO" "NOTES_FR"    "NOTES_MATH"   "PAR_IMM"
[25] "PAR_SEP"      "PR_AUTRES"    "SCOLMAX"      "SEVER03DICO"
[29] "SEVER36DICO"  "SEVER69DICO"  "SEVER912DICO" "SRDQ"
[33] "STATUT"       "TRAVAILM"     "TRAVAILP"     "X"
```

Models development from the training sample

We seek to avoid under- and over-adjustment of the model, meaning that we get an excellent classification with the training set, but a bad classification with new data. A solution to avoid this problem is to randomly divide the sample in two parts, 70% for the training data and 30% for the testing data (new data) This proportion is arbitrary, but the idea behind it is to keep the training set as large as possible to develop a good model while leaving enough testing data to validate it.

Thus, the sample is separated in two subsamples: 1. Training set (TRAIN.df ; N = 700) 2. Testing set (TEST.df ; N = 300)

```
> #Divide the data to create a training subsample and a testing subsample
> set.seed(1234)
> ECH.TRAIN <- sample(1:1000, 700)
> TRAIN.df <- SD.df[ECH.TRAIN,]
> TEST.df <- SD.df[-ECH.TRAIN,]
```

The variables of both data files are centered and standardized except the three first variables: ID, ECOLE (school), and STATUT.

```
> #Variables' standardization
> TRAIN.df[,-(2:4)]  <- scale(TRAIN.df[,-(2:4)])
> TEST.df[,-(2:4)]  <- scale(TEST.df[,-(2:4)])
>
> TRAIN.df <- as.data.frame(TRAIN.df)
> TEST.df <- as.data.frame(TEST.df)
```

```
> # Read the training data file to check the creation of the subsample
> ls (TRAIN.df)
```

```
 [1] "ADAPT"        "AGE"          "AIMES"        "AMB"
 [5] "CHRONSEVACT"  "DOUBLE"       "ECOLE"        "EVDISTMOD"
 [9] "EVDISTSEV"    "ID"           "IMP"          "LOW203DICO"
[13] "LOW236DICO"   "LOW269DICO"   "LOW2912DICO"  "MALE"
[17] "MINORITY"     "MODER203DICO" "MODER236DICO" "MODER269DICO"
[21] "MODER2912DICO" "NOTES_FR"    "NOTES_MATH"   "PAR_IMM"
[25] "PAR_SEP"      "PR_AUTRES"    "SCOLMAX"      "SEVER03DICO"
[29] "SEVER36DICO"  "SEVER69DICO"  "SEVER912DICO" "SRDQ"
[33] "STATUT"       "TRAVAILM"     "TRAVAILP"     "X"
```

```
> # Read the testing data file to check the creation of the subsample
> ls (TEST.df)
```

```
 [1] "ADAPT"        "AGE"          "AIMES"        "AMB"
 [5] "CHRONSEVACT"  "DOUBLE"       "ECOLE"        "EVDISTMOD"
 [9] "EVDISTSEV"    "ID"           "IMP"          "LOW203DICO"
[13] "LOW236DICO"   "LOW269DICO"   "LOW2912DICO"  "MALE"
[17] "MINORITY"     "MODER203DICO" "MODER236DICO" "MODER269DICO"
[21] "MODER2912DICO" "NOTES_FR"    "NOTES_MATH"   "PAR_IMM"
[25] "PAR_SEP"      "PR_AUTRES"    "SCOLMAX"      "SEVER03DICO"
[29] "SEVER36DICO"  "SEVER69DICO"  "SEVER912DICO" "SRDQ"
[33] "STATUT"       "TRAVAILM"     "TRAVAILP"     "X"
```

Next the training sample is randomly divided into ten subsamples of 70 units each to allow for 10-fold cross validation and selection of the alpha and lambda parameter for regularization.

```
> #The partitions and crossval function are built for cross validation procedures
> #Division of training data into 10 subsamples of 70 units
> PARTITION = sample(rep(1:10, rep(70,10)),700)
>
> crossval <- function(mod){
+   f1 <- function(x){
+     modi = update(mod, data = TRAIN.df[!(PARTITION %in% x),])
```

```
+     table(1*(predict(modi, newdata = TRAIN.df[PARTITION %in% x,],
+                      type = "resp")>0.5),
+           TRAIN.df[(PARTITION %in% x),"STATUT"])
+   }
+   CVT <- mapply(f1, x = 1:10)
+   as.table(matrix(apply(CVT, 1, sum), 2, 2,
+                   dimnames = list(c("P.ND","P.D"),
+                                   c("T.ND","T.D"))))
+ }
```

A good way to start the analysis is to estimate a classical logistic regression model using the 25 predictive variables.

```
> #Classical logistic regression model is adjusted to the training data
> var.model <- c("MALE", "AGE", "PAR_IMM", "MINORITY", "SCOLMAX", "TRAVAILM", "TRAVAILP", "PAR_SEP", "AI
+                "SRDQ", "EVDISTSEV", "EVDISTMOD","SEVER03DICO",
+                "SEVER36DICO", "SEVER69DICO", "SEVER912DICO",
+                "MODER203DICO", "MODER236DICO", "MODER269DICO",
+                "MODER2912DICO", "LOW203DICO", "LOW236DICO",
+                "LOW269DICO", "LOW2912DICO", "CHRONSEVACT")
> glm1 <- glmnet(x = TRAIN.df[, var.model] %>% as.matrix, y = TRAIN.df[,"STATUT"], lambda=0, family = "I
```

```
> #Visualize results
> print(glm1)
```

```
Call:  glmnet(x = TRAIN.df[, var.model] %>% as.matrix, y = TRAIN.df[,      "STATUT"], family = "binomial

  Df  %Dev Lambda
1 25 16.07      0
```

```
> predict(glm1, type="coef", "lambda.min", allCoef = TRUE)
```

```
26 x 1 sparse Matrix of class "dgCMatrix"
                       s0
(Intercept)    0.01223880
MALE           0.11619607
AGE            0.37832290
PAR_IMM       -0.02354991
MINORITY       0.01117619
SCOLMAX        0.28359589
TRAVAILM       0.11628193
TRAVAILP      -0.10514515
PAR_SEP        0.54567272
ADAPT         -0.12379524
SRDQ          -0.04120007
EVDISTSEV      0.26398912
EVDISTMOD     -0.49879199
SEVER03DICO    0.43484789
SEVER36DICO   -0.06016151
SEVER69DICO   -0.45572581
SEVER912DICO  -0.01289725
MODER203DICO  -0.36817345
```

```
MODER236DICO   0.36277321
MODER269DICO   0.44343925
MODER2912DICO  0.09907921
LOW203DICO     0.25546687
LOW236DICO     0.12153723
LOW269DICO    -0.23443896
LOW2912DICO   -0.17378213
CHRONSEVACT    0.60983017
```

```
> #Prediction with classical logistic regression
> glm1p <- predict(glm1, newx = TRAIN.df[,var.model] %>%
+                      as.matrix, s = "lambda.min")
```

```
> #Confusion matrix showing the model accuracy (frequency then percent)
> cv0 <- table(1*(glm1p>0), TRAIN.df$STATUT)
> cv0
```

```
      0   1
  0 245 105
  1 104 246
```

```
> prop.table(cv0)*100
```

```
           0        1
  0 35.00000 15.00000
  1 14.85714 35.14286
```

```
> sprintf("%.1f%% de bonne classification", sum(diag(prop.table(cv0)))*100)
```

```
[1] "70.1% de bonne classification"
```

We can see that the model has a predictive accuracy of 70,1% when using the training data set for the classical logistic regression model.

Now, let's examine regularization models.

##Regularized regression

Regularization or penalization consists in the addition of a penalty to the least square regression model to estimate the coefficients. Essentially, it estimates the coefficient while adding a penalty so as to reduce de regression dimension. A least square regression model has p parameters to estimate (number of variables), whereas a regularized regression has a number of degrees of liberty inferior to p, making to model more parsimonious. To estimate the coefficients of a regularized model, we use the following objective function:

$$RSS_{shrinkage} = (Y - BX) + \lambda f(B)$$

We use three methods corresponding to three different penalties:

1. The ridge regularization method uses a quadratic penalty

$$RSS_{shrinkage} = (Y - BX) + \lambda \sum_{i=1}^{p} \beta_i^2$$

2. The LASSO regularization method uses an absolute value penalty This penalty makes it so that if a coefficient if at 0 for a given λ, it is fozed at 0 for all $\lambda* > \lambda$.

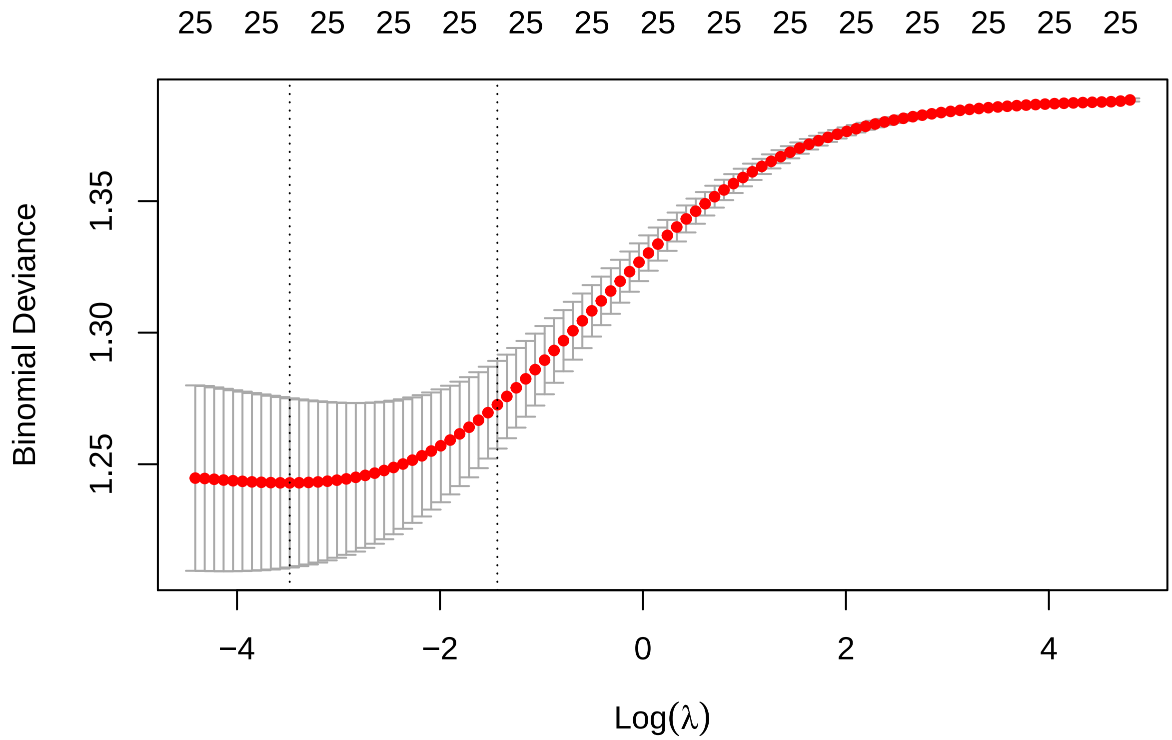
$$RSS_{shrinkage} = (Y - BX) + \lambda \sum_{i=1}^{p} |\beta_i|$$

3.The elastic-net regularization method uses a mix of the two last penalties. This method introduces a new parameter (α) to estimate in the model. By using this parameter, we end with $\alpha = 0$ corresponding to ridge regression, whereas $\alpha = 1$ corresponds to a LASSO regression. An alpha between those two values produces a mix of the two penalties.

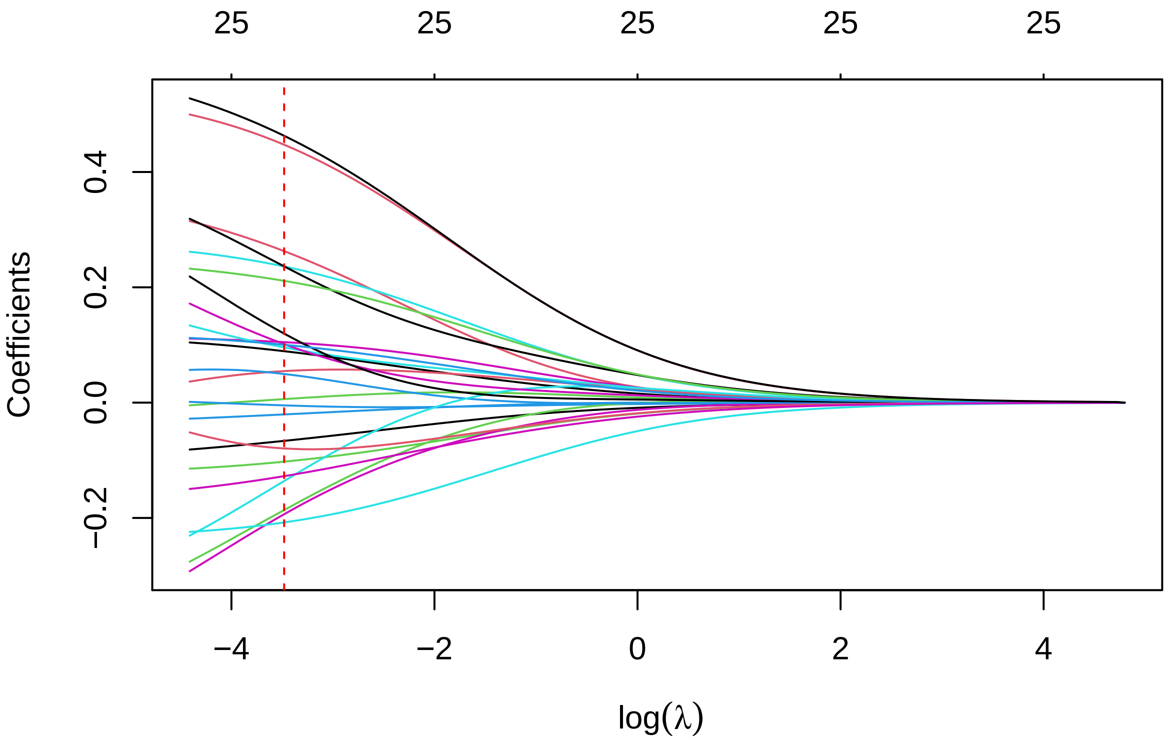
$$RSS_{shrinkage} = (Y - BX) + +(1 - \alpha)(\sum_{i=1}^{p} \beta_i^2) + (\alpha)(\lambda \sum_{i=1}^{p} |\beta_i|)$$

Let's start by trying ridge regularization.

```
> ##Logistic regression with ridge regularization
> #Selection of lambda with 10-fold cross validation
>
> var.model <- c("MALE", "AGE", "PAR_IMM", "MINORITY", "SCOLMAX", "TRAVAILM", "TRAVAILP", "PAR_SEP", "AB
+                 "SRDQ", "EVDISTSEV", "EVDISTMOD","SEVER03DICO",
+                 "SEVER36DICO", "SEVER69DICO", "SEVER912DICO",
+                 "MODER203DICO", "MODER236DICO", "MODER269DICO",
+                 "MODER2912DICO", "LOW203DICO", "LOW236DICO",
+                 "LOW269DICO", "LOW2912DICO", "CHRONSEVACT")
>
> cv.glmn1 <- cv.glmnet(x= TRAIN.df[,var.model] %>% as.matrix,
+                       y = TRAIN.df[,"STATUT"], alpha = 0, nfolds = 10,
+                       foldid = PARTITION, intercept= TRUE,
+                       family = "binomial", standardize = TRUE)
>
> #Visualize results (the line shows the optimal lambda value)
> plot(cv.glmn1)
```




```
> ##Logistic regression with ridge regularization
> glmn1.0 <- glmnet(x = TRAIN.df[, var.model] %>% as.matrix,
+                   y = TRAIN.df[,"STATUT"], alpha = 0, family = "binomial")
>
> #Visualize results: coefficient evolution depending on the selected lambda
> plot(glmn1.0, xvar = "lambda", label = FALSE, xlab = ~ log(lambda))
> abline( v = log(cv.glmn1$lambda.min), col = "red", lty = 2)
```



Cross-validation was applied to find the lambda parameter. We make a prediction based on this model, but generalization of the prediction based on the model will be confirmed with the testing data set later on.

```
> #Prediction from logistic regression with ridge regularization
> glmn1p <- predict(cv.glmn1, newx = TRAIN.df[,var.model] %>%
+                     as.matrix, s = "lambda.min")
```

```
> ##Confusion matrix showing the model accuracy (frequency then percent)
> cv2 <- table(1*(glmn1p>0), TRAIN.df$STATUT)
> cv2
```

```
      0   1
  0 243 100
  1 106 251
```

```
> prop.table(cv2)*100
```

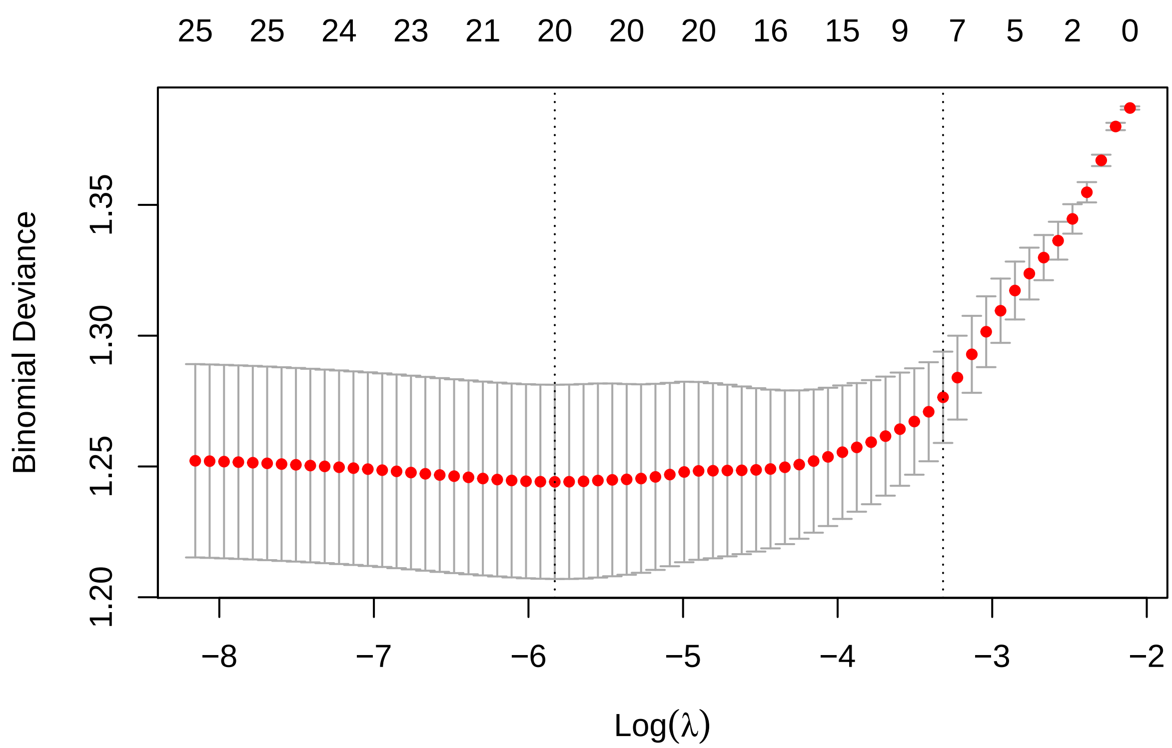
```
           0        1
  0 34.71429 14.28571
  1 15.14286 35.85714
```

```
> sprintf("%.1f%% de bonne classification", sum(diag(prop.table(cv2)))*100)
```

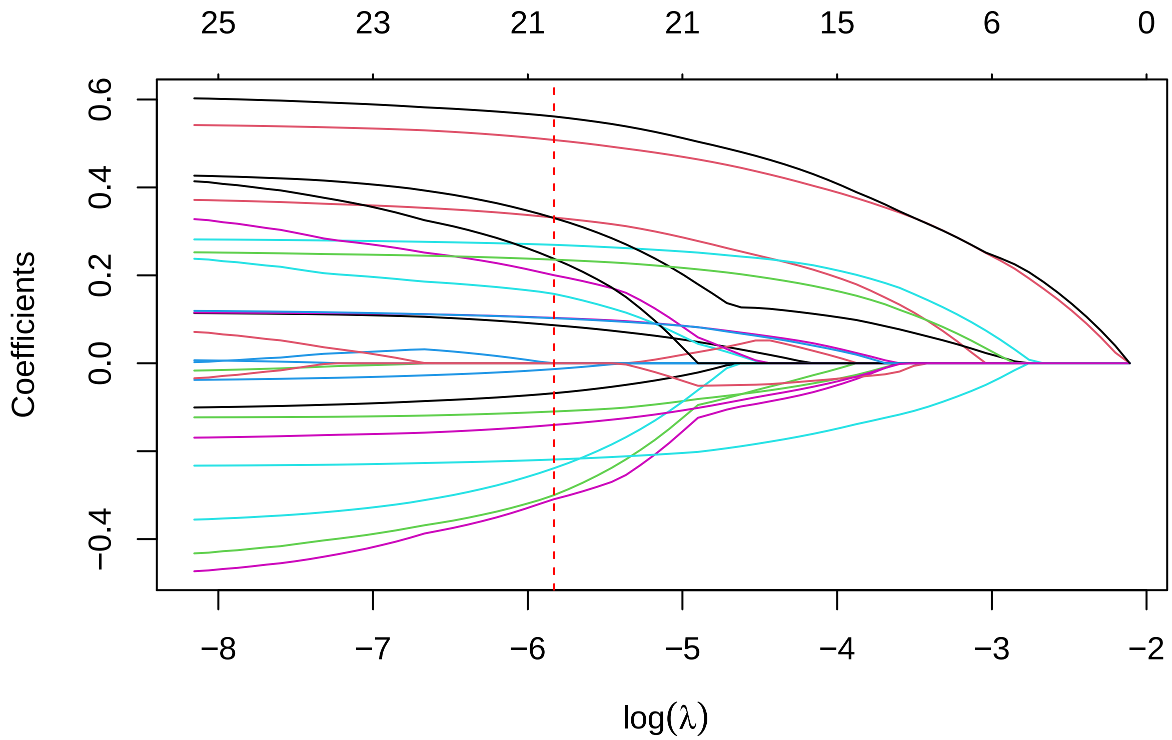
```
[1] "70.6% de bonne classification"
```

We now go to lasso regularization

```r
> ##Logistic regression with lasso regularization
> #Selection of lambda with 10-fold cross validation
> cv.glmn2 <- cv.glmnet(x = TRAIN.df[,var.model] %>% as.matrix,
+                       y = TRAIN.df[,"STATUT"], alpha = 1, nfolds = 10,
+                       foldid = PARTITION, family = "binomial")
>
> glmn2 <- glmnet(x = TRAIN.df[,var.model] %>%
+                   as.matrix, y = TRAIN.df[,"STATUT"], alpha = 1, family = "binomial",
+                   lambda = cv.glmn2$lambda.min)
>
> #Visualize results (the line shows the optimal lambda value)
> plot(cv.glmn2)
```




```r
> ##Logistic regression with lasso regularization
> glmn2.0 <- glmnet(x = TRAIN.df[,var.model] %>% as.matrix,
+                   y = TRAIN.df[,"STATUT"], alpha = 1, family = "binomial")
>
> #Visualize results: coefficient evolution depending on the selected lambda
> plot(glmn2.0, xvar = "lambda", label = FALSE, xlab = ~log(lambda))
> abline(v = log(cv.glmn2$lambda.min), lty = 2, col = "red")
```



Cross-validation was applied to find the lambda parameter. We make a prediction based on this model, but generalization of the prediction based on the model will be confirmed with the testing data set later on.

```
> #Prediction from logistic regression with lasso regularization
> glmn2p <- predict(cv.glmn2, newx = TRAIN.df[,var.model] %>%
+                    as.matrix, s = "lambda.min")
```

```
> #Confusion matrix showing the model accuracy (frequency then percent)
> cv3 <- table(1*(glmn2p>0), TRAIN.df$STATUT)
> cv3
```

```
      0   1
  0 242 104
  1 107 247
```

```
> prop.table(cv3)*100
```

```
           0        1
  0 34.57143 14.85714
  1 15.28571 35.28571
```

```
> sprintf("%.1f%% de bonne classification", sum(diag(prop.table(cv3)))*100)
```

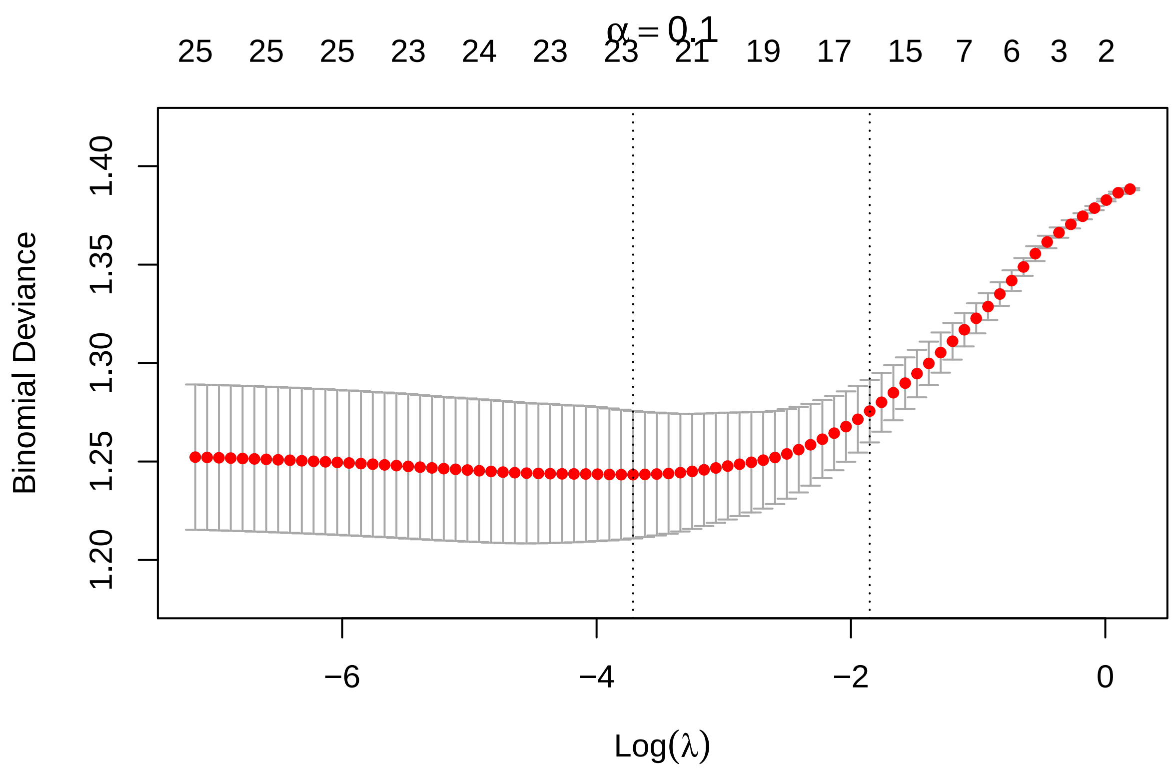
```
[1] "69.9% de bonne classification"
```

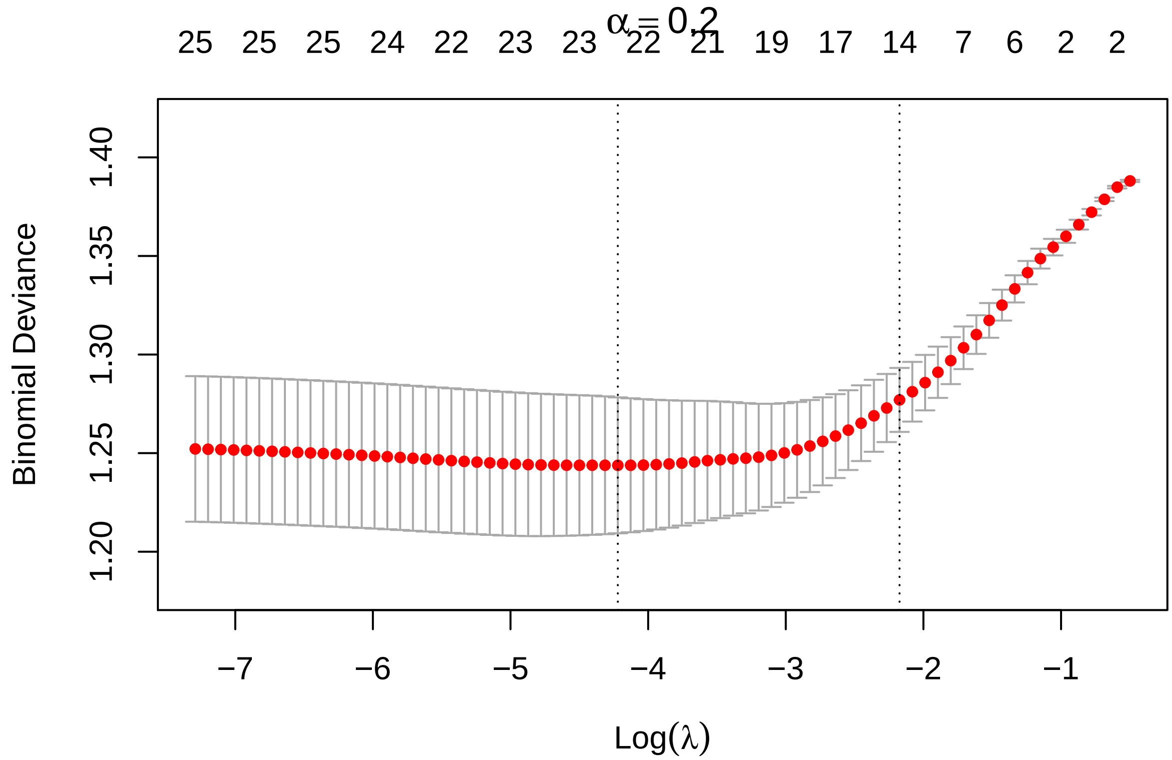
We now complete by trying elastic-net regularization, combining the two previous methods. For this last section, we want to find a compromise between ridge and lasso regression. Thus, we need to estimate a new parameter ($\alpha \in (0, 1)$).

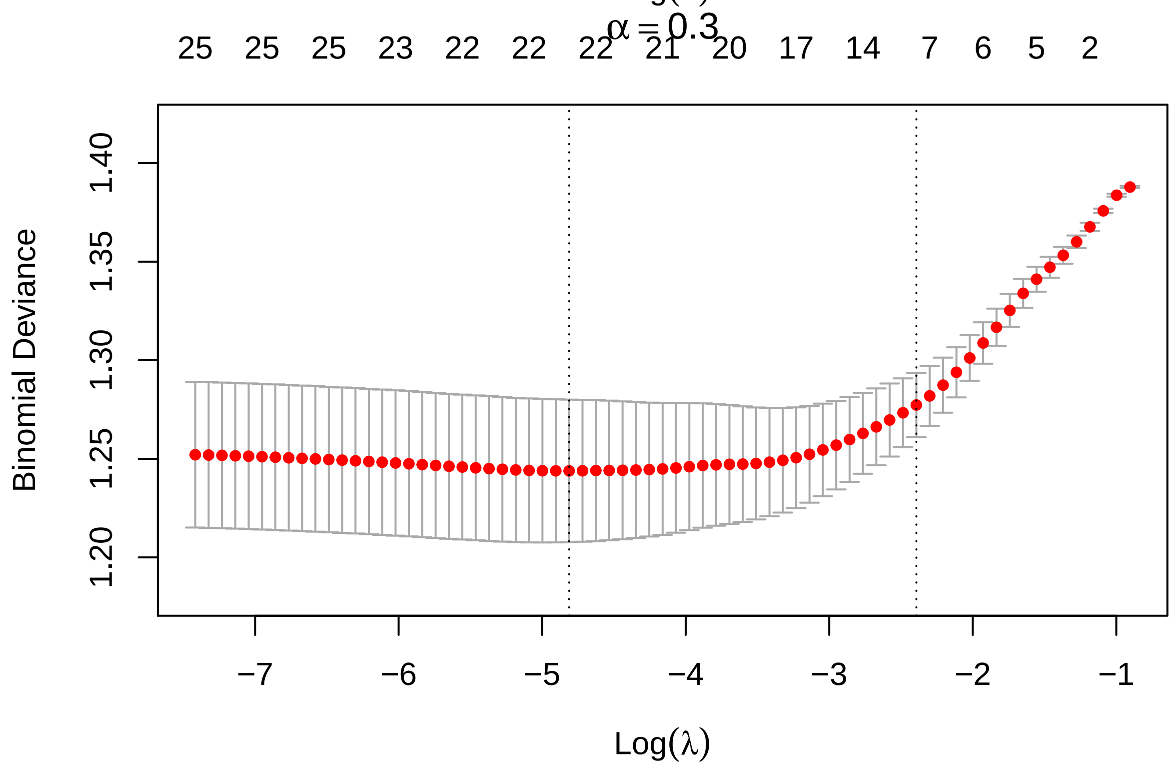
```
> #Matrix adjustment
> layout(matrix(1:10,3,3, byrow = TRUE))
```

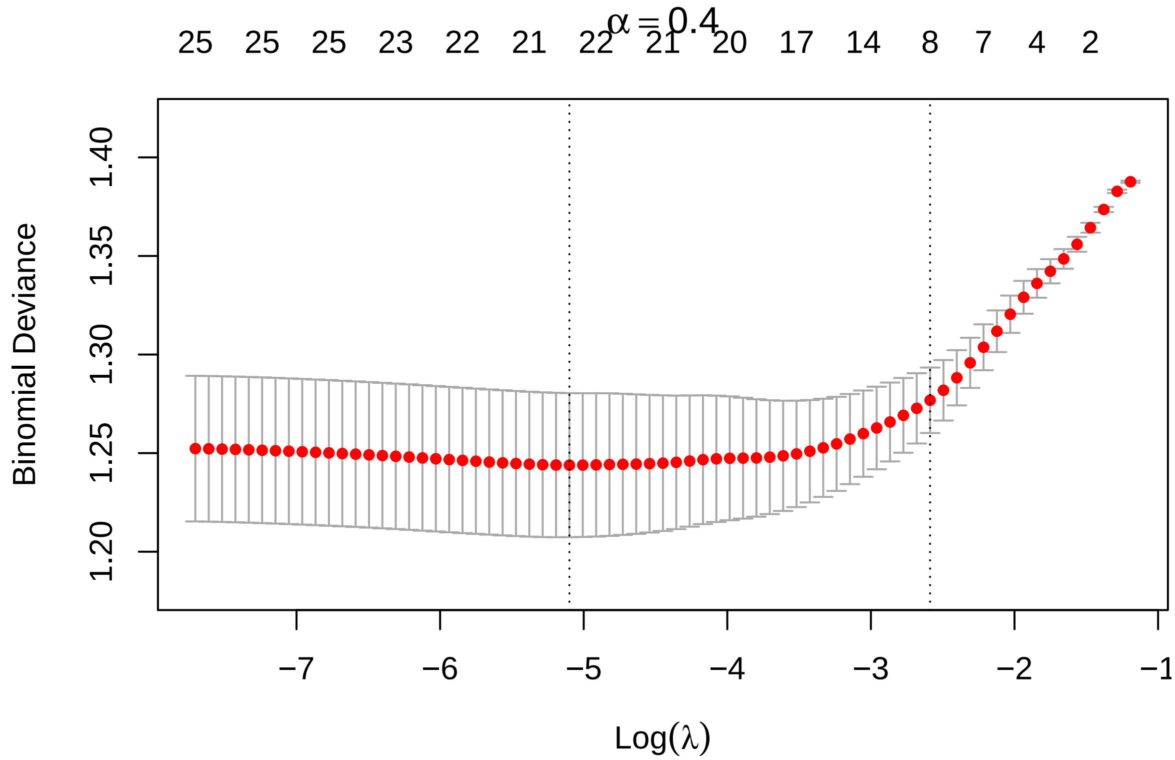
```
Warning in matrix(1:10, 3, 3, byrow = TRUE): data length [10] is not a sub-
multiple or multiple of the number of rows [3]
```

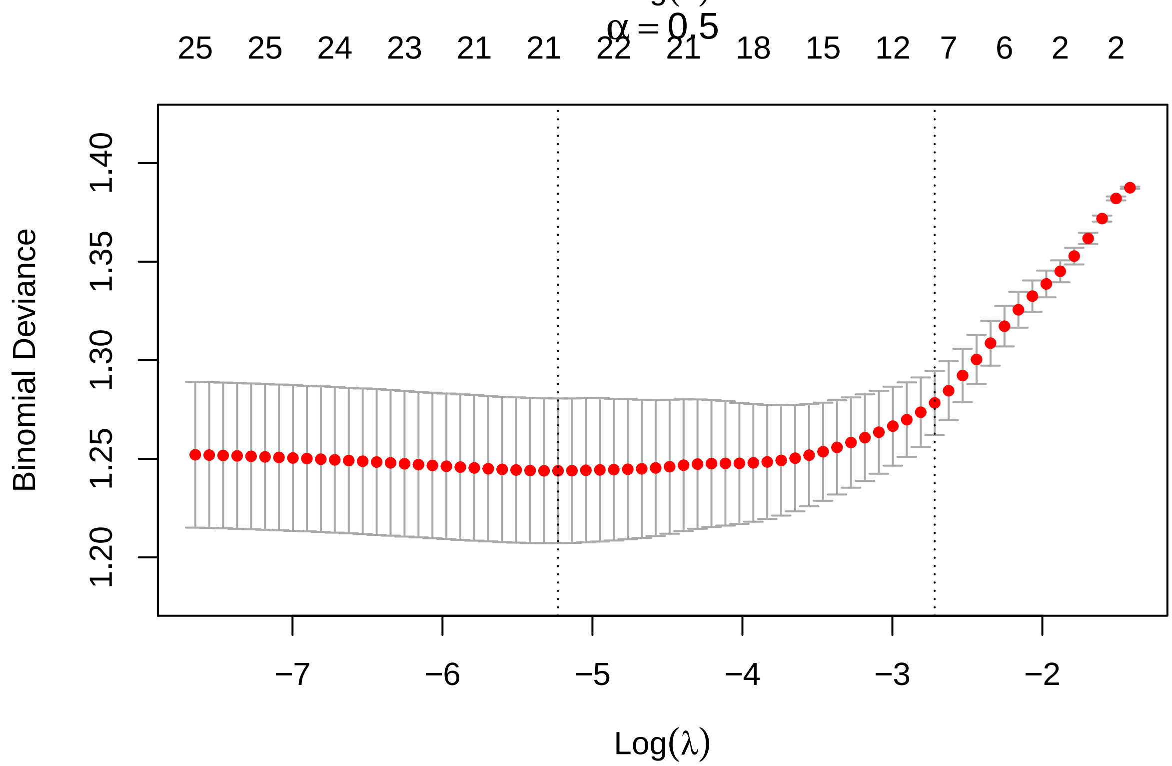
```
> #Selection of alpha with 10-fold cross validation
> cv.glmn3 <- list()
>
> for(al in seq(0.1,0.9,0.1)){
+     cv.glmn3[[sprintf("%.1f",al)]] <-
+     cv.glmnet(x = TRAIN.df[,var.model] %>% as.matrix,
+     y = TRAIN.df[,"STATUT"], nfolds = 10, foldid = PARTITION,
+     alpha = al, family = "binomial")
+ plot(cv.glmn3[[sprintf("%.1f",al)]],
+ main = latex2exp::TeX(sprintf("$\\alpha = %.1f$",al)), ylim = c(1.18, 1.42))
+ }
```

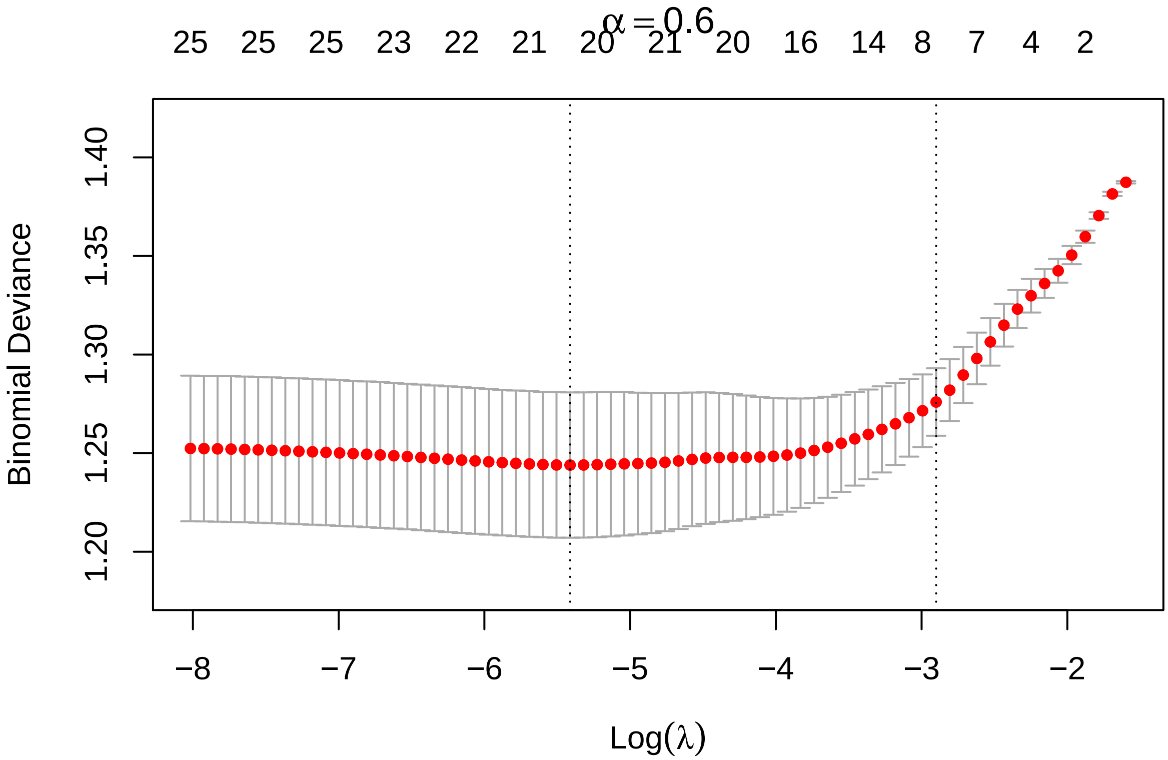


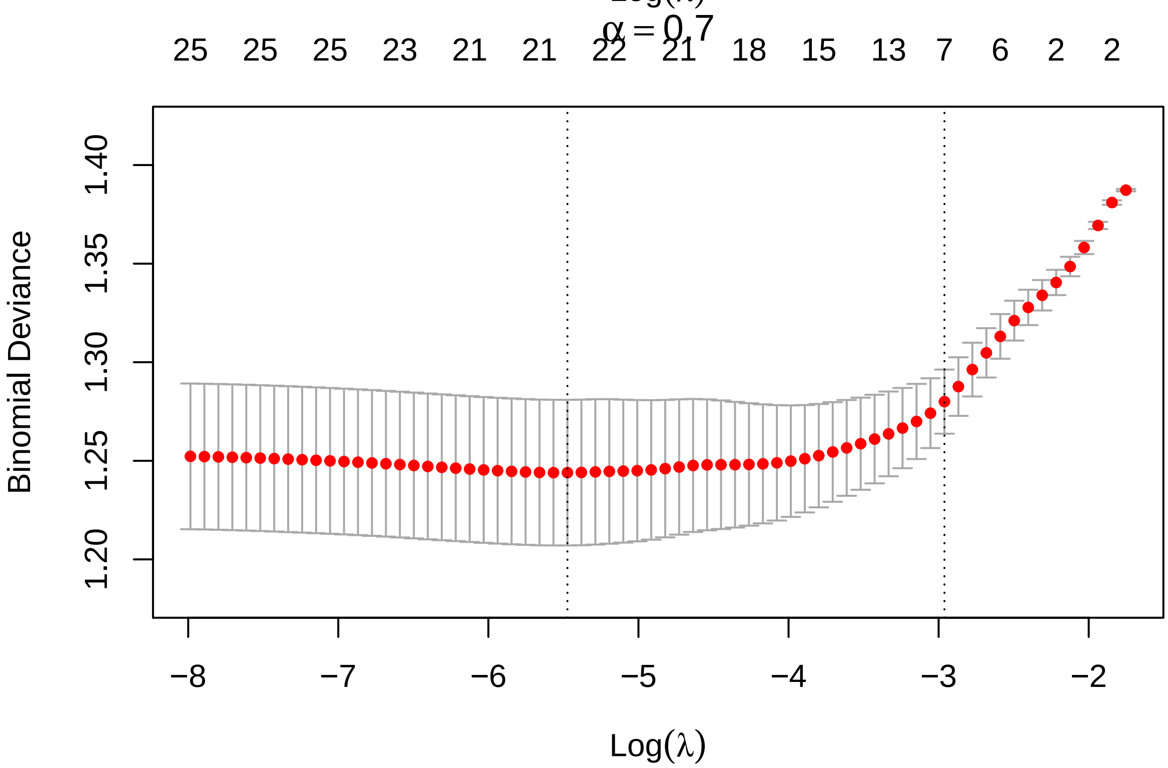

α = 0.2
25 25 25 24 22 23 23 22 21 19 17 14 7 6 2 2
Binomial Deviance
1.20
1.25
1.30
1.35
1.40
−7 −6 −5 −4 −3 −2 −1
Log(λ)

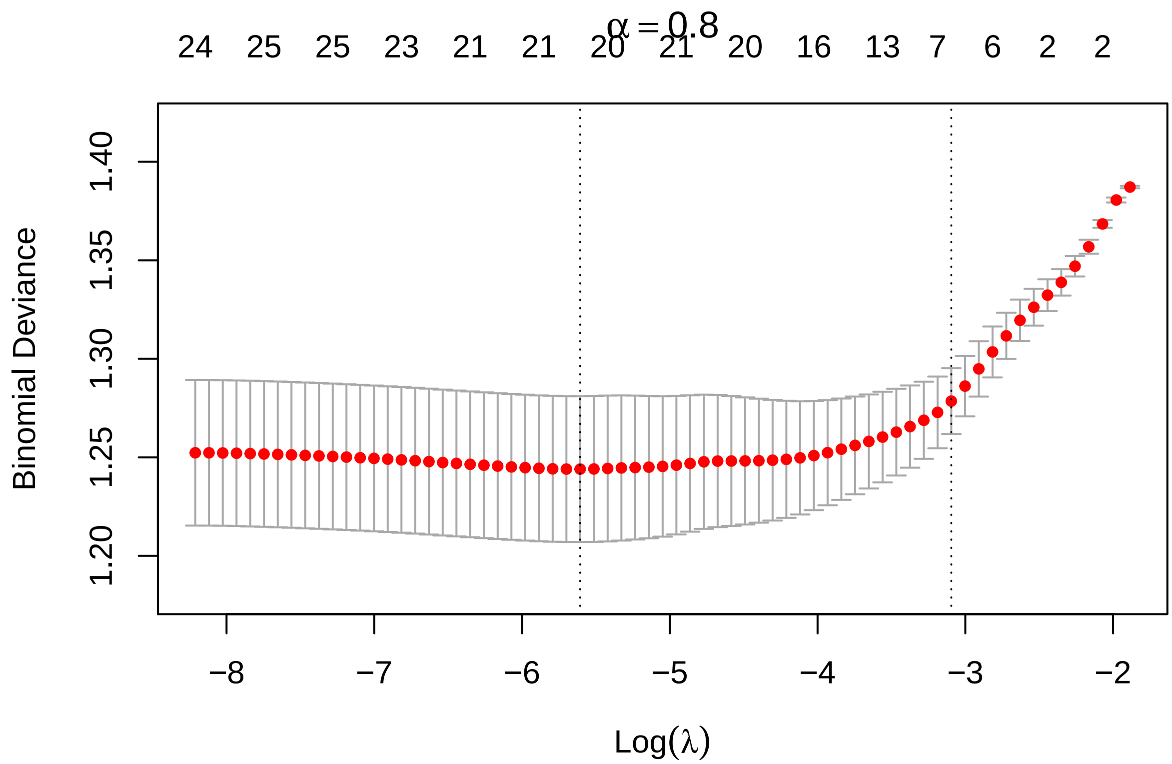

α = 0.3
25 25 25 23 22 22 22 21 20 17 14 7 6 5 2
Binomial Deviance
1.20
1.25
1.30
1.35
1.40
−7 −6 −5 −4 −3 −2 −1
Log(λ)

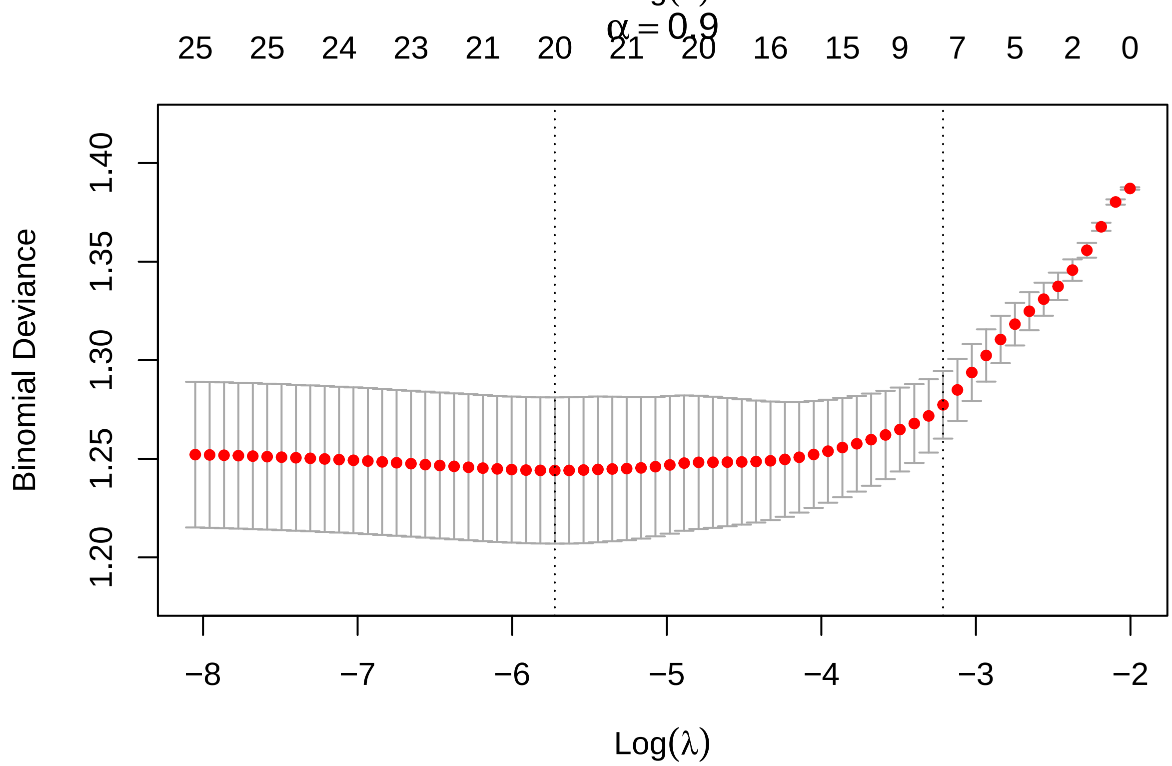

α = 0.4
25 25 25 23 22 21 22 21 20 17 14 8 7 4 2
Binomial Deviance
1.20
1.25
1.30
1.35
1.40
−7 −6 −5 −4 −3 −2 −1
Log(λ)


α = 0.5
25 25 24 23 21 21 22 21 18 15 12 7 6 2 2
Binomial Deviance
1.20
1.25
1.30
1.35
1.40
−7 −6 −5 −4 −3 −2
Log(λ)


α = 0.6
25 25 25 23 22 21 20 21 20 16 14 8 7 4 2
Binomial Deviance
1.20 1.25 1.30 1.35 1.40
−8 −7 −6 −5 −4 −3 −2
Log(λ)


α = 0.7
25 25 25 23 21 21 22 21 18 15 13 7 6 2 2
Binomial Deviance
1.20 1.25 1.30 1.35 1.40
−8 −7 −6 −5 −4 −3 −2
Log(λ)






```
> #Summary: optimal lambda for each value of alpha
> layout (1)
> lapply(cv.glmn3, function(x) c(x$cvm[x$lambda == x$lambda.min],
+ + x$cvsd[x$lambda == x$lambda.min]))
```

```
$`0.1`
```

```
[1] 1.24334596 0.03240747

$`0.2`
[1] 1.24380465 0.03446807

$`0.3`
[1] 1.24387059 0.03614771

$`0.4`
[1] 1.24390553 0.03652575

$`0.5`
[1] 1.24393115 0.03668735

$`0.6`
[1] 1.24394632 0.03684874

$`0.7`
[1] 1.24399564 0.03693823

$`0.8`
[1] 1.24402662 0.03701833

$`0.9`
[1] 1.24406738 0.03707612
```

```
> ##Logistic regression with elastic-net regularization
> glmn3 <- glmnet(x = TRAIN.df[,var.model] %>% as.matrix,
+            y = TRAIN.df[,"STATUT"], alpha = 0.1, family = "binomial",
+            lambda = cv.glmn3[[9]]$lambda.min)
```

```
> #Prediction from logistic regression with elastic-net regularization
> glmn3p <- predict(cv.glmn3[[9]], newx = TRAIN.df[,var.model] %>% as.matrix)
```

```
> #Confusion matrix showing the model accuracy (frequency then percent)
> cv4 <- table(1*(glmn3p>0), TRAIN.df$STATUT)
> cv4
```

```

      0   1
  0 240 117
  1 109 234
```

```
> prop.table(cv4)*100
```

```

           0        1
  0 34.28571 16.71429
  1 15.57143 33.42857
```

```
> sprintf("%.1f%% de bonne classification", sum(diag(prop.table(cv4)))*100)
```

```
[1] "67.7% de bonne classification"
```

Validating models with the testing set

We now take the testing sample and validate our prediction based on the models built with the training sample.

```
> #Prediction with testing data
> glm1tp <- predict(glm1, newx = TEST.df[,var.model] %>% as.matrix, s = "lambda.min")
> glmn1tp <- predict(cv.glmn1, newx = TEST.df[,var.model] %>% as.matrix, s = "lambda.min")
> glmn2tp <- predict(cv.glmn2, newx = TEST.df[,var.model] %>% as.matrix, s = "lambda.min")
> glmn3tp <- predict(cv.glmn3[[9]], newx = TEST.df[,var.model] %>%as.matrix, s = "lambda.min")
```

```
> #Confusion matrix showing accuracy of the classical logistic regression model on testing data
> cvt1 <- table(1*(glm1tp>0), TEST.df$STATUT)
> cvt1
```

```
     0   1
  0 100  49
  1  51 100
```

```
> prop.table(cvt1)*100
```

```
           0        1
  0 33.33333 16.33333
  1 17.00000 33.33333
```

```
> sprintf("%.1f%% de bonne classification", sum(diag(prop.table(cvt1)))*100)
```

```
[1] "66.7% de bonne classification"
```

```
> #Confusion matrix showing accuracy of the  logistic regression with ridge regularization model on tes
> cvt2 <- table(1*(glm1tp>0), TEST.df$STATUT)
> cvt2
```

```
     0   1
  0 100  49
  1  51 100
```

```
> prop.table(cvt2)*100
```

```
           0        1
  0 33.33333 16.33333
  1 17.00000 33.33333
```

```
> sprintf("%.1f%% de bonne classification", sum(diag(prop.table(cvt2)))*100)
```

```
[1] "66.7% de bonne classification"
```

```
> #Confusion matrix showing accuracy of the  logistic regression with lasso regularization model on tes
> cvt3 <- table(1*(glmn2tp>0), TEST.df$STATUT)
> cvt3
```

```
     0   1
  0 101  49
  1  50 100
```

```
> prop.table(cvt3)*100
```

```
           0        1
  0 33.66667 16.33333
  1 16.66667 33.33333
```

```
> sprintf("%.1f%% de bonne classification", sum(diag(prop.table(cvt3)))*100)
```

```
[1] "67.0% de bonne classification"
```

```
> #Confusion matrix showing accuracy of the  logistic regression with elastic-net regularization model
> cvt4 <- table(1*(glmn3tp>0), TEST.df$STATUT)
> cvt4
```

```
     0   1
  0 101  49
  1  50 100
```

```
> prop.table(cvt4)*100
```

```
           0        1
  0 33.66667 16.33333
  1 16.66667 33.33333
```

```
> sprintf("%.1f%% de bonne classification", sum(diag(prop.table(cvt4)))*100)
```

```
[1] "67.0% de bonne classification"
```

```
> #Bootstrap
> set.seed(1234)
> good.class <- function(model, i ){
+             if("glm" %in% class(model)){
```

```
+              glm1tp <- predict(glm1, newdata = TEST.df[i,])
+              cvt1 <- table(1*(glm1tp>0), TEST.df$STATUT[i])
+              sum(diag(prop.table(cvt1)))*100
+ }
+   else {glmn3tp <- predict(model,
+                           newx = TEST.df[i,var.model] %>% as.matrix, s = "lambda.min")
+   (cvt4 <- table(1*(glmn3tp>0), TEST.df$STATUT)) %>%
+ sum(diag(prop.table(cvt4)))*100
+   }
+ }
>
> sd(replicate(1000,good.class(glm1, sample(1:300, 300, TRUE))))
```

```
[1] 2.860233
```

```
> # Comparison table of the four models
> length(drop(coef(cv.glmn3[[6]],
+        s = "lambda.min",allCoef = TRUE)))
```

```
[1] 26
```

```
> coef(cv.glmn3[[6]], s = "lambda.min",allCoef = TRUE)
```

```
26 x 1 sparse Matrix of class "dgCMatrix"
                        1
(Intercept)    0.01013546
MALE           0.08685561
AGE            0.32715857
PAR_IMM        .
MINORITY       .
SCOLMAX        0.26739839
TRAVAILM       0.10375917
TRAVAILP      -0.06856472
PAR_SEP        0.50403987
ADAPT         -0.10908915
SRDQ          -0.01417881
EVDISTSEV      0.15589798
EVDISTMOD     -0.30191322
SEVER03DICO    0.32154633
SEVER36DICO    .
SEVER69DICO   -0.29121185
SEVER912DICO   .
MODER203DICO  -0.22970279
MODER236DICO   0.19622058
MODER269DICO   0.22864685
MODER2912DICO  .
LOW203DICO     0.23449993
LOW236DICO     0.10313554
LOW269DICO    -0.21882702
LOW2912DICO   -0.13987804
CHRONSEVACT    0.55478994
```

```
> cf <- data.frame(VAR = c("Int.", var.model),
+                  OLS = drop(coef(glm1, s = "lambda.min",allCoef = TRUE)),
+                  RIDGE = drop(coef(cv.glmn1,s = "lambda.min",allCoef = TRUE)),
+                  LASSO = drop(coef(cv.glmn2,s = "lambda.min",allCoef = TRUE)),
+                  `ELASTIC NET` = drop(coef(cv.glmn3[[6]],
+                                         s = "lambda.min",allCoef = TRUE)))
> kable(cf, digits = 2,row.names = FALSE)
```

VAR	OLS	RIDGE	LASSO	ELASTIC.NET
Int.	0.01	0.01	0.01	0.01
MALE	0.12	0.09	0.09	0.09
AGE	0.38	0.26	0.33	0.33
PAR_IMM	-0.02	0.01	0.00	0.00
MINORITY	0.01	0.00	0.00	0.00
SCOLMAX	0.28	0.24	0.27	0.27
TRAVAILM	0.12	0.10	0.10	0.10
TRAVAILP	-0.11	-0.07	-0.07	-0.07
PAR_SEP	0.55	0.45	0.51	0.50
ADAPT	-0.12	-0.10	-0.11	-0.11
SRDQ	-0.04	-0.02	-0.01	-0.01
EVDISTSEV	0.26	0.10	0.16	0.16
EVDISTMOD	-0.50	-0.19	-0.31	-0.30
SEVER03DICO	0.43	0.24	0.33	0.32
SEVER36DICO	-0.06	0.05	0.00	0.00
SEVER69DICO	-0.46	-0.19	-0.30	-0.29
SEVER912DICO	-0.01	0.05	0.00	0.00
MODER203DICO	-0.37	-0.14	-0.24	-0.23
MODER236DICO	0.36	0.10	0.20	0.20
MODER269DICO	0.44	0.12	0.24	0.23
MODER2912DICO	0.10	-0.08	0.00	0.00
LOW203DICO	0.26	0.21	0.24	0.23
LOW236DICO	0.12	0.10	0.10	0.10
LOW269DICO	-0.23	-0.21	-0.22	-0.22
LOW2912DICO	-0.17	-0.13	-0.14	-0.14
CHRONSEVACT	0.61	0.46	0.56	0.55

Summary of models' accuracy:

```
Classical logistic regression model:
  Accuracy of 70.1% with training data
  Accuracy of 66.7% with testing data
Logistic regression with ridge regularization model:
  Accuracy of 70.6% with training data
  Accuracy of 66.7% with testing data
Logistic regression with lasso regularization model:
  Accuracy of 69.9% with training data
  Accuracy of 67% with testing data
Logistic regression with elastic-net regularization model:
  Accuracy of 67.7% with training data
  Accuracy of 67% with testing data
```