Addressing the Principal-Agent Problem in Public Private Partnerships via Mixed-Integer Bilevel Linear Programming

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Abstract

Public Private Partnerships (PPPs) are a mechanism through which governments associate with private parties to deliver public assets and services. While PPPs are often advantageous, they are prone to potential conflicts of interest as a result of trade-offs between the service level expected of a public good and the profitability required for a contractor to partake in the project. We propose a bilevel optimization approach to study the coupled decision process of both parties, aiming to identify conditions and strategies that determine win-win outcomes not captured by unilateral analyses. We illustrate our approach in the context of a PPP focused on road maintenance, considering intervention decisions of a private contractor over a time horizon and inspection decisions of a public entity. We adopt a Mixed Integer Bilevel Linear Programming (MIBLP) framework, aiming to account for a two-player decision problem with combinatorial decisions that are typical in projects involving operations. In this paper we study how exact optimization approaches can help explore a joint-decision space that is also non-convex. Specifically, we use a Branch Cut strategy that offers bilevel feasible solutions (i.e., maintenance and inspection plans) within reasonable computation times. This allows to evaluate trade-offs for parties and opportunities to improve the definition, negotiation, and execution of PPPs, based on analyses that are rooted in how operational decisions unfold throughout a planning horizon.

Keywords: Public Private Partnerships, Infrastructure Maintenance, PA problem, Bilevel Programming, Branch & Cut.

1 Introduction

Public-Private-Partnerships (PPPs) often imply long-term projects in which a private contractor provides some public good or service. This poses interesting challenges in terms of decision support, as determining the conditions that meet the interests of the public and private parties throughout the development of complex projects is far from trivial. The decision space in such problems is difficult to explore due to: (i) the multi-actor nature of the problem; (ii) the dynamics of the project being carried out (e.g., decisions, constraints); and (iii) the uncertainties associated to the long-term impact of the project and externalities (e.g., risks). In this paper, we explore exact optimization techniques to address items (i) and (ii) as a step towards a decision support tool to explore these complex decision spaces, aiming to identify win-win solutions (or convenient trade-offs) that inform the definition, negotiation, and execution of PPP projects. Overall, this effort advances the use of technological tools to improve the value of critical societal projects (e.g., infrastructure procurement and maintenance).

While PPPs have shown advantages in bringing expertise and financial leverage to public projects (Trebilcock and Rosenstock, 2015), they are prone to conflicts of interest due to potential diverging objectives between the public and private parties (Páez-Pérez and Sánchez-Silva, 2016). For instance, the public party may focus on the project's performance and *societal value*; for infrastructure assets, reliability, service levels, or other metrics of their physical condition may be of interest. The private party, on the other hand, may focus on cost-effectiveness in delivering the requested good or service to protect the interests of their shareholders. The fact that the project is ultimately executed by the private party may lead to a Principal-Agent problem (Jensen and Meckling, 1976), in which the public party (the principal) assigns a task to the private party (the agent) but lacks the means to determine whether the agent would take advantage of asymmetry of information to pursue their interest in detriment of the project. Our claim is that, even in the absence of malicious behavior, unilateral solutions to the problem miss the opportunity to identify collectively good solutions (i.e., where one party's interest may be improved without affecting the other's, one party's gain may subsidize the other's loss, or convenient trade-offs may be pursued).

We propose the use of bilevel optimization as a suitable framework to model two-level decision problems involving a leader and a follower; in this case, the public and private parties, respectively. The leader's decisions influence the follower's solution space, while the followers decisions affect the leader's objective (e.g., a company decides about product pricing but their sales and revenue depend on users' reaction to the defined prices). In the context of PPPs, bilevel optimization could be used as a means to evaluate how a rational agent (the private) may respond to the conditions stated by a PPP owner, and how such conditions may be adjusted to improve the outcome for the players. Bilevel optimization, thus, offers an analysis tool that can enrich the decision process of public entities proposing PPPs, private contractors entrusted with PPP projects, or higher-level policy agencies defining the terms of a PPP.

In order to test our bilevel optimization approach for decision support in PPPs, we consider a maintenance problem of a public road, assigned to a private contractor. The road deteriorates over time and the contractor is responsible for applying maintenance actions that will temporarily bring the road's service level to its maximum value, assuming that different discrete service levels can be generated as a function of a continuous performance metric of the road. These service levels are associated with different values of societal benefit derived from the asset (road), which the principal seeks to maximize and for which a minimum threshold value must be satisfied at every period of a planning horizon. However, maintaining higher service levels is costly for the private contractor, thus, compromising their profitability. Finding optimal maintenance plans for each player (unilaterally) provides a notion of how divergent the parties' objectives may be (in terms of maintenance actions and overall outcomes for both parties), as explored in Gomez et al., 2020. We address the underlying question of how the principal can influence the contractor's actions to be more aligned with the interest of the public (even if that requires transferring part of the gained benefits back to the agent to make their operation viable). The public entity, thus, can perform inspections (at a cost) and apply a positive or negative incentive for the contractor depending on the road's service level. The value added with bilevel optimization in this PPP problem is the possibility to explore how the agent's maintenance plan responds to the principal's incentive and inspection strategies, allowing to analyze trade-offs of different possible settings (e.g., acceptable performance threshold, magnitude of reward or penalties).

PPPs typically involve long-term projects in which the overall outcome depends not only on higher-level strategical decisions, but also on the strategy translating to operational decisions that determine the execution of the project (e.g., the actual scheduling of tasks, or allocation of resources). The use of optimization models can provide decision support that accounts for the actual implementation of the project at a more granular level than a macroscopic analysis (e.g., closed-form analyses of broad cost functions in net-present-value). However, this specificity comes at the cost of computational burden, not only because of the actual modeling of the involved processes in the project, but because operational problems often involve combinatorial variables that complicate the solution of the models; in the proposed study, players face binary decisions regarding whether or not to do maintenance or inspection actions. While bilevel optimization problems are relatively well solved for the case of continuous variables, the case of mixed-integer problems is much more challenging due to their non-convex/combinatorial nature. We adopt a state-of-the-art technique to solve MIBLPs based on a Branch & Cut approach.

The contribution of this research is twofold. First, we propose a novel analysis framework to explore the joint decision space of multi-actor decision problems as complex and critical as the PA in PPPs, considering the underlying operational decisions faced by players; this allows to study interactions, trade-offs, and non-intuitive win-win opportunities that may inform the strategy for the parties involved in a PPP, or of those in charge of generating policies or regulation in this regard. Second, we study a realistic use-case for a challenging family of problems in OR (the MIBLP), putting state-of-the-art solution techniques to test; this allows to understand the advantages and limits of exact optimization approaches and guide/inspire future methodological contributions in MIBLP. Along these lines, our model finds bilevel feasible maintenance and inspection policies within reasonable computation times, finds bounds on the objective function for the associated MIBLP, and offers trade-off analysis to overcome drawbacks like the PA problem, guiding decision-making among conflicting interests at both a strategic and operational level.

This document is organized as follows. Section **2** presents the literature review related to previous approaches used to address and analyze conflicts within PPPs and the potential PA problem, as well as an overview on bilevel optimization. Moreover, section **3** provides an in-depth definition of the PPP context under study, as well as its parameters and the associated mathematical model. Section **4** explains the solution approach we follow to solve our MIBLP; a Branch & Cut approach. Section **5** presents the results for a variety of testbed instances from the literature considered in our computational study, whereas section **6** presents an illustrative example and results analysis for our PPP problem. Section **7** synthesizes the main contents of our research while presenting a brief analysis of the overall results of our instances testbed, while we explain the limitations, assumptions, and future work regarding our implementation. Finally, the appendix presents the mathematical formulation for our small instances testbed.

2 Literature Review

In order to mitigate the potential drawbacks akin to the PA problem in PPPs, various methodologies could be utilized, including the incorporation of game theoretical frameworks and Operations Research techniques. These mathematical-driven approaches aim to create mutually beneficial or "fair-enough" scenarios for both parties, promoting 'win-win' outcomes in the PPP dynamics (Eshun et al., 2020; Glumac et al., 2015).

Feng et al., 2018 develop a multi-objective optimization model to optimize and balance a set of the partnership's elements (i.e., planning horizon length, payment values to recover initial investment, debt-equity ratio and equity participation between the public and the private parties, public's subsidy to make the project attractive enough for the private) on PPP projects by making use of Non-dominated Sorting Genetic Algorithm (NSGA-II), in which the objectives are the maximization of the net present value of the private party, and maximization of societal welfare for the public.

Moreover, inspection mechanisms on the project assets or performance have been recently proposed as alternatives to address the PA drawbacks in infrastructure projects maintenance and PPPs. These inspection-based approaches seem to be motivated by 'Inspection Games'; a two-player zero-sum multistage game (Ferguson and Melolidakis, n.d.) for which inspection strategies are implemented to mitigate possible smuggling situations in customs control processes (Hohzaki and Maehara, 2010; Thomas and Nisgav, 1976). Corotis et al., 2005 incorporate a Partially Observable Markov Decision Process (POMDP) to determine optimal inspection and maintenance strategies for an infrastructure asset prone to deterioration with respect to the the system's life-cycle costs, and the expected present worth of the operational costs (i.e., inspection, maintenance). They propose in-service inspections to the infrastructure system to mitigate the impact that partial observability on the system's states has on making maintenance, repair, and rehabilitation decisions, as well as to update the values for the underlying uncertain parameters of the model. Additionally, in a PPP context, inspections are to be carried out by the public entity to impose incentives or penalizations to the private party as a function of the project's performance at the time of inspection.

Furthermore, Páez-Pérez and Sánchez-Silva, 2016 offer an agent-based simulation methodology that incorporates inspections as a mechanism for the private party to estimate the utility (i.e., social benefit) generated from the project's performance, while it offers the possibility to determine the private's compliance of the maintenance actions by applying fines or penalties when the performance passes a certain threshold. Lozano and Sánchez-Silva, 2019 delve into the 'win-win' scenario by incorporating the combination of agent-base modeling, optimization, and simulation on the contract's parameters s (i.e., minimum performance threshold, average frequency of public inspections to the system, and penalty fees in case of undesirable system's performance as a consequence of the privates operations on the infrastructure asset) in order to improve the respective objective function values of both parties. Their model also considers inspection actions on the project's performance in order to guarantee the private's compliance on the project's interests.

Although, these research projects offer excellent means to analyze the PPP dynamics within the involved parties, theoretical bounds on the quality of the potential solutions retrieved to mitigate PA problem seem to be absent as well as tractable means to analyze them. Gomez et al., 2020 research examined the relationship between a principal and an agent in the context of maintenance planning in PPPs. First, they represent the PPP as a MILP and then run sensitivity analysis on some of their model's parameters (i.e., threshold for the system's performance, return rate for the private's maintenance operations on the system) to determine the joint feasible region for both parties and set negotiation bounds. Second, the impact of diverging objectives between the public and the private is assessed using 4 formulation cases for their model: i) impose a minimum threshold constraint on the system's performance and provide economic incentives to the private as a function of the system's performance, ii) impose a minimum threshold constraint on the system's performance but not to provide economic incentives to the private as a function of the system's performance, iii) not to impose a minimum threshold constraint on the system's performance but to provide economic incentives to the private as a function of the system's performance, and iv) not to impose a minimum threshold constraint on the system's performance nor to provide economic incentives to the private as a function of the system's performance. Finally, win-win configurations were explored through a lexicographic optimization process to find the best compensation scheme, which consisted on finding the best unilateral solution for both parties and iteratively adding constraints to drive the private towards better-performing solutions for the public. Even though, while their solution process is exact, the compensation finding process is heuristic as it does not consider the parties' coupled decisions nor interests in simultaneous.

In an effort to address tractability and analyze the joint decision space of a Principal-Agent scheme simultaneously, Cecchini et al., 2013 consider a principal being a car dealership owner that sells one type of car, and the agent as a salesperson. The goal of the principal is to maximize the expected profits from the agent selling the cars, but the principal cannot directly observe the agent's actions, say, investing effort on selling the cars. These actions affect the agent's utilities by receiving compensations/bonuses from the principal for successful sales but disutilities also arise from the effort needed to perform the associated selling action. This PA problem is addressed using a nonlinear bilevel optimization approach using the Ellipsoid algorithm (Bland et al., 1981); linearity assumptions on the agent's attributes are relaxed (hence the non-linearity) and mathematical optimization is utilized to procure tractability.

Bilevel programming (BLP) seems to be a good mechanism to address PA problems in order to consider both parties coupled decisions and interests in simultaneous and, to the best of our knowledge, for the first time in a PPPs context. BLPs are optimization problems that contain a nested optimization problem in the constraints of an outer optimization problem (Sinha et al., 2017) and they are classified as NP-hard problems (Ben-Ayed and Blair, 1990; Kovács, 2018), meaning that the required time to find a solution for them grows exponentially as a function on the input size of the problem. In BLPs, the outer problem is known as the leader problem, while the nested problem is known as the follower problem. A MIBLP is a BLP that has some (if not all) their decision variables belonging to an integer or binary domain. An example of a MIBLP formulation is shown as follows:

$$\min_{x,y} c_x^T x + c_y^T y \tag{1}$$

$$G_x x + G_y y \leqslant q \tag{2}$$

$$x_j \text{ integer}, \; \forall\, j \in J_x \tag{3}$$

$$x^- \leqslant x \leqslant x^+ \tag{4}$$

$$y^- \leqslant y \leqslant y^+ \tag{5}$$

$$\begin{aligned} y \in \arg\min_{y \in \mathbb{R}^{n_2}} \{ & d^T y : Ax + By \leqslant b, \; l \leqslant y \leqslant u, \\ & y_j \text{ integer}, \; \forall\, j \in J_y \} \end{aligned} \tag{6}$$

$$J_x \subseteq N_x \coloneqq \{1, \ldots, n_1\}, \; J_y \subseteq N_y \coloneqq \{1, \ldots, n_2\} \tag{7}$$

In the model above, $x \in \mathbb{R}^{n_1}$, $y \in \mathbb{R}^{n_2}$, while c_x, c_y, G_x, G_y, q, d, A, B, b, l, u, x^-, x^+, y^- and y^+ are given matrices/vectors of appropriate size. Also, sets J_x and J_y represent the indexes for the integer-constrained x (leader) and y (follower) variables, respectively. Expressions (1)–(5) correspond to the leader optimization problem while expression (6) corresponds to the follower optimization problem. Expressions (4) & (5) are often included in the constraints of expression (2) (if any lower and/or upper bounds for the respective variables are given) (Fischetti et al., 2017).

MIBLPs have been granted special attention in recent years due to their challenging nature and applicability, depending on which level of the MIBLP (leader or follower) presents the mixed-integer requirements for the corresponding decision variables. Several approaches have been proposed to solve BLPs and MIBLPs to optimality or to retrieve a solution close to it. Some of these include single reduction techniques, which consist on reformulating the two-level problem into a single-level problem by exploiting its Karush–Kuhn–Tucker (KKT) conditions (Tuy et al., 1993); single reduction applying Branch & Bound methods (Bard and Moore, 1990); descent methods, which consist on decreasing the leader's objective function value (with respect to minimization sense) while keeping the new point follower-optimal (Kolstad and Lasdon, 1990); among others. For the rest of this research, we assume an optimistic position for MIBLPs, meaning that given multiple follower optimal solutions, the leader expects the follower to choose the solution that leads to the best objective function value for the leader. The reader can refer to (Sinha et al., 2017) for an extended in-depth survey on BLPs and MIBLPs.

Fischetti et al., 2017 proposed new approaches for solving MIBLPs to optimality using Branch and Bound (Fischetti et al., 2016) and Branch & Cut methods. The latter incorporates ideas of the corner polyhedron (Conforti et al., 2010; Gomory, 1969) and Intersection Cuts (ICs) (Balas, 1971). They framed their Branch & Cut methodology following the "disjunctive interpretation" (Glover, 1974; Glover and Klingman, 1976) for generating numerically reliable ICs.

3 PPP Instance

This section formally describes the PPP problem description and introduces its mathematical model.

A public entity awards a private company with a contract to maintain an infrastructure system (e.g., a road), which follows a known deterioration function. The payments for the company are split through each of the periods of an established horizon, whereas the public's expectation is to guarantee that a system's performance (i.e., physical conditions or reliability) to be as high as possible. The decision problem for the private party consists in determining when to apply maintenance actions (which restore the system to a good-as-new state) in a way that satisfies its business requirements (utility, Net Present Value or NPV), considering that the public party may impose rewards or penalties, if and when inspections occur, as a function of the system's performance. Moreover, the decision problem for the public party consists in defining an inspection scheme that effectively induces the private party to maintain the system at the performance level that maximizes the derived public infrastructure asset's benefits for the society.

Table 2 summarizes the main decision that each party has to make in every moment of the planning horizon and their associated joint outcomes, whereas 1 summarizes the mathematical programming model elements associated to the problem:

Table 1: Summary of sets, parameters and variables in the mathematical model.

Sets	T:	set of periods in planning horizon
	L:	set of discrete service-levels
Parameters	γ_t:	performance obtained after t periods without restoration
	$c^{(f)}$:	fix cost for restoration action
	$c^{(u)}$:	unit cost to restore a performance unit
	a:	fixed income from the government to the private party
	ϵ:	target return rate for the private party
	$\breve{\xi}_l$:	lower bound for service level $l \in L$
	$\hat{\xi}_l$:	upper bound for service level $l \in L$
	k_l:	penalty/reward from the road being at service-level $l \in L$
	g_l:	social benefit obtained from the system at service-level $l \in L$
	g^*:	target benefit
	$c^{(i)}$:	cost of inspection
Variables	q_t:	whether an inspection action is applied at period $t \in T$
	x_t:	whether a maintenance action is applied at period $t \in T$
	y_t:	number of periods elapsed after last restoration
	$b_{t,\tau}$:	whether $y_t = \tau$ for $\tau \in T$
	$z_{t,l}$:	whether system is at service level $l \in L$ at period $t \in T$
	v_t:	performance at period $t \in T$
	$p_t^{(+)}$:	earnings at period $t \in T$
	$p_t^{(-)}$:	expenditures at period $t \in T$
	$p_t^{(\cdot)}$:	available budget at period $t \in T$
	w_t:	linearization of $y_{t-1} \cdot x_t$
	u_t:	linearization for $v_t \cdot x_t$
	$\sigma_{t,l}$:	linearization of $z_{t,l} \cdot q_t$

For the parameters presented in 1, let $T = \{1, \ldots, |T|\}$ be the set of time periods of projec's planing horizon. Let $L = \{5, \ldots, 1\}$ be the set of discrete performance levels for the road, where 5 means the road is at optimal conditions while 1 means the road is at the worst conditions. Let γ_t be the performance obtained after t periods without restoration. Let $c_t^{(f)}$ be the fixed cost for restoration and $c_t^{(v)}$ be the unit cost to restore a performance unit at period $t \in T$. Let a be a fixed income from the public to the private party. Let ϵ be the target return rate for the private party. Let $\breve{\xi}_l$ be the lower bound and $\hat{\xi}_l$ be the upper bound for service level $l \in L$. Let k_l be the penalty or reward given to the private party by the public if the road's performance is at service level $l \in L$. Let g_l be the social benefit perceived by the government for the operation of the road for a performance level $l \in L$ and let g^* be a social benefit target. Finally, let $c^{(i)}$ be the cost incurred by the public regarding inspection mechanisms performed at any given period.

Variables for both parties are defined as follows. Let q_t be a binary variable to denote if an inspection action is performed at time $t \in T$. Let x_t be a binary variable to denote whether a maintenance action is carried out at time $t \in T$. Let y_t be an integer variable to denote the number of periods elapsed after last restoration at time $t \in T$. Let $b_{t,\tau}$ be a binary variable to denote whether $y_t = \tau$. Let $z_{t,l}$ be a binary variable to denote whether the system is at service level $l \in L$ at time $t \in T$. Let v_t be a continuous variable to capture the road's performance at time $t \in T$. Let $p_t^{(+)}$ be a continuous variable to denote the private party's earnings at time $t \in T$. Let $p_t^{(-)}$ be a continuous variable to represent the private party's expenses at time $t \in T$. Let $p_t^{(\cdot)}$ be a continuous variable to capture the available cash of the private party at time $t \in T$. Let w_t be an integer variable to capture the linearization of the product $y_{t-1} \cdot x_t$ at time $t \in T$. Let u_t be a continuous variable to capture the linearization of the product $v_t \cdot x_t$ at time $t \in T$. Finally, let $\sigma_{t,l}$ be an integer variable to capture the linearization of $z_{t,l} \cdot q_t$ at time $t \in T$ for service level $l \in L$.

Table 2: Summary of the decisions that can be performed by each party at every moment of the planning horizon, represented as a 2x2 game table.

Actions on the road	**Public inspects** ($q_t = 1$)	**Public does not inspect** ($q_t = 0$)
Private does maintenance ($x_t = 1$)	Best road's performance, best reward for the private ($q_t \cdot z_{t5} \cdot k_5$)	no incentives given
Private does not do maintenance ($x_t = 0$)	reward/penalty as a function of the road's performance ($q_t \cdot z_{tl} \cdot k_l$)	no incentives given

The PPP MIBLP formulation is presented as follows:

$$\begin{aligned} \min \sum_{t \in T} q_t \cdot c^{(i)} + \sum_{l \in L} \sum_{t \in T} k_l \cdot \sigma_{t,l} \\ + a \cdot |T| - \sum_{l \in L} \sum_{t \in T} g_l \cdot z_{l,t} \end{aligned} \tag{8}$$

s.t.

$$\textstyle\sum_{l \in L} g_l \cdot z_{t,l} \geqslant g^* \quad \forall\, t \in T \tag{9}$$

$$\begin{aligned} &x_t,\, y_t,\, b_{t,\tau},\, z_{l,t},\, v_t,\, p_t^{(+)},\, p_t^{(-)},\, p_t^{(\cdot)},\, w_t,\, u_t, \\ &\sigma_{t,l} \in \arg\min\{\text{Private party MILP}\} \end{aligned} \quad \forall\, t,\, \tau \in T,\, l \in L \tag{10}$$

Expressions (8)–(10) represent the public's (leader) problem, while expressions (11)–(36) denote the private party's (follower) one. Both parties objective functions are represented in a minimization fashion in order to follow the notation in section **4**. Expression (8) is consists of the costs of inspection at every period, the reward/penalty given to the private party with respect to the road's service level at the moment of inspection, the fixed payment per period to the private party and, finally, the social benefit perceived from the road being at every service level through the planning horizon. Expression (9) seeks to satisfy the minimum social benefit the road produces at the corresponding service level at every period. Expression (10) denotes the inner optimization problem for the private party as a constraint for the public's optimization problem. Expression (11) is composed of the expenses and profits for the private party.

$$\text{Private party MILP} = \min \sum_{t \in T} p_t^{(-)} - p_t^{(+)} \tag{11}$$

s.t.

$$y_1 = 0 \tag{12}$$

$$w_1 = 0 \tag{13}$$

$$u_1 = 0 \tag{14}$$

$$p_1^{(\cdot)} = p_1^{(+)} - p_1^{(-)} \tag{15}$$

Expressions (12)–(15) initializes the corresponding variables for the first period of the planning horizon.

$$y_t = y_{t-1} + 1 - w_t - x_t \quad \forall\ t\ \in\ T \mid t > 1 \tag{16}$$

Expression (16) captures the periods after last restoration as an inventory.

$$w_t \leqslant y_{t-1} \quad \forall\ t\ \in\ T \mid t > 1 \tag{17}$$

$$w_t \geqslant y_{t-1} - |T| \cdot (1 - x_t) \quad \forall\ t\ \in\ T \mid t > 1 \tag{18}$$

$$w_t \leqslant |T| \cdot x_t \quad \forall\ t\ \in\ T \mid t > 1 \tag{19}$$

Expressions (17)–(19) capture the linearization for the performance.

$$u_t \leqslant v_{t-1} \quad \forall\ t\ \in\ T \mid t > 1 \tag{20}$$

$$u_t \geqslant v_t - (1 - x_t) \quad \forall\ t\ \in\ T \mid t > 1 \tag{21}$$

$$u_t \leqslant x_t \quad \forall\ t\ \in\ T \mid t > 1 \tag{22}$$

Expressions (20)–(22) capture the linearization expressions to represent the private party's objective function appropriately.

$$p_t^{(\cdot)} = p_{t-1}^{(\cdot)} + p_t^{(+)} - p_t^{(-)} \quad \forall\ t\ \in\ T \mid t > 1 \tag{23}$$

Expression (23) updates the available cash for the private party at every period.

$$y_t = \textstyle\sum_{\tau \in T} \tau \cdot b_{t,\tau} \quad \forall\ t\ \in\ T \tag{24}$$

$$\textstyle\sum_{\tau \in T} b_{t,\tau} = 1 \quad \forall\ t\ \in\ T \tag{25}$$

Expressions (24) and (25) linearizes the restoration inventory to retrieve the road's performance.

$$v_t = \textstyle\sum_{\tau \in T} \gamma_\tau \cdot b_{t,\tau} \quad \forall\ t\ \in\ T \tag{26}$$

Expression (26) appropriately quantifies the road's performance.

$$v_t \leqslant \textstyle\sum_{l \in L} \hat{\xi}_l \cdot z_{t,l} \quad \forall\ t\ \in\ T \tag{27}$$

$$v_t \geqslant \textstyle\sum_{l \in L} \breve{\xi}_l \cdot z_{t,l} \quad \forall\ t\ \in\ T \tag{28}$$

Expressions (27)–(28) capture the upper and lower bounds for the service level of the road, respectively, given the road's performance at every period.

$$\textstyle\sum_{l \in L} z_{t,l} = 1 \quad \forall\ t\ \in\ T \tag{29}$$

Expression (29) denotes that, for every period, the road can only have one service level.

$$p_t^{(-)} = (c_t^{(f)} + c_t^{(v)}) \cdot x_t - c_t^{(v)} \cdot u_t \quad \forall\ t\ \in\ T \tag{30}$$

$$p_t^{(-)} \leqslant p_t^{(\cdot)} \quad \forall\, t \in T \tag{31}$$

Expressions (30) and (31) denote the expenses and budget balance for the implementation of maintenance actions to the road, respectively, by the private party.

$$\textstyle\sum_{t \in T} p_t^{(+)} \geqslant (1+\epsilon) \cdot \sum_{t \in T} p_t^{(-)} \tag{32}$$

Expression (32) denotes the monetary return for the private party as a factor of their expenses through time.

$$\sigma_{t,l} \leqslant q_t \quad \forall\, t \in T,\, l \in L \tag{33}$$

$$\sigma_{t,l} \leqslant z_{t,l} \quad \forall\, t \in T,\, l \in L \tag{34}$$

$$\sigma_{t,l} \geqslant q_t + z_{t,l} - 1 \quad \forall\, t \in T,\, l \in L \tag{35}$$

Expressions (33)–(35) capture the linearization of whether an inspection is occurs at any period when the road is on a given service level.

$$p_t^{(+)} = a + \textstyle\sum_{l \in L} k_l \cdot \sigma_{t,l} \quad \forall\, t \in T \tag{36}$$

Expression (36) captures the private party's profits for every period, adding the fixed income paid by the public and the reward/penalty given if an inspection is performed and the road is at a given service level.

$$q_t,\; x_t \in \{0,1\} \quad \forall\, t \in T \tag{37}$$

$$b_{t,\tau} \in \{0,1\} \quad \forall\, t,\, \tau \in T \tag{38}$$

$$z_{t,l},\; \sigma_{t,l} \in \{0,1\} \quad \forall\, t \in T,\, l \in L \tag{39}$$

$$y_t,\; w_t \in \mathbb{Z}^{+} \cup \{0\} \quad \forall\, t \in T \tag{40}$$

$$v_t,\; p_t^{(+)},\; p_t^{(-)},\; p_t^{(\cdot)},\; u_t \geqslant 0 \quad \forall\, t \in T \tag{41}$$

Finally, expressions (37)–(41) denote the variables domain.

4 Methodology

In this research paper we implement a Branch & Cut approach, as stated in Fischetti et al., 2016 and Fischetti et al., 2017, whose goal is to derive valid inequalities that are violated by a solution point. In this case, after retrieving a bilevel infeasible $\overline{\text{HPR}}$ solution point, say (x^*, y^*), we would want to enforce a cutting plane, named an IC, that cuts off this point, while keeping the rest of *bilevel feasible* solutions intact.

To make proper use of the Branch & Cut, an alternative way to formulate the MIBLP is presented and some necessary concepts are described.

value function formulation:

$$\min_{x,y} c_x^T x + c_y^T y \tag{42}$$

$$G_x x + G_y y \leqslant q \tag{43}$$

$$A_x x + B_y y \leqslant b \tag{44}$$

$$l \leqslant y \leqslant u \tag{45}$$

$$x_j \text{ integer}, \ \forall\ j \in J_x \tag{46}$$

$$y_j \text{ integer}, \ \forall\ j \in J_y \tag{47}$$

$$d^T y \leqslant \Phi(x) \tag{48}$$

In expression (48), $\Phi(x)$ is known as the *follower value function*; it returns the value of the follower's MILP optimal objective value as a function of a given leader variable solution vector $x^* \in \mathbb{R}^{n_1}$:

$$\begin{aligned} \Phi(x^*) \coloneqq \min_{y \in \mathbb{R}^{n_2}} \{ & d^T y : By \leqslant\ b - Ax^*,\ l \leqslant y \leqslant u, \\ & y_j \text{ integer},\ \forall\ j \in J_y \} \end{aligned} \tag{49}$$

Dropping expression (48) from the *value function formulation* model (42)-(48) results in a MILP known as the *high-point-relaxation* (HPR), whose Linear Programming (LP) is can be represented by $\overline{\text{HPR}}$. The following notation:

$$J_F \coloneqq \{ j \in N_x : A_j \neq 0 \} \tag{50}$$

is also proposed in the work of Fischetti et al. to denote the index set of leader variables x_j that appear in the follower problem. In expression (50), A_j represents the j-th column of matrix A.

Moreover, bilevel feasibility does not hold for a solution point (x, y) if it violates expression (48), whereas bilevel feasibility holds if (x, y) satisfies conditions (42)–(48),.

A valid IC that is violated by (x^*, y^*) if:

1. A cone pointed at (x^*, y^*) contains all *bi-level feasible* solutions inside it.

2. An convex set S contains (x^*, y^*) in its interior while leaving all other *bi-level feasible* solutions out of it.

To generate the ICs from a Hypercube convex set we assume $J_F \subseteq J_x$. Starting from a given HPR solution point (x^*, y^*), we define $(\hat{x}, \hat{y})$ as the best bilevel feasible solution point satisfying $\hat{x}_j = x_j^* \ \forall\ j \in J_F$.

1. We solve the follower MILP from expression (49) for $x = x^*$ and compute $\Phi(x^*)$.
2. We add the constraints (51) (52), to the HPR.

$$x_j = x_j^* \tag{51}$$

$$d^T y \leqslant \Phi(x^*) \tag{52}$$

3. We solve the restricted HPR and set $(\hat{x}, \hat{y})$ as the best bilevel solution given x^*.
4. Then, the Hypercube:

$$\mathrm{HC}^+(x^*) = \left\{(x, y) \in \mathbb{R}^n : x_j^* - 1 \leqslant x_j \leqslant x_j^* + 1,\ \forall\ j \in J_F\right\} \tag{53}$$

   results in the convex set S of interest.

Some definitions to generate ICs derived from the set S, as mentioned in Fischetti et al., 2017, are presented as follows.

1. Let ξ represent the whole variable vector $(x, y) \in \mathbb{R}^n$.
2. Let the $\overline{\mathrm{HPR}}$ at the given Branch & Bound node to be formulated in its standard form:

$$\min\left\{\hat{c}^T \xi : \hat{A}\xi = \hat{b},\ \xi \geqslant 0\right\} \tag{54}$$

3. Let ξ^* be an optimal vertex for the above $\overline{\mathrm{HPR}}$, associated to an optimal basis, say $\hat{B}$.
4. Let the convex set S of interest be defined as:

$$S = \left\{\xi : g_i^T \xi \leqslant g_{0i},\ i = 1, \ldots, k\right\} \tag{55}$$

5. To derive a valid IC violated by ξ^*, satisfying that all other feasible ξ must not belong to the inside of S, expression (54) can be restated to its k-term disjunction (see Glover, 1974 & Glover and Klingman, 1976):

$$\bigvee_{i=1}^{k} (g_i^T \xi \geqslant g_{0i}) \tag{56}$$

   It is important to note that in expression (56), the $\geqslant$ permits the feasible point ξ to belong to any facet of S (in our case, the set denoted by expression (53)).

6. Solve the above $\overline{\mathrm{HPR}}$ to retrieve an optimal vertex ξ^* along with its associated basis $\hat{B}$.

In expression (55), g_i^T represents the coefficient vector for all variables in the $i-th$ row of the convex set S. By construction, the Hypercube only spans the x (leader) variables space. This is, for every leader variable $x_j \; \forall \; j \; \in \; J_F$ there is going to be two associated rows (one for the condition $x_j^* - 1$ and one for the condition $x_j^* + 1$), in which its coefficients will be 1 for this specific variable, and 0 otherwise. Similarly, g_{0i} represents the right hand side of the $i-th$ row of S. In the Hypercube, for every $x_j \; \forall \; j \; \in \; J_F$ and for each of the two rows associated to x_j, the right hand side will constitute of $x_j^* - 1$ and $x_j^* + 1$, in the corresponding row. Moreover, in the Branch & Cut algorithm that generates ICs, the expression $(g_i)_{\hat{B}}^T$ in (57) represents the coefficient vector for the $i-th$ row of the associated basis for the vertex ξ^* in S. Analogously to g_i^T, the coefficient $(g_{i,j})_{\hat{B}}$ is takes the value of 1 for the leader variables $x_j \; \forall \; j \; \in \; J_F$ and 0 otherwise, if and only if the variable $x_j \; \forall \; j \; \in \; J_F$ is part of the basis at the current solution vertex ξ^*.

Algorithm 1 Branch & Cut Algorithm

Input: An optimal $\overline{\text{HPR}}$ vertex ξ^* and the associated convex *bilevel-free* set S.
Output: An IC violated by ξ^*.
Require: Branch as usual on fractional integer-constrained variables.

```
for i := 1 to k do
    (ḡ^T, ḡ_i0) := (g_i^T, g_i0) − (g_i)^T_B̂ B̂^{-1}(Â, B̂)        (57)
end for
for j := 1 to n do
    γ_j := max{ḡ_ij/ḡ_i0 : i ∈ {1,...,k}}                          (58)
end for
if γ ⩾ 0 and ξ is integer constrained then
    for for j := 1 to n do
        γ_j := min{γ_j, 1}                                         (59)
    end for
end if
Return the violated intersection cut γ^T ξ ⩾ 1
```

$$(\bar{g}^T, \bar{g}_{i0}) \coloneqq (g_i^T, g_{i0}) - (g_i)_{\hat{B}}^T \hat{B}^{-1}(\hat{A}, \hat{B}) \tag{57}$$

$$\gamma_j \coloneqq \max\{\bar{g}_{ij}/\bar{g}_{i0} : \; i \in \{1, \ldots, k\}\} \tag{58}$$

$$\gamma_j \coloneqq \min\{\gamma_j, 1\} \tag{59}$$

For the implementation details for the Branch & Cut approach, we built from zero our own Branch & Bound exploration algorithm. The reason for this is, although we used **Gurobi Optimizer 9.0.1 in Sublime-Python IDE**, we were not able to retrieve information about the inherited cuts from "parent" nodes for every node in the Branch & Bound tree. We ran our experiment over the $\overline{\text{HPR}}$ in its standard form. After finding any integer (x^*, y^*), we implemented a bilevel feasibility check and, if this point did not satisfy bilevel feasibility, we computed $\Phi(x^*)$ to capture the best bilevel feasible $(x^*, \hat{y})$ solution for the current node.

5 Testbed results

We present in **Table 3** the computational results for a small instance set of problems, which we solved using the presented methodology. The reported times are given in seconds.

Table 3: Small instance results table

Instance	(x^*, y^*)	z_l^*	z_f^*	Time B&C
1	(4,4)	-12.00	4.00	0.45
2	(6,2)	12.00	-2.00	0.89
3	(2,2)	-22.00	2.00	1.41
4	(3,1)	5.00	-1.00	0.37
5	(2,0,1,0)	-3.50	-4.00	0.15
6	(1,9,0)	-19.00	-9.00	0.49
7	(0,1,0,1,0,75,21.67)	-1,1011.67	-4,673.33	0.10
8	(17,10)	-76.00	30.00	5.09
9	(1,75,21.67)	-961.67	-4,673.33	0.06
10	(12,3)	27.00; 28.72*	-3.00; -3.47*	36,050.17*
11	(3,4,2,0,3,0)	49.00	-27.00	22.17

These 11 instance problems are found in the MIBLP literature (Bard, 1991; Li et al., 2014). A special consideration must be addressed with respect to instance 10, where we encountered a high execution time, most likely due to this instance's leader and follower variables belonging to the real numbers $\mathbb{R}^{\geqslant 0}$ domain, leading to non-optimal but *bilevel-feasible* solutions when the Branch & Cut is applied to it. This situation poses an interesting example for the specific characteristics that a bilevel instance must follow in order for the Branch & Cut methodology to be of practical use when applied to it. As mentioned in Fischetti et al., ICs effectiveness decreases over time when they are generated iteratively over the same Branch & Bound node. The main difference of generating ICs on LPs rather than MILPs arises since, for the former's, Branch & Bound cuts are not a present mechanism to speed up the feasible region exploration. Hence, ICs violation for a given vertex, say (x^*, y^*), is going to get weaker every time an intersection cut is generated. The number of ICs generated before stopping the methodology implementation on instance 10, after approximately 10 hours of running time, was 587 cuts with a 6.37% gap from the incumbent with respect to the optimal solution. It is worth noting that we did not implement any stopping criteria for our Branch & Cut implementation over the testbed instances. Since all these problems were relatively small, we allowed the exploration to visit every possible node in the Branch & Bound tree, hence, resulting in a variant of exhaustive enumeration on the Branch & Bound tree.

6 PPP results

For the solution of the PPP instance using the Branch & Cut methodology, and considering the ICs weakening feature when generated iteratively on the same LP, we defined a maximum number of 10 ICs per Branch & Bound node. We present figures **1**, **2**, and **4** as means to visualize the solution for the PPP PA problem. These graphs show the road's performance, the inspection and maintenance actions, how they vary with respect to the public's optimal solution and the subsequent private's optimal response (i.e., optimizing the problem under the public's objective and then solving the private party's MILP as a function of the inspections solution). The black-stripped line segment represents the road's performance, the red squares denote when an inspection was performed, the blue triangles represent the moments in which maintenance actions were performed, the continuous gold-colored stripped line represents the minimum required social benefit at every time of the planning horizon, and the magenta stripped line represents the solution's respective social benefit generated by the road's service level at every moment in time. Figure **1** shows what the inspection and maintenance actions would look like if they were solely performed by the public by maximizing the social benefit, under the assumption that the public is accountable for the maintenance actions. In this scenario, the public strives to maintain a high road performance, resulting in a maximum social benefit that does not decline over time.

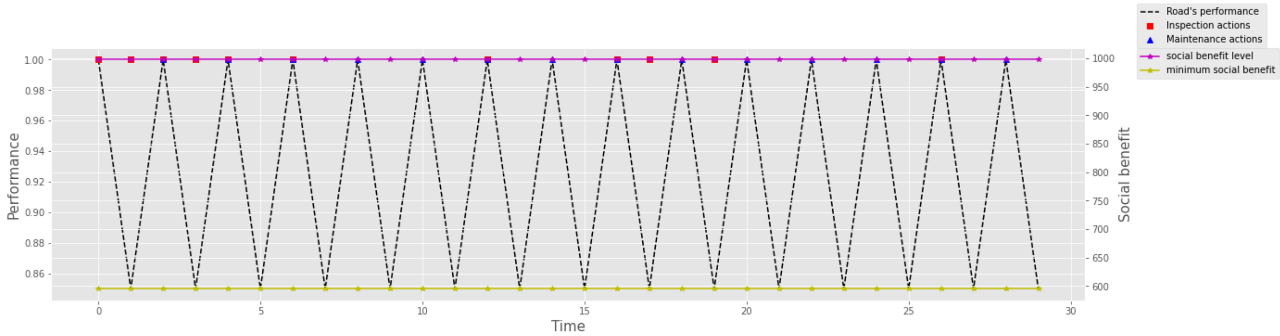

Figure 1: Solution under the public's perspective

Nonetheless, under this inspection setup, the private party, being responsible for the road's maintenance, seems to arrange maintenance actions to maximize incentives and minimize penalties based on the road's service level at inspection times.

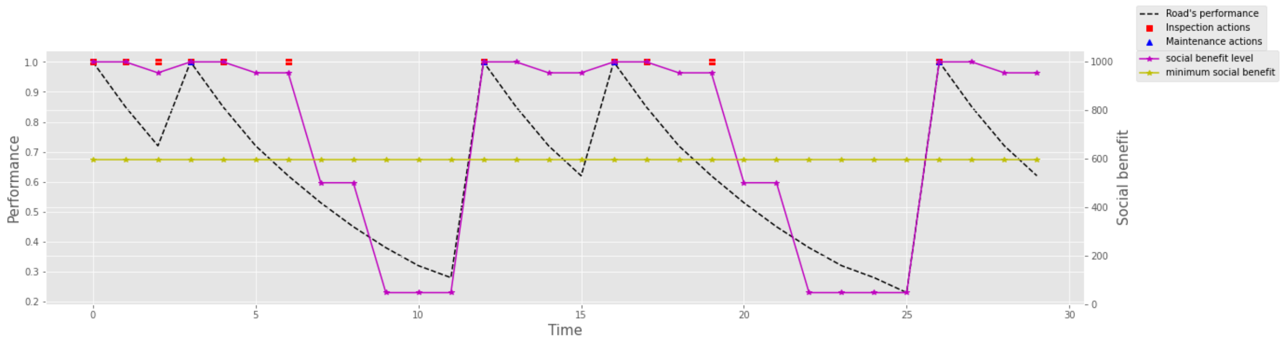

Figure 2: Private's reaction solution to the public's

This private party's maintenance plan response, however, is not feasible for the public as constraint (9) is not met. This means that the combined public and private inspection-maintenance decisions cannot ensure minimum social benefit over the planning horizon, as seen in **2** where the magenta striped line dips below the gold striped line at certain points in time. After enforcing appropriate ICs that violate such infeasible solution points, we found a set of bilevel feasible solution and a best bilevel feasible solution (incumbent) after an exploration of the Branch & Bound tree. Given the size of the problem, let alone 2^30 combinations for inspection actions only, we considered the following stopping criteria for the IC's generation algorithm: i) we established the optimal objective function value for the HPR under the private's perspective as a best-case scenario, and we chose a 5% gap distance from the incumbent with respect to it, and ii) if the number of nodes in the Branch & Bound tree exceeded a certain number, in our case $10,000$.

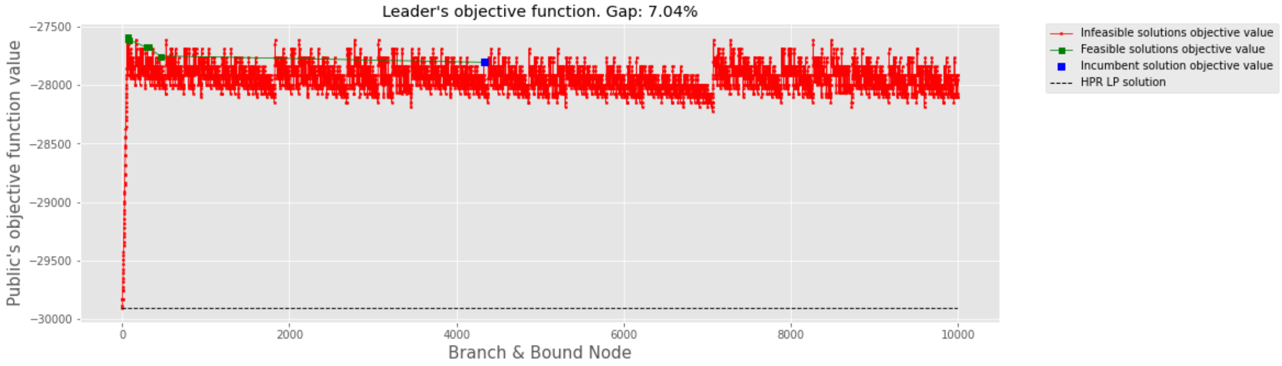

Figure 3: Public's objective function's behavior as the Branch & Bound tree is explored for bilevel feasible solutions

Figure 3 shows how the objective function of the public party behaves from bilevel infeasible (red squares) to bilevel feasible (green squares) solutions with respect to the number of nodes explored in the Branch & Bound tree; the graph depicts the incumbent solution node (blue square) and the associated incumbent's optimallity gap.

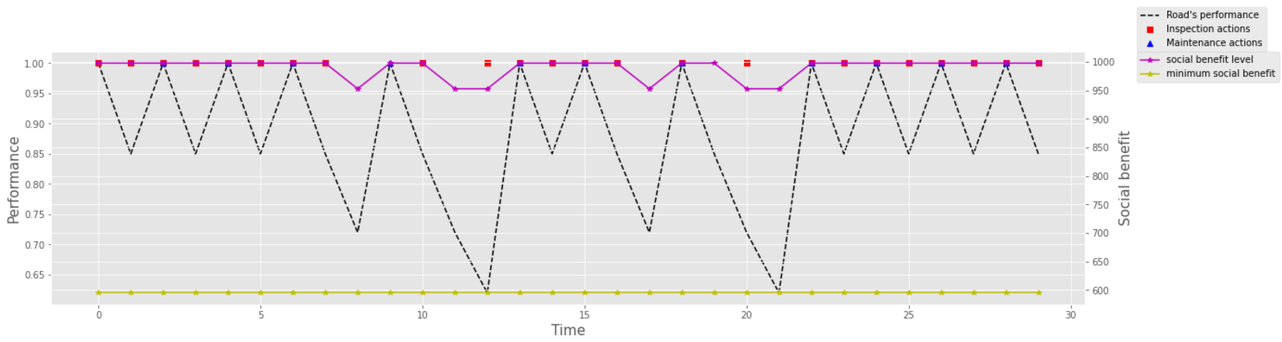

Figure 4: Incumbent bilevel solution

Figure 4 represents the incumbent solution for the PPP problem in mention. In the first place, the maintenance actions seem to match the inspection periods, making the road's performance to take its highest value by accommodating the maintenance mechanisms to the moments when inspections are performed, which derives in higher incentives from the public to the private party in parallel with a high social benefit, under the assumption that the private has early access to the inspection information from the public. This trade-off solution balances the private and public's interests by fulfilling the minimum social benefit requirement. Essentially, the public may have to compromise on their optimal, but unrealistic, desired outcome. In exchange, the private party is given incentives or penalties based on periodic evaluations of the road's performance, ensuring alignment with the project goals.

Moreover, Figure 5 shows how both parties objectives evolve with respect to the bilevel feasible nodes exploration, suggesting that an update on the public's incumbent solution implies and increase on the private's objective, exemplifying the conflict of interest between both parties objectives.

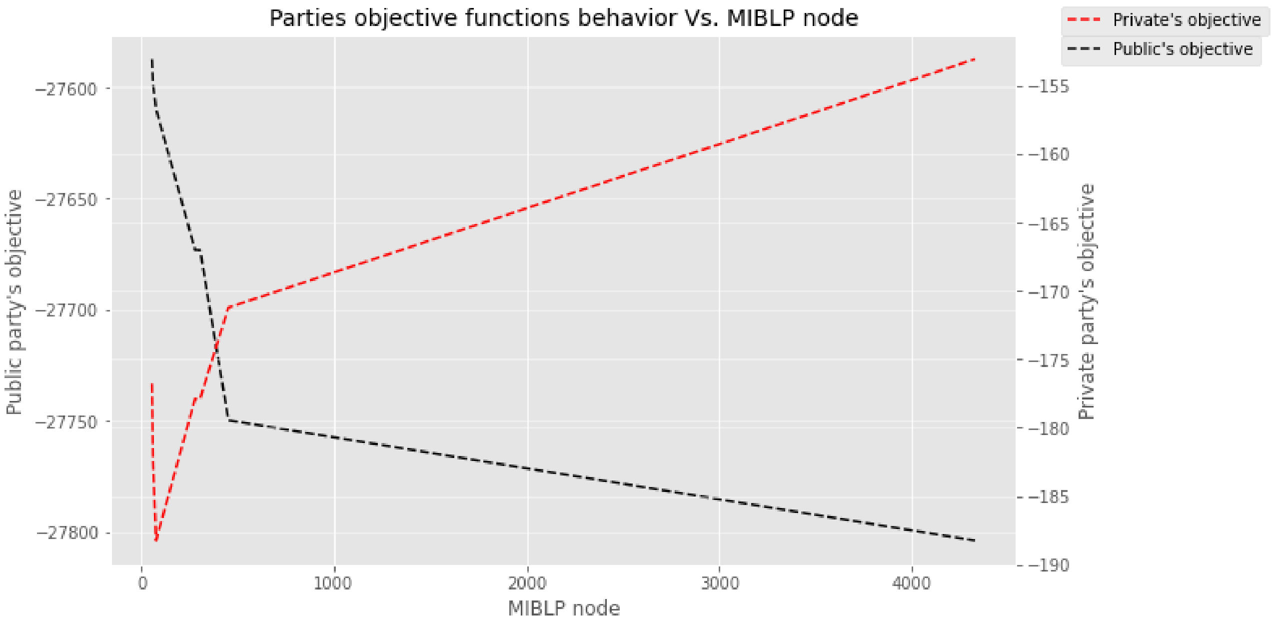

Figure 5: Public and private parties objective function's behavior with respect to the Branch & Bound tree's bilevel feasible nodes

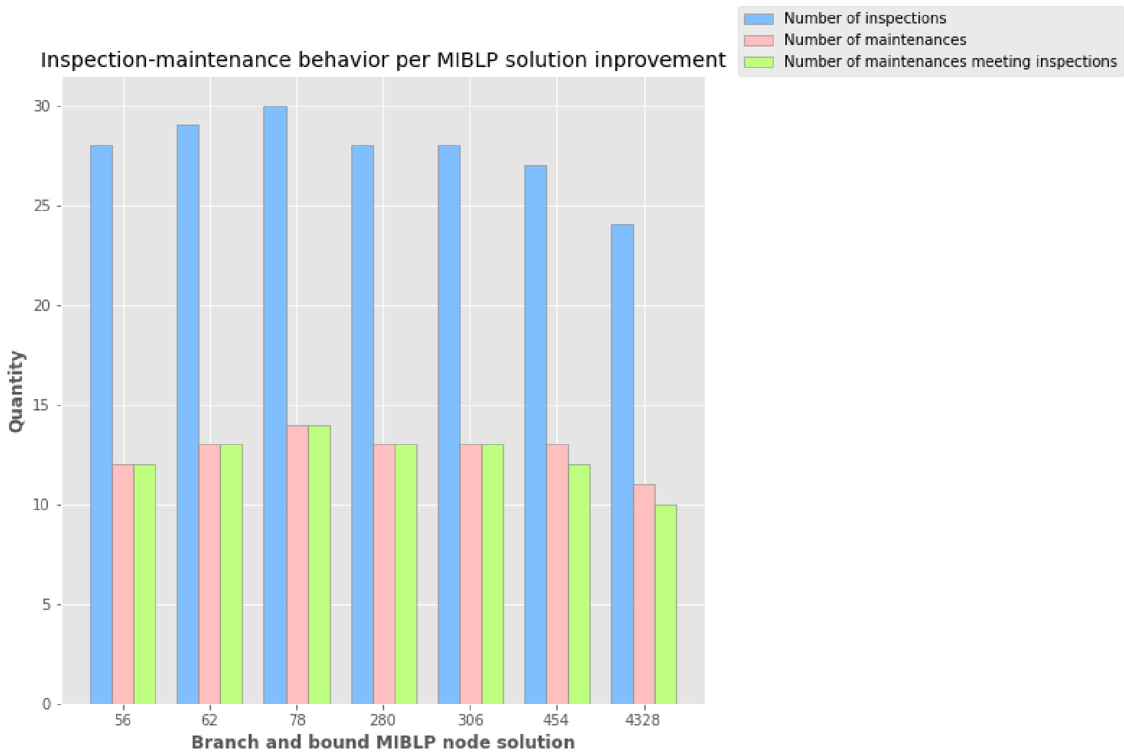

Figure 6: Inspection-maintenance behaviour with respect to the bilevel feasible Branch & Bound tree nodes

Furthermore, Figure 6 shows how the number of inspection and maintenance actions for the public and private parties, respectively, vary with respect to updates on the incumbent solution. In the first place, the inspection actions seem to increase, as well as the maintenance actions which always match the times of inspections (i.e., resulting on monetary rewards from the public to the private). Nonetheless, as the incumbent solution is updated, there seems to be better inspection configurations for the public that not only improve its objective function value while also decreasing the number of maintenances performed by the private during the planning horizon but, for some configurations, the inspection times distribution also seem to serve as mechanism to motivate the private party to perform maintenance actions at times when the public is not inspecting, resulting in no monetary rewards received at these times (e.g., see time 9 on Figure 4). The latter private behavior could be interpreted by they knowing the road's discrete service level value is not going to change with respect to changes on the continuous road's performance from one period to another, given the deterministic performance deterioration function. Hence, if they perform maintenance at time t, and an inspection is expected at $t+1$, the service level would not change from times t to $t+1$.

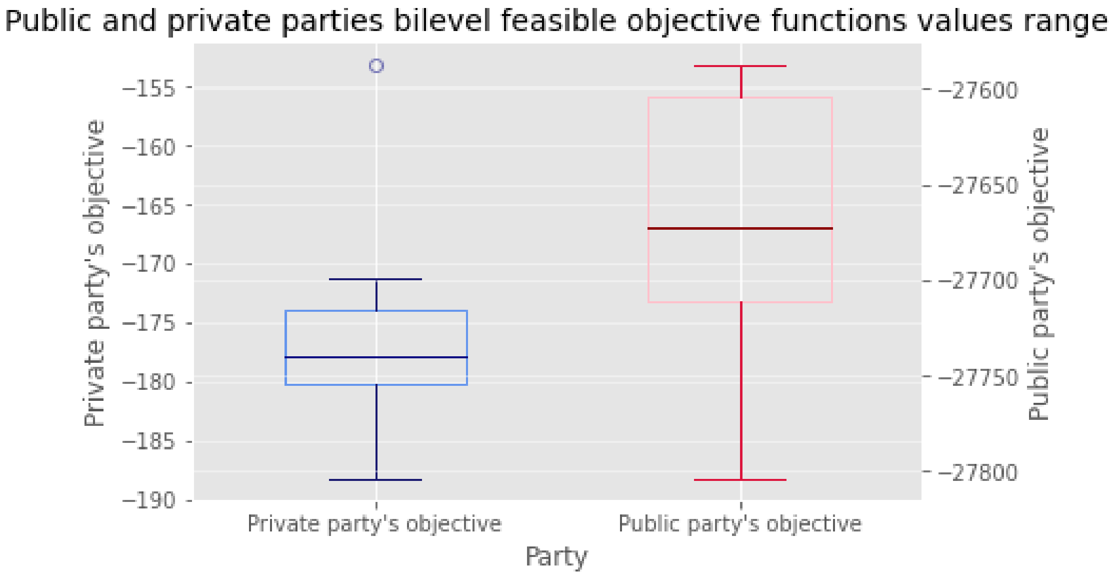

Figure 7: Public and private parties bilevel feasible objective function's box-plots

Also, Figure 7 shows the distribution for the feasible objective function values for both parties, suggesting that the distribution of values for the public's objective is significantly wider than that of the private, relatively speaking.

These results are a demonstration of the theoretical analysis capabilities that the MIBLP framework has over the joint decision space of the PPPs involved parties.

7 Conclusions

We study PPPs infrastructure projects with a focus on road maintenance. We acknowledge the advantages of PPPs (e.g., privates financial leverage and/or technical expertise) but also their vulnerability to conflicting interests, or PA problem, between the public and the private parties. To address this issue, we make use of MIBLPs to model the theoretical joint decision space of both parties while considering their divergent objectives simultaneously. The resulting optimization problem is combinatorial and difficult to solve, for which we use a Branch & Cut state-of-the-art method to address such complexities. We tested the solution methodology over a small set of MIBLP instances from the literature, and over a PPP road maintenance, which is the main focus of the research, but it could be extended to other operations contexts of application that incorporate combinatorial decision spaces.

The question addressed in this research project is: how can operations research methods be articulated into a methodology to provide quantitative and analytical support for decisions in infrastructure engineering projects in which conflicts of interest such as the PA problem are a matter of concern to guarantee acceptable outcomes? Our study is useful to estimate a trade-off solution for such potential conflict of interests that may arise between the public and private parties by incorporating conditions (e.g., inspections deriving in penalties or rewards given a determined system's performance and consequential service level) that aid their contractual or policy settlements. We developed an application for stakeholders in PPPs to perform analysis on the system's dynamics or the participant's behaviors by changing design parameters such as performance thresholds for which inspection actions derive in rewards/penalties, size or magnitude of reward/penalties given from the public to the private, planning horizon length, initial private party's budget, fix and variable inspection/maintenance costs, minimum system's performance thresholds, performance deterioration functions, etc.

The computational experiments conducted on the PPP instance demonstrate the effectiveness of MIBLPs in modeling the behavior of both parties, both individually and in a coupled decision-making context. The public prioritizes high road performance levels, which results in greater social benefit, without the limitations of budget constraints or incurring in maintenance costs. In response, the private party optimizes the project's cost-efficiency based on the public's latter inspection policy, which turns out to be infeasible for the public since the resulting performance from the private's maintenance actions violates maintaining a minimum social benefit over time. Our methodology balances both parties' interests, achieving an equilibrium or trade-off for each. The public allocates inspections to ensure that the private party performs maintenance in a way that maximizes overall social benefit and aligns their interests with the project's.

This research project has several limitations and areas for future work. Our PPP model currently only considers the deterministic case of road performance decay, ignoring the stochastic nature of natural phenomena and man-made activities that can contribute to performance deterioration. To address this limitation, future work could incorporate uncertainty into the deterioration functions by modeling them as random shocks that suddenly lower the road's performance level. Robust optimization methods could also provide a risk-averse analysis when considering uncertainty in the probability distributions of these shocks. Additionally, our model relies on the assumption of perfect information sharing between the parties, which is often not the case in practice. Improved mechanisms to address this limitation could be valuable to consider in future work. Additionally, since the reward/penalties given to the private as a function of the road's performance when and if inspected were selected as unique values, it would be worth evaluating what the impacts of different reward/penalty functions would have on the system's performance and the coupled decision-making process theoretical joint solution space.

Moreover, future work could also focus on improving the MIBLP methodology used in this project. Our methodology considered the optimistic position for MIBLPs and the optimallity gap was based on the HPR MILP solution from the leader's perspective, without considering the non-convex nature of MIBLPs. Incorporating the follower upper bound (Fischetti et al., 2017) or metaheuristic approaches, could help to select valid dual bounds to further reduce the optimallity gap for a given incumbent bilevel feasible solution.

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Appendices

Instance problems:

1.
$$\min x - 4y$$
s.t.,
$$\min y$$
s.t.,
$$-x - y \leqslant -3$$
$$-2x + y \leqslant 1$$
$$2x + y \leqslant 12$$
$$3x - 2y \leqslant 4$$
$$x,\ y\ \in\ \mathbb{Z}^{+}\ \cup\ \{0\}$$

2.
$$\min x + 3y$$
s.t.,
$$x \geqslant 0$$
$$x \leqslant 6$$
$$\min -y$$
s.t.,
$$x + y \leqslant 8$$
$$x + 4y \geqslant 8$$
$$x + 2y \geqslant 13$$
$$x,\ y\ \in\ \mathbb{Z}^{+}\ \cup\ \{0\}$$

3.
$$\min -x - 10y$$
s.t.,
$$\min y$$
s.t.,
$$-25x + 20y \leqslant 30$$
$$x + 2y \leqslant 10$$
$$2x - y \leqslant 15$$
$$2x + 10y \geqslant 15$$
$$x,\ y\ \in\ \mathbb{Z}^{+}\ \cup\ \{0\}$$

4.

$$\min x + 2y$$

s.t.,

$$\min -y$$

s.t.,

$$-x + 2.5y \leqslant 3.75$$
$$x + 2.5y \geqslant 3.75$$
$$2.5x + y \leqslant 8.75$$
$$x,\ y\ \in\ \mathbb{Z}^{+}\ \cup\ \{0\}$$

5.

$$\min -2x_1 + x_2 + 0.5y_1$$

s.t.,

$$x_1 + x_2 \leqslant 2$$
$$\min -4y_1 + y_2$$

s.t.,

$$2x_1 - y_1 + y_2 \geqslant 2.5$$
$$-x_1 + 3x_2 - y_2 \geqslant -2$$
$$x_1,\ x_2,\ y_1,\ y_2\ \in\ \mathbb{Z}^{+}\ \cup\ \{0\}$$

6.

$$\min -x - 2y_1 - 3y_2$$

s.t.,

$$\min -y_1 + y_2$$

s.t.,

$$x + y_1 + y_2 \leqslant 10$$
$$x,\ y_1,\ y_2\ \in\ \mathbb{Z}^{+}\ \cup\ \{0\}$$

7.

$$\min -20x_1 - 60x_2 - 30x_3 - 50x_4 - 15y_1 - 10y_2 - 7y_3$$

s.t.,

$$\min -20y_1 - 60y_2 - 8y_3$$

s.t.,

$$5x_1 + 10x_2 + 30x_3 + 5x_4 + 8y_1 + 2y_2 + 3y_3 \leqslant 230$$
$$20x_1 + 5x_2 + 10x_3 + 10x_4 + 4y_1 + 3y_2 \leqslant 240$$
$$5x_1 + 5x_2 + 10x_3 + 5x_4 + 2y_1 + y_3 \leqslant 90$$
$$x_1,\ x_2,\ x_3,\ x_4\ \in\ \{0, 1\}$$
$$y_1,\ y_2,\ y_3\ \geqslant\ 0$$

8.

$$\min 2x - 11y$$

s.t.,

$$\min x + 3y$$

s.t.,

$$x - 2y \leqslant 4$$

$$2x - y \leqslant 24$$

$$3x + 4y \leqslant 96$$

$$x + 7y \leqslant 126$$

$$-4x + 5y \leqslant 65$$

$$-x - 4y \leqslant -8$$

$$x,\ y\ \in\ \mathbb{Z}^+\ \cup\ \{0\}$$

9.

$$\min -60x - 10y_1 - 7y_2$$

s.t.,

$$\min -60y_1 - 8y_3$$

s.t.,

$$10x + 2y_1 + 3y_2 \leqslant 225$$

$$5x + 3y_1 \leqslant 230$$

$$5x + 3y_2 \leqslant 85$$

$$x\ \in\ \{0, 1\}$$

$$y_1,\ y_2\ \geqslant 0$$

10.

$$\min x + 5y$$

s.t.,

$$\min -y$$

s.t.,

$$-x - y \leqslant -8$$

$$-3x + 2y \leqslant 6$$

$$3x + 4y \leqslant 48$$

$$2x - 5y \leqslant 9$$

$$x,\ y\ \geqslant 0$$

11.

$$\min -4x_1 + 8x_2 + x_3 - x_4 + 9y_1 - 9y_2$$

s.t.,

$$-9x_1 + 3x_2 - 8x_3 + 3x_4 + 3y_1 \leqslant 1$$

$$4x_1 - 10x_2 + 3x_3 + 5x_4 + 8y_1 + 8y_2 \leqslant 25$$

$$4x_1 - 2x_2 - 2x_3 + 10x_4 - 5y_1 + 8y_2 \leqslant 21$$

$$9x_1 - 9x_2 + 4x_3 - 3x_4 - y_1 - 9y_2 \leqslant -1$$

$$-2x_1 - 2x_2 + 8x_3 - 5x_4 + 5y_1 + 8y_2 \leqslant 20$$

$$7x_1 + 2x_2 - 5x_3 + 4x_4 - 5y_1 \leqslant 11$$

$$\min -9y_1 + 9y_2$$

s.t.,

$$-6x_1 + x_2 + x_3 - 3x_4 - 9y_1 - 7y_2 \leqslant -15$$

$$4x_2 + 5x_3 + 10x_4 \leqslant 26$$

$$-9x_1 + 9x_2 - 9x_3 + 5x_4 - 5y_1 - 4y_2 \leqslant -5$$

$$5x_1 + 3x_2 + x_3 + 9x_4 + y_1 + 5y_2 \leqslant 32$$

$$x_1,\ x_2,\ x_3,\ x_4,\ y_1\ y_2 \in \mathbb{Z}^+ \cup \{0\}$$