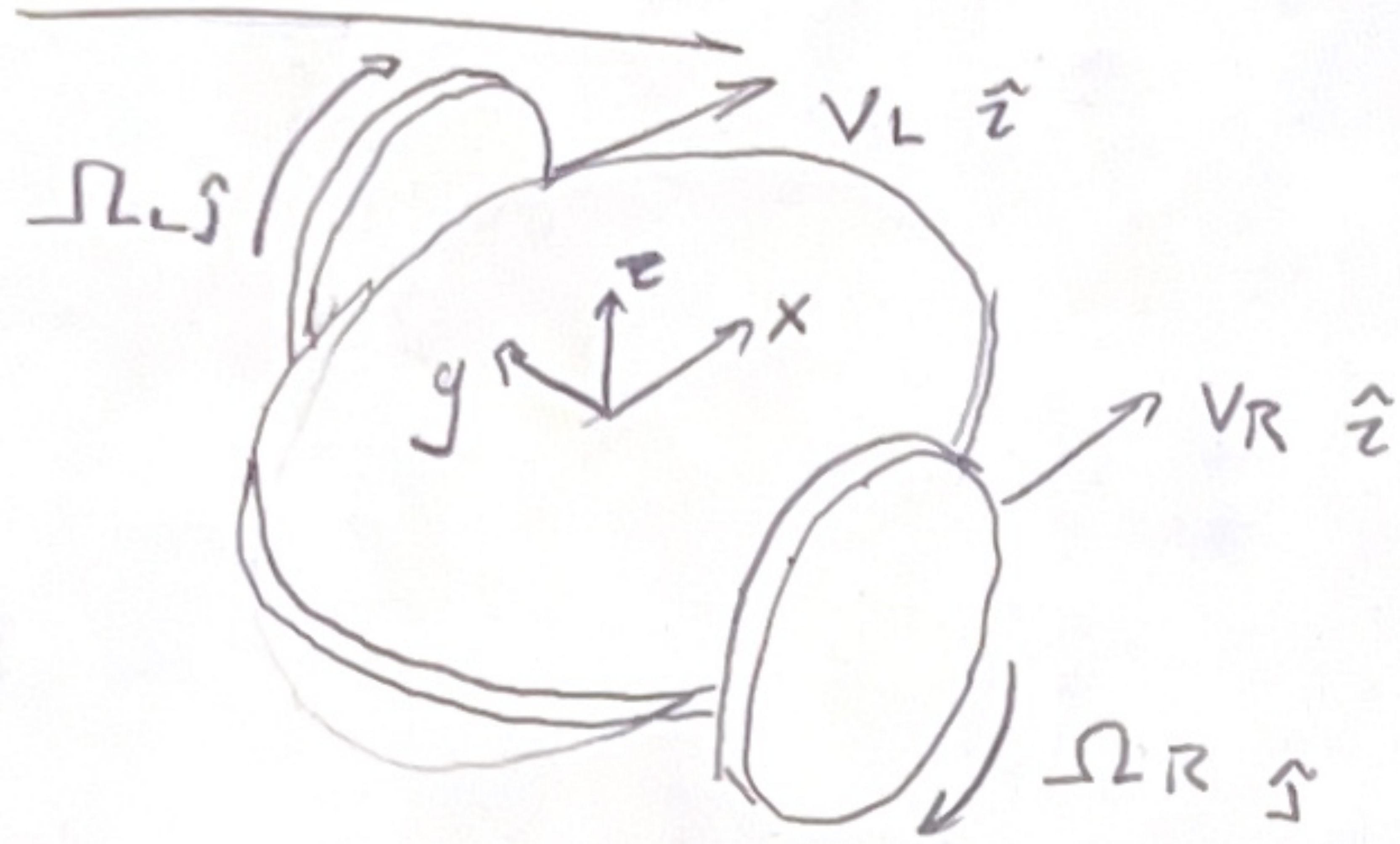
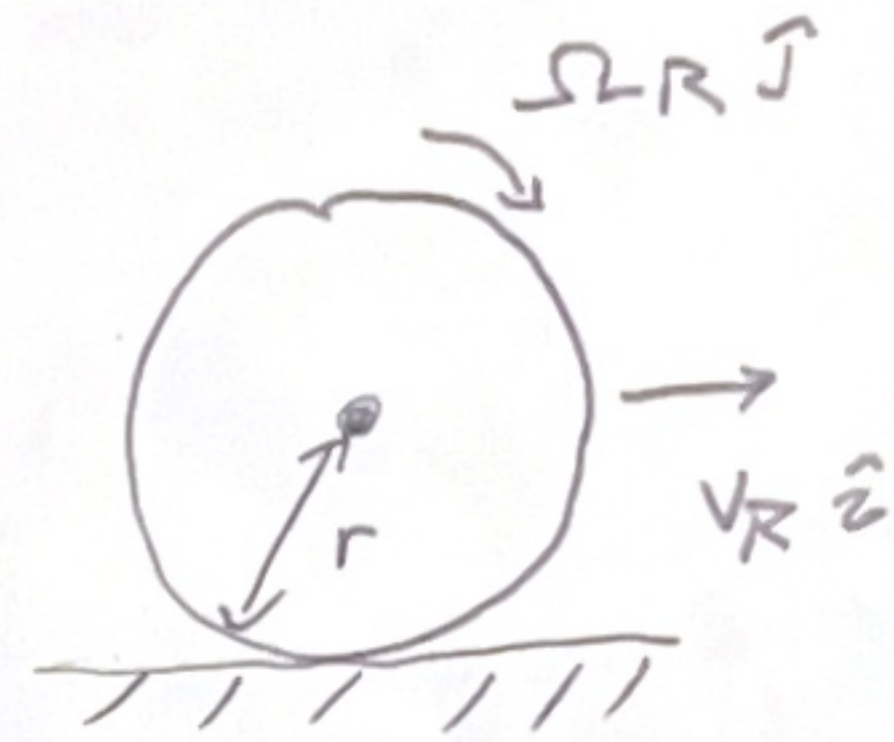
INTRODUCTION:

THESE CALCS WILL CONSIDER THE DYNAMICS OF THE 2-WHEEL ROBOT, ROMI. A KINEMATIC MODEL IS DEVELOPED w/ THE INTENTION OF CREATING A SIMULATION. THE SIMULATION WILL BE USED TO TEST CLOSED-LOOP (CL) CONTROL ON THE ROBOT.

SCHEMATIC ①:





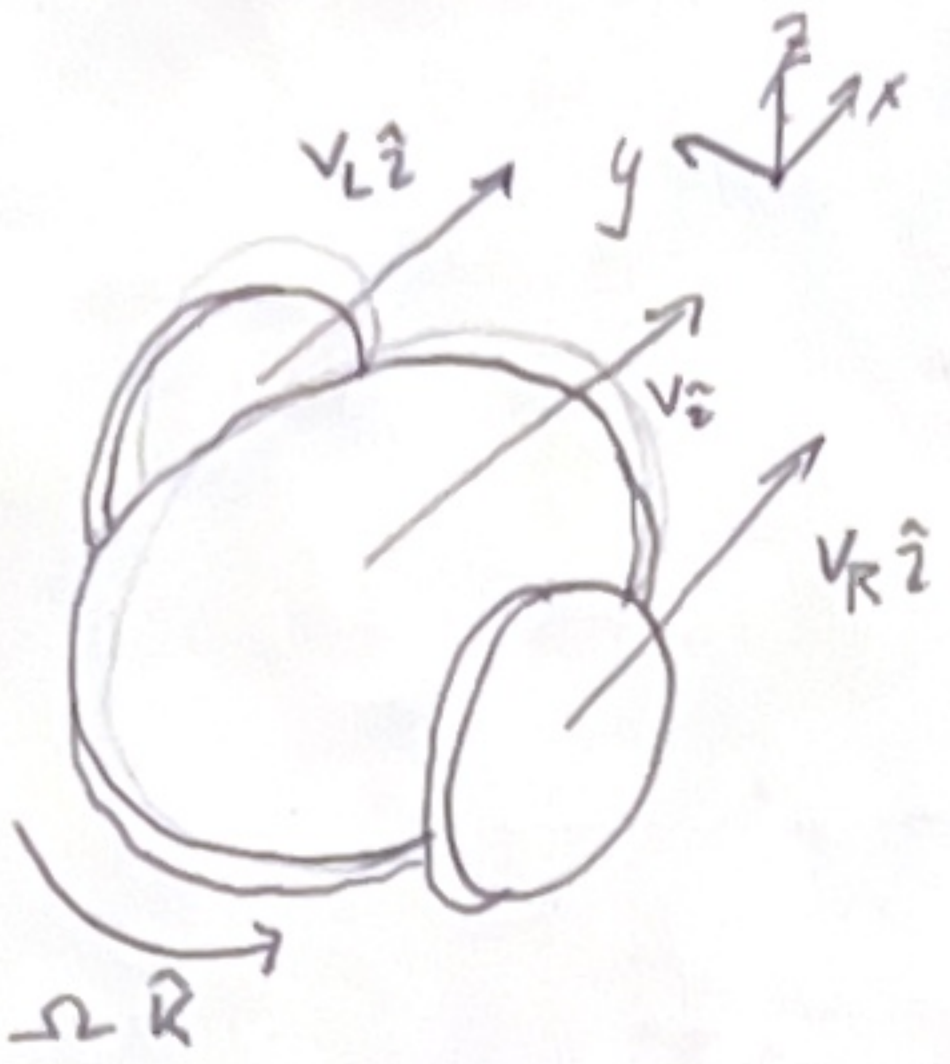
ANALYSIS:

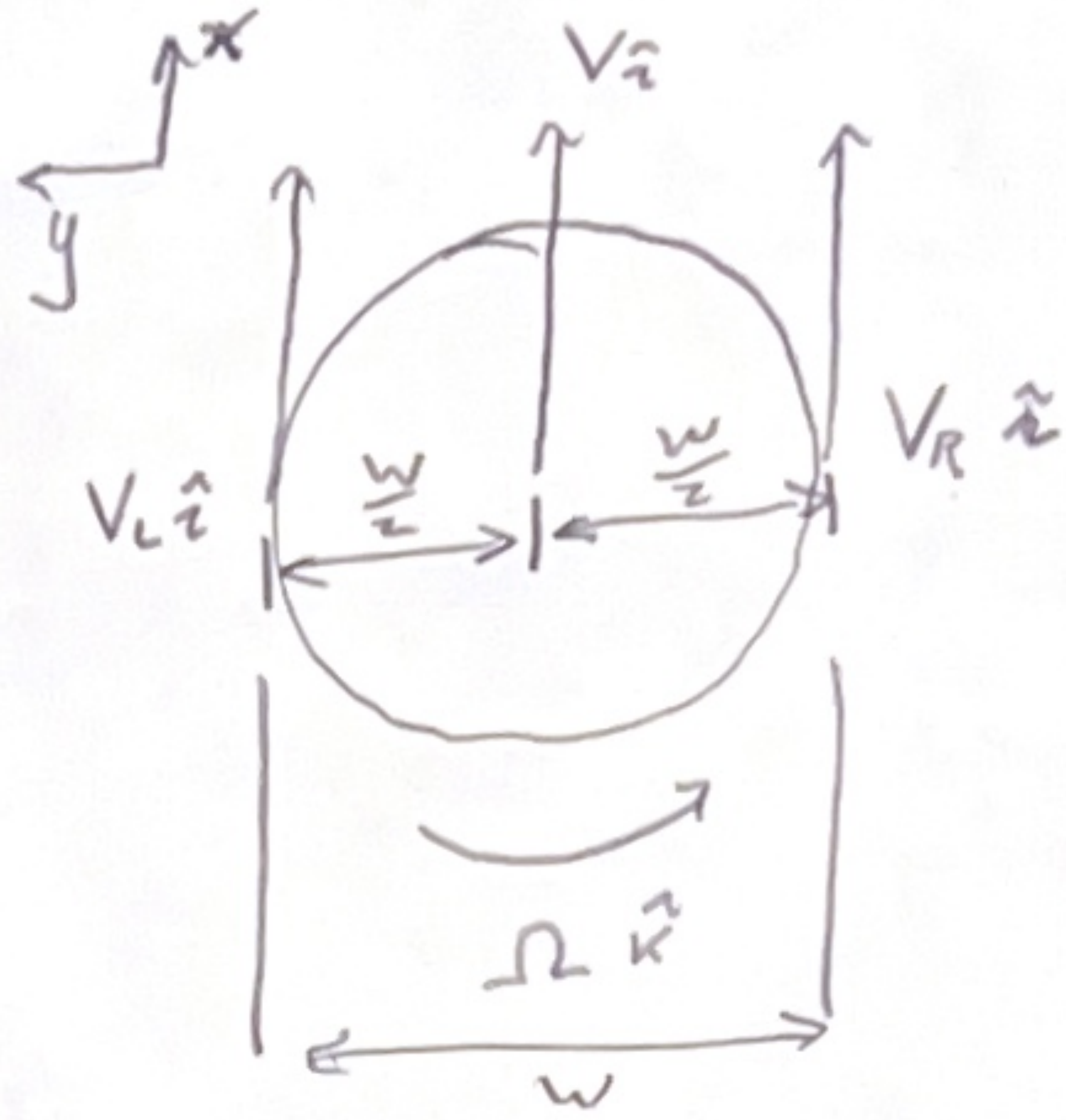
$$V_R \hat{i} = \Omega_R \hat{j} \times r\hat{k}$$
$$= \Omega_R r \hat{i}$$
$$\boxed{V_R = \Omega_R r}$$

NOTE! THIS ASSUMES NO SLIP

$$V_L \hat{i} = \Omega_L \hat{j} \times r\hat{k}$$
$$= \Omega_L r \hat{i}$$
$$\boxed{V_L = \Omega_L r}$$

SCHEMATIC ②:





ANALYSIS:

RIGID BODY USING TRANSLATING COORD SYS:

$$\vec{V}_A = \vec{V}_B + \vec{\omega} \times \vec{r}_{A/B}$$

$$\vec{V}_R = \vec{V}_L + \vec{\Omega} \times \vec{W}$$

PLUG IN FOR Ω

$$V_R\hat{i} = V_L\hat{i} + \Omega\hat{k} \times -W\hat{j}$$

$$\boxed{\Omega = \frac{(V_R - V_L)}{W}}$$

$$\vec{V} = \vec{V}_L + \vec{\Omega} \times \frac{\vec{W}}{2}$$

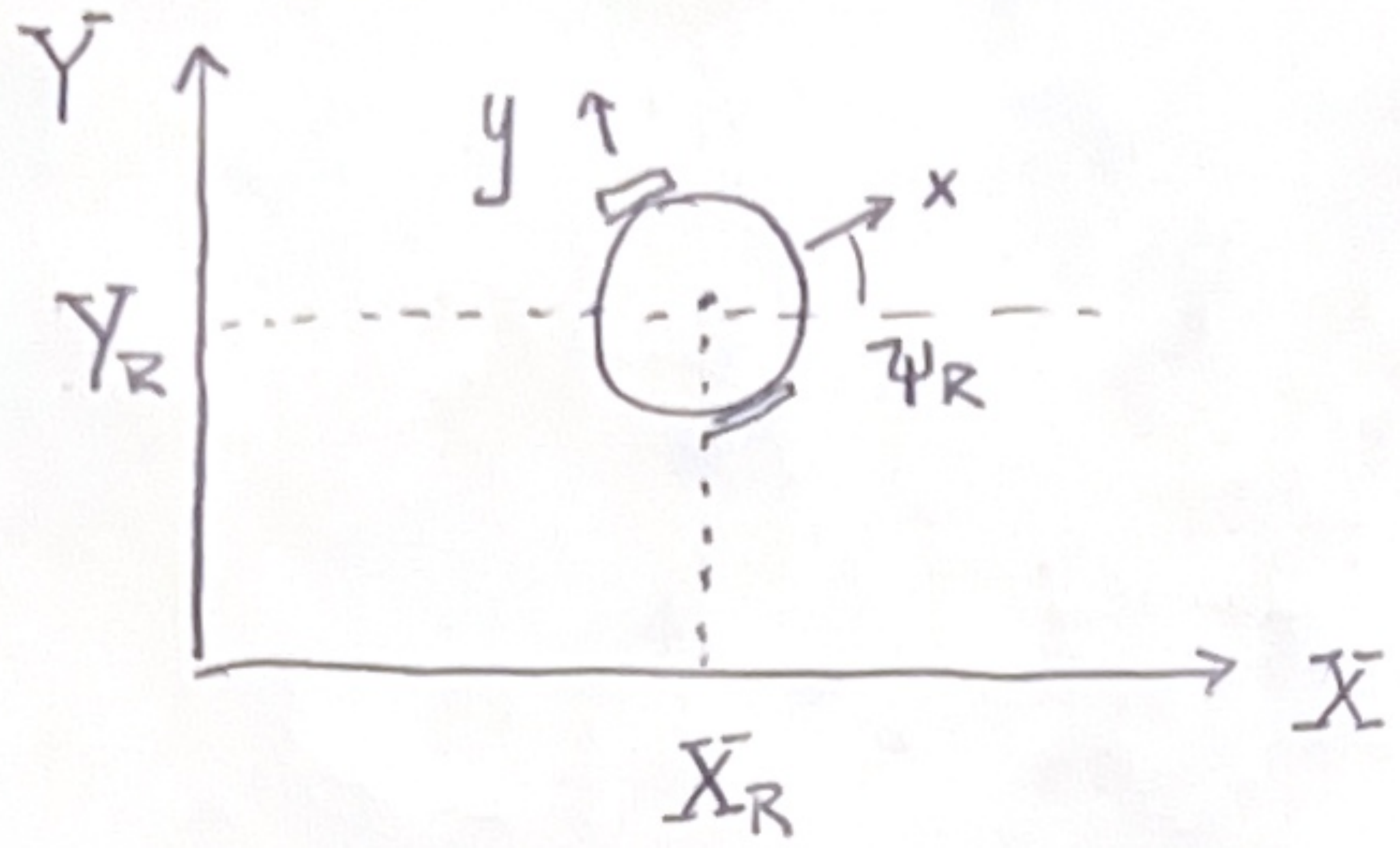
PLUG IN FOR V

$$V\hat{i} = V_L\hat{i} + \frac{\Omega W}{2}(\hat{k} \times -\hat{j})$$

$$V = V_L + \frac{(V_R - V_L)}{2}$$

$$\boxed{V = \frac{V_L + V_R}{2}}$$

SCHEMATIC ③:



NOTE: WE HAVE DERIVED THE KINEMATICS IN A FRAME FIXED TO ROMI.
NOW, LET US MOVED TO AN INERTIAL FRAME.

ANALYSIS:

APPLY ROTATION MATRIX

$$\begin{bmatrix} \hat{I} \\ \hat{J} \end{bmatrix} = \begin{bmatrix} c\psi_R & -s\psi_R \\ s\psi_R & c\psi_R \end{bmatrix} \begin{bmatrix} \hat{i} \\ \hat{j} \end{bmatrix}$$

$$\dot{X}_R \hat{I} + \dot{Y}_R \hat{J} = v\hat{i}$$

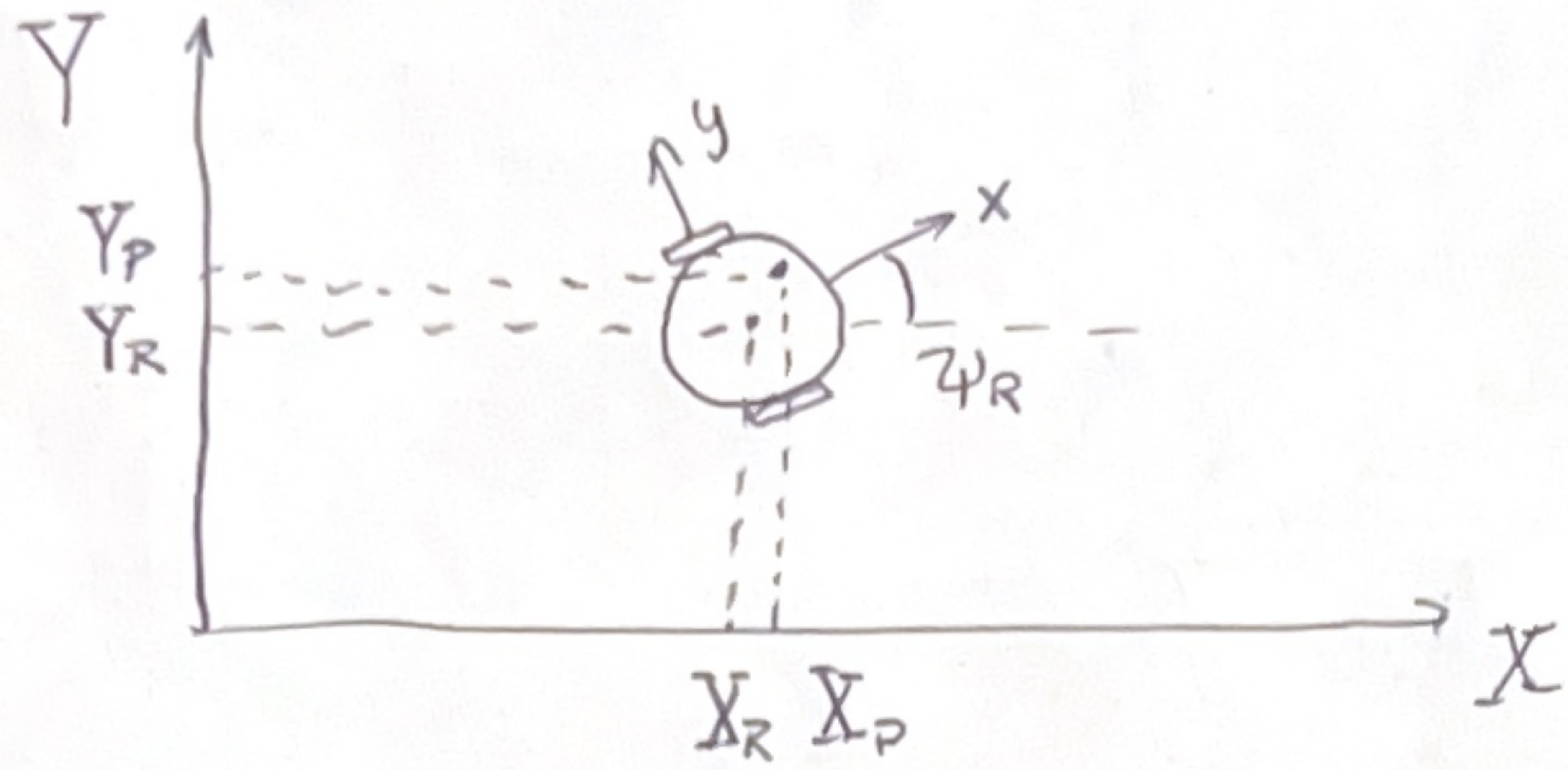
$$= v\left(c\psi_R \hat{I} + s\psi_R \hat{J}\right)$$

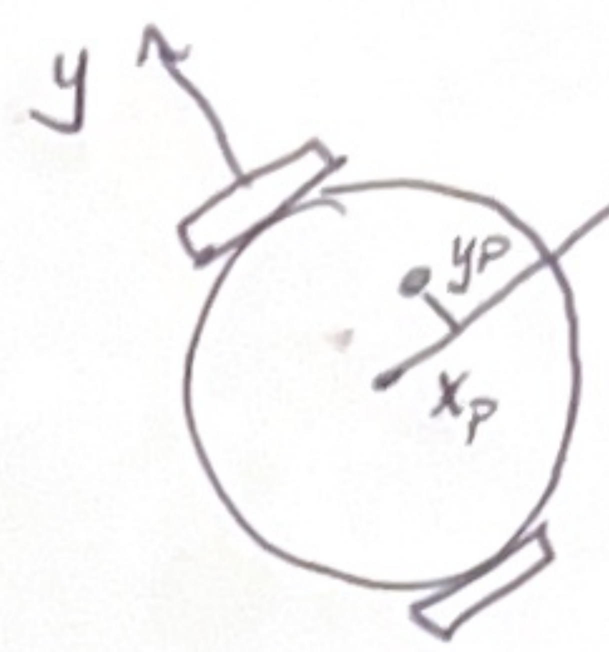
$$\boxed{\begin{aligned} \dot{X} &= v\, c\psi_R \\ \dot{Y} &= v\, s\psi_R \end{aligned}}$$

AND SINCE $\hat{K} = \hat{k}$

$$\boxed{\dot{\psi}_R = \Omega}$$

SCHEMATIC:





NOTE:

NOW WE MUST CONSIDER POINTS OF INTEREST ON THE ROMI (SENSORS) WRT THE INERTIAL FRAME.

ANALYSIS:

$$X_p \hat{I} + Y_p \hat{J} = X_R \hat{I} + Y_R \hat{J} + x_p \hat{i} + y_p \hat{j}$$

APPLY ROTATION MATRIX

$$X_p \hat{I} = X_R \hat{I} + x_p \cos \psi_R \hat{I} - y_p \sin \psi_R \hat{I}$$
$$Y_p \hat{J} = Y_R \hat{J} + x_p \sin \psi \hat{J} + y_p \cos \psi_R \hat{J}$$

$$\boxed{\begin{aligned} X_p &= X_R + x_p \cos \psi_R - y_p \sin \psi_R \\ Y_p &= Y_R + x_p \sin \psi_R + y_p \cos \psi_R \end{aligned}}$$

NOTE:

NOW THAT WE HAVE OBTAINED KINEMATIC EQS, WE CAN CREATE A STATE-SPACE MODEL FOR THE COMPUTER SIMULATION. THE INPUTS WILL BE u_L & u_R, THE VOLTAGE TO EACH MOTOR. THE FUNCTIONS $\Omega_R(u_R)$ & $\Omega_L(u_L)$ HAVE BEEN DETERMINED EXPERIMENTALLY.

ANALYSIS:

INPUT: $\vec{u} = \begin{bmatrix} u_L \\ u_R \end{bmatrix}$

STATE VARS: $\vec{x} = \begin{bmatrix} X_R \\ Y_R \\ \psi_R \\ s \\ v \\ \Omega \end{bmatrix}$

NOTE: S IS THE ARC LENGTH ROMI HAS DRIVEN

OUTPUTS: $\vec{y} = \begin{bmatrix} X_R \\ Y_R \\ \psi_R \\ X_P \\ Y_P \\ s \\ v \\ \Omega \end{bmatrix}$

WE HAVE v & Ω BUT NEED $\dot{v}$ & $\dot{\Omega}$

FOR A DC MOTOR, RECALL THAT

$$\dot{\Omega} = \frac{1}{\tau}(Ku - \Omega)$$

WHERE τ IS THE TIME CONST & K IS THE MOTOR GAIN. BOTH HAVE BEEN FOUND EMPIRICALLY.

ANALYSIS:

$$V = \frac{1}{2}(V_R + V_L)$$

$$V = \frac{r}{2}(\Omega_R + \Omega_L)$$

$$\dot{V} = \frac{r}{2}(\dot{\Omega}_R + \dot{\Omega}_L)$$

$$\dot{V} = \frac{r}{2}\left(\frac{1}{\tau}(K u_R - \Omega_R) + \frac{1}{\tau}(K u_L - \Omega_L)\right)$$

$$\dot{V} = \frac{rK}{2\tau}(u_L + u_R) - \frac{r}{2\tau}(\Omega_R + \Omega_L)$$

$$\boxed{\dot{V} = \frac{rK}{\tau} u_1 - \frac{1}{\tau} V}$$

WHERE $u_1 = \frac{1}{2}(u_R + u_L)$

$$\Omega = \frac{1}{w}(V_R - V_L)$$

$$\Omega = \frac{r}{w}(\Omega_R - \Omega_L)$$

$$\dot{\Omega} = \frac{r}{w}(\dot{\Omega}_R - \dot{\Omega}_L)$$

$$\dot{\Omega} = \frac{r}{w}\left(\frac{1}{\tau}(K u_r - \Omega_R) - \frac{1}{\tau}(K u_L - \Omega_L)\right)$$

$$\dot{\Omega} = \frac{rK}{w\tau}(u_R - u_L) - \frac{r}{w\tau}(\Omega_R - \Omega_L)$$

$$\boxed{\dot{\Omega} = \frac{rK}{w\tau} u_2 - \frac{1}{\tau}\Omega}$$

WHERE $u_2 = u_R - u_L$

ANALYSIS:

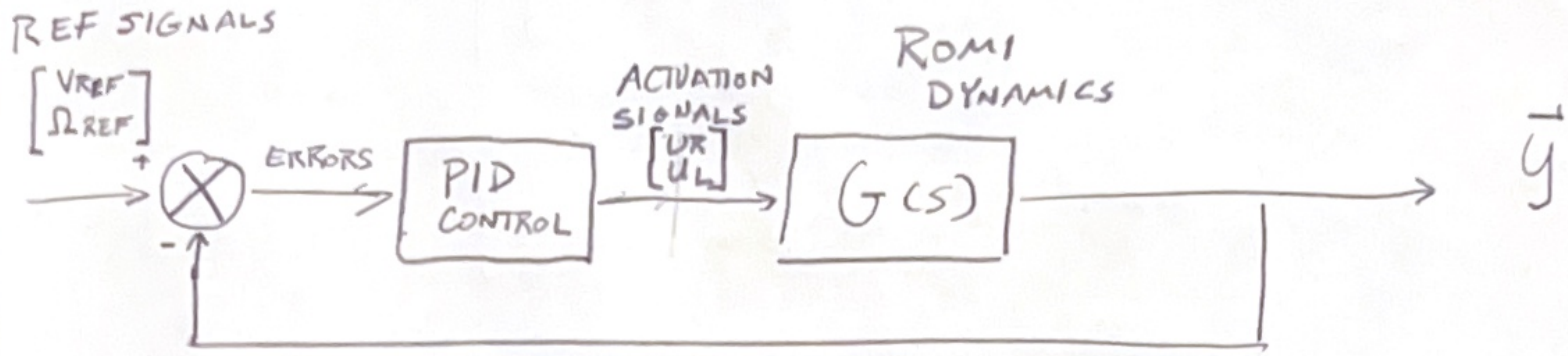
NOW TIME FOR THE STATE SPACE EQS:

$$\frac{d}{dt}\begin{bmatrix} X_R \\ Y_R \\ \psi_R \\ s \\ v \\ \Omega \end{bmatrix} = \begin{bmatrix} v\cos\psi_R \\ v\sin\psi_R \\ \Omega \\ v \\ \frac{rK}{\tau}u_1 - \frac{1}{\tau}v \\ \frac{rK}{\tau}u_2 - \frac{1}{\tau}\Omega \end{bmatrix}$$

OUTPUTS:

$$\begin{bmatrix} X_R \\ Y_R \\ \psi_R \\ X_P \\ Y_P \\ s \\ v \\ \Omega \end{bmatrix} = \begin{bmatrix} X_R \\ Y_R \\ \psi_R \\ X_R + x_p\cos\psi_R - y_p\sin\psi_R \\ Y_R + x_p\sin\psi_R + y_p\cos\psi_R \\ s \\ v \\ \Omega \end{bmatrix}$$

BD:


REF SIGNALS
$\begin{bmatrix} V_{REF} \\ \Omega_{REF} \end{bmatrix}$
+
-
ERRORS
PID CONTROL
ACTIVATION SIGNALS
$\begin{bmatrix} U_R \\ U_L \end{bmatrix}$
ROMI DYNAMICS
G(s)
$\vec{y}$